

Course Name: Power Electronics Applications in Power Systems

Course Instructor: Dr. Sanjib Ganguly

**Department of Electronics and Electrical Engineering,
Indian Institute of Technology Guwahati**

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Power Electronics Applications in Power Systems

Course Instructor: Dr. Sanjib Ganguly

Associate Professor,

Department of Electronics and Electrical Engineering,
IIT Guwahati

(Email: sganguly@iitg.ac.in)

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Lec 11: Numerical example of mid-point compensation: Part A

Welcome again in my course Power Electronics Applications in Power Systems. In the last lecture, I derived the expressions for the voltage-current power of the midpoint of a symmetrical, long, lossless transmission line. And I concluded at the end that this midpoint is most vulnerable to experience overvoltage and undervoltage, specifically during light load conditions and the rated load conditions respectively. So, this over-voltage and under-voltage need mitigation. So, in this particular lecture, we will learn how do we mitigate this over voltage and under voltage by placing compensators at the midpoint. And in this particular lecture, we will also take a numerical example to understand the size of the compensator which is going to be required for mitigating this over-voltage and under-voltage.

Also, we will try to understand the various practical aspects of it. So, let us proceed. So, the goal of this lecture is to understand this midpoint compensation. Now, what do you mean by midpoint compensation? It is basically the undervoltage and overvoltage mitigation at the midpoint due to this different loading of a long symmetrical lossless transmission line.

So, here also we will consider the assumptions are. So, here also our assumptions are we will consider we will be having a symmetrical, lossless, long transmission line. We will

be having a symmetrical lossless long transmission line. And we will also keep on using whatever notations we have already used to model a long lossless symmetrical transmission line. We will keep all these notations intact. Now what we will do is that let us plot. Let us draw a symmetrical lossless long transmission line. It simply represents a line between two buses like this. And as we know, this voltages at this bus is regulated as V_s is equal to V at an angle 0. Voltage at this bus is regulated as V_r is equal to V at an angle minus delta.

So, this is our sending end side, this is our receiving end side and let us consider that this is the midpoint; this is the midpoint. We derive the expression of active voltage at this midpoint, which is coming out to be, these are all phasors, I missed that notation, but you should not miss it. So, this V_m is, will be equal to V at an angle $\delta/2$, and I_m at this point will be equal to I at an angle minus $\delta/2$. Now what we will do is that we will place a compensator over here. Let us consider this is a compensator.

This is a compensator. Now, what actually it will compensate? The compensator will actually compensate the reactive power. So, basically, in my previous lectures also, I tried to explain that this over-voltage and under-voltage happen due to the different, in different loading also. This happens due to the mismatch of the reactive power. The system sometimes generates reactive power more, system sometimes generates reactive or absorbs reactive power more.

Now, in order to alleviate that, we will put a compensator, we will connect at the midpoint like this. And, we will consider that this compensator will be responsible to maintain a certain voltage at the midpoint. So, here our goal is to design a compensator such that it regulates the midpoint voltage and thereby it will mitigate the over-voltage and under-voltage. Thereby it will mitigate the over-voltage and under-voltage. So what we will do is that we will consider that at this midpoint we will maintain a voltage magnitude of V_m irrespective of the loading.

And this will be done by this particular compensator by injecting or absorbing the reactive power whenever would be required. So, that is what the goal is, that is what the goal of this particular lecture is and we will also show the numerical example, we will show a numerical example to understand it more generous way. So, our goal is to maintain the compensator. So, here also we will design a compensator such that the compensator will maintain the midpoint voltage constant at V_m irrespective of the loading, irrespective of the loading. So, what we will do is here this compensator will inject or absorb certain amount of reactive power to the both side of the line. Let us consider that this will inject a reactive power to the both side of the line to maintain the voltage at the midpoint to this particular value that V_m . Now, V_m can be of 1 per unit can be of little bit of higher than 1 per unit or so. But if this compensator would

be designed such that it will always maintain a constant voltage at the midpoint. So, that is the goal of this particular lecture. Now, when you have so, then what would be the magnitude of this Q_m and what would be the total compensation required? Now, here the total compensation required is basically Q_c , total compensation required that will be Q_c .

