**Course Name: Power Electronics Applications in Power Systems**

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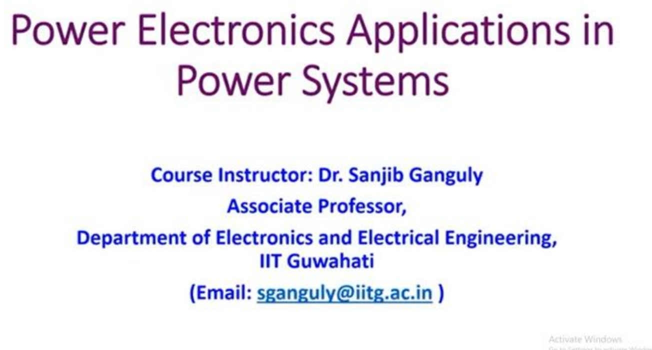
**Lecture: 02**

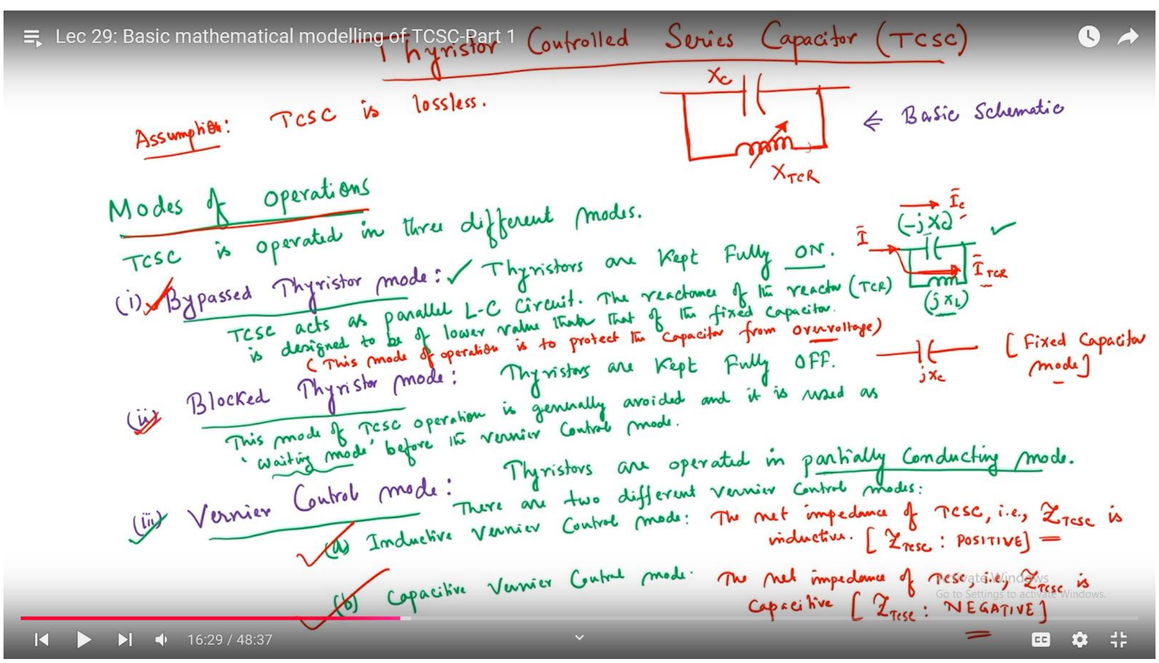



Lec 29: Basic mathematical modelling of TCSC-Part 1

Welcome again to my course Power Electronics Applications in Power Systems. In the last lecture, I started the discussion on a different type of power electronic-based compensator which is a series type of compensator and its name is thyristor control series capacitor, right? So, in the last lecture, I discussed the basic operating principle of the compensator that is TCSC and also I discussed the different possible operating conditions for TCSC and also I discuss the practical schematics of a TCSC those things. So, in this particular lecture, I will again start with the different modes of operation of TCSC and I will also develop the mathematical model for TCSC which is very important. So, if you have followed my previous lecture, you have seen that in developing the concept of any type of compensator, one needs to understand the mathematical modeling, right? So, therefore, here also in TCSC, we will develop detailed mathematical modeling, so that things will be or concepts will be understood by you. So, let us proceed. So, in this lecture also, we will consider thyristor controlled series or it is most famous with its acronyms TCSC.

