**Course Name: Power Electronics Applications in Power Systems**

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**Week: 10**

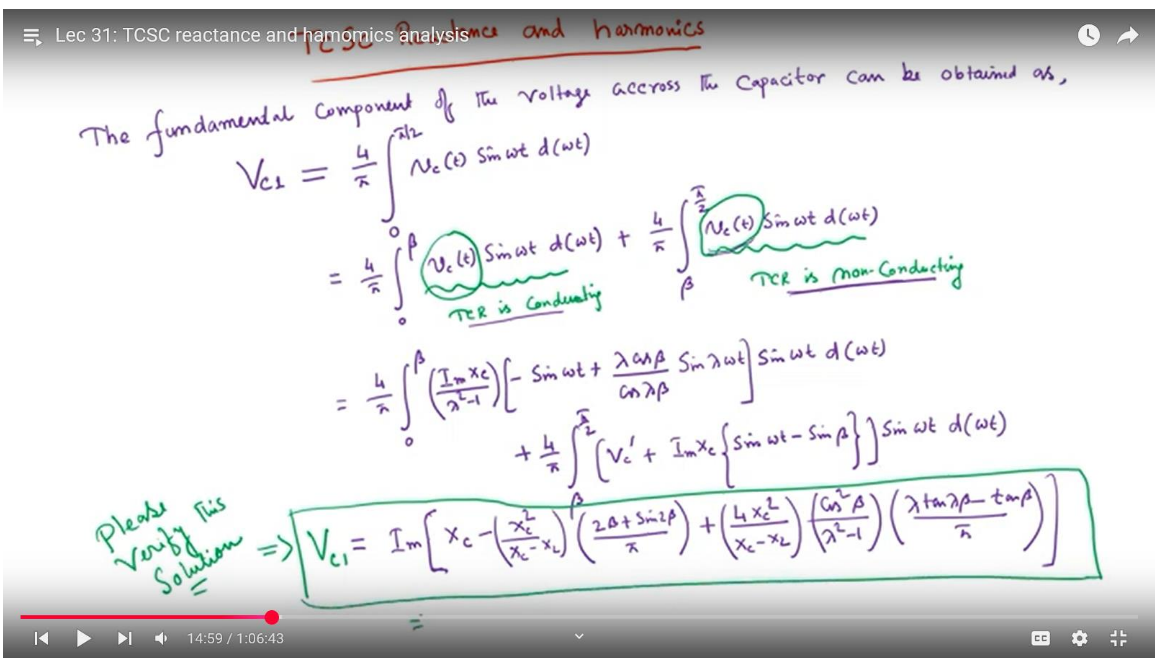
**Lecture: 01**


Lec 31: TCSC reactance and hamomics analysis

So welcome again to my course Power Electronics Applications in Power Systems. In last three lectures, I was discussing the basic operation of a series connected power electronic based compensators that is TCSC which is used in power system for various applications or various reasons. So, in this particular lecture also, I will discuss or rather I will finish the discussion on the analysis of TCSC. So, let us proceed. So, in last lecture, I stopped at this point, deriving the expressions for voltage across the capacitor. As you know, the basic schematic diagram of this TCSC is something like this.

We have a fixed capacitor, this fixed capacitor and we have a variable reactor, we call it a TCR, whose reactance should be controlled by the firing angle control of the thyristors. So voltage across capacitor means voltage across this particular series unit and which is of course the voltage across this TCR unit as well. So the expression for voltage across the capacitor we derive. There are two equations when this TCR is conducting the voltage across capacitor is this when TCR is non-conducting that is I TCR is 0. So, voltage across the capacitor is that. So, this is voltage across the capacitor that expression is when TCR is conducting and this is the voltage across the capacitor that expression when TCR is non-conducting. So, you can see, if you look at both the equations, this equation as well as this equation, you can see that these are not ideally sinusoid. So, this would be some kind of distorted sinusoid. So, therefore, some amount of harmonics would be there within the expression. And we will analyze this harmonic as well. So, in this particular lecture, basically, I will discuss TCSC reactance and harmonics. So, the goal of today's lecture is to discuss the TCSC reactants and the harmonics. Now, what do you mean by TCSC reactants? Already I explained one of my previous lectures, when I discussed this basic operating principle of TCSC, that this TCSC reactants is effectively Z-TCSC, which is a parallel combination of this Xc and Xtc. Now, it could be positive. When it is positive, then it is inductive. It could be negative as well. When it is negative, it is capacitive mode of operation. So, we have two Vernier control modes, which I already explained. So, for these two Vernier control modes, the TCSC reactants would be in different Or TCSC reactants will have different sign, one is positive when it is operating at inductive vernier control and it is negative when it is operating at capacitive vernier control.

So, these are the two operating modes of TCSC and we will determine the expression of TCSC in particular as a function of this parameter beta. As I explained this beta is the angle of advance, it is an important parameter for TCSC analysis. And this beta can be varied. Now, with the variation of the beta, what would be the impedance or what would be the expression of the impedance of the TCSC that we are trying to determine today. Now, in order to determine that, as I explained, the voltage across this capacitor is non-sinusoidal.




So, therefore, we have to find out the fundamental component of the voltage across the capacitor. So, let us find out this. So, the fundamental component of the voltage across the capacitor can be obtained as. This is we can obtain from Fourier series analysis as you know. So, this is V c 1, 1 stands for the fundamental, V c stands for voltage across the capacitor. So, this is equal to 4 by pi integration of 0 to pi by 2 V c t sin omega t d omega t. Now, you know that why we keep this limit 0 to pi by 2 because we have some symmetry of this capacitor voltage as well. So, look at this voltage waveform. So, here basically we restrict our limit from this omega t is equal to 0 to omega t is equal to pi by 2 for this two limit. Now, within this two limit we have these two different expressions for VCT. As we know that this VCT expression when TCR is non-conducting would be something else and rather than when TCR is conducting.

Now you can see that here TCR will conduct from the time interval 0 to beta. So this is the angle beta. So up to this, this TCR will conduct. And then in this particular interval that is beta to pi by 2, TCR will not conduct. So, TCR will be non-conducting. So, therefore, I can write, I can split this interval of this integration into two parts, one is 4 by 0, 0 to beta VCT sin omega t d of omega t. Another is 4 by pi integration beta 2 pi by 2 v c t, v c of t that is voltage across the capacitor instantaneous voltage, of course, sin omega t d omega t. Now, you understand that in this particular interval, TCR is conducting. In this particular interval, TCR is non-conducting. So, therefore, this VCT expression whichever we will be using in this particular interval that is 0 to beta would not be applicable when we use the interval beta to pi by 2.

So, if I write the actual expressions for VCT for these two intervals, so it will be something like this 4 by pi 0 to beta. Now, if you go back and see what the expression for VCT was when TCR was conducting, TCR is conducting. So, this is the expression, this is the expression. So, let us copy it and write over here, it will be equal to IMXC divided by IMXC divided by lambda square minus 1 multiplied with minus sin omega t. minus sin omega t plus lambda cos beta divided by cos lambda beta lambda cos beta divided by cos lambda beta multiplied by sin lambda omega t multiplied by sin lambda omega t. So, this is the expression for VCT when TCR is conducting if you look at. So, this is what the expression of VCT is when TCR is conducting. So, therefore, we will use it and this would be multiplied with sin omega t d of omega t. So, this is one part of this expression. Another part would be 4 by pi integration beta to pi by 2. Now, in this particular interval, TCR is non-conducting. So, therefore, the expressions for VCT would be what when it was applicable for TCS, TCR is non-conducting, that means this expression, that means this expression. So I will write this directly. So this will be vc dash. Vc dash is some parameter which is independent of time.

