Course Name: Power Electronics Applications in Power Systems

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Week: 09

Lecture: 03

Lec 30: Basic mathematical modelling of TCSC-Part 2

So welcome again to another lecture of my course Power Electronics Applications in Power Systems. In the last lecture, I started discussion on the mathematical modeling of thyristor-controlled series capacitors. In short, it is known as TCSC and it is popularly named as TCSC in the literature of this power electronics compensator used in power systems. Now in this particular lecture, we will continue the mathematical modeling of the TCSC. Let us quickly recapitulate what we have learned in the last lecture. So in the last lecture, we have learned the modes of operation of TCSC and then I started the mathematical modeling of this TCSC.

You can see this is the very basic schematic of a TCSC which I had drawn in the last lecture. Then we apply this very basic law of electrical engineering, Kirchhoff's current law, and Kirchhoff's voltage law to find out a set of equations. This is one equation. This is one equation. This is another equation that we developed by applying this Kirchhoff's voltage law. And this is another equation we developed by using Kirchhoff's voltage law. Now, with these equations, we will get a second-order differential equation that is this. This is the equation that we got. And once solving this equation, we have come out with the instantaneous value of the current flowing through the TCR, that is this, this current, this arrow.

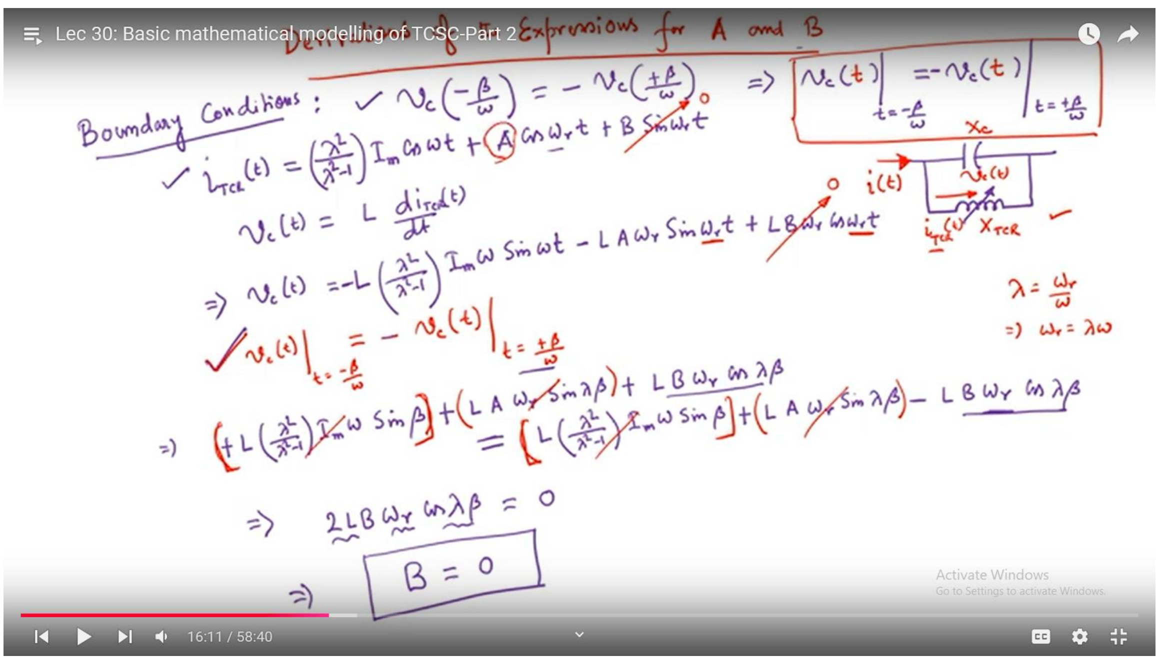
It is showing the instantaneous current flowing through the reactor. As I said, if you look at this expression, you will see that the lambda we already defined, it is the ratio of the omega r to omega and A and B are the two, arbitrary constants, and omega as you know is power frequency, I m is the peak value of the system voltage which already I have taken this and you know that omega r comes due to the resonance which is equal to 1 upon root over LC. So, A and B are the two arbitrary constants. We need to find out the expressions for A and B through the boundary conditions. These boundary conditions, we get by applying from this particular waveform. So, these are the two boundary conditions. One is this, another is that. So, we will apply these two boundary conditions today to find out the expressions for the arbitrary constants A and B. So, today we will start with derivations of the expressions for for A and B. We will do with these two arbitrary constants, two boundary conditions that we have developed in the last lecture. One is this, another is that. So, let us write these two boundary conditions. One is this. So, what are the boundary conditions we developed in the last lecture? This is the boundary condition. So, I will start with the second boundary condition first.

So, it is v of c minus beta by omega. So, v of c minus beta by omega is equal to minus v of c plus beta by omega or alternatively, we can write these equations like this v c t where t is equal to minus beta by omega is equal to v c of t where is equal to minus of v c omega t where t is equal to plus beta by omega. Now, if you look at this boundary condition to find this, you know, to put this boundary conditions in a mathematical equation, one needs to find out the expression for v c of t. So, you know, this already we determined the expression for i t c r. And we also know the relationship of this VCT with this ITCR.

So, what we can do? Let us write this expression for I of TCR T. What is this basically? This is the instantaneous current flowing through the TCR unit of the TCSC. So, this is the I of TCR, or alternatively on that particular page, I can draw it basic schematic diagram once again. This is our fixed capacitor. Here we have this variable reactor, which we are calling as TCR, right? We have this fixed capacitor, it is let us represent it by xc, here we have a variable reactor, let us represent it by Xtcr.