So, what is our assumption? Our assumption is this TCSC is lossless. So, in fact, this same assumption we have taken for this static var compensator as well, we consider the device is lossless. So, not only our transmission system is assumed to be lossless, this device compensators are also considered to be lossless. So, this will hold to be true for all my lectures in this particular course. This is just to avoid some more complexities and to make it more simplistic form. Now, what is that basic schematic diagram? So, in the last class, I have shown that the basic schematic of the TCSC is something like that. We have a fixed capacitor and we have a variable reactor and this reactor reactance will be varied through this thyristor control. So, this is what the basic schematic diagram. So, this is what basic schematic or circuit diagram.

Now, next is, we will try to understand what are the modes of operation of TCSC are, i.e., modes of operations. So, basically this TCSC operates in three different modes. I will discuss each of them. So, TCSC is operated in three different modes. What are the three different modes? Number one is called bypassed thyristor mode, bypassed thyristor mode. So, what it is actually I will come to that. Now, the second mode is blocked thyristor mode. And the third one is Vernier control mode. So, these are the three different modes of operation. Usually, a TCSC is operated in these three different modes and I will discuss them one by one. Now, if you can remember my discussion on static var compensator or thyristor control reactor to be more precise, I discussed that a thyristor can be either operated in fully on mode or it can be operated in fully off mode or it can be operated as partially on mode. So, based upon that only, we have these three different modes of operation of TCSC. So, basically in this bypass thyristor mode, these thyristors are kept fully on.




And block thyristor mode, thyristors are kept fully off and in Vernier control mode, these thyristors are operated in partially conducting mode. Now, why we require these three different modes of operation, this I will come to that and then the things will be clear to you. Now, look at this block thyristor mode, when thyristors are fully on, then this according to this schematic diagram, this TCSC will look like this, we will have this capacitor, we will have this TCR. Now, this will be the reactance that is minus j xe, which is the reactance of the capacitor and the reactance of the inductor will be j xl. Now, in this particular mode, so TCSC acts as a parallel LC circuit.

So, TCSC act as a parallel LC circuit and if you look at this basic schematic diagram, here we have the reactance of the capacitor. So, out of these, the reactance of the reactor is kept lower. So, the reactance of the reactor that is TCR is designed to be of lower value or lesser value than that of the fixed capacitor, ok. So, when you have this Jxl that is reactance of the reactor is lower than this reactance of the capacitor, then what will essentially happen? The current flowing through this, suppose this is ITCR would be higher than the current flowing through the capacitor which is let us say IC. So, most of the current will flow through the ITCR.

So, out of this line current I, where this I represents the line current of the transmission line. So, most of the current will flow through this reactor rather than this capacitor. So, that is why it is called bypass thyristor mode. But obviously, this is a not normal operating mode of TCSC. Rather, this is, this mode of operation is used when this capacitor is to be saved from the abnormal higher voltage or abnormal over voltage. So, this mode of operation, this mode of operation is to protect the capacitor from over voltage. So, when over voltage may happens, so that is of course not a healthy condition. So, when it happens, when there is a possibility of having over-voltage across the capacitor, this mode of operation is initiated, that is bypass thyristor mode. Now, the next mode is block thyristor mode. In block thyristor mode, thyristors are kept fully off.