You can look at this expression is time-independent. So this expression is time-invariant. So, therefore, this is something like a constant, you can find this value and put it over here directly. But this part, this part is a part which is time variant at the sin omega t component. So, therefore, this plus I m x e sin omega t minus sin beta this is the expression or I should use a different parenthesis for this, because already the square parenthesis is used. So, I will use this curly bracket. So, this is the expression for this

VCT when TCR is non-conducting, right? This is already we determined in the last lecture. So, when we put this, then this has to be multiplied with sin omega t d omega t, ok. Now, this is a very big integration. I am leaving to you all the learners to do it by yourself. In fact, in my live classes, I usually tell my students to derive it and then I will match this result that I am having with them. Because this is a very long integration, you need to put considerable effort into solving this. Therefore, we will be coming up with an expression which will be also very large. Do not remember this particular equation, there will be no use of it. You have to understand the concept underlying this equation that is all.

I will never ask you to derive this expression in your examination or I will not set any questions regarding this particular explanation which you need to remember. So, there is no need of that, but this underline concept should be understood. Then this is the basic goal of this particular lecture. Now, I have the solution with me, I can directly write over here and I will leave it with you to match the solution with your solution. So, the solution that I have is Im multiplied by xc minus xc square divided by xc minus xl like this, 2 beta plus sin 2 beta divided by pi plus 4 x square x e minus x l multiplied by cos square beta divided by lambda square minus 1 multiplied by lambda tan lambda beta minus tan beta divided by pi. This is what the expression which you need to verify. Now, I will leave it to you to verify. Please verify this solution. So, I have this solution which I can show to you, but what you need to do is you need to verify it.

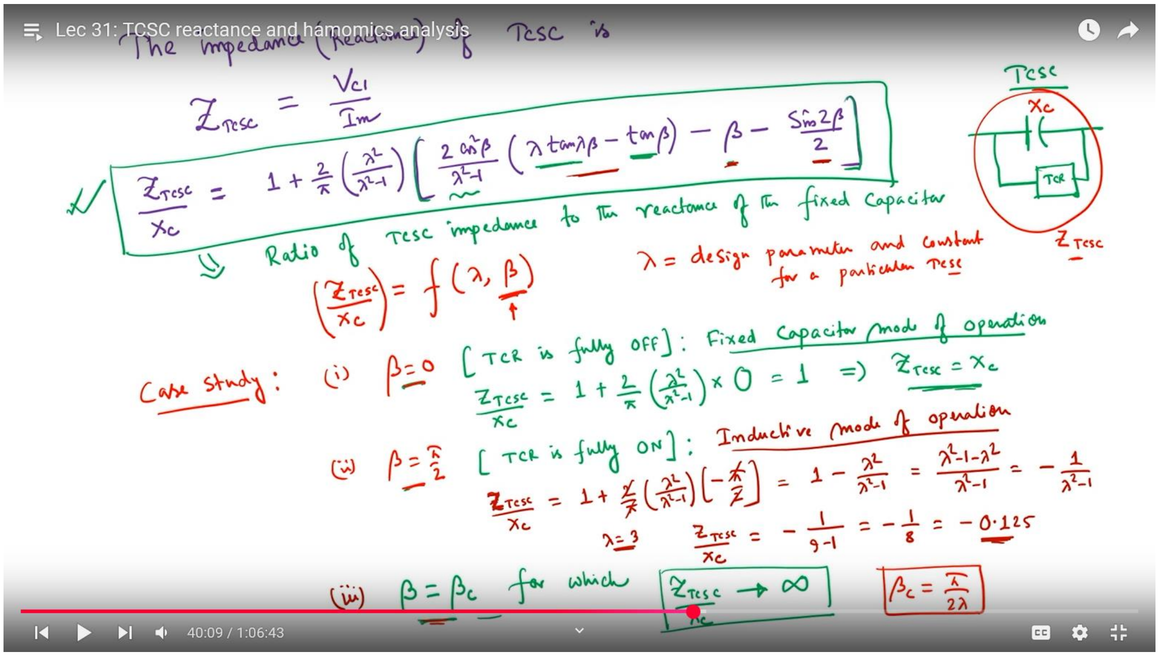
It is a very long equation. As far as this power system learners is concerned, because we do not have such a very big equations having so many you know components, so many things within this equation usually in power system. So, therefore, this is relatively a very long equation. You need to carefully verify it, but I think that you should be able to derive it even though you do not remember this, but you should be able to derive this equation whenever is required. So, this is something is the goal of this particular lecture. Now, what is this V c 1? V c 1 is the fundamental component of the voltage across the capacitor. And what was our goal? Our goal is to find out the TCSC reactance, right? Now, how can we find out this TCSC reactance? We can find out you by taking the ratio of the voltage across the capacitor and this Im, Im is the current flowing through this particular line, because we are interested to find out the this impedance of the TCSC. Now, what would be the impedance of the TCSC? Obviously, the impedance of the TCSC would be the voltage across the TCSC and current flowing through the TCSC. So, voltage across the TCSC is V c 1, current flowing is basically this I and the maximum value of it is I m. So, that is why it is this Im is the Im is representation of the current that is the peak value of the current flowing through the TCSC and Vc1 is the representation of the voltage across the TCSC. Their ratio would be of course the representation of the overall impedance of the TCSC.

So, therefore the impedance or one may tell that this is also reactance because both are representing same thing because we are assuming that the TCSC is lossless. So, therefore, the impedance of TCSC is ZTCSC. And, this is, it is found out to be the ratio of this V c 1 to I m. And, in, if you look at this expression that we derived right now, this is, the left hand side we have V c 1, right hand side we have I m. So, therefore, this ratio can be easily determined from that particular expression. And, this ratio is the representation of this impedance or reactance of the TCSC.

Now, if you find this and do some further simplification, then the expression we get of ZTCSC as a ratio of Xc. Now, what is Xc? Xc is the reactance of the fixed capacitor. So, this expression after doing some simplification is coming out to be 1 plus 2 by pi multiplied by lambda square divided by lambda square minus 1 multiplied by a bracket 2 cos square beta divided by lambda square minus 1, then tan lambda beta minus tan beta minus beta minus sin 2 beta by 2. Now, what is that ratio? This ratio is the, this is basically representing, this is basically representing ratio TCSC impedance to the reactance of the fixed capacitor or capacitor of TCSC. Now, you know that at this point this TCSC you know schematic diagram is something like that we have a fixed capacitor in parallel to a TCR.

Now, if you consider the reactance of this particular fixed capacitor is Xc and overall impedance of this is ZTCSC. Then the ratio of this ZTCSC to XC is found out to be this and this we obtain from this previous expression, from this expression after doing some sort of simplification. We converted this XCXL in terms of lambda and wherever is possible that we convert all this parameter into lambda and beta. So therefore, this expression if you look at then you can see ZTCSC to x e ratio is function of lambda beta, only function of lambda and beta. Now what is lambda, if I hope that you can remember it appropriately, this lambda already we derived in this one of this expression, it is the ratio of the omega R to omega, where omega R is equal to 1 upon root over LC, which is resonating frequency. So it is basically representation of that ratio of the omega R to omega that means omega R is what times of this power frequency that is what. So, basically lambda is a constant because it is a design parameter. It depends upon this 1 upon L and C which are fixed. So, lambda is constant. Only parameter here is variable is beta. So, if we go back and see. So, therefore, lambda is design parameter and constant for a particular TCSC. So, therefore, this ratio is variable only with this beta. Now, what is beta? Beta is the angle of advance as I said this is a very important concept and we will derive all this later on parameter in terms of beta and the choice of beta is important for controlling the TCSC. So, this ratio is an important parameter which can be varied or controlled with the appropriate choice of the beta depending upon the requirement.

