Course Name: Power Electronics Applications in Power Systems

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Week: 02

Lecture: 01

Lec 3: Basic mathematical modelling of power transmission systems

So welcome again in my course power electronics application in power system. In the last two lectures, I taught you the basic concepts of active and reactive power of single-phase as well as three-phase circuits and the concept of reactive power compensation. Also, I discussed the significance of reactive power compensation. So, in this lecture, I will start teaching the basic mathematical modeling of power transmission lines, which is very essential to learn in order to understand the applications of different types of power electronic compensators for reactive power compensation in power systems. So, let us proceed. So, today's lecture is all about transmission line modeling.

So, the goal of this lecture is (number 1), the review of concepts of power transmission systems. Number 2 is the mathematical modeling of long transmission. So, these are the goals. So, the concepts of power transmission system is already taught in basic power system course, which is one of the prerequisites for this course.

And, this mathematical modeling of these power transmission lines, in particular long transmission lines, is an essential part to understand the further derivation of this compensator modeling and compensator applications. So I will go in very slow space so that it is understandable to all the learners. So first of all, what is the purpose of transmission line? Purpose of transmission line in electrical power system is to evacuate power from the generating station to the load centers, to evacuate power from the generating stations to the load centers. So, this transmission line acts as a transmission medium of power to bring power towards the load center. Load center stands for the localities where the customers live. And in general, in India, this generating stations stands for this thermal power generating Although we have a considerable share of renewable nowadays, but major amount of power more than 60 percent of power is coming from the conventional thermal power stations. And these thermal power stations are usually located far away from the cities or localities where customers live. So, therefore, to bring power from those generating stations to the cities, localities, where the customers live, where we have the industries also, we need long transmission lines. And in India, we have several hundred 100,000 kilometers of transmission lines in primarily 3 different voltage levels 220 kV, 765 kV, and 400 kV. This is I am talking about AC transmission lines.

Now, second thing that I will discuss that the parameters of transmission lines. So there are three important parameters in a typical transmission line. One is number one is line resistance, number two is line inductance or inductive reactance and number 3 is capacitances. So, this R that is resistance, inductance, and capacitance are three linear circuit elements that is well known by whom who have already taught those who have been already taught this basic electrical engineering. So here also in power transmission lines, we have three important parameters.

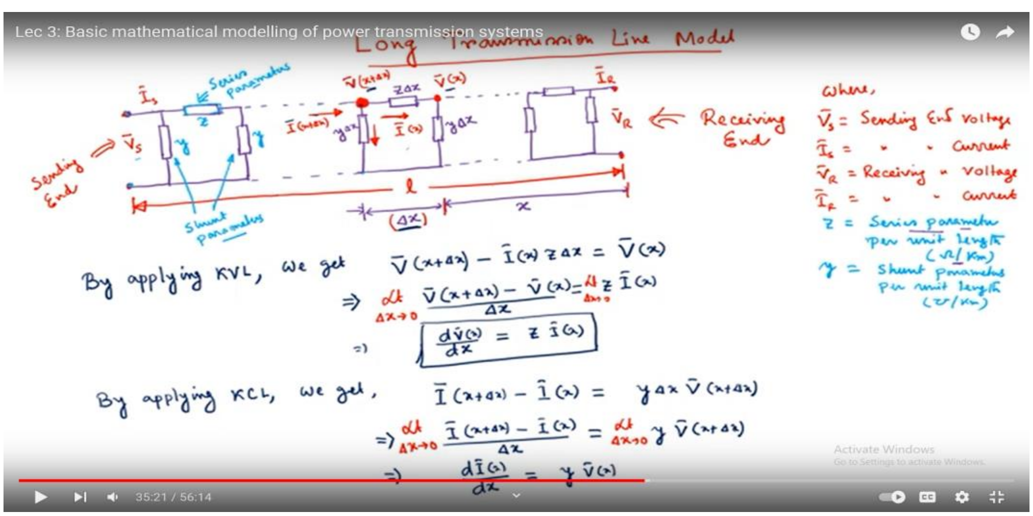
One is line resistance, because these lines are usually constructed of this aluminum or its alloy, this wire. So these have some finite resistance. And due to this line spacing, there is some inductance and also line is having some sort of capacitance. Now, when you talk about this transmission line models, these models essentially consist of these three important parameters, line resistance, line inductance and line capacitance. Another important thing is that these parameters, these parameters are usually in nature.