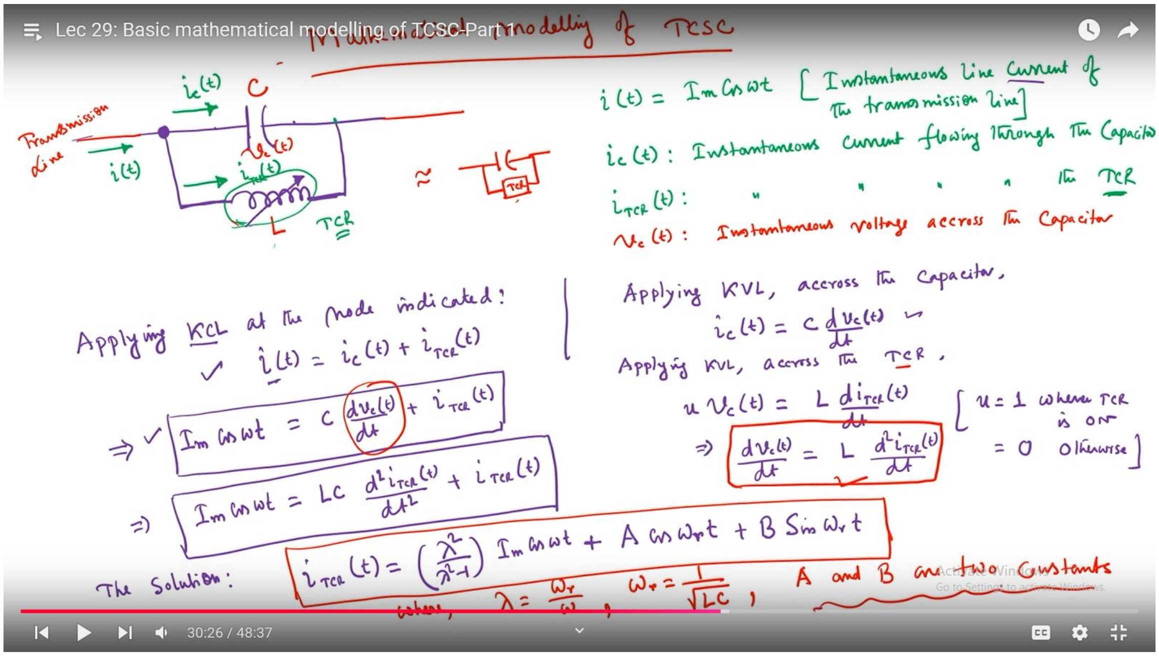
Now, if thyristor is kept fully off, then as if this path which is parallel path of the fixed capacitor is getting disconnected, because there is a switch here which will isolate this reactor from the parallel circuit of the fixed capacitor. So, therefore, the circuit will look like this. So, it will have a just a capacitor and it is also known as a fixed capacitor mode of operation. It is also known as a fixed capacitor mode of operation. Now, the question is when we require this? Of course, we do not require this or we do not intend to have this mode of operation during healthy conditions. Rather, we will be using this mode of operation in specific conditions. Now, what is that condition? So, this mode of operation of TCSC operation is generally avoided and it is used as a waiting mode before the Vernier control mode. So, this mode is called waiting mode and this is operated just before you bring this thyristor this third mode of operation which is Vernier control mode. Now in Wiener control mode, you know, it is used during healthy operating conditions, and in this particular mode of operation thyristors are operated in partially conducting mode. So, that means we have the controllability feature here, we can control the overall impedance of the circuit through this mode of operation and thereby you have already seen there are two possible cases, one is the inductive mode of operation of TCSC, another is the capacitive mode of operation of TCSC.

That means in one mode of operation, the effective impedance of the TCSC would be inductive impedance and in another mode of operation, the effective impedance or the net impedance of the TCSC would be capacitive impedance. So, we have two different types of this Vernier control modes. We have two different Vernier control modes. Number one is inductive Vernier control, I should write it A inductive Vernier control mode. Now, B is capacitive vernier control mode.

Now, what happens in this inductive vernier control? As you know, we have given conditions. So, here the net impedance impedance of TCSC that is ZTCSC is inductive. So, this I already have shown the four different cases in the last lecture. Now, in this capacitive vernier control mode the net impedance of TCSC that is ZTCSC is capacitive. That means here ZTCSC is positive and here ZTCSC is negative.