This is something one needs to understand. Now we will be doing some case study here. What is the case study? First case study is let us consider beta is equal to 0. Now what will happen? And second case study is let us consider beta is equal to pi by 2. Because if you look back and see the waveforms that I derived, this possible value of this beta can be 0 to pi by 2. When beta is equal to 0, that means TCR is non-conducting. When beta is equal to pi by 2, then that means TCR is fully conducting. So, that means beta is equal to 0 means TCR is non-conducting or TCR is fully off. Now, what mode of operation it is? It is a fixed capacitor mode of operation. It is a fixed capacitor mode of operation because if your TCR is fully off that means this TCSC is nothing but a fixed capacitor. So, overall impedance of the TCSC should be equal to the reactance of the fixed capacitor or impedance of the fixed capacitor.




Does it happen? Let us see. Now, if we put beta is equal to 0, you look at this cos beta will be equal to 1. 10 lambda beta this part would be 0. So, 10 beta will be also 0. So, if 0 multiplied with this would be 0. So, therefore, this would be also 0, this would be also 0. So, inside this bracket whatever term we will be having corresponding to beta is equal to 0 will be 0. So, that means this ratio there Tc Sc by Xc will be equal to 1 plus 2 divided by pi lambda square divided by lambda square minus 1 multiplied by 0, which is equal to 1. So, this gives that ZTCSC is equal to XC, which is the fixed capacitor mode of operation. So, this gives one indication that this expression is correct, but you need to also verify whether it is true or not. Now, the second case study is when beta is equal to pi by 2.

Now, what do you mean by beta is equal to pi by 2? At this case, TCR is fully on. So therefore, what mode of operation it is, if you look back at this waveform, which we had drawn earlier, this is what the, you know, this TCR current for this value of beta. Now, if beta is equal to pi by 2, then this current would be somewhere like this, up to So, I can draw this by some other color, let us say here, when this beta, this will be equal to I TCR, this would be equal to I TCR of t corresponds to beta is equal to pi by 2. So, then this would be equal to the I TCR current. So, when it happens that means TCR is fully conducting. So, in that case there would be no harmonics of course, that is one of the advantages. But when it happens then what would be the value of this ZTCSC that is what something important to us. So, this is you know fixed capacitor mode and when TCR is fully conducting we have to find out what mode of operation it is. So, if we put beta is equal to pi by 2. Then what we will get? You can see this part would be equal to 0 and now cos beta cos pi by 2 is what? It is 0.

So, this has to be multiplied with 0 multiplied this entire block. So, this would be 0, this would be 0. So, only this beta will remain. So, then this equation according to me will be equal to equal to Z T C S C divided by X C is equal to or I should write it here ZTCSC divided by XC is equal to 1 plus 2 by pi lambda square lambda square minus 1 and then within this bracket it will be multiplication with pi by 2. Then, what we will get? This pi by 2 and this would be cancelled out. So, this will be equal to 1 minus lambda square divided by lambda square minus 1. So, this is lambda square minus this, this is minus 1 divided by lambda square minus 1. So, this ZTCSC and XCSC will be this. Now, what we can interpret from it? Let us consider lambda is equal to 3. That means, this omega r is 3 times of the power frequency. So, then this ZTCSC to this XC ratio would be equal to minus 1 upon 3 square, that is 9 minus 1. So, that is minus 1 divided by 8. Now, minus 1 by 8 is a fractional number, my point around 0.125 or something like that.

So, therefore, it is a fractional number, but negative. Now, what this negative sign signify? This negative sign signify that this overall impedance of T c s c would be minus of this x c. Now, x c is a capacitive impedance. So, negative of that would be an inductive mode. So, therefore, this is a inductive mode of operation. This is an inductive mode of operation. Now, there would be another you know value of beta which will be also important to us. This beta is the value of this beta c which is called the cutoff value of this beta for which this ratio that is ZTCSC to XC ratio would be infinite. That is why this resonance will happen. This is also an important case study that we can do. Now, from this, we can find out at what value of this beta, this ratio would be infinitely large.

In a practical sense, it cannot be infinite but it can be infinitely large. That means the resonance will happen which makes it this isolated from the system and that means this will create a discontinuity. The placement of this TCSC will cause a discontinuity at the point where it is placed in the transmission line. So, we have also this point is interest to us. Now, in order to find this, what we will do is, we have to do some more derivations. So, let us do some derivation to arrive at the conclusion that what could be the value of beta c to guess at least what could be the value of this beta for which this would be equal to infinite. Now, we can see this mode of operation is prohibited and this we have to identify before and we should control or tune the parameter of TCSC such that this situation or this case will never appear. So, therefore, to find this what we can do, let us do some derivations of ZTCSC to Xc. So, I am just rewriting this expressions once again. So, 1 plus 2 by pi lambda square divided by lambda square minus 1 multiplied by now you can see here we have two terms; one is lambda tan lambda beta; another is tan beta.

So, what we can write this is 2 cos square beta divided by lambda square minus 1 lambda tan lambda beta minus 2 cos square beta divided by lambda square minus 1 tan beta. We did nothing but we just multiplied this multiplier, this multiplier one with this lambda tan beta and another with tan beta. Now, we write this minus beta as it is and minus sin 2 beta by 2 as it is. Now, what we will do? We will go ahead with this derivation once again. So, this will be lambda square minus 1. Now, this can be written as 2 cos square beta lambda tan lambda beta divided by lambda square minus 1 minus. Now, you can see we can write this tan beta by sin beta by cos beta. So, if we write it, if we just write tan beta sin beta divided by cos beta, then this cos beta and this 1 cos beta in the numerator will be canceled out. So, in the numerator we will have 2 cos beta sin beta which can be written as sin 2 beta divided by lambda square minus 1 minus beta minus sin 2 beta by 2.

