

Course Name: Power Electronics Applications in Power Systems

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Power Electronics Applications in Power Systems

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Lec 13: Effect of Mid-point compensation on power flow of transmission lines

So welcome again in my course power electronics applications to power systems. So in the last lecture, or in fact in the last couple of lectures, I discuss a numerical problem in which I discuss how a midpoint compensation can improve the voltage profile of a symmetrical long transmission line. So, also we have seen that when we have a midpoint compensation for a symmetrical lossless long transmission line, there is a change of active power flow through the line. In general for any lossless transmission line, you have seen that the power flow expression for each and every point of the line will remain same. Because there is no power loss, so power flow remains constant in each and every point of the line. The line might be 1000 kilometer long, but the power flow through each and every point of the line will remain same for a lossless transmission line.

However, if there is a midpoint compensation, then the power flow expression will get changed. So, these things we have seen from the numerical problem I discussed in my couple of last lectures. Now, in this particular lecture, I will see or you will see that the how a midpoint compensation effects on the power flow of a transmission line. So, let us begin with that. So, this expression the topic would be the effect of midpoint compensation on power flow. So, this is the topic which I am going to discuss today. This

is in general discussion I will show you how a midpoint compensation effect on the power flow capacity or power flow amount of a transmission line. So, here our assumptions are to discuss this our assumptions are we will consider a short transmission line model for simplicity. Then we will consider a symmetrical transmission line and we will also consider the transmission line to be lossless.

So these are our assumptions. So we will consider the transmission line to be lossless, we will consider it is a short transmission line model and we will also consider the line to be symmetrical. Now we know there are two types of, there could be two types of midpoint compensation. 1) shunt compensation or if I put number one as series compensation. Then number two would be shunt compensation.

So, these are typically two different types of compensation and in general, in future lecture also whatever power electronic compensator I will discuss they will either belong to series compensation or shunt compensation. So, here in this particular lecture, I will simply use both the type of compensation separately and then find out the expression of the change of active power flow through this unit compensation that we are providing at the midpoint. So, let us begin with series compensation. Let us begin with series compensation at midpoint. Now, for this time being you understand that this midpoint means midpoint of a typical transmission line.

If we represent this short line, then its representation is something like that. We have a line reactance like this. Since, we consider the line to be lossless, so only series parameter that we will have is the line reactance. Now, if we consider the whole line is having reactance as x , then x by 2, x by 2 would be the reactance up to this midpoint. So, x represents line reactance.

Now, let us consider that this sending end voltage is V at an angle δ and receiving end voltage is V at an angle 0 . Here is our midpoint, so this is midpoint and the voltage at this midpoint is V_m at an angle δ by 2. Now, since the line is symmetrical, so you have understood that for symmetrical line, the magnitude of the sending end voltage and the magnitude of the receiving end voltage are equal. So, that is why I consider both are same, right? But this midpoint voltage would be something different, that is V_m . So, here V_m is basically the midpoint voltage.

And δ is your load angle. Now, we know the expression of the midpoint voltage for a symmetrical lossless long transmission line. So, we already derived the expression of midpoint voltage magnitude for a symmetrical lossless long transmission line couple of lectures back. And, if you remember the expression, this expression is something like that, V_m is equal to $V \cos \delta$ by 2 divided by $\cos \beta$ l by 2. This is the expression of midpoint voltage for symmetrical lossless long transmission line.

Now, from this expression, can we derive the expression of this midpoint voltage for symmetrical lossless short transmission line? Yes, we can. So, for short line, for short line, we know that, this we can consider that βL is very near to 0. So, therefore, $\cos \beta L$ is close to 1. So, from that we can find out this V_m is equal to $V \cos \frac{\delta}{2}$. This we can verify also from this from the phasor diagram as well.

This we can verify from the phasor diagram as well. Now, if we need to draw the phasor diagram for this circuit, then this phasor diagram will be something like that. Here we have this voltage. This voltage V at an angle 0, and here we have this voltage at the sending end side, which is V at an angle δ . So, this is the angle δ .

So, the difference of these two voltages, one is the voltage at the sending end, another is voltage at the receiving end, is the voltage drop that is happening through the line. So, that is the voltage drop happening through the line. Now, what we consider that, we will consider there is a midpoint compensator somewhere here. So, this represents a midpoint compensator. So, this represents a series compensator series compensator at midpoint.