Now, the voltage across this fixed capacitor is vct and the current flowing through this Xtcr is itcr of t. And, this is what the line current is, it is the transmission line current, I am representing it by I of t. So, if I draw this, things would be clear to you. Now, we know the expressions of I TCRT, which we have already determined in the last lecture. This is the equation. So, I will just copy this equation in this particular page. So, what would be that equation? This equation would be lambda square divided by lambda square minus 1 I m cos omega t. This is what the power frequency term. But apart from that, we will have this resonant frequency term which is cos omega RT plus B sin omega RT. From the KVL equation, we also get a relationship that is between the voltage across the capacitor, the instantaneous voltage across the capacitor, and the instantaneous current flowing through the reactor or the TCR.

So, we know that VCT is equal to L di TCR dt. Now, from this particular equation, we can derive this expression of this. So, V c of t is equal to this L, this would be lambda square lambda square minus 1 I m. Now, if we differentiate cos omega t with respect to t, then what we will get? We will get minus omega sin omega t. Now, we have to differentiate this component with respect to t and multiply it with L. So, this will be equal to minus, because if we differentiate cos omega r t with respect to t, it will be L A omega r sin omega r t. And if you differentiate this again with respect to t, L would be of course there, L B sin omega r cos omega r t. So, that is what this v c t. Now, what we have to do is, we have to put these conditions over here. We have to put this condition over here. So, if we put this condition over here, and if we write that v c t when t is equal to minus beta by omega is equal to minus v c t, where t is equal to plus beta by omega. Now, if you put this equation over here, what we will get is, so left hand side I will put this equation, so this will be minus L lambda square, lambda square minus i m omega.

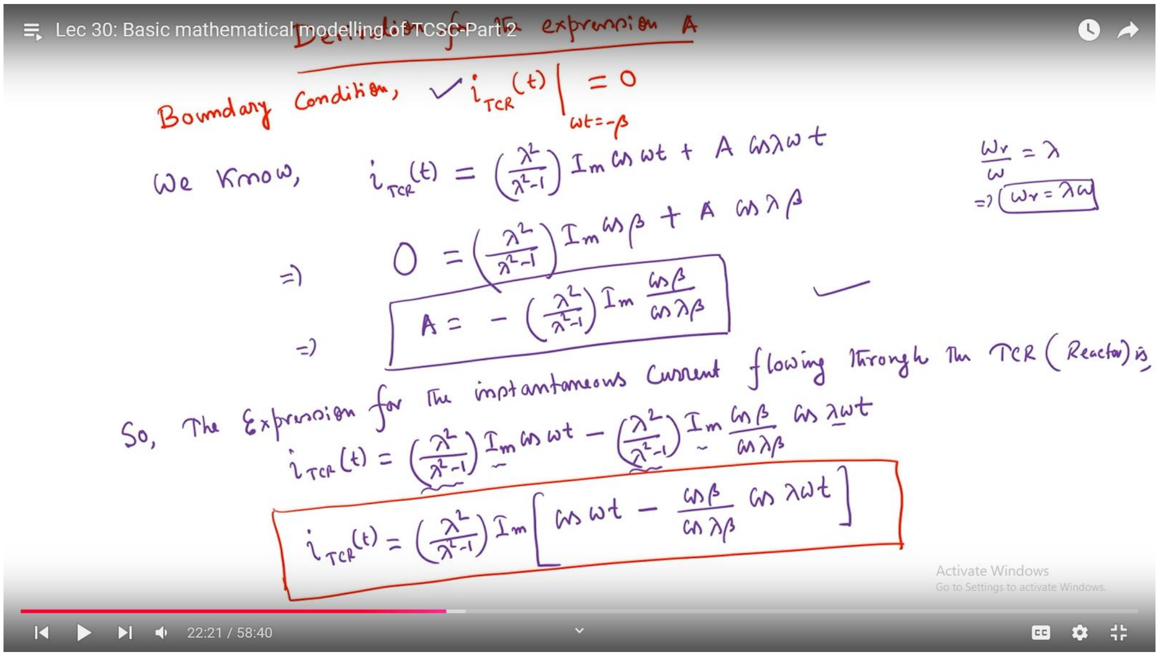


Now, if we put this expression over here, so t is equal to or omega t is equal to minus beta, so this will be plus. So, because omega t is equal to minus beta means sin of minus beta, which is minus sin beta. So, this we can write as sin beta. So, this negative sign I multiply it with this negative, it would get a positive value over here and then minus L A omega r. Now, sine omega t, if we put omega t is equal to minus beta. Now, you know the relationship of omega r to omega, already we determined that the ratio of omega r to omega is equal to lambda. So, since lambda is equal to omega r 2 omega, we can write omega r is equal to lambda omega. So, this omega r we can write as a lambda omega. So, this would be equal to L A omega r sin minus lambda beta. So, I am just replacing this omega r as lambda omega. So, omega t is now beta. So, it will be minus lambda beta. So, sin minus lambda beta will be sin lambda beta. Now, next, if I put over the here, so it will be L constant B omega r. So, again this if we put this cos omega r t as cos lambda omega t, then it will be cos lambda beta, if I am not wrong. So, this is what the left-hand side of this, this is what the left-hand side of the boundary conditions.