So, there are different types of transmission line models considering that different types of considerations of these parameters for simplicity and without sacrificing the essential or important concept. we have three different types of transmission lines. So, let us write types of transmission line models. So, if I consider this line resistance is represented by R, line inductance is represented by L, line capacitance is represented by C. So, how this whether these three different parameters have been considered or not based upon that and whether how the consideration is whether this parameters are considered to be lumped or distributed based upon that we have three different transmission line models.

These models are number one short line model, number two medium line model, number 3 long line model. Now, what is the basis of these three models? Of course, the line length is one of the basis of this categorization of these three different models. So, transmission line length is an important parameter based upon which we can categorize which type of model we will be using to mathematically model that particular transmission line. So, based upon the short, this line length, this short transmission line is usually classified those line which are below this 80 kilometer length. So, if line length is below this 80 kilometer, we will consider it is a short transmission line model. This 80 kilometer is basically equal to somewhat 50 mile. So, any line which is whose length is below this 80 kilometer is categorized in short transmission line model. In this model, we only consider this line resistance and line inductance and we ignore the capacitance. Why we ignore this line capacitance? Because it has been seen that the total line capacity VAR generation is not significantly high when line length is below 80 kilometers. So, that is why capacitance is neglected. So, this makes overall model is simpler. And another important thing is that this R and L are considered to be lumped. So, this is a kind of lump modeling, although you should understand at this point that this line, when we have a long transmission line, this line resistance is not concentrated at a particular point. So, we cannot consider the whole resistance like a single resistance. So, it is a distributed in space.

So, if we ignore that distribution, and consider that the whole resistance to be concentrated to be lumped at a particular point, then it is called lumped parameter representation. And for short line model, we consider lumped representation because we do not lose any important concept by considering the lumped parameter over here. For medium line model, we consider all these three parameters R, L and C, but we consider them to be lumped again. So, that is the difference between this medium line model to short line model. Now, to whom we will classify as the medium line model, when a line length is higher than 80 kilometer, but lower than 240 kilometer, 80 kilometer, but lower than 240 kilometer. So, 240 kilometer is somewhat similar to 150 mile. So, when we have this line length in between this 80 kilometer to 240 kilometer, we categorize this transmission line to be medium transmission line. In medium transmission line model, we considered all these three parameters that is R, L and C, resistance, line inductance, line capacitance and they are considered to be lumped. So, the third model that is long transmission line model are those models whose length is usually higher than 240 kilometer or 150 mile. So, here we also consider all these three parameters R, L and C and we consider these parameters to be distributed in space that is the difference between this medium line model and the long line model. So, we consider this line parameters to be distributed in space in long line model. And in this lecture, I will basically discuss this long line model because this short and medium line models are simpler and one can easily study from this basic power system course. So, we will directly discuss this long transmission line model in this particular lecture. Long transmission line model; So the difference of this long transmission line and the medium transmission line as I have pointed out in the last slide that in medium transmission line we consider parameters to be concentrated at a particular point or they are considered to be lumped. Whereas in long transmission line we consider them to be distributed. Now the question is when the line length is less, somewhat lesser than this 240 kilometer or so, then if we consider that lamp parameter model, we generally do not sacrifice much of accuracy in modeling. However, when we go for this long transmission line, where your length is more than 240 kilometer and above, we need to consider the parameters to be distributed. Now, what we will gain when we consider these parameters to be distributed in long transmission line model, this also I will discuss in due course. So, in long transmission line model, let us consider that this circuit is like this, we have a sending end site, this one is sending end site and we represent this circuital model with a pi section like this.