Now normally what the series compensator does is a series compensator changes the reactance of the line. So therefore it impacts the current flowing through this line. Thereby it also impacts the power flow. Now how do we find out this current? So from this expression of this midpoint voltage, we can find out the midpoint current also. Now to do that, So, midpoint power flow, power flow expression is P_m is equal to V_m multiplied by I_m and since you know that the line is lossless, so this midpoint power flow is same as the active power flow from the sending end and the active power flow near to the receiving end.

So, this expression is as you know, this is equal to v^2 , this v multiplied by this v that is v^2 divided by whole line reactance that is x multiplied by $\sin \delta$. So, this is what the power flow of the uncompensated line and this will be also power flow expression at the midpoint. So, therefore, this midpoint current, so here I should write that V_m is the midpoint voltage, I_m is midpoint current. Now, From this expression, we can find out this midpoint current is equal to v , v^2 divided by $x \sin \delta$. You know that the phase difference from the sending end to the receiving end is δ .

So, that is why it is δ . So, this divided by V_m . So, which we know that V_m is equal to this. So, we can put it over here. So, this will be v^2 by $x \sin \delta$ by $V \cos \frac{\delta}{2}$. Now, we can write this $\sin \delta$ as $2 \sin \frac{\delta}{2} \cos \frac{\delta}{2}$ and this v and this v will be cancelled out. So, this will be $2 v \sin \frac{\delta}{2} \cos \frac{\delta}{2}$ divided by $x \cos \frac{\delta}{2}$; $\cos \frac{\delta}{2} \cos \frac{\delta}{2}$ will be cancelled out. So, therefore, what we get I_m magnitude as $2 v$ divided by $x \sin \delta$ that is what the expression


So, therefore, this I_m will get change for the change of x . So, from this expression, we can write, we can write the change of ΔI_m is equal to $2V / 2V$ this if I differentiate with respect to x , then this will be $x^2 \sin \delta$ minus $x^2 \sin \delta$ multiplied by Δx . So, this is what we get. So, this is why, this is what we get the unit change of this midpoint current due to this unit change of this line reactance or X , unit change of the line reactance. Now, if we have a series compensator over here, then what would be the unit change of the series compensation reactive power that the compensator will provide? This is something you need to know.

Lec 13: Effect of Mid-point compensation on power flow in transmission lines

Assumptions: (a) Short line, (b) Symmetrical, (c) Lossless

There could be two types mid-point Compensation: (i) Series Compensation
(ii) Shunt Compensation

Series Compensation at mid-point



x : Line reactance
 V_m : Mid-point voltage
 δ : Load angle

③: Series compensator at mid-point
 I_m : Mid-point current

Mid point power flow
 $P_m = V_m I_m = \frac{V^2 \sin \delta}{X}$

$I_m = \frac{V^2 \sin \delta}{X} \times \frac{1}{V_m} = \frac{V^2 \sin \delta}{X} \times \frac{1}{V \cos \frac{\delta}{2}} = \frac{2V \sin \frac{\delta}{2} \cos \frac{\delta}{2}}{X \cos \frac{\delta}{2}} = \frac{2V \sin \frac{\delta}{2}}{X}$


$I_m = \frac{2V \sin \frac{\delta}{2}}{X} \Rightarrow \Delta I_m = -\frac{2V \sin \frac{\delta}{2}}{X^2} \Delta X$

$\Delta P_m = -\frac{V^2 \sin \delta}{X^2} \Delta X$

So, $\frac{\Delta P_m}{\Delta X} = -\frac{V^2 \sin \delta}{X^2} \Rightarrow \frac{\Delta P_m}{\Delta Q_{sc}} = \frac{1}{2 \tan \frac{\delta}{2}} \left(\frac{\Delta P_m}{\Delta Q_{sc}} \right)$

Mid-point voltage for Symmetrical lossless, long line
 $V_m = \frac{V \cos \frac{\delta}{2}}{\cos \frac{\delta}{2}}$

For short line $\delta \approx 0$, $\cos \frac{\delta}{2} \approx 1$
 $V_m = V \cos \frac{\delta}{2}$