Now, what would be the right-hand side? This would be equated with the right-hand side. Right hand side already look at, we have a negative sign. So, then this negative, this negative would be positive. So, it would be L lambda square, lambda square minus I m omega. Now, here omega t is equal to plus beta. So, simply I will write sin beta over Now, look at this again I will write minus L A omega r, so sin omega t is again sin lambda omega t, so that means I can write it as a sin lambda beta, sin lambda beta. We have already a negative sign over here. So, we already have negative sign over here, this negative and that negative will make it positive. Now, so this negative again I will write here, this is equal to L b omega r. So, cos omega r t we can write is cos lambda omega t.

So, this is cos lambda beta because here this omega t is equal to beta plus beta. Now, this is what the relationship we get from this particular boundary conditions. Now, if you look at this relationship, this and this will be cancelled out, because they are identical. So, lambda L lambda square divided by lambda square minus 1 I m omega sin beta are common to both our side, both right hand side and left hand side. So, I will just simply cut it. Similarly, look at this L A omega r sin lambda beta, come on to both side, again I can cut this. So, what we will get is, if we bring this right-hand side to the left-hand side, so what we will get is 2 L B omega r cos lambda beta, because this again is identical with this, but their sign is different, this is equal to 0. So, if it is equal to 0, you can see 2 L cannot be equal to 0, omega 8 cannot be equal to 0. Similarly, cos lambda beta cannot be equal to 0. So, therefore, this gives the constant B is equal to 0.

So, because you can see that this cos beta is here, this beta is a finite value, it is not equal to pi by 2. So, it is not equal to 0. So, that is why B is equal to 0. So, we get the expressions of for B. So, that means from this particular equations, from this particular equation, this part, this part would be equal to 0. Similarly, from this particular equation, this part will be equal to 0. Then we have to find out this, the expressions for A as well. So, to find out this A, constant A, expression for constant A, we will again apply some boundary conditions. Already we have defined this boundary conditions over here, i t c r minus, where omega t is equal to this minus beta is equal to 0. So, it will continue the derivation for the expression A. And to find this expression of A, that arbitrary constant A, what do we do? We will use this boundary condition I t c r t, where omega t is equal to minus beta is equal to 0. So, we will put this boundary condition I T c r of t, where omega t is equal to minus beta is equal to 0. Now, already we know that what is the expression of I T c r t, it will be reduced to this, it will be reduced to this. Now, let us write it over here. So we know I TCR of T which is current instantaneous current flowing through the TCR is equal to lambda square divided by lambda square minus 1 IM IM cos omega T cos omega t plus A cos omega r t.



Again, we know this omega r to omega, this ratio is equal to lambda. So, we can replace this omega r by lambda omega. So, we can do so. We have done it earlier. So, let us represent it with lambda omega t. Now, if we put this expression over here, that i t c r, i t c r t, when omega t is equal to minus b t is equal to 0. So, here left-hand side would be 0. So, right-hand side would be lambda square, lambda square minus 1, i m cos beta, right, plus this a cos lambda beta. So, therefore, A is equal to minus this lambda square divided by lambda square minus 1 I m cos beta divided by cos lambda beta. So, this is what the expressions for A we derived, and of course, we already derived the expressions for B.

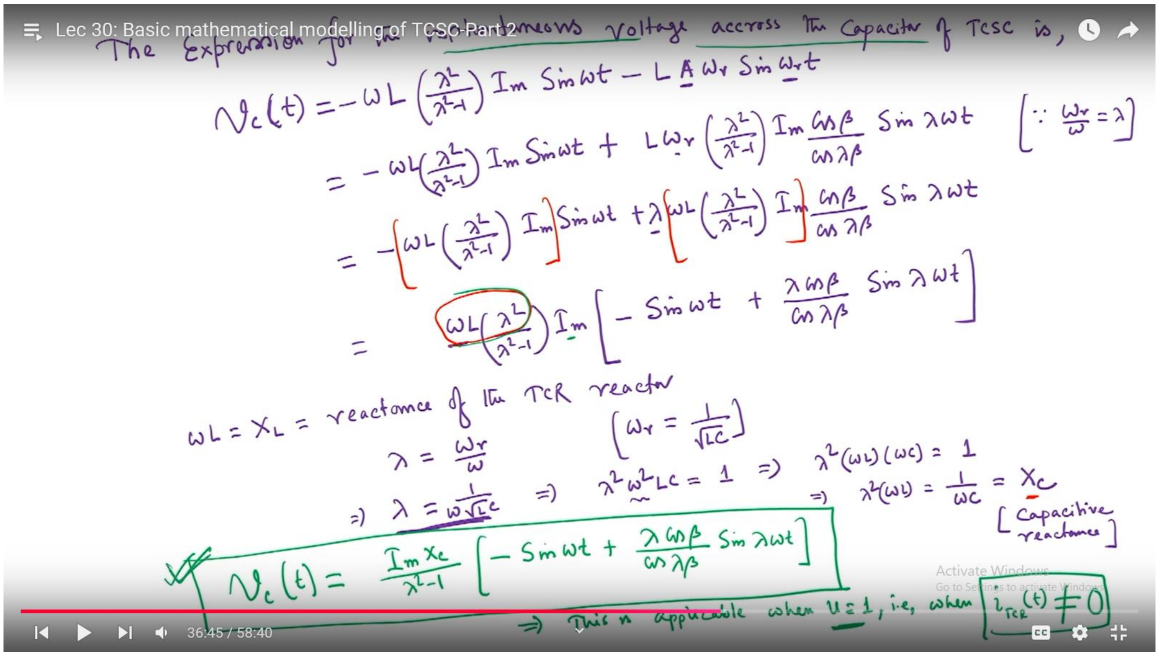
So, we got both the expressions for A and B and we will put them over here. So, if we put over here, so then, so the expression for the instantaneous current flowing through the TCR or the reactor is, so if I put this, then I T c r t is equal to lambda square minus 1 I m cos omega t. Now, I will put this expression a over here. So, it will be, there is a negative sign. So, this will be minus lambda square, lambda square minus 1 I m cos beta divided by cos lambda beta multiplied by cos omega t, or we can write cos lambda omega t. This is the expressions we get. Now, if we just take this lambda square minus 1 common from both side because this is common and I m is also common. So, our expression will look like lambda square lambda square minus 1 Im. Let us put outside of the bracket, then what we will get is cos omega t here minus cos beta divided by cos lambda beta multiplied by cos omega t or you can write lambda omega t. So, this is what the expressions for ITCR of t. So this is the complete expression for ITCR of T.