So, this will be equal to twice of Q_m , because suppose from this side to this sending end, this is our sending end side and this is our receiving end side. Now, suppose from midpoint to the sending end site, we are injecting a reactive power Q_m and from midpoint to the receiving end site, we are injecting a reactive power Q_m , same. Then net reactive power we are injecting through this compensator is twice of this Q_m . Now, we know that in order to hold this midpoint voltage V_m to C , the expression of this Q_m is equal to $V_m^2 \cos^2 \frac{\beta l}{2}$. This we will consider line to line or if we consider that not line to line, then this will be multiplied with 3.

So, $V_m^2 \cos^2 \frac{\beta l}{2}$ by 2 minus $V_m^2 \cos^2 \frac{\delta}{2}$ divided by $Z_c \sin \frac{\beta l}{2}$ by 2. So, that much of reactive power we need to inject to both side. So, if we include these three within this, then this expression would be twice of $V_m^2 \cos^2 \frac{\beta l}{2}$ by 2 minus $V_{L-L}^2 \cos^2 \frac{\delta}{2}$ divided by $Z_c \sin \frac{\beta l}{2}$ by 2, where this L to L represents line to line voltage. And as you know, this line-to-line voltage, this $V_m^2 \cos^2 \frac{\beta l}{2}$ is basically root 3 times of V_m^2 then since there is a square over here and also we know that V_{L-L} is equal to root 3 times of V , then this 3 is basically absorbed here in this particular expression because this is a square. When power expression you are writing as a square of the voltage, then line to line voltage can absorb that this 3 terms.

$$Q_m = \frac{V_m^2 \cos^2 \frac{\beta l}{2} - V_m^2 \cos^2 \frac{\delta}{2}}{Z_c \sin \frac{\beta l}{2}}$$

$$Q_c = 2 \times \left(\frac{(V_m^2)_{L-L} \cos^2 \frac{\beta l}{2} - (V)_{L-L} (V_m^2)_{L-L} \cos^2 \frac{\delta}{2}}{Z_c \sin \frac{\beta l}{2}} \right)$$

$$P_{comp} = \frac{V V_m \sin \frac{\delta}{2}}{Z_c \sin \frac{\beta l}{2}}$$

So, this is what the thing is. Now, you should understand that we consider that receiving end voltage and sending end voltages are regulated to V . So, what we will have that in order to maintain the midpoint voltage at a voltage V_m , that voltage we are writing as V_m . So, V_m is the voltage which is maintained at the midpoint by this particular compensator irrespective of the loading. So, to do that we have to inject that much of reactive power at the midpoint.

Lec 11: Numerical example of mid-point compensation: Part A

Assumptions: Symmetrical, loss-less, long transmission line

Goal: To design a compensator such that it regulates the mid-point voltage

+ Compensator will maintain the mid-point voltage constant at $(V_m)_L$ irrespective of the loading

$$Q_C = 2 Q_m$$

$$= 2 \cdot 3 \left[\frac{(V_m)_L^2 \omega \frac{L}{2} - V(V_m)_L \omega \frac{L}{2}}{Z_c \sin \frac{\omega L}{2}} \right]$$

$$= 2 \left[\frac{(V_m)_L^2 \omega \frac{L}{2} - (V)_{L-L} (V_m)_L \omega \frac{L}{2}}{Z_c \sin \frac{\omega L}{2}} \right]$$

$L-L$: Line to line voltage

$$(V_m)_{L-L} = \sqrt{3} V_m$$

$$(V)_{L-L} = \sqrt{3} V$$

So, the compensator rating should be this, compensator reactive power because compensator means at the very beginning I said that is a reactive power compensator. So, its rating is usually represented by volt-ampere reactive, or even though it is not mentioned, if we assume that the system is lossless, so volt-ampere is equal to volt-ampere reactive for this reactive power source. That is something one needs to understand. What we will do, let us have a numerical problem to understand it more generously. So, in that particular numerical problem, we will consider there is a symmetrical lossless long transmission line with the following parameters.