The unit change of Series Compensation
 $\Delta Q_{sc} = -I_m \Delta X = -\left(\frac{2V}{X}\right) \sin \frac{\delta}{2} \Delta X$

$\frac{\Delta P_m}{\Delta Q_{sc}} = \frac{1}{2 \tan \frac{\delta}{2}} \left(\frac{\Delta P_m}{\Delta Q_{sc}} \right)$

Mid-point voltage for symmetrical lossless, long line

$$V_m = \frac{V \cos \frac{\delta}{2}}{\cos \frac{\beta l}{2}}$$

For short line $\beta l \cong 0$, $\cos \frac{\beta l}{2} \cong 1$

$$V_m = V \cos \frac{\delta}{2}$$

Mid-point power flow (P)

$$P = V_m I_l = \frac{V_S V_R \sin \delta}{X_l} = \frac{V^2 \sin \delta}{X_l}$$

$$I_l = \frac{V^2 \sin \delta}{X_l} \times \frac{1}{V_m}$$

$$I_l = \frac{V^2 \sin \delta}{X_l} \times \frac{1}{V \cos \frac{\delta}{2}}$$

$$I_l = \frac{2V}{X} \sin \frac{\delta}{2}$$

$$\frac{dI_l}{dX_l} = \frac{\Delta I_l}{\Delta X_l} = \frac{-2V}{X_l^2} \sin \frac{\delta}{2}$$

$$\Delta I_l = \left(\frac{-2V}{X_l^2} \sin \frac{\delta}{2} \right) \Delta X_l$$

$$\frac{\Delta I_l}{\Delta X_l} = \left(\frac{-2V}{X_l^2} \sin \frac{\delta}{2} \right)$$

$$\frac{\Delta P}{\Delta X_l} = \frac{-V^2}{X_l} \sin \delta$$

$$\Delta P = \left(\frac{-V^2}{X_l} \sin \delta \right) \Delta X_l$$

$$\frac{\Delta P}{I_l^2} = \left\{ \left(\frac{-V^2}{X_l} \sin \delta \right) \Delta X_l \right\} \frac{1}{\left(\frac{2V}{X_l^2} \sin \frac{\delta}{2} \right)^2}$$

$$\Delta P = \frac{1}{2 \tan \frac{\delta}{2}} (-I_l^2 \Delta X_l)$$

$$\frac{\Delta P}{\Delta Q_{se}} = \frac{1}{2 \tan \frac{\delta}{2}}$$

$\Rightarrow \delta$ varies with line loading

So, we can also put a negative sign over here. So, that is what the thing and if we take a ratio of this ΔP_m to ΔQ_{se} that means, this ΔP_m to ΔQ_{se} gives us the ratio of the unit change of active power flow due to the unit change of the or the rather I should say the change of active power flow due to the unit change of the series compensation. So, this will can be obtained as minus V^2 divided by $X^2 \sin \delta$ multiplied by 1 upon ΔQ_{ac} is this. So, minus I_m^2 square; Now, I can write that this minus I_m^2 square is this. I_m is equal to that. So, I_m^2 square will be minus $2V$ by $X^2 \sin^2 \delta$ by 2 , then ΔX should be there. So, I will simply put this over here in the denominator. So, what I get is $2V$ divided by $X^2 \sin^2 \delta$ by $2 \Delta X$. So, this will be also negative. So, this negative, this negative will be cancelled out.

Now, if we just simplify this expression what we get that V by X^2 and this V by X^2 will be cancelled out only 1 upon 4 will be left. Now, similarly this $\sin \delta$ this ΔX ΔX will be cancelled out $\sin \delta$ we can split it like $2 \sin \delta$ by $2 \cos \delta$ by 2 divided by this $\sin^2 \delta$. Now, this $\sin^2 \delta$ by 2 will cancel one denominator $\sin \delta$ by 2 . So, what we will get, this 2 will be divided by 4 .

So, there will be 2 over here. So, this is 1 upon $2 \tan \delta$ by 2 . So, this is what we got. So, this is what we got. So, this we can write as a change of this active power flow due to the unit change of the series compensation of a short symmetrical lossless transmission line. And these equations we will revisit again So, this equation, the importance of this equation we will revisit again, once I also discuss the shunt compensation.