So this is the complete expression for the instantaneous current flowing through the TCR unit or flowing through the reactor of the TCSC. Now again, we also need to determine the expression for VCT as well. In order to find that, so let us copy these expressions once again. This expression what we get here, let us copy these expressions once again in another page. So, the expression for the instantaneous voltage across the capacitor of TCSC is v c of t is equal to, let me copy this thing once again. So, this is minus L, minus L, then lambda square divided by lambda square minus 1 I m. There would be an omega term here. This is an omega term which I want to get multiplied with L. So, I should write this omega here before I write L. So, this is omega L I m sin omega t. Look at this, whether this I wrote correctly or not. So, the only thing is that this omega I brought to here, this omega I brought to here, just to multiply it with L. The Rest of the equations are the same I believe. So, this is equal to this.

Now, that minus we have L A omega r sin omega t. Minus L A omega r sin omega r t that was the expression we got L A L A omega r sin omega r t. Now, we already determined the expression for A, which will be put over here as well. So, if we put, then this would be as it is minus omega L lambda square lambda square minus 1 I m sin omega t right minus L omega r along with this expression for A. Now, what was the expressions for A? Expressions for A we determined here.

It is a big expression. So, since there it is having a negative value, so it will be multiplied with here. So, this will be positive and then, we will be having again this lambda square divided by lambda square minus 1, this factor, this factor would be here. Then we have I m, we have cos beta divided by cos lambda beta, so cos beta divided by cos lambda beta. Now, this along with this sin omega t, now omega r again I replace it with lambda omega t, as we did earlier in this case, since we know the ratio of omega r to omega is equal to lambda, since the ratio of omega r to omega is equal to lambda.

Now, let us do some more derivations over here. So, this side now we will have minus omega L, here lambda square, lambda square minus 1, Im sine omega t. Here we have L, now omega we know, lambda omega, so this is lambda omega L, lambda omega lambda square lambda square minus 1 I m cos beta a very big equation cos lambda beta then sin lambda omega t. Now there are some parts which are common in the both the terms here we have you know two terms where some of the parts are common like this is a common part this is omega l multiplied by this is common. In fact, I m is also common. I m is also common. I can put it inside the bracket and bring it outside. So, then what will it be? Let us see. So, this will be minus omega L lambda square divided by lambda square minus 1 I m, if I put outside the bracket, let us do not put this negative sign at the very beginning, so that I can write this is minus sin omega t. Now, plus this lambda cos beta, because this lambda will be outside the bracket, so lambda cos beta divided by cos lambda beta, then sin lambda omega t.



This is what the expressions for VCT. Now, we will put some more derivation over here. One is what is omega L? Omega L nothing but we can put at x L. Now, so omega L is nothing but x L. Now, what is the x L? x L is the reactance of the TCR reactor. Now, when you multiply this omega L with lambda square, what may happen? Let us see.

So, actually we know lambda is equal to omega r 2 omega, and we know omega r is equal to 1 upon root over L C, which is the resonance frequency, which we know, already we explained over here. This gives the solution. Now, if it is so, then let us put this over here. So, then what we will get? We will get lambda is equal to 1 upon root over L c omega. So, from this, we can write lambda square omega square L C if we just square both the left-hand side and right-hand side. So, what we will get is lambda square omega square multiplied with L of c is equal to 1. So, from this we can write that lambda square multiplied by omega L, so this is omega square, so I take 1 omega. So, it is multiplied with omega c is equal to 1 or from this we can write, so lambda square omega L is equal to 1 by omega c. Now, what is 1 upon omega c? It is nothing but the reactance of the capacitor. So, this is x of c, which is the capacitive reactance of the capacitor.