This is already I explained and these two are two modes of operation of TCSC during healthy operating conditions. These two modes are the two modes of operation which are only exercised during abnormal operating conditions or before you go for this vernier control mode. So, these are the modes of operation of TCSC. Now, next, I will start explaining this basic mathematical modeling of TCSC, mathematical modeling of TCSC. So, this is one of the important aspects of understanding the basic concept of the compensator or TCSC. So, as this mathematical modeling of different types of static var compensators, which I discussed before. Now, before we do this mathematical modeling, so let us again draw this equivalent circuit diagram for TCSC. Here we have this fixed capacitor, here we have variable reactor. This reactance of the variable reactor would be varied through the firing angle control of the thyristors and that is what I hope you understood. So, this is our fixed capacitor, here we have a fixed inductor and this is basically connected to the transmission line.

So, these are transmission lines. Now, suppose the current flowing through the transmission line is represented with I of t. So, looking at this representation you can understand this I of t is equal to this Im let us say cos omega t which represents the instantaneous line current of the transmission line. Now, suppose the current flowing through this capacitor is represented by I of c t. So, I of c t is instantaneous current flowing through the capacitor, and suppose the current flowing through this reactor having this inductance L is represented by I L T or we will let us represent it I TCR T. So, where this I TCR T represents instantaneous current flowing through the TCR. Now what is TCR? TCR is thyristor control reactor. This is the TCR. This is representing a TCR. So only difference of the usual TCR that you know is that this TCR is coming in parallel to the capacitor.




**Mathematical modelling of TCSC:**

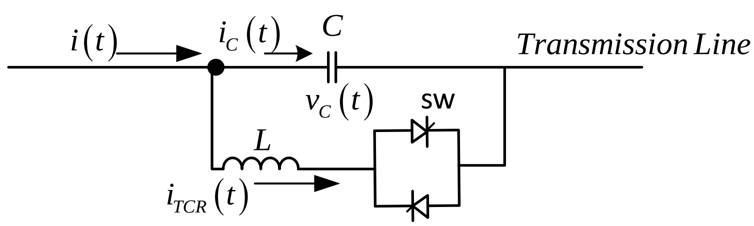



**Fig.1.** Equivalent circuit diagram of a TCSC

In Fig.11,

$i(t)$:The instantaneous current flowing through the transmission line

$$i(t) = I_m \cos(\omega t) \tag{1}$$

$i_c(t)$: The instantaneous current flowing through the capacitor

$v_c(t)$: The instantaneous voltage across through the capacitor

$i_{TCR}(t)$: The instantaneous current flowing through the TCR

Applying KCL at the node indicated in Fig.1

$$i(t) = i_c(t) + i_{TCR}(t) \tag{2}$$

Applying KVL, across the capacitor, $i_c(t) = \frac{C\, dv_C(t)}{dt}$ (3)

Applying KVL, across the TCR $uv_c(t) = L\frac{di_{TCR}(t)}{dt}$ (4)

[Where, $u = 1$, when TCR is ON

$u = 0$ , otherwise]

Substituting Eq$^n$ (1) and Eq$^n$ (3), in Eq$^n$ (2)

$$I_m \cos(\omega t) = \frac{C\, dv_C(t)}{dt} + i_{TCR}(t) \quad (5)$$

Now differentiating both sides of Eq$^n$ (4)

$$\frac{dv_C(t)}{dt} = L\frac{d^2 i_{TCR}(t)}{dt^2} \quad (6)$$

Substituting the value of Eq$^n$ (6), in Eq$^n$ (5)

$$I_m \cos(\omega t) = LC\frac{d^2 i_{TCR}(t)}{dt^2} + i_{TCR}(t) \quad (7)$$

By solving Eq$^n$ (7), we get:

$$i_{TCR}(t) = \underbrace{\left(\frac{\lambda^2}{\lambda^2-1}\right) I_m \cos(\omega t)}_{\text{Transient Part}} + \underbrace{A\cos(\omega_r t) + B\sin(\omega_r t)}_{\text{Steady State part}}$$

Where, $\lambda = \frac{\omega_r}{\omega}$ and $\omega_r = \frac{1}{\sqrt{LC}}$ (Resonant frequency) and A and B are two constants.