So, but one thing should be clear to use that, that we arrived at the expression of ΔP_m to ΔQ_{su} is equal to 1 upon $2 \tan \delta$ by 2 , where this ratio that ΔP_m by ΔQ_{su} , ΔQ_{se} , it is, it represents the change of active power flow through the line with the unit change of the series compensation. And it is coming out to be a function of only this line load angle. So, if I write this in words, then we can write that from this derivation that is ΔP_m divided by ΔQ_{se} is equal to 1 upon $2 \tan \delta$ by 2 . That is, so we get the ratio of change of active power flow, power flow of a symmetrical lossless short transmission line to the unit change of midpoint series compensation.

So that is our goal to find out. We are interested to find out the change of active power flow, change of or effect of active power flow for this midpoint compensation. Here we consider the series compensation at the midpoint and from that you can find out That, we come out with the expression of this, which shows that when we have a unit change of this series compensation, there is a change in active power flow through the transmission line, and that can be represented by this ratio, $\Delta P_m / \Delta Q_{ac}$, and that is coming out to be a function of the line load angle, that is δ . So, that is what our goal was. Now, next

is we will also analyze the shunt compensation. So, here also our network will remain same, we will consider a symmetrical lossless short transmission line.

So, here also we will consider this asymmetrical lossless short transmission line. Now let us draw the circuit diagram for the shunt compensation at the midpoint of a symmetrical lossless short transmission line model. So this will be something like that. We have, this is the line reactance up to this midpoint and this is the line reactance from the midpoint to the receiving end. So, this is the model of the short transmission line and here itself we have a series compensator, sorry, shunt compensator.

Here only we will have a shunt compensator. Now, suppose this voltage at this sending end side is V at an angle δ . The voltage at the receiving end side is V at an angle 0 . Since they are symmetrical line, we consider the voltage magnitude at the sending end and receiving end are similar or same. And, this is, this x by 2 and x by 2 are the line reactants which I already discussed in the last slide that this x represents the line reactants. Now, we need to see that what would be the effect of this shunt compensator placed at the midpoint on the power flow through this line.

So, that is what we are interested to find out. Now, voltage at this point is V_m at an angle δ by 2 and as we know for uncompensated line, for uncompensated line, we know the expression of V_m is equal to $V \cos \delta$ by 2 . That is what we also obtained in the last slide, that is this. We simply copied this thing. Now, the difference between a series compensation and shunt compensation is that the presence of the series compensation changes the line reactance.

However, the presence of a shunt compensation changes the voltage of the midpoint. So, the effect of the shunt compensation is that it will definitely change the midpoint voltage. And that is what we have understood in the last numerical problem that you have seen without the shunt compensation, the area was a significant over voltage or under voltage. However, the goal of having a shunt compensation at the midpoint was to mitigate this over-voltage and under-voltage. So, of course, you should understand at this point that the presence of a shunt compensation definitely changes the midpoint voltage.

Now, let us draw the phasor diagram for this. So, suppose this is our receiving end voltage, which is represented by V at an angle 0 . This is what our sending end voltage, which represents V at an angle δ . And this is the angle δ . This is the angle δ . So, this is the difference of the sending end voltage and receiving end voltage is represented by this dotted line.

Now, you know that this current which is flowing through this line without having a compensation that is I_m and without this compensation this is also I_m . So, without compensation or for uncompensated line, the current also, so I_m represents the midpoint current for uncompensated line. Now, what do you mean by uncompensated line? When

we do not have this uncompensation or any type of compensation of a transmission line that is normally the uncompensated line. Now, when we have a compensated line. This I_m will change and you have seen that we also derived that this V_m is having angle $\delta/2$ and as we know for symmetrical line I_m is also having same phase with V_m .

So, I_m magnitude is also determined over here from this particular expression I_m magnitude is also determined and angle also we know that it is $\delta/2$. So, we know that I_m magnitude is $2V/x$ divided by $\sin(\delta/2)$. So, this is we already derived in the last page here. So, I_m is equal to $2V/x \sin(\delta/2)$. So, $\sin(\delta/2)$; And if you consider this is a phasor, then it will also having angle of $\delta/2$ similar to this V_m .