Now, this is whatever this diagram I am going to draw right now, it is basically a single-phase representation or single line representation of a three-phase transmission line. I hope that in this basic power system course one get an idea that what is single line model is. So, a single line model is an equivalent one line representation of a three-phase transmission line. So, what circuit I am going to draw is a single line diagram. Single line diagram is a very common terminology used in power system. So, this is sending end pi section, then we consider that we have a long transmission line and somewhere there is one particular pi section we considered in the middle. These dotted lines are not discontinuity, rather they represent that we have infinite number of such pi section to represent from sending n to this length. Then, we will also represent another pi section at the receiving end site like this. And then, we have this receiving end parameter here. So, now, this is basically sending end side of the line and this is basically receiving end side of the line.



The parameters in the sending end are named as sending end parameters. So the voltage across these two terminals is Vs. Here V is the voltage across these two terminal, this terminal and that terminal. And Vs stands for sending end. And the current flowing through here is represented Is. Or I should not give a direction over here. It can be, the direction can be erased over here. So it can be any direction. but whatever current is you are measuring at the sending end site that is Is. Similarly, in receiving end side, the voltage across these two terminal let us termed as Vr and the current here let us termed as Ir, where Vs is sending end voltage voltage, Is is sending end current, Vr is receiving end voltage, Ir is receiving end current. And let us consider this line length is l, it could be l kilometer or it could be l meter as well. Now you should understand looking at this single line diagram is, this long transmission line is usually modeled with infinite number of such pi sections. Now what this pi section is representing, I am coming to that. Now this long transmission line is representing infinite number of pi sections from sending end to receiving end. Now this rectangular box which is in series in the pi section is termed as series parameters. And this rectangular box two boxes in the two sides of the series parameters are representing shunt parameters. Now, what do you mean by series parameters of a typical transmission line? A series parameter of a typical transmission line represents this line resistance and line inductance. So this series parameter is representing line resistance and inductance in short together we call it line impedance. So for a small pi section let us say that impedance of this series parameter is represented by small z, where small z is representing, small z is representing parameters per unit length. Now suppose this line, if this line is of 1000 kilometers and if you consider that line to be modeled with 1000 such pi sections, then each pi section will have this series parameter representation with that much of, that much magnitude of series impedance per length. So usually it is, its unit is ohm per kilometer or ohm per meter. Similarly, this shunt parameters are represented by small y and shunt parameter is represented by small y parameters per unit length is represented by small y since they are in shunt. So, we consider this y is the admittance of the line and it is represented by this mo per kilometer or something like that. Now, what we will do is. we need to understand that what difference we will get from this distributed representation of this transmission line parameters in comparison with the lump parameters model that is short and medium line model I discussed.

So, when we have such a distributed parameter model, so you should understand that when you are energizing this one end with a voltage So, what is actually happening, these small pi sections what we have considered, suppose per kilometer we consider one pi section, they are sequentially to be energized. And then energy will take some finite amount of time to reach from sending end to receiving end. So, that travel time might be in nanoseconds, might be in microseconds or very low, but that cannot be 0. So, this information you will never get if you model it in lumped parameter.

$$\bar{V}(x + \Delta x) - \bar{V}(x) = \bar{I}(x) z \Delta x$$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{\bar{V}(x + \Delta x) - \bar{V}(x)}{\Delta x} = \bar{I}(x) z$$

$$\Rightarrow \frac{d\bar{V}(x)}{dx} = \bar{I}(x) z$$

This is one thing. important thing is that this voltage, whatever we are talking about, so this voltage we have usually considered time varying voltage, but this voltage will getting change over every, each and every point of the line starting from sending end to receiving end. So, this voltage will have some space dependency, some space dependency. Those things you will never get if you model this transmission line as a lumped parameter model. in order to derive the mathematical modeling of this long transmission line, what we will do is, we will consider that from the receiving end at a distance of x meter or x kilometer, let us take an infinitely small length of the line, whose length is represented by del x; whose length is represented by del x. Now as I said this series parameters, series parameter is representing the series impedance per unit length. So what would be the impedance that we will see in this particular box? It will be small z times del x because z is representing ohm per this unit length and the length of this small section we consider del x. Similarly, here also we will consider this is y del x and this is also y del x. Also, what we will consider that at this particular point, what is that particular point? This particular point is x kilometer or x meter f on the receiving end. And we consider that voltage at this point is v x.