**Voltage and current waveforms**

$\beta$: Angle of advance

$\sigma$: Conduction angle

$\alpha$: Firing angle [Measured from zero crossing line current]

$\omega t_1 = -\beta$ [TCR current starts]

$\omega t_2 = \omega t_1 + \sigma = +\beta$

$\omega t_3 = \pi - \beta$ [TCR current starts]

$\omega t_4 = \omega t_3 + \sigma$

$\sigma = \pi$ [Thyristors are fully turned ON]

$= 0$ [Thyristors are fully turned ON]

Vernier control mode of operation, i.e, partial conduction of thyristors will create harmonics.

Boundary conditions to determine the constants A and B

Condition 1: $i_{TCR}(t)]_{\omega t_1=-\beta} = i_{TCR}(t)]_{\omega t_2=\beta} = 0$

$$i_{TCR}\left(\frac{-\beta}{\omega}\right) = i_{TCR}\left(\frac{+\beta}{\omega}\right) = 0$$

Condition 2: $v_c\left(\frac{-\beta}{\omega}\right) = -v_c\left(\frac{+\beta}{\omega}\right)$

So this TCR does not need to have the system voltage level rather it would be its voltage level would be much lower than the voltage level of the overall this transmission line. And if this voltage level of this TCR will be similar to the voltage across the capacitor, right? Now with this let us apply this our basic electrical engineering laws that we know. Now what are the laws we know? One is Kirchhoff s current law, another is Kirchhoff s voltage law. Now suppose this voltage across this capacitor at any instant of time is representing V c  or I should draw it here. So, voltage across the capacitor, let us consider represented by Vct. So, Vct represents instantaneous voltage across the capacitor. Now, if we apply Kirchhoff's current law here, so applying KCL, KCL at the node indicated what we get? We will get it if we apply KCL at this point, so you know that the incoming current should be the summation of the outgoing current. So, we can write I of t is equal to I C of t plus I TCR of t, ok. So, this is the by applying this KCL, ok. Now, if we apply KVL across this capacitor.

So, what would be the KVL equation? Applying KVL, catcher's voltage law at across the capacitor, what do we get? We get this ICT is equal to C dVc dt, where Vc is also instantaneous voltage. So, this we get by applying Kirchhoff's voltage law across this capacitor. Similarly, this applying KVL across this inductor, applying KVL across the TCR what we get because as if you can understand that this is why we are considering TCR because we are considering that this is equivalent to we have a fixed capacitor and a TCR. Now, what is the TCR? This TCR is basically there is a bidirectional switch here of thyristor and the inductor or reactor in series. So, when we have a bidirectional thyristors in series with a reactor, this forms a TCR. So, this is what we know. Now, if we apply KVL across the TCR, what we will get? We will get the voltage across this inductor is equal to the voltage across the capacitor. This is equal to V CT, V C of t is equal to L, we know that this voltage across this inductor is represented by L di dt. So, this is equal to L di dt TCR t of t. However, as you know this current flowing through this reactor is basically controlled by this TCR switching. So, there is to be one variable that should be multiplied with this v c t, where u is equal to 1 when TCR is on, okay? Otherwise, u would be equal to 0, 0 otherwise.

So, that is something we should understand. Now, we need to aggregate all these equations, one is KCL equation, another is 2 KVL equation. Now, if I put this equation ICT is equal to this in this equation and we know IFT is equal to IM cos omega t. So, what we can write I m cos omega t that is left hand side which is the basically the instantaneous line current, this current flowing through this transmission line. Now, here we do not assume that this TCR is located or this TCSC is located at the midpoint, it can be located at any point of the transmission line, it can be at the sending end or at the receiving end or at the any point of the transmission line.