Number 1 is L is equal to 0.932 milli Henry per kilometer. Capacitor is equal to 12.2 nanofarad per kilometer. Then line length is 800 kilometer. System voltage is or the base voltage is 735 kilo-Volt and the frequency is 60 hertz that is what are given here. And this problem I sincerely acknowledge with this Mathur Verma book which is considered to be a textbook for this particular this lecture. So, for this particular transmission line what do we need to do? We need to design a compensator to hold the midpoint voltage to 1.05 per unit irrespective of loading. So, this is our goal is to design a compensator at the midpoint to hold the midpoint voltage at 1.05 per unit. It means that actual voltage is 1.05 times of that 735 kV that much amount at the midpoint irrespective of the loading. It means that loading can be 0 or loading can be up to the rated loading, but the voltage profile of the midpoint will remain constant to be 1.05 per unit or 1.05 multiplied by 735 kV. Now, let us see the solution. So what is to be done first from this parameter L and C , what we have to find out? We have to find out Z_c . So Z_c as we know, it will be equal to

$\sqrt{L/C}$. This is already I have shown you in one of my previous lectures that this Z_c will be coming out to be 276.4 ohm. So Z_c is our surge impedance. So Z_c is our surge impedance. And this βl is coming out to be, in degree it is coming out to be 58.2 degree. So, this β we find out already as an $\omega \sqrt{LC}$, where ω is equal to 2π . So, this is equal to 2π multiplied by 60 multiplied by $\sqrt{L/C}$, whatever values are given. If you do so, then it will coming out to be 1.26 multiplied by 10 to the power minus 3 radians per kilometer. Now, when you multiply this β that is the phase constant with this line length given as 800 kilometers, then whatever you will get that is in radians and then let us convert it to degrees, then whatever you will get that is coming out to be βl that is 58.2 degree. So, several times we need to find out βl by 2, so which will be nothing but 29.1 degree. Now what we will find out in order to design the compensator, let us find out that what would be the, this surge impedance loading of the line. So, surge impedance loading SIL, so the surge impedance loading of the line will be equal to, as we know that V line to line square divided by Z_c , V line to line is given out to be 735 kV square. Remember, this 735 kV, 220 kV, 400 kV, 765 kV, whatever is given in the problem, even though it is not mentioned, you have to assume that they are line-to-line quantities.

So, this I understand that you know, because any basic power system course teaches that. So, 735 kV square divided by this Z_c , which is given out to be 276.4. So, since it is kV square, so whatever will come, that will be in terms of this 10 to the power 6. So, I write it simply in megawatt. So, it is coming out to be 1954 megawatt, 1954 megawatt. Now, I derived this surge impedance loading also. Now, one thing we should understand that this midpoint voltage during this no-load conditions can be also determined because we already know this midpoint voltage magnitude expression. So, this will be equal to $V \cos \delta$ by divided by $\cos \beta l$ by 2. So, at no load you know δ is equal to 0. So, V_m by V which is representing that V_m per unit will be equal to 1 upon $\cos \beta l$ by 2. So, which is coming out to be 1 upon $\cos 29.1$ degree which is definitely higher than 1 and which will be definitely higher than 1.05. So, that means that at no load, this midpoint voltage is significantly higher than the system actual voltage, which needs to be mitigated.

Now, we also determine that this active power flow at the midpoint. So, P_m is basically equal to as we know, equal to this active power flowing to this midpoint, which is equal to this V square. Actually, it is equal to V multiplied by V_m divided by $Z_c \sin \beta l$ by 2. So, I should tell you that before that this P whatever we are deriving that is without compensation. Without compensation, the active power flow is basically equal to V square divided by $Z_c \sin \beta l$ multiplied by $\sin \delta$, where V is the system voltage that is 735 kV and V has to be line to line. So, this is equal to 735 square divided by Z_c that is given out to be 276.5 $4 \sin \beta l$, $\sin \beta l$ is equal to $\sin 58.2$ multiplied by $\sin \delta$.