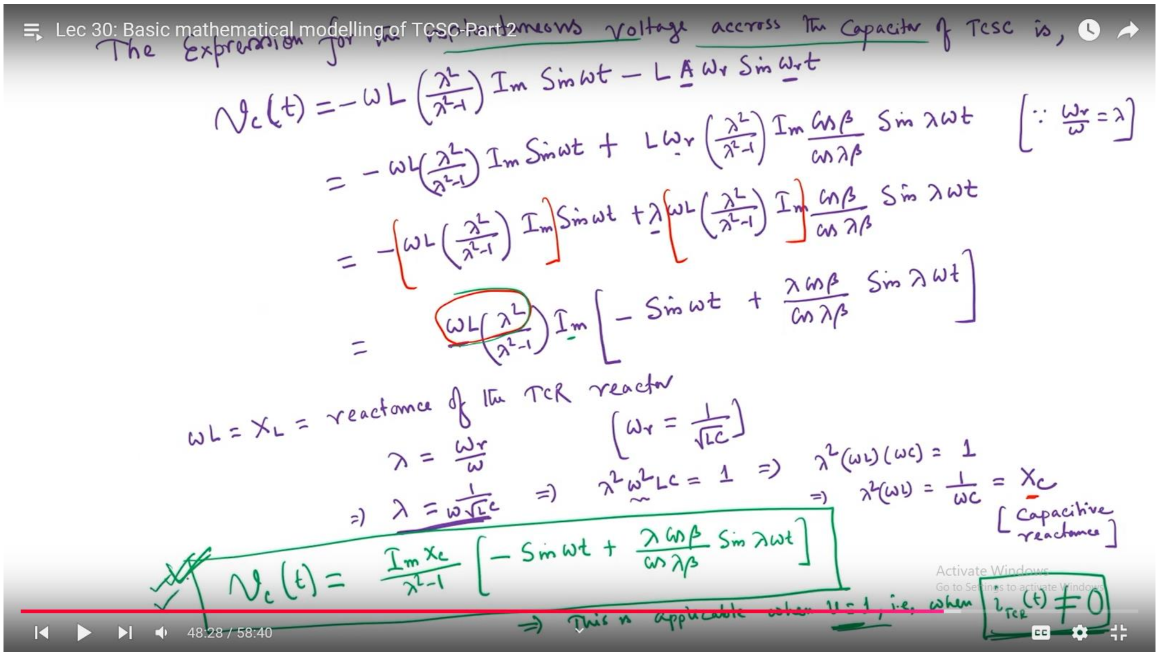
So, therefore, I can do one thing, I can replace this omega L lambda square, this part with the x c. So, then we will get the expression for this v c of t is equal to this lambda square omega l we are replacing with x c. So, this will be I m multiplied by x c divided by lambda square minus 1 I am, I am just multiplying with this because omega L multiplied by lambda square is nothing but x c, then this will be as it is minus sin omega t plus lambda cos beta divided by cos lambda beta sin lambda omega t. All right, so this is what the expressions of voltage, instantaneous voltage, and instantaneous voltage across the capacitor. Similarly, that was the expression for instantaneous current flowing through the TCR.

Once we develop these two, then this could be useful for our, you know, future derivations. So, we first derive this expression and you can plot this to eventually match with whether this plot, this waveforms for, of this TCSC parameters would match with thought. So that you can eventually verify by using any either in MATLAB or any software that you know. So, you can verify this. Here we will further use these expressions for further derivations of some of the parameters of TCSC and to understand the concept of TCSC in more detail. For example, let us go back and see this particular waveform. So, what you can see over here is I say that there are four instances, one is this, another is this, another is this, and another is this. One instance is omega t1, another is omega t2, another is omega t3, and another is omega t4. Now when you measure these four instances, what they stand for? This omega t1 is when the TCR current starts and omega t2 when it again comes back to 0. Omega t3 again when TCR current starts in the other half cycle and omega t4 when it again returns back to 0. Now you can see due to this particular nature of this TCR current since it is of harmonic in nature then it will create the harmonics in the line current or in the capacitive current also and in most importantly the voltage across the capacitor which depends upon the current flowing through the capacitor will also be harmonic in nature.

Now, this expression of ITCR we derive, what we derive, this ITCR is will be applicable for the entire duration of the current flowing through the reactor. Whenever this current flowing through the reactor is non-zero, then it will follow this equation. However, this capacitor voltage that is this voltage will be only applicable for a specific period of time because you know this capacitor voltage, capacitor voltage what we derive only for from this particular expressions that is L d i T c r d t that is this expression when u is non-zero, when u is equal to 1. This means, that this expression of capacitor voltage what we derive right now will be applicable.

So, this is applicable when u is equal to 1. That means, that is when I T C R of T is non-zero, not equal to 0. Now, when u is equal to 1, you can see, go back and see that when u is equal to 1, the TCR is on. So, therefore, some current will flow through this TCR. Now, when TCR is off, that there is no current flowing through this ITCR, then U is equal to 0. When U is equal to 0, this equation will not be valid, this equation will not be valid. Rather, when U is equal to 0, this TCSC will act as a fixed capacitor. So, therefore, the current expression will not be valid, this voltage across the capacitor expression will not be valid. So, therefore, these expressions for voltage will be only applicable when this current flowing through the TCR is non-zero. This is something one needs to understand. Then, the question is when the TCR current is 0, what would be the expression of voltage across the capacitor? That means if I go back and see the waveform once again, so during this interval, during this interval here to here, what will be the expression of this VCT?

That means when this current is non-zero, then what would be the expressions of capacitor voltage? This is the, you know, the next part which I should discuss. Now when this current is zero, then what would be the voltage across capacitor? Basically when this Tcr is not conducting, then the whole unit act as a fixed capacitor.



So that means I should write when the TCR is non-conducting, the TCSC will act as a fixed capacitor. This is something you want to understand. So, when u is equal to 0, that means TCR is non-conducting, conducting non-conducting or I should write I TCR current is equal to 0, we need to derive the expression for the instantaneous voltage across the capacitor. So, you have to understand during this period, TCSC will act as a fixed capacitor. So, therefore, if we draw the TCSC diagram, it will be something like that, where this, you know, this is not conducting. So, therefore, this current, the entire line current, which is coming from this transmission line will flow through the capacitor. That is this and the capacitor voltage during this moment of time will follow these equations. Capacitor voltage will you know follow these equations. So, during this period of time the capacitor you know voltage would be equal to during this moment of time if we consider this is I of c t and this is v c t then we know I of c t is equal to this I of t which is the line current because there is no other current if you apply KCL over here this I of t is equal to I c t. If we apply KCL at this point, at this particular node, there is no other component of current because, during that time, these switches are non-conducting.