So, V_m is also having this magnitude at an angle $\delta/2$. And both I_m and V_m will be the same phase that is already established in my series of derivation in previous lectures, right. Now, what we have to see is that what will be the effect of the shunt compensator. Now, this shunt compensator what actually does is it draws certain amount of current. Let us consider that this current is I_{sh} , which is drawn by the shunt compensator through this midpoint.

So when it draws like this, so this current flowing through this, your left half of the shunt compensation and right half of the shunt compensation will get changed. So if this current is I_{sh} , so if I apply a KCL at this point, so then this current will get changed to I_m plus I_{sh} . Right? And this current will get change with I_m minus I_{sh} . So that means the current at the both sides of both sides are changed to change to this I_m plus minus I_{sh} , where I_{sh} represents the current drawn by shunt compensator. So, I_{sh} represents the current drawn by the shunt compensator.

So, that means, so without this presence of the shunt compensation, suppose the current which was also having a phase angle $\delta/2$, so this is $\delta/2$, suppose this is I_{sh} , sorry, this is I_m , then this I_m will get change due to this current drawn by the shunt compensator at the midpoint. So, one half would be plus I_m plus I_{sh} , another half would be I_m minus I_{sh} . So once it gets changed, once the currents get changed, so V_m will also get changed. So we have to find out that how much change happens to the V_m . So due to that, due to this, due to the change in current, V_m which is the midpoint voltage will also be changed to V_m' .

So, once V_m' will get changed, we need to find out that what is the magnitude of the V_m' . To find out, so we can write the KVL equation either in this particular loop or in this particular loop. So, in this particular loop, what would be the KVL equation? The KVL equation would be V_m' is equal to this V at an angle 0 minus V at an angle 0 multiplied by this, multiplied by this current multiplied by this $Jx/2$. So, that voltage drop due to this current is multiplied by this $x/2$. So, this is V plus; this I_m minus I_{sh} by $x/2$ multiplied by $x/2$.

So, if I simplify this, this will be V at an angle 0 plus I_m multiplied by x by 2 minus ΔI_{sh} multiplied by x divided by 4. Now, if you look at this expression, v plus im multiplied by $x/2$, this was our original V_m . So, therefore, this is what the change that will happen due to this ΔI_{sh} current. So, these changes can be represented by ΔV_m . So, if we do not have this shunt compensator at the midpoint, this was our midpoint voltage expression that is V_m .

So, and this point is the change of the V_m . So, we can write ΔV_m magnitude is basically equal to $\Delta I_{sh} x$ divided by 4. So, this is what the change of the midpoint voltage that we get from the you know, the current drawn by the shunt compensator. Now you know, I will discuss in the future lecture also, the shunt compensator, the effect of the shunt compensator is that it will act as a variable susceptance at the midpoint. So I can represent this I_{sh} , ΔI_{sh} as I can represent it this ΔI_{sh} as this, this is basically represented by this V_m multiplied by ΔB_{sh} , where this ΔB_{sh} is the change of susceptance of the shunt compensator. So, if we write so, then ΔQ_{sh} , which is the net change of the reactive power provided by the shunt compensator, can be represented as V_m multiplied by ΔI_{sh} , which is equal to $V_m^2 \Delta B_{sh}$.

So, that is what the that is what the expression that we get and this is basically representing the change of compensation compensation provided by the shunt compensator. So, what we will also see that we also know that this change of the shunt compensation reactive power will also impact in power flow. Now, the power flow expression for this equation will be at the midpoint. This can be written as P is equal to V_m divided by x by 2 $\sin \delta$ by 2. So, this expression we get, you may also get from this short line model, so that this $z \sin \beta L$ by 2 can be approximated to x by 2 during, for this short transmission line model.

$$\begin{aligned}\bar{V}'_m &= V \angle 0 + \left(\bar{I}_l - \frac{\Delta I_{sh}}{2} \right) \frac{x}{2} \\ &= V \angle 0 + \bar{I}_l \frac{x}{2} - \frac{\Delta I_{sh} x}{4} \\ &= \bar{V}_m - \Delta \bar{V}_m\end{aligned}$$

$$\Delta \bar{V}_m = \frac{\Delta I_{sh} x}{4}$$

$$\Delta I_{sh} = \bar{V}_m \Delta B_{sh}$$

ΔB_{sh} is the change of susceptance of the shunt compensator.