Now, since what will be the unit of this P? Since we consider that 735, it is kilovolt. So, the unit of this will be automatically megawatt.

Because if you multiply kilovolt, there will be 10 to the power 6 term that will come out to be as a multiplier. So, this gives this watt multiplied by 10 to the power 6, which means this is represented by a megawatt. Now, as per my calculation, this is coming out to be 2297.7 sine delta, that much of megawatt. So, this is without compensation. Now, this P will get change when we have a compensator placed at the midpoint, this P will also get change and I will come to that, the expression for this P. But without this compensation, this P expression will be valid for every, each and every point of the line, because we consider the line is lossless. Now with compensation, we know that the rating of the compensator Q_c already we have derived that is equal to twice. $V_{mL} \cos \beta L$ by 2 minus this $V \cos \delta$ by 2 divided by $Z_c \sin \beta L$ by 2. Now, we know everything except that we do not calculate it for a particular value of delta because we are considering this compensators to hold the midpoint voltage to this value V_{mL} irrespective of the loading.

Lec 11: Numerical example of mid-point compensation: Part A

Numerical Problem

A Symmetrical, lossless, long transmission line with following parameters
 $L = 0.932 \text{ mH/km}$, $C = 12.2 \text{ nF/km}$, Line length = 800 km, voltage = 735 kV, $f = 60 \text{ Hz}$
 We need to design a compensator to hold the midpoint voltage to 1.05 p.u.

Solution: Surge Imp. $Z_c = \sqrt{\frac{L}{C}} = 276.4 \Omega$, $\beta L = 58.2^\circ$, $\beta = \omega \sqrt{LC} = 2\pi \times 60 \sqrt{0.932 \times 10^{-3} \times 12.2 \times 10^{-9}} = 1.26 \times 10^{-3} \text{ rad/km}$
 $\frac{\beta L}{2} = 29.1^\circ$

SIL = Surge Impedance Loading = $\frac{V_{LL}^2}{Z_c} = \frac{735^2}{276.4} = 1954 \text{ MW}$

$V_m = \frac{V \sin \frac{\beta L}{2}}{\sin \frac{\beta L}{2}}$ At no load ($\delta = 0$), $\left(\frac{V_m}{V}\right)_{p.u.} = \frac{1}{\sin \frac{\beta L}{2}} = \frac{1}{\sin 29.1^\circ}$

Without Compensation $P = \frac{V_{LL}^2 \sin \delta}{Z_c \sin \beta L} = \frac{735^2 \times \sin \delta}{276.4 \sin 58.2^\circ} = 2297.7 \sin \delta \text{ MW}$

So, I will put all the values except this cos delta. So, what we will get? Let us see. So, this is 2 multiplied by this V_{mL} line to line is basically 1.05 multiplied by 735 square cos beta L by 2. Now cos beta L by 2 already we derived 29.1 degree. So, this is 29.1 minus this is V line to line is again 735 multiplied by V_{mL} line to line is 1.05 multiplied by 735 multiplied by cos delta by 2 divided by $Z_c \sin \beta L$ by 2. Z_c is coming, Z_c is, we know that it is 276.4 ohm. So, this is 276.4 ohm multiplied by sin 29.1 degree. So, this is our Q_c . So, this is our Q_c . Now, what would be the unit of this Q_c ? Look at this, here also

we consider 735 square, that is kilo volt square, here also 735 kilo volt multiplied by 735 kilo volt. So, we are basically accommodating this kilo volt square in the main time. So, this will be, the unit will be that much of MVAR, similar to megawatt. So, now, if you solve this, then this will be coming out to be 7742.97 minus 8439.57 cos delta by 2 that is the compensator rating we require, that is the compensator rating we require, that much of MVAR, that is what the compensator rating is required. So, our goal also was to determine the compensator rating irrespective, which will hold, which will be able to hold the midpoint voltage to 1.05 per unit of irrespective of the loading. So, that we determine. So, this is what the compensator, midpoint compensator rating.