And obviously, since this line this of del x, so this voltage at the other end of this pi section would be considered as Vx plus del x. And current flowing through this is suppose considered to be Ix. I x representing the current flowing through the infinitely small pi section that we considered which is located x kilometer or x meter away from the receiving end site. Now we will analyze this pi section. So with this analysis what we will do? We will apply KVL inside the loop of the small pi section that we consider, infinitely small pi section that we consider, that is in this particular loop. If we apply this KVL in this particular loop, then you can see the difference of this voltage vx plus del x and this voltage that is vx is due to the voltage drop at the series parameter, okay. So, I can write by applying KVL, what do we get? We get this equation that is v x plus del x minus this drop that is minus i x z del x is equal to the voltage at the other end of the pi section that is v x. So, this is the equation we get by using, by applying KVL inside the loop of the, this infinitely small pi section which we consider, which is located x kilometer or x meter away from the receiving end site. Now, what we will do is, we will simplify this. So, this is, we get vx plus del x minus v x is equal to, if I put this I x del x to the other side, that is right hand side and divide del x with both the side, then I will get this equation. Now, you remember that we consider this del x to be infinitely small. So, I can write this del x tends to 0, limit del x tends to 0, both the side. So, what we will get is, here this will represent d vx dx that is derivative of vx is equal to z ix. This is one equation I get. Now, what we will do? that for basic electrical engineering you know that we can apply or we can solve any electrical engineering problem either by applying KVL that is Kirchhoff s voltage law or by applying KCL that is Kirchhoff s current law.

So what we will do is that now we will apply Kirchhoff s current law. So, to apply Kirchhoff's current law, you know, that we consider the current flowing through this pi section is i x. So, current in this side would be i x plus del x, right. So, we will apply KCL at this particular node.

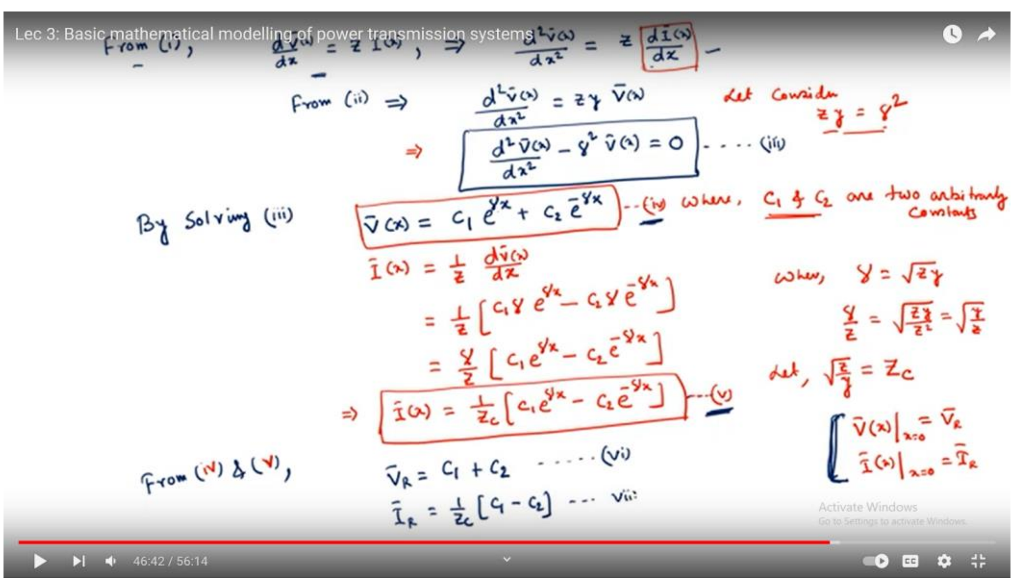
$$\bar{I}(x+\Delta x)-\bar{I}(x)=\bar{V}(x)y\Delta x$$
$$\Rightarrow \lim_{\Delta x\to 0}\frac{\bar{I}(x+\Delta x)-\bar{I}(x)}{\Delta x}=\bar{V}(x)y$$
$$\Rightarrow \frac{d\bar{I}(x)}{dx}=\bar{V}(x)y$$