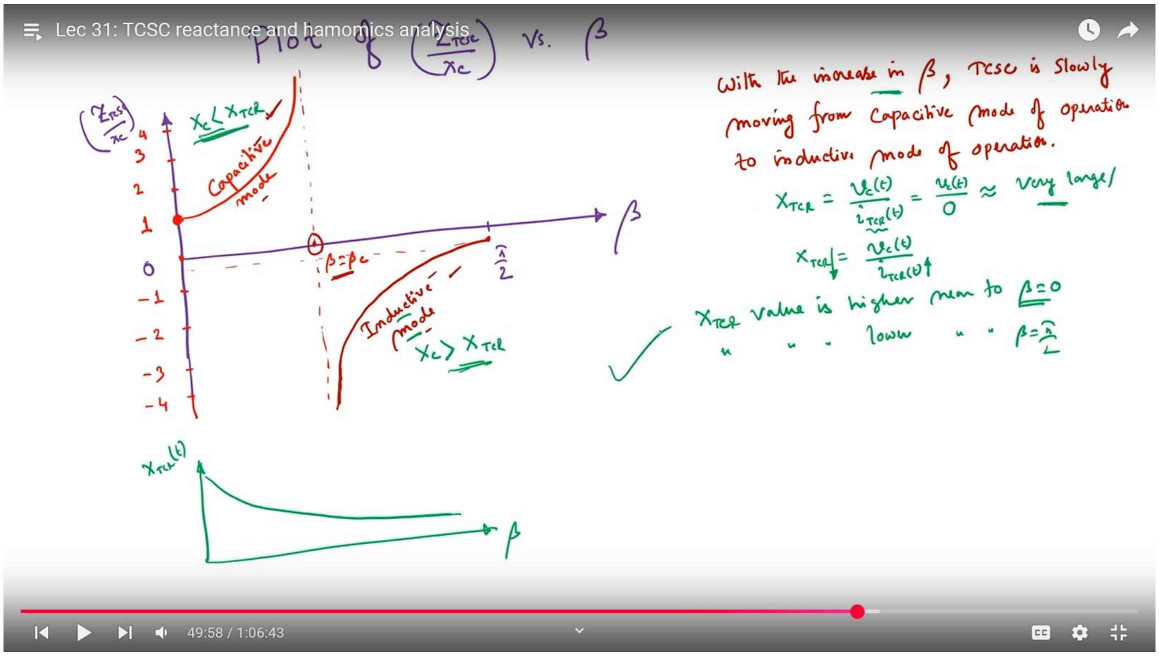
Now, we have two sin 2 beta term, one is this, another is this. So, let us aggregate these two. So, 1 plus 2 divided by pi lambda square lambda square minus 2 cos square beta lambda tan lambda beta divided by lambda square minus 1 minus, if I take sin 2 beta common outside a bracket, then here we will have 1 by lambda square minus 1, here we have 1 by 2, right. Then minus beta we, I am keeping as it is. Now let us write it again 2 by pi lambda square lambda square minus 1 multiplied by I will keep this as it is 2 cos square beta lambda tan lambda beta lambda square minus 1 minus the sin 2 beta I will write as it is. Now, if we just do this addition 1 upon lambda square minus 1 plus 1 upon 2, then it will be equal to 2 plus lambda square minus 1 divided by 2 lambda square minus 1 minus beta.

So, again we write 2 by pi lambda square divided by lambda square minus 1, this portion as it is 2 cos square beta lambda tan lambda beta divided by lambda square minus 1 minus sin 2 beta with the multiplication of this will be lambda square plus 1 divided by if I bring this two outside divided by lambda square minus 1 minus beta. So, this is the expression I wanted to derive. Now look at this particular expression. Here only parameter is varying, only variable parameter is beta. So, in this particular expression, so the only variable in this expression is beta. Apart from that, everything is constant. Lambda is a design parameter, it is constant. All other parameters are constant. So, only parameter which is can be varied is beta. Now, what is the range of the beta? Beta can be varied from 0 to pi by 2. Beta can be varied from 0 to pi by 2. Now, as you can already have seen neither beta is equal to 0 or beta is equal to pi by 2, two extreme limits gives the ratio ZTCSC to Xe infinite or infinitely large. Then what would be the value of beta for which this would be infinity? If you look at this particular expression, you can see this, if we vary beta from 0 to pi by 2, this minus beta, this component will never be infinite. There is no chance, it is a parameter. Now, sin 2 beta, so sin 2 beta cannot be infinite within this range, 0 to pi by 2. In fact, sin 2 beta can never be infinite, so as this cos square beta. So, what parameter can be infinitely large so that this ZTCSC to XC ratio will be infinite. So, if you look at then only this tan lambda beta is a parameter which can be infinite. Tan theta can be infinite. Under what condition tan theta can be infinite? You know when theta is equal to pi by 2. Then you know that this sin pi tan theta when theta is equal to pi by 2 that is tan pi by 2 is equal to sin pi by 2 divided by cos pi pi 2.

Now sin pi by 2 is equal to 1, cos pi pi by 2 is equal to 0, so this can be infinite. So therefore, in this particular expression, if this lambda beta is equal to pi by 2, then what will happen? Z T C S C by X C can be infinity. So, this is an important relation. This is an important relation and therefore, this beta C that is known as this cutoff value of this beta, which should never be appeared is equal to pi by 2 to lambda. This is an indirect way of determining it, but you can mathematically also derive this expression, this beta c is equal to pi by 2 lambda from this particular expression if you want to solve. Now what we will do next? We got these three important cases that at beta is equal to beta c, this is infinity and we find out this beta c is equal to pi by 2 lambda.

So, this what we derive. We know that what will happen beta is equal to 0, we also know that what will happen at beta is equal to pi by 2. Then what we can do is that we can plot this ZTCSC to XC ratio with respect to beta. Now, how would be the plot? Let us see. So, let us have this plot of this ratio ZTCSC to XC versus this beta. So, one axis I will write this particular ratio that is ZTCSC to XC. It can be positive, it can be negative as well. In another axis, we will keep beta. And beta can be varying from 0 to let us say pi by 2. And this is, suppose this beta is equal to beta C for which this resonance will happen. Now, as we know that when this beta is equal to 0, then this ratio will be equal to 1. So, this is 1. Let us say this is 2, this is 3, this is 4, and so on. And here also this is minus 1, this is minus 2. This is minus 3, this is minus 4, and so on.

Now at beta is equal to 0, this is our operating point. This is what would be that value of this ratio. Then what will happen if you take this particular expression and plot it by using any any coding language you know either in MATLAB or any C or C++, then this plot we will see something like this. This plot will be something like this. And as we know that when beta is equal to pi by 2, this would be negative, but this value would be something close to 0. So, this plot of this side would be something like that. So, the plot of this ZTCSC to Xc which is an important you know parameter would be something like this and when beta is equal to beta c their value will be infinitely large. So, that is why we keep it as a discontinuity there. So, this plot is also important for the application of TCSC in particular for controlling the TCSC parameters, so that our operation should be within this you know feasible range of these characteristics. Now the question is, we have two plots, one is this red, another is this. Which one is, what mode of operation? Now we can see that in this particular mode of operation, ZTCSC to XC ratio is positive.




That means ZTCSC is having same sign with XC. So therefore, it is a capacitive Vernier mode, capacitive mode of operation. Whereas, in this particular mode, when ZTCSC and XC ratio is negative, that means, the sign of this impedance, overall impedance of the TCSC is negative of the reactance of the fixed capacitor, which is eventually an inductive, you know, reactance. So, therefore, this would be inductive mode of operation. So, this is inductive mode of operation. So, in this Vernier control mode of operation of TCSC either the operation should be here or here. In general it is in capacitive mode. But this point has to be avoided. So, you should have a sufficient you know margin to avoid this state, where this ratio is infinite and this may cause a discontinuity in the transmission line, which is not acceptable at all. So, this is something I want to tell. In fact, what is actually happening over here is that, with the change, with the increase in this beta, what is actually happening is this ZTCSC is slowly moving from capacitive mode of operation to inductive mode of operation or inductive Vernier control mode of operation.

I am just writing in a short inductive mode of operation. Now, why it is actually happening if you look at this waveform once again which I have drawn. At the very beginning when I started the analysis, when beta is equal to 0 means, what does it mean actually? This TCR is non-conducting or TCR is fully off. When TCR is fully off, it is capacitive mode. Now, when TCR is slowly building its current like this, slowly building its current, then what is actually happening when this TCR current is 0, then the net reactance of the TCR is the voltage across TCR which is the voltage across this capacitor that is Vc divided by the current that is flowing through the TCR that is ITCR. That ratio would be infinitely large when it is operating, beta is operating near to 0. And therefore, in this particular, this interval when beta is very near to 0 and it is operating in capacitor mode.