$$Z_c = \sqrt{\frac{l}{c}} = 276.4 \Omega$$

$$\beta = \omega\sqrt{lc} = 1.27 \times 10^{-3} \text{ rad/km}$$

$$\beta l = 1.017 \text{ rad}$$

$$\beta l = 58.27^\circ$$

$$SIL = P_c = \frac{V^2}{Z_c} = 1954.57 \text{ MW}$$

$$P = \frac{V_{L-L}^2 \sin \delta}{z_c \sin \beta l} = \frac{735^2 \times \sin \delta}{276.4 \sin 58.27} \text{ MW} = 2297.97 \sin \delta \text{ MW}$$

$$V_m = \frac{V \cos \frac{\delta}{2}}{\cos \frac{\beta l}{2}}$$

$$\text{At no load } \delta = 0, \left(\frac{V_m}{V} \right) = (V_m)_{p.u.} = \frac{1}{\cos \frac{\beta l}{2}} = \frac{1}{\cos 29.135^\circ} = 1.14 \text{ p.u.}$$

14% overvoltage at mid-point at no load (No mid-point compensation)

Compensation,

$$Q_c = 2 \times \left(\frac{(V_{mc})_{L-L}^2 \cos \frac{\beta l}{2} - (V)_{L-L} (V_{mc})_{L-L} \cos \frac{\delta}{2}}{z_c \sin \frac{\beta l}{2}} \right)$$

$$Q_c = 2 \times \left(\frac{(1.05 \times 735)^2 \cos 29.135^\circ - 735 \times (1.05 \times 735) \times \cos \frac{\delta}{2}}{276.4 \times \sin 29.135^\circ} \right) \text{ MVar}$$

Mid-point compensator rating,

$$Q_c = 7742.97 - 8439.57 \cos \frac{\delta}{2} \text{ MVar}$$

The compensated line's active power

$$P_{comp} = \frac{VV_{mc} \sin \frac{\delta}{2}}{z_c \sin \frac{\beta l}{2}}$$

$$P_{comp} = 4215.157 \sin \frac{\delta}{2} \text{ MW}$$

This is the midpoint compensator rating. Now we will do some sort of analysis. Before that also we consider that for, if we have this compensator placed, so with compensation the active power flow of the line will get changed and the compensated line active power expression will be equal to this V multiplied by V_{mc} . This all will be line to line divided by $Z_c \sin \beta l$ by 2 multiplied by $\sin \delta$ by 2 because this, as you have seen that, this we can find out just by from the expression of this V_{mc} multiplied by that I_{mc} and if you put this then this expression is coming out to be 4219.78 sine delta by 2 Megawatt. Now, we will analyze this result and we will find some important remarks or observations from that.

Lec 11: Numerical example of mid-point compensation: Part A

With compensator, $Q_c = 2 \left[\frac{(V_m)^2 \cos \frac{\beta l}{2} - (V_{L-L})(V_m) \cos \frac{\delta}{2}}{Z_c \sin \frac{\beta l}{2}} \right]$

$= 2 \left[\frac{(1.05 \times 735)^2 \cos 29.1^\circ - 735 \times (1.05 \times 735) \cos \frac{\delta}{2}}{276.4 \times \sin 29.1^\circ} \right] \text{ MVAR}$

Mid-point Compensator Rating

$Q_c = 7742.97 - 8439.57 \cos \frac{\delta}{2} \text{ MVAR}$

The compensated line's active power $= \frac{V_L (V_m) \cos \frac{\delta}{2}}{Z_c \sin \frac{\beta l}{2}} = 4219.78 \sin \frac{\delta}{2} \text{ MW}$

Observations:

- (i) Line active power flow is modified due to the compensator placement.
- (ii) When the line is operated at 'No-load', $\delta = 0$, $Q_c = 7742.97 - 8439.57 = -696 \text{ MVAR}$
- (iii) When the line is operated at max^{imum} loading ($\frac{\delta}{2} = 90^\circ$), $Q_c = 7742.97 \text{ MVAR}$
- (iv) The range of compensation is required as, $(-696, 7742.97) \text{ MVAR}$
- (v) No practical compensator can provide such a wide range of compensation.