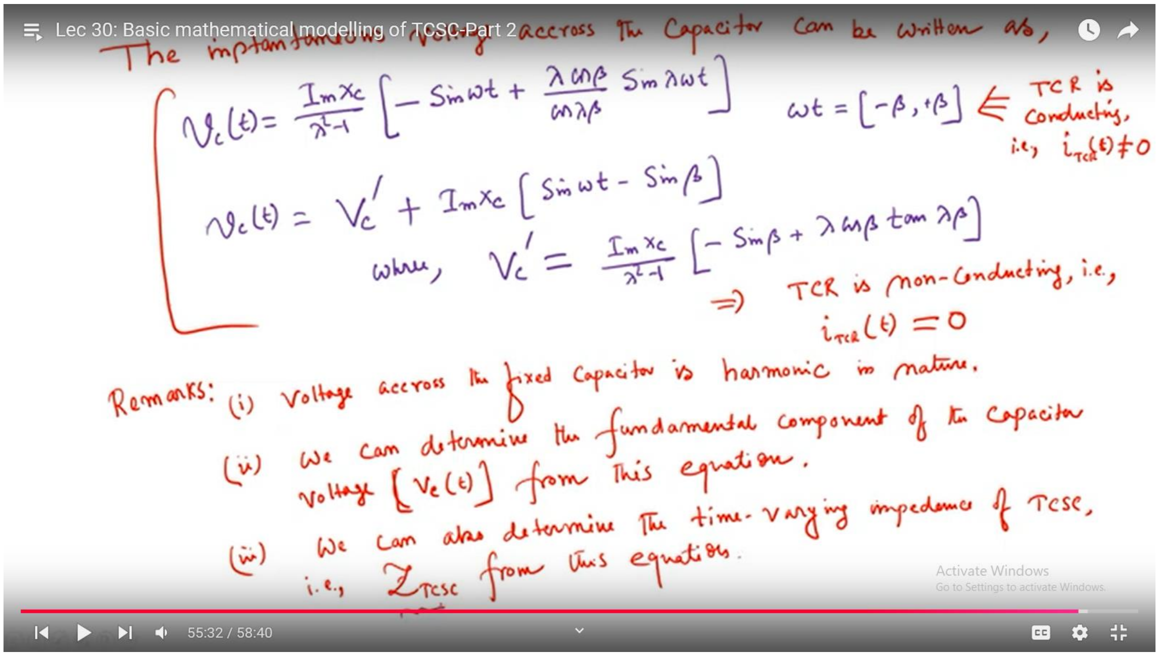
So, this current drawn by this TCR is 0. So, I of t is equal to ICT. So, therefore, we know that this equation, that ICT will follow this equation C d Vc(t) d t. Now, if we just use these expressions, then this, if you find the solution of these expressions, then if you find this v c t, then what would be the expression of v c t? It will be integration of 1 upon c, this I c t, which is equal to I of t dt plus this initial voltage across the capacitor that is vct when this fixed capacitor you know operation starts that means at what instant this fixed capacitor operation starts. So, this is the period where fixed capacitor operation of the TCSC will start. So, this is the fixed capacitor mode of operation.

So, just before this operation starts, the instant was this. During that period, that what was the v c t. So, this instant is nothing but instant when omega t is equal to beta. So, that means, this instance corresponds to this v c t when omega t is equal to plus beta. Let us consider this is, because this is, a single variable, this is not dependent on this thing, that is only parameter which will vary with this beta. So, if we find out that what will be that v c t, when omega t is equal to beta, then we have to apply this expression and put omega t is equal to beta.

So, we will apply this expression and put omega t is equal to beta. So, what we will get, will be I m x c lambda square minus 1 multiplied by, multiplied by minus sin omega t, now omega t is equal to beta, so minus sin beta plus this lambda cos beta divided by cos lambda beta, lambda cos beta divided by cos lambda beta multiplied with a sin lambda omega t. So, again if we have this omega t is equal to beta, so this will be equal to sin lambda beta. So, that means this is equal to I m x e lambda square minus 1 minus sin beta plus since in the numerator we have sin lambda beta and denominator we have cos lambda beta. So, the ratio of sin lambda beta to cos lambda beta will be tan lambda beta.

So, I am writing is lambda cos beta tan lambda beta. And, this voltage is just at the instant before the fixed capacitor operation starts. So, where that, this is the instant, at this particular instant, that is at instant omega t2. This is the instant corresponds to omega t2, which is equal to plus beta. So, at this particular instant, this VCT expression will not follow the derivation that we have done so far. So, therefore, at this particular instant it would be something else. So, let us consider this is that instant and let us consider this is equal to V c dash or let us put it V c dash, where V c dash is the voltage across the capacitor before the fixed capacitor mode starts. Now, we know this, this is VC, this VC2 we already derived, we have to derive this portion once again. So, then at fixed capacitor mode of operation, at fixed capacitor mode of operation, v c t is equal to, this v c dash will be as it is, already we derived it, plus, so this is 1 upon c integration of I of t. Now I of t is what? Already we know I of t is basically the current flowing through the line, the current flowing through the line.