So, basically what is actually happening is, since this impedance or reactance of this TCR is, because you can see as I show, show you that X TCR is basically equal to this voltage that is V CT divided by I TCR T. Now, when I TCR is T is close to 0, then this will become very large, very large or infinitely large, whatever the value of VCT might be or infinitely large. Now, when you move this value of the ITCR from 0 to M1 value, still this XTCR is usually large. So, therefore, in this particular mode, Xc is lower than XTCR, this condition is getting satisfied and that is why, you know, the capacitive of mode of operation is taking place. Whereas, if we keep on this increase in the value of beta, then this xtcr, which is the ratio of vct divided by itcrt, when itcrt is sufficiently large, then xtcr would be lower.

So, therefore, this will be the case when xc is greater than this xtcr. That is why it is operating in the inductive mode of operation. So, at this happens, so this xtcr value is higher than higher near to beta is equal to 0, what I have shown over here, and xtcr value is lower to beta is equal to pi by 2, which is happening over here. And that makes these two conditions satisfied, which I discussed long time before, when I started this discussion on TCSC. And that is why it is actually happening. Now, so therefore, with the increase of this beta, TCS is slowly moving capacitive mode, because actually this TCR reactance, if you plot this TCR reactance, it is getting reduced.

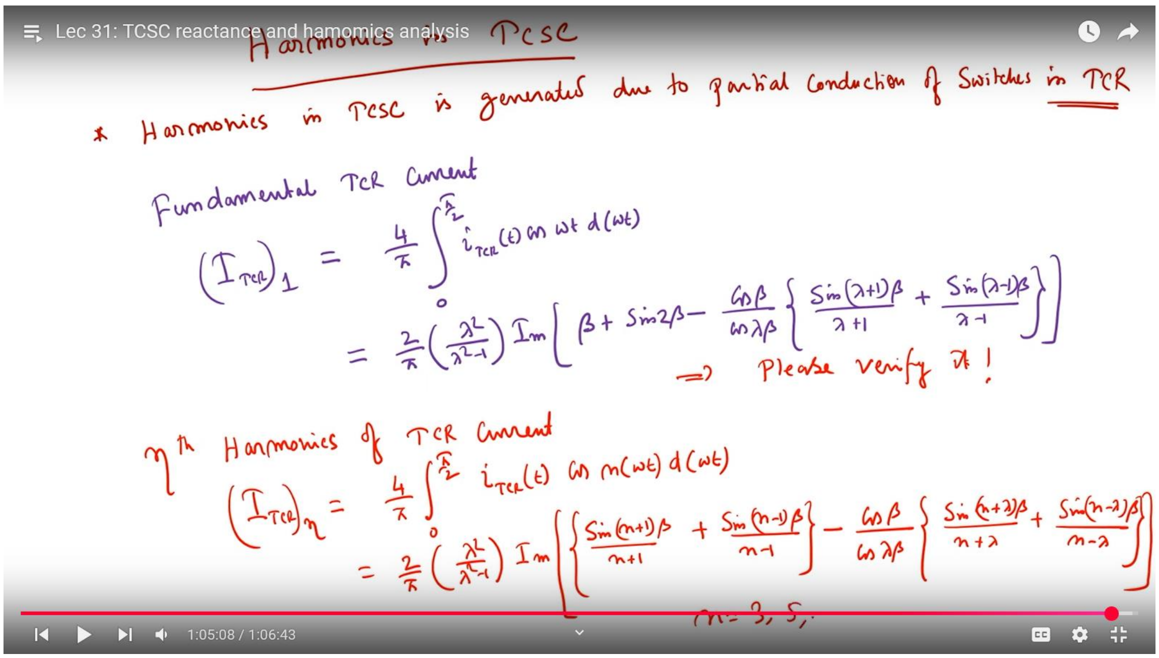
So, if this, if you plot this TCR reactance, X TCR of t with respect to this beta, then what we will see, its value is getting reduced. So, that is what is actually happening and this is what the reason is and that is why these two modes of operation is taking place. Now, next if you remember I explained that this when I discussed this waveform I said that this VCT will not be sinusoidal, TCSC it will be only sinusoidal when TCSC operation at fixed capacitor mode. Then the question would be how would be the characteristics of VCT or how would be the waveforms of VCT. So, that is also an important to us. So, this you can eventually verify by taking an example. So, what I can show over here is that VCT waveforms for capacitive and inductive mode of operations. Now, how would be, let us see. So, let us start with this inductive mode of operation. So, what is actually happening in the inductive mode of operation or let us start with this capacitive mode of operation.

So, in capacitive mode of operation, if I draw at this waveform once again. So, suppose Suppose this is our line current is, this is our line current I t, I of t that is line And, then this for this particular line current as you know when it is operating at this capacitive mode of operation. So, what we know is this would be if we go back and see. Suppose this is our line current and then this would be our, this green line is showing that this would be our, this ITCRT. So therefore ITCRT would be something like this. I T C R T will be something like this. This is minus beta, this is beta. So, this would be something like this. So, therefore, what would be the net, this current, as we know, if we just draw this, this schematic diagram of this, this is our fixed capacitor, that is the voltage across this is V c t, and this is our, this T c r unit, it is drawing a current, which is equal to I T c r t And, this is basically this I of t. Now, this the difference of this I of t, I of t minus this I TCR t is basically representation of current flowing through the capacitor. Now, how would be the you know this difference that you can see over here, if you have if you take the difference of this, then this would be something like this. Up to this it will be like this, then it would be something like this and then again this would be something like this. So, this green curve is basically representation of current flowing through the capacitor. And integration of this will representation of the voltage across the capacitor which would be you know that there would be a phase displacement of this pi by 2, but if you look back I already explained this, but this characteristics would be almost similar to this the current flowing through the capacitor. So, it will be having a phase displacement of this pi by 2. So therefore, so this is pi by 2. So therefore, this capacitor current, capacitor voltage will be something like this. It will be like this. Similarly, it will be like this. So this is what would be the capacitor voltage or V c of t.

Now, similarly, this will happen for the capacitive mode of operation. This is for the capacitive mode of TCSC operation. Now, similarly, we can also draw similar characteristics for inductive mode of operation, and similar waveforms. So, suppose this is what this line current that is I of t. Now, we know that we already explained at the very beginning, if you can remember that, for the inductive mode of operation, the direction of the line current and the direction of this TCR current would be different than this capacitive mode of operation.

Therefore, this current would be something like this, ITCR will be something like this. This will be ITCR or actually, I should draw it in a bigger way because here now ITCR will be the considerably higher magnitude, and there will be again this ITCR, this will be again ITCR. So, therefore, the net current, the difference of IT and ITCR which will be the current flowing through the capacitor would be something like this. it would be something it will be follow like this profile and then it will getting be reduced like this then it will have the same profile like this then it will follow this again then it will be again reduced it will be something like this. And accordingly this voltage across this capacitor or V c t will be something like this, it will follow it will there it will be. So, I although I have not written, but you understand this will be the expression for current flowing through the capacitor. So therefore, the capacitor voltage would be almost similar in nature.