So, what are our observations? First observation is we got a revised value of active power. So, lines active power is revised, so line active power flow is modified due to the compensator placement. Now, whether this is having some positive impact or negative impact, those things we will discuss in very detail in the future lectures, but this is happening. So, without compensator you can see the, this active power flow throughout the line was this 2297.7 sine delta megawatt and with compensator it is revised to 4219.78 multiplied by sine delta by 2 megawatt. So, there is a change in the active power flow and that change happens due to the placement of the compensator. So, this is our

first remark. Then the second remark is, you look at this compensator rating. Now, when the line is operated at no load, operated at no load. So, then we know that δ is equal to 0, δ is equal to 0. So, when the δ is equal to 0, so that Q_c would be equal to 7742.97 multiplied by this. If you put this $\cos \delta$ by 2 when δ is equal to 0 then this will be 1 actually. So, this will be 8439.57 which gives you my around minus 696 MVR. So that means at no-load condition, we need that much of MVR, that much amount of MVR to be supplied. Now when, what do you mean by negative MVR supplied? That means actually we need to absorb 696 MVR at the midpoint to hold the midpoint voltage constant at 1.05 per unit. So, this is our second observation. Now, our third observation is, that when the line is operated at this theoretically maximum loading, theoretically maximum loading, considering that δ by 2 is equal to 90 Then, this Q_c requirement would be equal to, so $\cos \delta$ by 2 90 degree means cosine 90 degree which is 0.

So, this is coming out to be 7742.97 MVR. So, the range of compensation is required. So, this is what the at no load you know that much of compensation we require and that maximum load that much of compensation is required. So, the range of compensation is required, range of compensation is required as minus 696 to 7742.97 MVR. So, this is what we require and we consider that this compensator will be rated such that it will be able to provide compensation from starting from this range to that range.

Now, minus 696 stands for the compensator needs to absorb some amount of reactive power. Because, we consider the reactive power is injected, so when injection is negative that means negative injection is as similar to the absorption. So, when there is a no load, so compensator needs to absorb that much amount of MVR minus 696 or 696 MVR of reactive power. Now, when the line is loaded at theoretically maximum loading, then the compensator needs to provide 7742.97. So, it is around 7743 MVAR, that amount of reactive part to be provided or to be injected to the midpoint. This range is significantly longer range and no practical compensator can provide that much amount of wide range of compensation. So, our last comment is no practical compensator can provide such a wide range of compensation. So, no practical compensator can provide such a wide range of compensation at a particular point to hold the midpoint voltage constant.

So, what is the remedial measure? This we will see in the next lecture. But of course, at this point one, you should understand that by considering this equation, whatever we develop, that is this equation and designing this compensator will come out to be an impractical range of compensation. And, you know that if the compensator rating is higher, the cost of the compensator will also be higher. So, higher rating stands for higher cost. So, a compensator at a range of 1000 MVR is a significantly high cost involved.

So, which is of course not practically feasible. So, we need to design a feasible compensator. And, thereby we have to see that what we have to sacrifice for that. So, this will be the part of the next lecture. And, this particular numerical example will give you

an idea that how to proceed with this compensator design and this what are the limitations of this design and how to mitigate that. Now, if one designs a compensator that can provide such a wide range of compensation, it might be very costly, extremely costly, but it will be able to maintain the midpoint voltage always at 1.05 per unit theoretically irrespective of the loading. But, since it requires extreme investment or it is extremely costly, so this is not practically feasible. Now, in the next lecture, I will discuss how do we design a practically feasible compensator and if we design, so what actually we need to sacrifice. So, thank you very much for attending this part of the lecture and look forward to see you in the next lecture.