This is I of t and this expression already we know. This is equal to Im cos omega t where Im is the peak value of the current and cos omega t represents the power frequency component. So, therefore, I should write this is Im cos omega t d t. Now, the question is how long it will go? So, what would be the limit? So, this limit of omega t starts from omega t is equal to beta to omega t. So, therefore, we can write it as v c dash plus. So, if you just integrate this cos omega t, it will be sin omega t. So, and one omega will be in the denominator, so that will be omega c, Im is constant, so this will be sin omega t. So now if we put this limit, so this will be sin omega t minus sin beta. So this would be expression of the voltage across the capacitor when it is operated as fixed capacitor mode of the TCSC. So, this is what the voltage across the capacitor.



So, during this period of time, this will be the expression of Vct. And during this period of time, the expressions of Vct already we derived, that is this. So, therefore, we should write the instantaneous voltage across the capacitor can be written as, rather expression for this instantaneous voltage across the capacitor can be written as v c t is equal to this expression, when this u 1 is equal to 0, that means, the T c are conducting. So, let me write this, I m x e divided by lambda square minus 1 I m x e divided by lambda square minus 1 multiplied by this minus sin omega t plus lambda cos beta divided by cos lambda beta multiplied by sin omega t or sin lambda omega t. So, this is for instance when omega t is in between minus beta to plus beta. And it will be the expressions for v c t will be equal to v c dash plus this one I already determine the last page that is this.

So, v c dash plus this I m c you can understand that I m 1 upon omega c we can write it as v c dash 1 upon omega c you can write is as x c. So, this is I m x c sin omega t minus sin beta. So, therefore, we can write it as v c dash plus I m x c sin omega t minus sin beta where v c dash value also we determine this is the expressions for this v c dash. So, this is equal to I m x e lambda square minus 1 multiplied with minus sin beta plus lambda cos beta tan lambda beta.

So, this will hold for this omega t. So, this as you know already we determined. So, this is the expression that would be applicable when this TCR is conducting that is I TCR of t not equal to 0 and this expression will work when TCR is non-conducting that is I TCR of t is equal to 0. So, altogether this is the instantaneous voltage across the capacitor. So, why we have derived this? Why we have derived this? Because this voltage expression is essential to understand the concept of TCSC operation. And also this expression is useful to find out the time-varying impedance of the TCSC as a whole.

So this expression will be also used to find out the time varying impedance. So what this expression shows? So let us put some remarks. So first remark we can write that the voltage across the fixed capacitor is harmonic in nature. So, why it is harmonic in nature? This is because of the TCR operation. Number 2, so we can determine, we can determine the fundamental component, fundamental component of the capacitor voltage in bracket this V c of t from these equations, which we will be doing in the next lecture from these equations. And number 3 is that we can also determine the impedance or the time-varying impedance of TCSC that is ZTCSC from this equation.

Basically, what we can do, we can find out the expression of Z TCSC as a function of this parameter beta. Now what is beta? Beta is the angle of advance which already I explained when I started this discussion of this. This, it is an important parameter, this beta is angle of advance. So this is an important parameter. And, we can find out the impedance of this TCSC as a function of this beta and we can choose the different value of beta to study how it will impact on the operation or rather we can also find out the range of the beta to find out different mode of operation. In particular, these two Vernier control modes, one is inductive Vernier control, another is capacitive Vernier.

So, this would be the, you know, topic of the discussion in my subsequent lecture. So, in this particular lecture what I did, let me summarize. So, what we do is that in the last lecture we determined the expression for instantaneous current flowing through the TCR of the TCSC and also we determined the expressions for voltage across the capacitor for a specific duration that means when this TCR is was conducting. Now, in this particular lecture, we derive the expressions for the arbitrary constants in the TCR current expression, which are A and B, their expression I derived. Also, I derived the whole expression of this voltage across the capacitor in both modes of operation, one is when the TCR is conducting, and the other is when the TCR is non-conducting.

So, this gives us expression, complete expression of voltage across the capacitor, which is this. And these equations would be useful. And this equation shows that this voltage across the capacitor is not a sinusoidal, because it will be harmonic in nature. And therefore, we need to find out the fundamental component of that. We will do that in the next lecture and also we will come up with the expressions for this ZTCSC as well. So, this will be the part of the next lecture. So, let us again meet in the next lecture. Till then, let me thank you for your attention in this particular lecture. I look forward to meet you in the next lecture again.

Boundary conditions to determine the constants A and B

Condition 1: $i_{TCR}(t)]_{\omega t_1=-\beta} = i_{TCR}(t)]_{\omega t_2=\beta} = 0$

$$i_{TCR}\left(\frac{-\beta}{\omega}\right) = i_{TCR}\left(\frac{+\beta}{\omega}\right) = 0$$

Condition 2: $v_c\left(\frac{-\beta}{\omega}\right) = -v_c\left(\frac{+\beta}{\omega}\right)$

Derivations of the expressions for A and B

$$v_c\left(\frac{-\beta}{\omega}\right) = -v_c\left(\frac{+\beta}{\omega}\right) \Rightarrow v_c(t)]_{t=\frac{-\beta}{\omega}} = -v_c(t)]_{t=\frac{+\beta}{\omega}}$$

$$i_{TCR}(t) = \left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \cos(\omega t) + Acos(\omega_r t) + Bsin(\omega_r t)$$

we know, $v_c(t) = L\frac{di_{TCR}(t)}{dt}$

$$v_c(t) = -L\left(\frac{\lambda^2}{\lambda^2-1}\right) I_m\omega \sin(\omega t) - LA\omega_r sin(\omega_r t) + LB\omega_r cos(\omega_r t)$$