So, it will be something like this or it will be something like that. So, this will be capacitor voltage. So, this will be the actual, you know, waveform for the capacitor voltage, which I told you that I will derive. So, and this would be this, you know, VCT waveform for inductive mode of operation, inductive mode of TCSC operation. Now you can see there is a difference of this waveforms in altogether and in between these two different modes of operation of TCS. Now one last thing that I will discuss in this particular lecture before I stop that is the harmonics in TCS. So you know that this harmonic is generated in TCSC because of the TCR operation. So I should write the harmonics in TCSC are generated due to the partial conduction of switches in TCR. This is very important to understand that it is because of because if TCR is responsible for this having harmonics in TCSC because and most importantly if the TCR is operated at partially conducting mode then this harmonics will be generated. But we need to find out this harmonics and we need to find out the mitigations of the harmonic as well.




But this harmonics will be less as compared to this normal operation of the TCR in shunt. But we need to analyze this harmonic. So, first we need to find out the fundamental TCR current which is ITCR, you can make it 1 to represent it is fundamental, it is equal to 4 by pi 0 to pi by 2 ITCR of t cos omega t d omega t. So, this is by using this Fourier series expression and you know that ITCRT expression already we have drawn, we have derived, this is what the expression for ITCRT. So, if I put it over here, then I will come up with the expression for this fundamental TCR current and that expression also you can derive, I ask you to derive and this expression of this is coming out to be 2 by pi lambda square divided by lambda square minus 1 I m peak value of this line current beta plus sin 2 beta minus cos beta divided by cos lambda beta. I am just writing whatever expressions I got, you also verify it, sin lambda plus 1 beta divided by lambda plus 1 plus sin lambda minus 1 beta divided by lambda minus 1.

So this expression I got, but please verify it. Similarly, this n-th harmonics of TCR current can be obtained as this I T c r of n-th is equal to 4 by pi integration 0 to pi by 2 I T c r t cos n omega t  d omega t, which is if you again I will ask you to verify it is coming out to be 2 by pi lambda square minus lambda square divided by lambda square minus 1 I m multiplied by sin n minus 1 beta divided by n plus 1 plus sin n minus 1 beta divided by n minus 1. So, let us use some different parenthesis. This is minus cos beta divided by cos lambda beta multiplied by sin n plus lambda beta divided by n plus lambda plus sin n minus lambda beta divided by n minus lambda.

Where n is equal to 3, 5, 7 all are harmonics. This is again I will ask you to verify. So, these are the different harmonics that we will have in the TCSC and we cannot do anything else in that we have to bear with this harmonics and but we should analyze it so that we can control this TCSC such that less harmonics would be generated in it. So, that is all about this TCSC analysis and its operation. So, in the next lecture, I will discuss the application of TCSC in power systems. So, till then thank you very much for attending this lecture. So, thank you very much for your attention, we look forward to see you in the next lecture.

## ❖ **TCSC Reactance and Harmonics**

As we know, the instantaneous voltage across the capacitor can be written as:

$v_c(t) = \frac{I_m X_C}{\lambda^2-1}\left[-sin\omega t + \frac{\lambda cos(\beta)}{cos(\lambda\beta)} sin(\lambda\omega t)\right], \quad \omega t \epsilon[-\beta, \beta]$ $\Leftarrow$ TCR is conducting, $i_{TCR}(t) \neq 0$

$v_c(t) = V_c' + I_m X_c[sin\omega t - sin\beta]$ $\Leftarrow$TCR is non-conducting, $i_{TCR}(t) = 0$

Where, $V_c' = \frac{I_m X_C}{\lambda^2-1}[-sin\beta + \lambda cos(\beta)tan(\lambda\beta)]$ $\Leftarrow$ Time-invariant

The fundamental component of the voltage across the capacitor can be obtained as,

$$V_{c1} = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} v_c(t) sin\omega t.\, d\omega t \tag{1}$$

$$= \underbrace{\frac{4}{\pi}\int_0^{\beta} v_c(t) sin\omega t.\, d\omega t}_{\text{TCR is conducting}} + \underbrace{\frac{4}{\pi}\int_{\beta}^{\frac{\pi}{2}} v_c(t) sin\omega t.\, d\omega t}_{\text{TCR is non-conducting}}$$

$$= \frac{4}{\pi}\int_0^{\beta} \frac{I_m X_C}{\lambda^2-1}\left[-sin\omega t + \frac{\lambda cos(\beta)}{cos(\lambda\beta)} sin(\lambda\omega t)\right] sin\omega t.\, d\omega t + \frac{4}{\pi}\int_{\beta}^{\frac{\pi}{2}} [V_c' + I_m X_c\{sin\omega t - sin\beta\}] sin\omega t.\, d\omega t$$

$$\Rightarrow V_{c1} = I_m \left[X_C - \left(\frac{X_C^2}{X_C - X_L}\right)\left(\frac{2\beta + sin2\beta}{\pi}\right) + \left(\frac{4X_C^2}{X_C - X_L}\right)\left(\frac{cos^2\beta}{\lambda^2-1}\right)\left(\frac{\lambda tan\lambda\beta - tan\beta}{\pi}\right)\right] \qquad (2)$$

The impedance (Reactance) of TCSC is $Z_{TCSC}$

$$Z_{TCSC} = \frac{V_{c1}}{I_m}$$

$$\Rightarrow Z_{TCSC} = X_C - \left(\frac{X_C^2}{X_C - X_L}\right)\left(\frac{2\beta + sin2\beta}{\pi}\right) + \left(\frac{4X_C^2}{X_C - X_L}\right)\left(\frac{cos^2\beta}{\lambda^2-1}\right)\left(\frac{\lambda tan\lambda\beta - tan\beta}{\pi}\right)$$

$$= X_C \left[1 - \left(\frac{X_C}{X_C - X_L}\right)\left(\frac{2\beta + sin2\beta}{\pi}\right) + \left(\frac{4X_C}{X_C - X_L}\right)\left(\frac{cos^2\beta}{\lambda^2-1}\right)\left(\frac{\lambda tan\lambda\beta - tan\beta}{\pi}\right)\right]$$

$$= 1 - \left(\frac{X_C}{X_C - X_L}\right)\left(\frac{2\beta + sin2\beta}{\pi}\right) + \left(\frac{4X_C}{X_C - X_L}\right)\left(\frac{cos^2\beta}{\lambda^2-1}\right)\left(\frac{\lambda tan\lambda\beta - tan\beta}{\pi}\right)$$

$$= 1 - \left(\frac{\frac{X_C}{X_L}}{\frac{X_C}{X_L} - 1}\right)\left(\frac{2\beta + sin2\beta}{\pi}\right) + \left(\frac{4\frac{X_C}{X_L}}{\frac{X_C}{X_L} - 1}\right)\left(\frac{cos^2\beta}{\lambda^2-1}\right)\left(\frac{\lambda tan\lambda\beta - tan\beta}{\pi}\right)$$