So, we assume that current flowing through this transmission line is IM cos omega t, wherever it is placed, then this will be equal to, we know ICT is equal to this. So, let us put it there, ICT is equal to CDVCT dt plus this ITCR of t. Now, what we can also write? So, this is one equation we get. Now, from this equation again we can write one thing that this d if we just differentiate this v c t again with respect to t both sides then it will be d v c of t dt will be equal to L d 2 i T c r t dt of course, when u is equal to 1. Now, if we put this over here, then this equation will be Im cos omega t is equal to, if you put this equation here, that dvc dt, I am just simply replacing this dvc dt with this, then what I will get is, this will be equal to d 2 i t c r of t d t 2 plus i t c r t. So, this is what the equations, final equations we are trying to arrive at. Now, look at this equation, what sort of equation it is? It is a second order equation. So, this is having an equation with second-order differential form or it is a second-order differential equation. Now, we need to solve that. In the classroom, I asked my students to solve it.

So, here I can show you the direct solution. All of you, all of these learners can solve it personally. So, if you solve it, then we will get the solution of this equation, we will get I T c R T is equal to some factor lambda square divided by lambda square minus 1 I m cos omega t plus some arbitrary constant A cos omega r t, cos omega r t is another frequency component plus B sin omega t. Again, this B is an arbitrary constant. So, this is the solution of this differential equation. Here this lambda is the ratio of omega r to omega and omega r is the frequency other than the power frequency which is the frequency of resonance as we know and A and B are to arbitrary constants. So, when we have two arbitrary constants like this, we need to find out the expressions of this A and B, which we will be determining by applying the boundary conditions, similar to this what we did before also. So, let me draw the waveform. Suppose this is the profile of I of t, although it is cos omega t, but do not assume that this corresponds to omega 2 is equal to 0. So, then this omega t is equal to 0 would be somewhere else, but this will be like a sinusoid.

So, this is the line current. So, this is the line current I of t. Now if this line current is this then this could be our axis corresponds to omega t is equal to 0. This could be our axis corresponds to omega t is equal to 0. I will come to that. Now, if it is the profile of I of t, then how this v c t will be? So, if it is so, then this our v c t profile would be like this. So, v c t profile will be like this. This is the VCT profile, but actual VCT will be something else, which I am coming to that later on. But this VCT profile would be something like that. For certain conditions, this will get changed, which I will discuss later on. So, this is the profile of VC of t. Then, where would be the waveforms of this, the current flowing through the TCR or pion flowing through this reactor? So, that is ITCR that is this current.

So, this current profile would be somewhere like this and this current you know, it can be distorted current because it can be fully sinusoid or it can be distorted because this TCR depending upon the mode of operation of these switches can provide you a sinusoidal or can draw a sinusoidal current or a distorted current. So, this I already discussed. So, this is a harmonical current as you can understand and you know that here there are four different instants one is let us say this instant this is omega t 1 let us say this is another instance let us say this is omega t 2. And, this instance is, let us say, omega t 3 and this instance, let us say, is represented by omega t 4. What are those things? I will come to that. Now, this is representing I of TCR t, instantaneous current flowing through the TCR. This is I TCR t. The angle for which this will conduct is the conduction angle that is sigma. So this is sigma, this is sigma.