$$\Delta \bar{V}_m = \frac{\bar{V}_m \Delta B_{sh} x}{4}$$

The change of compensation:

$$\Delta Q_{sh} = \bar{V}_m \Delta I_{sh} = V_m^2 \Delta B_{sh}$$

The power flow, at mid-point

$$P_m = \frac{V V_m}{X} \sin \frac{\delta}{2}$$

$$\Delta P_m = \frac{2 V \Delta V_m}{X} \sin \frac{\delta}{2}$$

From the diagram, $\Delta Q_{sh} = 2 \tan \frac{\delta}{2}$ is the ratio of change of active power flow of a Symmetrical, lossless, Short transmission line to the unit change of mid-point Series Compensation.

Shunt Compensation [Symmetrical, lossless, Short transmission line]

For uncompensated line, $V_m = V \sin \frac{\delta}{2}$

$\bar{V}_m = V_m \angle \frac{\delta}{2}$

$\bar{I}_m = \left(\frac{2V}{X} \sin \frac{\delta}{2} \right) \angle \frac{\delta}{2}$

Where ΔI_{sh} : The current drawn by Shunt Compensation

$\Delta I_{sh} = \frac{V_m \Delta B_{sh}}{4}$

ΔB_{sh} : Change of Susceptance of the Shunt Compensation

$\Delta Q_{sh} = V_m \Delta I_{sh} = V_m^2 \Delta B_{sh}$

The current at the both sides are changed to $\left(\bar{I}_m \pm \frac{\Delta I_{sh}}{2} \right)$

Due to the change in current \bar{V}_m will also be changed to \bar{V}_m'

$\bar{V}_m' = V \angle 0 + \left(\bar{I}_m - \frac{\Delta I_{sh}}{2} \right) \frac{X}{2}$

$= \frac{V \angle 0 + \bar{I}_m \frac{X}{2}}{2} - \frac{\Delta I_{sh} X}{4}$

The change of Compensation

$\Delta V_m = \frac{\Delta I_{sh} X}{4}$

$\Delta Q_{sh} = V_m \Delta I_{sh} = V_m^2 \Delta B_{sh}$

The power flow, at mid-point

$P_m = \frac{V V_m \sin \frac{\delta}{2}}{X}$

$\Delta P_m = \frac{2 V \Delta V_m \sin \frac{\delta}{2}}{X}$

So, therefore, when there is a unit change in the midpoint voltage, it will also impact this midpoint power. So, therefore, from this equation, I can write ΔP_m is equal to $V \Delta V_m$ divided by X , this $2 V_m$ multiplying in the numerator $\sin \frac{\delta}{2}$ by 2. So, this is the expression we get for this change of the active power at the midpoint and that is also the change of the active power throughout the line because we consider it is a lossless line. So that expression we get.

Now what we will do? We will take the ratio of these two. So the ratio of change in power flow change in power flow to the change in shunt compensation can be written as ΔP_m divided by ΔQ_{sh} . Now, I know the expression of ΔP_m , I will put it here, ΔP_m expression is already obtained from here, let us put it there, that is $2 V \Delta V_m \sin \frac{\delta}{2}$ divided by X multiplied by $\sin \frac{\delta}{2}$ and ΔQ_{sh} , so this is multiplied by, this is multiplication \sin , do not mix with X , X is our line reactance. So, this is multiplied by 1 upon $V_m^2 \Delta B_{sh}$. So, 1 upon $V_m^2 \Delta B_{sh}$; Now, we know that V_m is equal to, V_m also we obtained that the expression of this V_m also we obtained as this, right? So, what we will put it over here is that this is $V_m^2 \Delta B_{sh}$ that is ΔQ_{sh} and this is what ΔV_m .

So, what we will do is that we will put ΔV_m there. So, we will keep everything as it is, only we will put ΔV_m expression over there. So, $\sin \frac{\delta}{2}$ multiplied by this ΔV_m , ΔV_m we already obtained that, ΔV_m is equal to this, ΔI_{sh} multiplied by x divided by 4, this is what the expression that we obtained and ΔI_{sh} is represented by this. So, what we can write from this expression and this expression, what we can write ΔV_m is equal to ΔI_{sh} which is equal to V_m multiplied by ΔV_{sh} multiplied by x divided by 4. So, this expression I am going to write over here. So, ΔV_m is now this, V_m multiplied by ΔV_{sh} multiplied by x divided by 4.