Applying the boundary condition $\left(v_c(t)]_{t=\frac{-\beta}{\omega}} = -v_c(t)]_{t=\frac{+\beta}{\omega}}\right)$

$$\Rightarrow L\left(\frac{\lambda^2}{\lambda^2-1}\right) I_m\omega sin\beta + LA\omega_r sin\lambda\beta + LB\omega_r cos\lambda\beta = L\left(\frac{\lambda^2}{\lambda^2-1}\right) I_m\omega in\beta + LA\omega_r sin\lambda\beta - LB\omega_r cos\lambda\beta$$

$$\Rightarrow 2LB\omega_r cos\lambda\beta = 0$$

$$\Rightarrow B = 0$$

Derivation for the expression of A

Boundary condition: $i_{TCR}(t) = 0]_{\omega t=-\beta} = 0$

We know, $i_{TCR}(t) = \left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \cos(\omega t) + Acos(\lambda\omega t)$ [As = 0]

$$\Rightarrow 0 = \left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \cos(\omega t) + Acos(\lambda\beta)$$

$$\Rightarrow A = -\left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \frac{cos(\beta)}{cos(\lambda\beta)}$$

So, the expression for the instantaneous current flowing through the TCR (Reactor) is,

$$i_{TCR}(t) = \left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \cos(\omega t) - \left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \frac{cos(\beta)}{cos(\lambda\beta)} cos(\lambda\omega t)$$

$$\Rightarrow i_{TCR}(t) = \left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \left[\cos(\omega t) - \frac{cos(\beta)}{cos(\lambda\beta)} cos(\lambda\omega t)\right]$$

The expression for the instantaneous voltage across the capacitor of TCSC is,

$$v_c(t) = -\omega L \left(\frac{\lambda^2}{\lambda^2 - 1}\right) I_m \sin(\omega t) - LA\omega_r sin(\omega_r t)$$

$$= -\omega L \left(\frac{\lambda^2}{\lambda^2 - 1}\right) I_m \sin(\omega t) + L\omega_r \left(\frac{\lambda^2}{\lambda^2 - 1}\right) I_m \frac{cos(\beta)}{cos(\lambda\beta)} sin(\lambda\omega t)$$

$$= -\omega L \left(\frac{\lambda^2}{\lambda^2 - 1}\right) I_m \sin(\omega t) + \lambda\, \omega L \left(\frac{\lambda^2}{\lambda^2 - 1}\right) I_m \frac{cos(\beta)}{cos(\lambda\beta)} sin(\lambda\omega t)$$

$$\Rightarrow v_c(t) = \omega L \left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \left[- \sin(\omega t) + \frac{\lambda cos(\beta)}{cos(\lambda\beta)} sin(\lambda\omega t)\right]$$

Now, $\omega L = X_L$ = reactance of the TCR reactor

$\lambda = \frac{\omega_r}{\omega}, \omega_r = \frac{1}{\sqrt{LC}}$

$\Rightarrow \lambda = \frac{1}{\omega\sqrt{LC}} \Rightarrow \lambda^2\omega^2 LC = 1 \Rightarrow \lambda^2(\omega L)(\omega C) = 1$

$\Rightarrow \lambda^2(\omega L) = \frac{1}{(\omega C)} = X_C$ [Capacitive reactance]

$\Rightarrow v_c(t) = \frac{I_m X_C}{\lambda^2-1}\left[-sin\omega t + \frac{\lambda cos(\beta)}{cos(\lambda\beta)} sin(\lambda\omega t)\right]$

This equation is applicable when $u = 1$ i.e., $when\ \ i_{TCR}(t) \neq 0$

When the TCR is non-conducting, the TCSC will act as a fixed capacitor.

So, when $u = 0$ [TCR is non-conducting, $i_{TCR}(t) = 0$], we need to derive the expression for the instantaneous voltage across the capacitor.

Thus, $i_c(t) = i(t) = \frac{C\, dv_C(t)}{dt}$

$$\Rightarrow v_c(t) = \frac{1}{C}\int i(t)\, dt + v_c(t)]_{\omega t=+\beta}$$

$$v_c(t)]_{\omega t=+\beta} = \frac{I_m X_C}{\lambda^2 - 1}\left[-sin\beta + \frac{\lambda cos(\beta)}{cos(\lambda\beta)} sin(\lambda\beta)\right]$$

$$v_c(t)]_{\omega t=+\beta} = \frac{I_m X_C}{\lambda^2-1}[-sin\beta + \lambda cos(\beta) tan(\lambda\beta)] = V_c'$$

[V_c': Voltage across the capacitor before the fixed capacitor mode starts]

At fixed capacitor mode of operation,

$$v_c(t) = V_c' + \frac{1}{C}\int I_m \cos(\omega t)\, dt$$

$$\Rightarrow v_c(t) = V_c' + \frac{1}{\omega C} I_m[sin\omega t - sin\beta] = V_c' + I_m X_c[sin\omega t - sin\beta]$$

<u>The instantaneous voltage across the capacitor can be written as</u>

$v_c(t) = \frac{I_m X_C}{\lambda^2 - 1}\left[-sin\omega t + \frac{\lambda cos(\beta)}{cos(\lambda\beta)} sin(\lambda\omega t)\right], \quad \omega t \epsilon[-\beta, \beta]$ $\Leftarrow$ TCR is conducting, $i_{TCR}(t) \neq 0$

$v_c(t) = V_c' + I_m X_c[sin\omega t - sin\beta]$ $\Leftarrow$TCR is non-conducting, $i_{TCR}(t) = 0$

Where, $V_c' = \frac{I_m X_C}{\lambda^2 - 1}[-sin\beta + \lambda cos(\beta) tan(\lambda\beta)]$ $\Leftarrow$ Time-invariant.