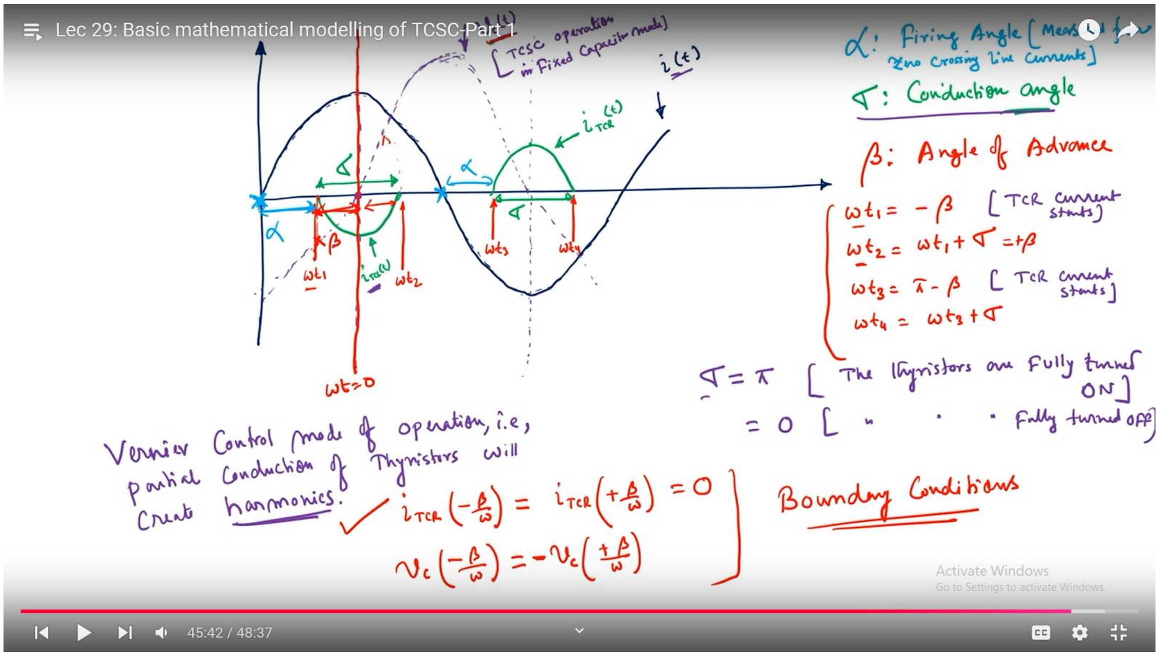
So as you know sigma is the conduction angle. And this VCT profile as I said this VCT profile is basically the voltage across the capacitor when it will be operated as a fixed this capacitor mode. So, this VCT profile is rather this in this particular condition this voltage across the capacitor when it is operated like a fixed capacitor in block thyristor mode of operation. But, when it is operated in vernier control mode, this VCT would be different. I am coming to that. I will slowly try to develop this waveform for this VCT, that is voltage across the capacitor, when the TCSC would be operating in either of these vernier control modes.

It can be operated in inductive vernier control or capacitive vernier control. Now, what is this omega t omega t 2? There is another terminology that would be exclusive for this TCSC which is this angle, this angle is called angle beta. So, if this is beta, so this will be also beta. So, this here, this beta is called angle of advance. Why it is called the angle of advance? Because you see this corresponds to this curve, this line, this vertical line corresponds to the instance, let us say omega t is equal to 0. So, if it is so, then this conduction starts beta angle before and that is why it is called angle of advance. Now we have as I said this omega t1 which is equal to minus beta. Now what is then omega t2? So omega t2 will be equal to omega t1 plus sigma. Then what is omega t3? This omega t3 will be again, so as you know from here to here, it will be pi angle, so this is pi minus beta and this omega t 4 is equal to omega t 3 plus sigma.

So, these are the four instances, ok. So, this is the instance where conduction starts, this is the instance where conduction, the TCR current starts, TCR current starts and this is also the instant when the TCR current starts. Now, you know that depending upon the value of this conduction angle, this conduction angle can be this pi or it can be 0. So, the limit of this conduction angle is pi to 0. So, when this conduction angle, this sigma is equal to pi, that means this starting from here to here, then it means this thyristors are fully turned on, thyristors are fully turned on. When this is 0 that means thyristors are fully turned off. So, when sigma is in between these values that means thyristors are partially conducting. So, any value of the sigma in between pi to 0 implies to the thyristors are partially conducting. They are partially turned on. So, when the thyristor is partially turned on, there would be harmonics. So, that means, this Vernier control mode of operation, Vernier control mode of operation that is partial conduction of thyristors will create harmonics.

So that is something you should understand. So here also there is a problem of harmonic when they are operating in this Vernier control mode. But this Vernier control mode is the actual mode of operations of TCSC. So obviously that means that similar to TCR or similar to any kind of static bar compensator in TCSC also there would be some harmonical current. Now, you may ask one thing that this is ITCR and this is what the line current. So, therefore, if this line current let us assume perfectly sinusoid, because of the non sinusoidal or distorted sinusoidal of this TCR current, this current flowing through the capacitor that is ICT would be also distorted.