The ratio of change in power flow to the change in shunt compensation

$$\begin{aligned}\frac{\Delta P_m}{\Delta Q_{sh}} &= \frac{2 V \Delta V_m}{x} \sin \frac{\delta}{2} \times \frac{1}{V_m^2 \Delta B_{sh}} \\ &= \left(\frac{2 V}{x} \sin \frac{\delta}{2} \right) \times \left(\frac{V_m \Delta B_{sh}}{4} \right) \times \frac{1}{V_m^2 \Delta B_{sh}} \\ \frac{\Delta P_m}{\Delta Q_{sh}} &= \frac{2 V \sin \frac{\delta}{2}}{2 V_m} = \frac{1}{2} \tan \frac{\delta}{2}\end{aligned}$$

Comparison of the series and shunt compensations

$$\frac{\Delta P_m}{\Delta Q_{sh}} \times \frac{\Delta Q_{se}}{\Delta P_m} = \frac{1}{2} \tan \frac{\delta}{2} \times 2 \tan \frac{\delta}{2}$$

$$\frac{\Delta Q_{se}}{\Delta Q_{sh}} = \tan^2 \frac{\delta}{2}$$

This is what the expression for ΔV_m , right? So, this is we already obtained in the last page. This is what the expression is that I simply copied and put it over here. And then we will have the expression of 1 upon V_m square ΔV_{sh} . So, now what we do is that, now we will simplify this, okay. So, to simplify this, this you know that this V_m and this square will cancel out, ΔV , ΔV_{sh} , ΔV_{sh} will be cancelled out, Δ , this x and this x will be cancelled out. So, this 2 and this 4 if you cut there will be 2 here over, then what we will get in the numerator that is $V \sin \frac{\delta}{2}$ and the denominator we have 2, this V_m we will also have and that is all. Now, we know that V_m is equal to $V \cos \frac{\delta}{2}$. We will simply put it over here. So, this will be then $V \sin \frac{\delta}{2}$ divided by $2 V \cos \frac{\delta}{2}$, which is where V again will cancel out.

So, which can be written as $\frac{1}{2} \tan \frac{\delta}{2}$, because $\sin \frac{\delta}{2}$ and $\cos \frac{\delta}{2}$, if you take the ratio, this will be $\tan \frac{\delta}{2}$. So, what we get is P_m divided by ΔQ_{sh} , this ratio is coming out to be half $\tan \frac{\delta}{2}$. So, this is again you can see that this ratio gives the change of power flow due to the change of the shunt compensation. So, shunt compensation has the impact of the change in power flow as well and this is what the expression that we receive and that expression is applicable for any symmetrical short lossless transmission line. Now, what we will do with this

expression? We may have a comparison with the series compensation to the shunt compensation.

Lec 13: Effect of Mid-point compensation on power flow of transmission lines. Shunt Compensation.

The ratio of change in power flow

$$\frac{\Delta P_m}{\Delta Q_{sh}} = \frac{2V(\Delta V_m) \sin \frac{\delta}{2}}{x} \times \frac{1}{V_m^2 \Delta B_{sh}}$$

$$= \left(\frac{2V \sin \frac{\delta}{2}}{x} \right) \times \left(\frac{V_m \Delta B_{sh}}{2} \right) \times \frac{1}{V_m^2 \Delta B_{sh}}$$

$$= \frac{V \sin \frac{\delta}{2}}{2 V_m}$$

$$= \frac{V \sin \frac{\delta}{2}}{2 V \cos \frac{\delta}{2}} = \frac{1}{2} \tan \frac{\delta}{2}$$

$$\boxed{\frac{\Delta P_m}{\Delta Q_{sh}} = \frac{1}{2} \tan \frac{\delta}{2}}$$

$$\boxed{\frac{\Delta P_m}{\Delta Q_{se}} = \frac{1}{2 \tan \frac{\delta}{2}}}$$

We can, therefore, compare the Series and Shunt Compensations

$$\frac{\Delta Q_{se}}{\Delta Q_{sh}} = \frac{\Delta P_m}{\Delta Q_{sh}} \times \frac{\Delta Q_{se}}{\Delta P_m} = \frac{1}{2} \tan \frac{\delta}{2} \times 2 \tan \frac{\delta}{2} = \tan^2 \frac{\delta}{2}$$