Now, $\lambda = \frac{\omega_r}{\omega}, \omega_r = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \lambda = \frac{1}{\omega\sqrt{LC}} \Rightarrow \lambda^2\omega^2 LC = 1 \Rightarrow \lambda^2(\omega L)(\omega C) = 1$$

$$\Rightarrow \lambda^2(\omega L) = \frac{1}{(\omega C)} \Rightarrow \lambda^2 = \frac{\frac{1}{(\omega C)}}{(\omega L)} \Rightarrow \lambda^2 = \frac{X_C}{X_L}$$

$$\Rightarrow Z_{TCSC} = 1 - \left(\frac{\lambda^2}{\lambda^2-1}\right)\left(\frac{2\beta + sin2\beta}{\pi}\right) + \left(\frac{4\lambda^2}{\lambda^2-1}\right)\left(\frac{cos^2\beta}{\lambda^2-1}\right)\left(\frac{\lambda tan\lambda\beta - tan\beta}{\pi}\right)$$

$$= 1 - \left(\frac{2}{\pi}\right)\left(\frac{\lambda^2}{\lambda^2-1}\right)\left(\beta + \frac{sin2\beta}{2}\right) + \left(\frac{2}{\pi}\right)\left(\frac{\lambda^2}{\lambda^2-1}\right)\left(\frac{2cos^2\beta}{\lambda^2-1}\right)(\lambda tan\lambda\beta - tan\beta)$$

$$= 1 + \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right)\left[\frac{2cos^2\beta}{\lambda^2-1}(\lambda tan\lambda\beta - tan\beta) - \beta - \frac{sin2\beta}{2}\right]$$

$$\Rightarrow \underset{\Downarrow}{\frac{Z_{TCSC}}{X_C}} = 1 + \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right)\left[\frac{2cos^2\beta}{\lambda^2-1}(\lambda tan\lambda\beta - tan\beta) - \beta - \frac{sin2\beta}{2}\right] \qquad (3)$$

Ratio of TCSC impedance to the reactance of the fixed capacitor.

$$\frac{Z_{TCSC}}{X_C} = f(\lambda, \beta)$$

$\lambda$ =Design parameter and constant for a particular TCSC

**Case study:**

(i) $\beta = 0$ [ TCR is fully OFF]: Fixed capacitor mode of operation

$$\frac{Z_{TCSC}}{X_C} = 1 + \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right) \times 0 = 1 \Rightarrow Z_{TCSC} = X_C$$

(ii) $\beta = \frac{\pi}{2}$ [ TCR is fully ON]: Inductive mode of operation

$$\frac{Z_{TCSC}}{X_C} = 1 + \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right)\left(\frac{-\pi}{2}\right) = 1 - \frac{\lambda^2}{\lambda^2-1} = \frac{-1}{\lambda^2-1}$$

$$\lambda = 3, \frac{Z_{TCSC}}{X_C} = -\frac{1}{9-1} = \frac{-1}{8} = -0.125$$

(iii) $\beta = \beta_C$ for which $\frac{Z_{TCSC}}{X_C} \to \infty$

From equation (3) :

$$\frac{Z_{TCSC}}{X_C} = 1 + \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right)\left[\left(\frac{2cos^2\beta}{\lambda^2-1}\right)\lambda tan\lambda\beta - \left(\frac{2cos^2\beta}{\lambda^2-1}\right)tan\beta - \beta - \frac{sin2\beta}{2}\right]$$

$$= 1 + \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right)\left[\left(\frac{2cos^2\beta}{\lambda^2-1}\right)\lambda tan\lambda\beta - \left(\frac{2cos^2\beta}{\lambda^2-1}\right)\frac{sin\beta}{cos\beta} - \beta - \frac{sin2\beta}{2}\right]$$

$$= 1 + \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right)\left[\frac{2cos^2\beta\ \lambda tan\lambda\beta}{\lambda^2-1} - \frac{sin2\beta}{\lambda^2-1} - \beta - \frac{sin2\beta}{2}\right]$$

$$= 1 + \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right)\left[\frac{2cos^2\beta\ \lambda tan\lambda\beta}{\lambda^2-1} - sin2\beta\left\{\frac{1}{\lambda^2-1} + \frac{1}{2}\right\} - \beta\right]$$

$$= 1 + \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right)\left[\frac{2cos^2\beta\ \lambda tan\lambda\beta}{\lambda^2-1} - sin2\beta\left\{\frac{2+\lambda^2-1}{2(\lambda^2-1)}\right\} - \beta\right]$$

$$\Rightarrow \frac{Z_{TCSC}}{X_C} = 1 + \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right)\left[\frac{2cos^2\beta\ \lambda tan\lambda\beta}{\lambda^2-1} - \frac{sin2\beta}{2}\left\{\frac{\lambda^2+1}{(\lambda^2-1)}\right\} - \beta\right] \qquad (4)$$

The only variable in the above expression is $\beta$ and $\beta \in \left[0, \frac{\pi}{2}\right]$

$$if\left(\lambda\beta = \frac{\pi}{2}\right) \Rightarrow \frac{Z_{TCSC}}{X_C} \approx \infty$$

$$\Rightarrow \beta_C = \frac{\pi}{2\lambda}$$

**Plot of $\left(\frac{Z_{TCSC}}{X_C}\right) vs\ \beta$ :**

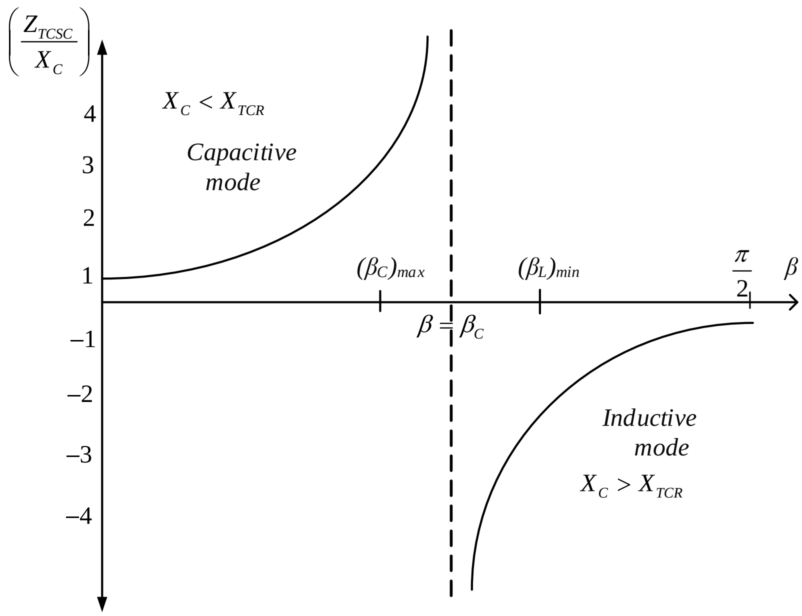



**Fig..1** Variation of $(X_{TCSC}/X_C)$ as a function of $\beta$

$X_C < X_{TCR}$ : Capacitive operation

$X_C > X_{TCR}$ : Inductive operation

From Fig.1, it is observed that with the increase in $\beta$, TCSC is slowly moving from capacitive Vernier control mode of operation to inductive Vernier control mode of operation.

$$X_{TCR} = \frac{v_c(t)}{i_{TCR}(t)}$$

When, $i_{TCR}(t) \approx 0 \Rightarrow X_{TCR} = Very\ large\ value$

When, $i_{TCR}(t) \uparrow \Rightarrow X_{TCR} \downarrow$

The variation of $X_{TCR}$ with $\beta$ is shown in Fig.2.