So, therefore the voltage across the capacitor will also be distorted. So, this voltage VCT waveform whatever I have shown you, this is the voltage across this capacitor profile considering that this TCSC operation is in fixed capacitor mode. So other than that, this VGT actual instantaneous voltage across the capacitor would be also distorted, would be also harmonics. So how this would be, I will draw it later on. But this is what the, this situation is. Now, one thing I will also discuss that we will work on this particular solution in very detail. We will derive the expressions for this A and B in subsequent lecture through this boundary conditions by using this boundary conditions. Now, what should be this boundary conditions that we can develop right now from this particular waveform? Now, what would be this boundary conditions for this ITCRT? Now, one thing that you can see this ITCRT, ITCRT. When it is equal to this T 1 that is minus beta by omega. So, as you can see omega T 1 is equal to beta.




So, T 1 is equal to minus beta by omega. So, this is equal to this I TCR this t 2 which is equal to minus beta or it can be written as plus beta by omega. So, this can be also written as, omega t can be also written as since this is the angle of advance. So, this will be equal to plus beta. So, this is plus beta omega is equal to 0. So, this could be one boundary condition. So, this could be one boundary conditions. Another is you can see if you look at v c t, so at this value at omega t 1, so that means v c t when this is equal to t is equal to minus beta by omega, so this will be equal to this V c that is this beta by this beta divided by omega. So, the voltage at this point and voltage at this point would be equal, but opposite sign. So, these are the two boundary conditions. From these boundary conditions, we will proceed further and develop these expressions for these two arbitrary constants. One is A, another is B in the next lecture. But we have these boundary conditions. Now, one thing that I also discuss over here is how to this consider this firing angle control in TCSC. In TCSC firing angle control, usually this zero crossing of this line current is considered. So, either this or this. So, this is considered the firing angle alpha. This is also considered the firing angle alpha. So, here alpha is the firing angle which is measured from 0 crossing line currents. So, this alpha is the firing angle. What is a firing angle? The firing angle is the angle which is required to be set to bring the thyristors in conduction. And this is done by considering this zero crossing, it is measured from this zero crossing line currents.

One is this, another is this. So, this is how it works. Briefly, this is the mathematical modeling of TCSC. We will continue this mathematical modeling in the subsequent lecture. We will derive the expressions of this arbitrary constants A and B. And we also derive the actual expression of this VCT, that is this VCT. As I said, this VCT profile is not the actual, this voltage profile across the capacitor, because the voltage across the capacitor will depend upon the current flowing through the capacitor that is ICT. This ICT is basically, as you see from this KCL equation, the difference between the line current, this line current, and this TCR current. Now, when you have this TCR current distorted, it is non-sinusoid or it is of harmonic, then even if this line current is perfectly sinusoidal, then ICT will be also non-sinusoid which makes this voltage across the capacitor which already depends upon this voltage. So, the voltage also subsequently depends upon the current. So, this voltage across the capacitor will be also distorted.

It will be non-sinusoidal or it would be harmonic. Now, what would be the actual expression of this instantaneous voltage across the capacitor? That I will derive from the mathematical equations and then we will try to plot it. So, this we will do and another thing we will do also that we will try to develop the expressions for actual impedance which is ZTCSC of this overall system in a function of this angle of advance that is beta which is very important. So, here as I said angle of advance or beta is having a importance important parameters. So, depending upon this value by changing this value we can control this ITCR current and subsequently we can control the capacitor voltage as well. So, therefore, this angle of beta is an important parameter and we will also try to develop the expression for the impedance of the overall circuit as a function of this angle of advance.

So, this will develop in the subsequent lecture. So, up to this today. So, let me thank you for your attention in this particular lecture. So, thank you very much for attending this lecture. I am looking forward for the next lecture. Thank you very much. Thank you.