You look at my cursor. At this particular node, we will apply KCL. by applying KCL, we get So, here you can see this is incoming current to this particular small infinitely small pi section, this is the incoming current Ix plus del x and this is the outgoing current that is Ix. So, the difference of these two that is Ix plus del x minus Ix. So, the difference of these two would be the current drawn by this shunt parameter that is this current. What is that current? That current will be this admittance multiplied by this voltage at this particular node. So therefore this would be equal to y del x multiplied by v x plus del x. Now, we will also simplify this equation. So, we will divide both sides with del x. So, what we will get? Let us see that is i x plus del x minus i x divided by del x is equal to y v x plus del x. Now, again we since already we have considered that this del x is a infinitely small section, pi section of this transmission line. So, we will again apply this limit del x tends to 0, del x tends to 0. Then what we will get here also this left-hand side we will get this is a derivative of this i x. And, right hand side what we will get this is y of Vx already if you put del x tends to 0. So, this will be eliminated and we will get this another equation. So, this is suppose our first equation and this will be our second equation.

Now, we need to solve this differential equation. In order to solve this differential equation, what we will do, we will further differentiate this Vx, then we will see. So, from 1, what we will get? We will get, this is basically dvx is equal to z of ix that we already have seen or derived. Now, what we will do is we will differentiate. So, from this we will get d2 Vx dx2 differentiating further both side is equal to z dix dx. So, this is we get further differentiating this equation that is from one in both side we are differentiating with respect to x we will get this equation.

Now, let us see that we know that our second equation is. that is d i x dx is equal to y v x. So, if I put this to this equation, so from 2, what we will get is d 2 v x dx 2. I am just replacing this d i dx from this equation 2. So, what we will get? This will be equal to z y So, I just replacing this dy dx, dy dx from the previous equation, this dy dx from the equation 2, I am just replacing this 2 over here and I will get this equation. Now, let us consider, let consider this small z y is a parameter which is gamma square.

Now, what is that gamma? I will come to that. So, what we will get from this? We will replace this z y with gamma square and we will get this equation is equal to 0. So, this is the main differential equation that we are trying to derive. Now we need to solve this equation to find out what is the expression of Vx. Now what is basically Vx? Vx is the voltage at the point, this particular point, if you look at my cursor at this particular point, which is located x kilometer or x meter away from the receiving end. by solving this equation, solving, suppose if we consider that is equation 3, by solving equation 3, what we get? We get this expression of Vx.



Now, what would be the expression of Vx? Vx will be some arbitrary constant c1 multiplied by e to the power gamma x plus some arbitrary constant c2 multiplied by e to the power minus gamma x. This is the solution of this, where c1 and c2 are two arbitrary constants and we will derive the expression of this c1, c2 through boundary condition later on. But this is the expression we get. Now from this expression can we find out this current expression as well that is ix as well? Yes, we can find. because we know that this Vx and Ix, they are related to each other from this equation 1 and 2.

So, what we can see from this equation 1, I can write ix is equal to 1 upon z dvx dx. So, already we get this Vx, we can differentiate it. So if we differentiate this, then what we will get is, this will be c1 gamma e to the power gamma x plus c2 minus gamma. So this plus will be changed to negative.So c2 gamma e to the power minus gamma x. Now, since gamma are common in both the terms, so I will bring it outside. So, this is gamma divided by z multiplied by c1 e to the power gamma x minus c2 e to the power minus gamma x. Now as we know this gamma square, we already considered that is z y. So we can write here gamma is equal to root over z y. Now if it is so, then gamma by z will be equal to root over z y divided by z square.
So this z becomes square inside the square root. So it will be equal to root over y by z. Now, root over z by y, we consider another parameter. Let this is considered another parameter that is called zc. What is the significance of that? I will come to later on.