\Rightarrow To have same change in active power flow, what is the ratio of ΔQ_{se} to ΔQ_{sh}

$\delta \approx 30^\circ$ $\left(\frac{\Delta Q_{se}}{\Delta Q_{sh}} \right) = 0.07$ We need only 7% ratio Series comp. as compared to shunt compensation to have same change in power flow.

So, we may have a comparison. So, what type of comparison we can make? So, what we can do is that we can therefore compare the series and fund compensation. So, therefore, we can compare the series and shunt compensation. So, what comparison we can do? We can get the ratio of the change in power flow due to the unit shunt compensation. We can also get the change in this power flow with respect to the series compensation.

So, we will have these two and we will put it side by side. So, this is one expression and that was another expression del P m divided by del Q ac, which is coming out to be 1 upon 2 tan delta by 2. So, this will, this we get. So, here del Q s e represents the series change in series compensation, del Q s h represents the change in shunt compensation. Both the change impacts on the power flow of the line, that is visible, that is understandable. Now, if we take the, if we compare these two compensation, just by taking a ratio of the just by dividing this expression to that ratio.

So, then what we will write is del P m del Q s h multiplied by del Q s e 2 del P m. I am just inverting this. I am taking reciprocal of that. So, this will be equal to half tan delta by 2 multiplied by 2 tan delta by 2. So, 2 2 will be cancelled out. So, it will be tan square delta by 2. So, this is again an important, which is again an important relation which gives that for having the same change in the active power flow, what is the ratio of the series compensation to shunt compensation is required. So, that is very important to understand. So, this this gives to have to have same change in active power flow, what is the ratio of series compensation because this is ultimately giving you the ratio of del Q se to del Q sh, the ratio of del Q se to del Q sh. del Q sh. So, del Q sh represents the unit,

the change of the series compensation, ΔQ_{sh} represents the change of the shunt compensation.

So, this we can find out from this expression $\tan^2 \delta$ by 2. Now, if we take value of this, realistic value of the load angle, that is suppose around 30 then what we will get this ΔQ_{SE} to ΔQ_{SH} is coming out to be 0.07. It means that to have the same change in the active power flow, the shunt compensation rating requirement will be only 7 percent of the series compensator which is very important remark that you can find out. This ratio gives that because this ratio is the ratio of the rating requirement also for the shunt series compensation as compared to the shunt compensation. So, when we have δ is equal to 30 degree, this ratio gives a value that is $10, 15$ degree square which is coming out to be around 0.07. It means that only 7 percent rated series compensator can impact the same amount of power flow which a shunt compensation can impact. So, this gives some sort of superiority of the series compensation, but of course, this rating requirement is one aspect. And there are other aspects also. In general, this series compensator used to be more than, more costlier, at least twice as costlier as the shunt compensator.

Because of various reasons, I will discuss in my future lecture. But one thing that you can find out for this theoretical derivation, that series compensation is more effective in terms of the control of the flow of the power through a particular transmission line and it requires very little change as compared to the shunt compensation to have a same change in the power flow of the line. So, this is an important remark. So, this gives the important remark is that we need only 7 percent rated series compensator as compared to shunt compensator to have same change in power flow. So, in this, with this remark, it may appear to you that series compensation is more economic, but it is actually not because series compensation compensator cost is much higher than the shunt compensator.

And there are various other aspects. So, here we analyze the aspect of the compensation in view of only the change of active power flow through the line. But there are other aspects which are more, which are also more important in terms of the power system point of view. And those things I will discuss in my future lecture. But this lecture will give you an idea the relative performance of the series and shunt compensator at the midpoint of a transmission line and also will give you the idea how they can effect on the power flow through the transmission line.

So, this is what our goal off course to discuss this particular lecture. And, in next lecture I will start with a specific type of shunt compensator is called as static var compensator. For this time being, thank you very much for your attention. Thank you very much for joining. Thank you.