$X_{TCR}$ value is higher near to $\beta = 0$

$X_{TCR}$ value is lower near to $\beta = \frac{\pi}{2}$

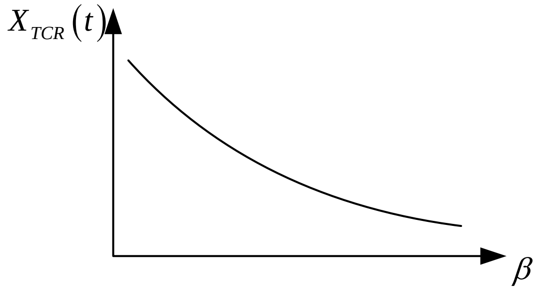



**Fig.2.** Variation of $X_{TCR}$ with respect to $\beta$

***$v_c(t)$* waveforms for capacitive and inductive mode of operations:**

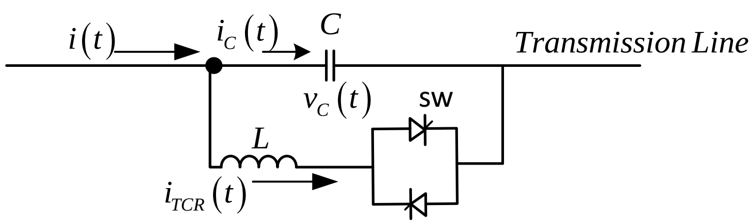



**Fig.3.** Equivalent circuit diagram of TCSC

From Fig.3, $i(t) - i_{TCR}(t) = i_c(t) =$ Current flowing through the capacitor

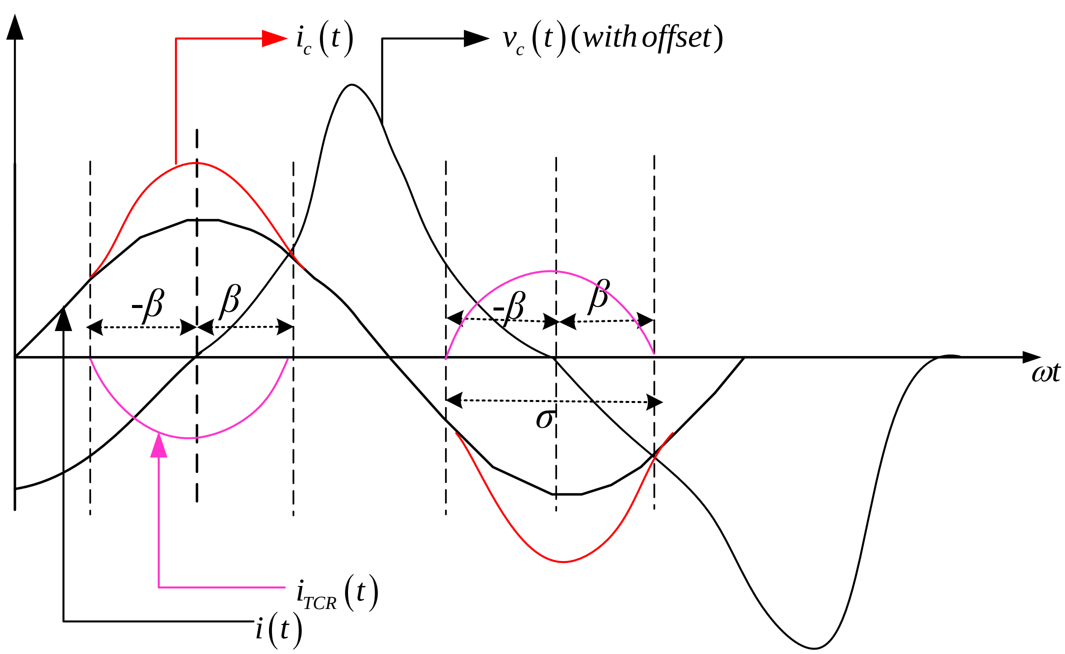



**Fig.4.** Waveforms of $i_c(t), i_{TCR}(t), v_c(t)$ for capacitive mode of TCSC operation

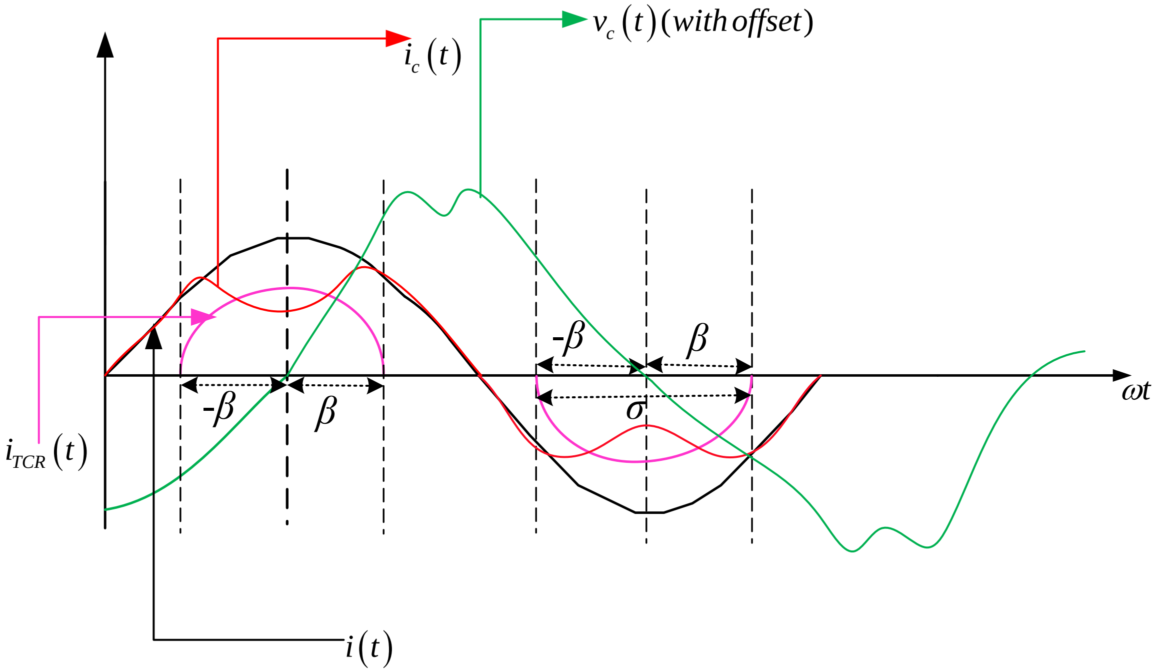



**Fig.5.** Waveforms of $i_c(t), i_{TCR}(t), v_c(t)$ for inductive mode of TCSC operation

The waveforms of $i_c(t), i_{TCR}(t), v_c(t)$ for capacitive and inductive mode of TCSC operation are shown in Fig.4 and Fig.5 respectively.

**<u>Harmonics in TCSC</u>**

The harmonics in TCSC is generated due to partial conduction of switches in TCR.

Fundamental TCR current

$$(I_{TCR})_1 = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} i_{TCR}(t)\, cos\omega t\; d(\omega t)$$

$$(I_{TCR})_1 = \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \left[\beta + sin2\beta - \frac{cos\beta}{cos\lambda\beta}\left\{\frac{sin(\lambda+1)\beta}{\lambda+1} + \frac{sin(\lambda-1)\beta}{\lambda-1}\right\}\right] \qquad (5)$$

$n^{th}$ harmonics of TCR current

$$(I_{TCR})_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} i_{TCR}(t)\, cosn(\omega t)\; d(\omega t)$$

$$(I_{TCR})_1 = \frac{2}{\pi}\left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \left[\left\{\frac{\sin(n+1)\beta}{n+1} + \frac{\sin(n-1)\beta}{n-1}\right\} - \frac{cos\beta}{cos\lambda\beta}\left\{\frac{sin(n+\lambda)\beta}{n+\lambda} + \frac{sin(n-\lambda)\beta}{n-\lambda}\right\}\right] \qquad (6)$$

$$n = 3,5,7, \dots \dots$$