But let us consider this. So, I can write then that Ix is equal to 1 upon zc c1 e to the power gamma x minus c2 e to the power minus gamma x. So, this is another equation we get. Now, as I said this c1, c2 are two arbitrary constants. So we need to solve this, we need to derive this c1, c2, the expressions of c1, c2 by using some boundary conditions. Now what would be the boundary condition that we could apply over here? Let us go back and see the single line diagram of the circuit once again.

So you can see the circuit when this x basically is considered to be a measurement starting from the receiving end. So, x can vary from 0 to l. So, when x is equal to 0, then basically that Vx is equal to Vr and Ir and when x is equal to l, Vx will be Vs and ix will be il. So, what I can write over here that these boundary conditions that when x is equal to 0, Vx is equal to Vr, ix is equal to ir. And when x is equal to l, that is the line length, Vx is equal to Vs, Ix is equal So, we will apply this boundary out of these two, one boundary condition will apply this boundary condition. So, we know that at this Vx with x is equal to 0 is equal to Vr and ix with x is equal to 0 is equal to ir, we will put over there. So, what we will get? this is equation 4 and this is equation 5. So, what we will do is, we will apply this boundary condition in this equations 4 and 5. So, what we will get is, from equation 4, we will get that Vx when x is equal to 0 is equal to Vr. So, Vr is equal to, so since I put this x is equal to 0, so e to the power gamma x will be 1. So, this is c1 and this will be c2. So, this is I applied this boundary condition in this particular equation 4. Similarly, we will apply this boundary condition again in this equation that is equation 5.

At $x = 0$,

$\bar{V}(x) = \bar{V}_R, \ \bar{I}(x) = \bar{I}_R$

$\bar{V}_R = c_1 + c_2$

$\bar{I}_R = \left(\frac{1}{z_c}\right)(c_1 - c_2)$

$\Rightarrow c_1 = \frac{\bar{V}_R + z_c \bar{I}_R}{2}$ and $c_2 = \frac{\bar{V}_R - z_c \bar{I}_R}{2}$

So, what we will get? We will get ir is equal to 1 upon zc c1 minus c2. Now, we will have another two sets of equation that is equation 6 and equation 7. We can solve these two equations to find out the expressions of c1 and c2 easily. So, let us do this again. So, what we will get from this equations 6, 7 that we will get c1 plus c2 is equal to Vr and c1 minus c2 is equal to from this equation 7, from this equation 7 if we multiply Zc to that side, left hand side, so I get c1 minus c2 is equal to Ir multiplied by zc.

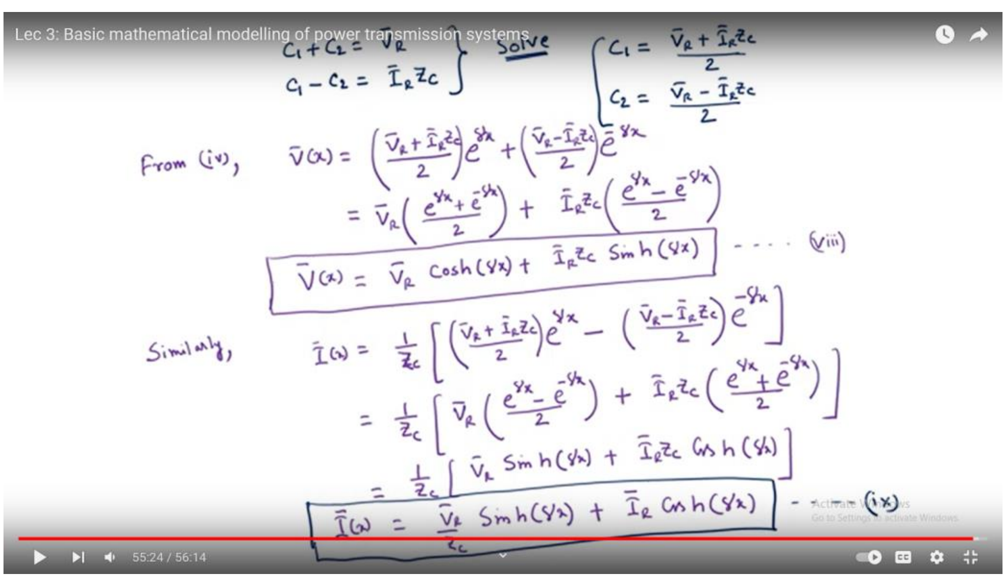
So, this is Ir multiplied by zc. Now, we will solve this. We will solve this. This is an easy to solve this 2-variable algebraic equation. So, what we will get by solving this that c1 will be equal to, if we add this together, then what we will get c1 will be equal to Vr plus I r Z c divided by 2 because if you add these two equations together, then 2 C 1 will be left hand side, so right hand side will be V r plus I r Z c. So, C 1 will be this and C 2 will be equal to V r minus I r Z c divided by 2. Now we already got this expression for c1 and c2 and what we will do? We will put this in this particular equation that is equation 4 and equation 5. So, from equation 4 what we will get? From equation 4 what we will get? Vx is equal to c1 that is Vr plus irzc divided by 2 e to the power gamma x plus Vr minus irzc divided by 2 e to the power minus gamma x.

So, from this I can further simplify because this here we have Vr term, here also we have Vr term, let us consider that Vr as a common and then we can write this as a e to the power gamma x and this as a plus e to the power minus gamma x divided by 2 plus this Irzc we will take a common. So, what we will get? Here we will get e to the power gamma x, here we will get minus e to the power minus gamma x divided by 2. So, this is we got. Now, we will write this e to the power gamma x plus e to the power minus gamma x by 2 as cosine hyperbolic gamma x plus this, we know that e to the power gamma x minus e to the power minus gamma x by 2 can be written as sin hyperbolic gamma x.

$$\bar{V}(x) = \bar{V}_R \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) + \bar{I}_R z_c \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)$$

$$\bar{V}(x) = \bar{V}_R \cosh(\gamma x) + z_c \bar{I}_R \sinh(\gamma x)$$

So, that is what our Vx is. So, this is another equation we get. So, in continuation to this, we will write that is equation 8. So, previously we had seventh equation, so this is 8. So, similarly we will also put this c1, c2 expression over here in this equation 5. So, what we will get is this will be equal to 1 upon, so ix will be equal to 1 upon zc multiplied by c1 e to the power gamma x. that is c 1 is this V r plus I r Z c divided by 2 e to the power gamma x minus this I put inside a bracket minus this V r; this is c2 that is V r minus I r Z c divided by 2 e to the power minus gamma x. So, again we will do similar simplification. So, what we will get 1 upon Z c. So, we just separate this V r and I r Z c like this. So, if we separate V r, so what we will get this V r multiplied by e to the power gamma x minus e to the power minus gamma x divided by 2 minus or plus this I r z c multiplication of e to the power gamma x this minus and that minus will be plus e to the power minus gamma x divided by 2. So, what we can write this as a 1 upon zc Vr, this is sin hyperbolic gamma x and this is cos hyperbolic gamma x. Now, we can further simplify this, we can further simplify this as a vr divided by zc sin hyperbolic gamma x plus this ir, this zc and that zc will be cancelled So this will be equal to Ir cosine hyperbolic gamma x. This is what the expression of Ix we derived. So this is our equation 9. So now we will come up to the expressions of this Vx and Ix. What are this Vx, Ix stand for? Go back and look at, you see that this vx and ix are the voltage and the current at this particular point.



What is that point? This point is basically the point which is located x kilometer or x meter or x distance away from the receiving end side of the transmission line and we derive the expression of this voltage Vx and Ix. So, with these voltages, with this voltage expression, we will do further simplification and further derivation to derive the power flow at the point of this x also will derive the power flow at the sending end and receiving end side and this will be part of the study of the next lecture. So, up to this today. Thank you for your attention. Thank you.