**Course Name: Power Electronics Applications in Power Systems**

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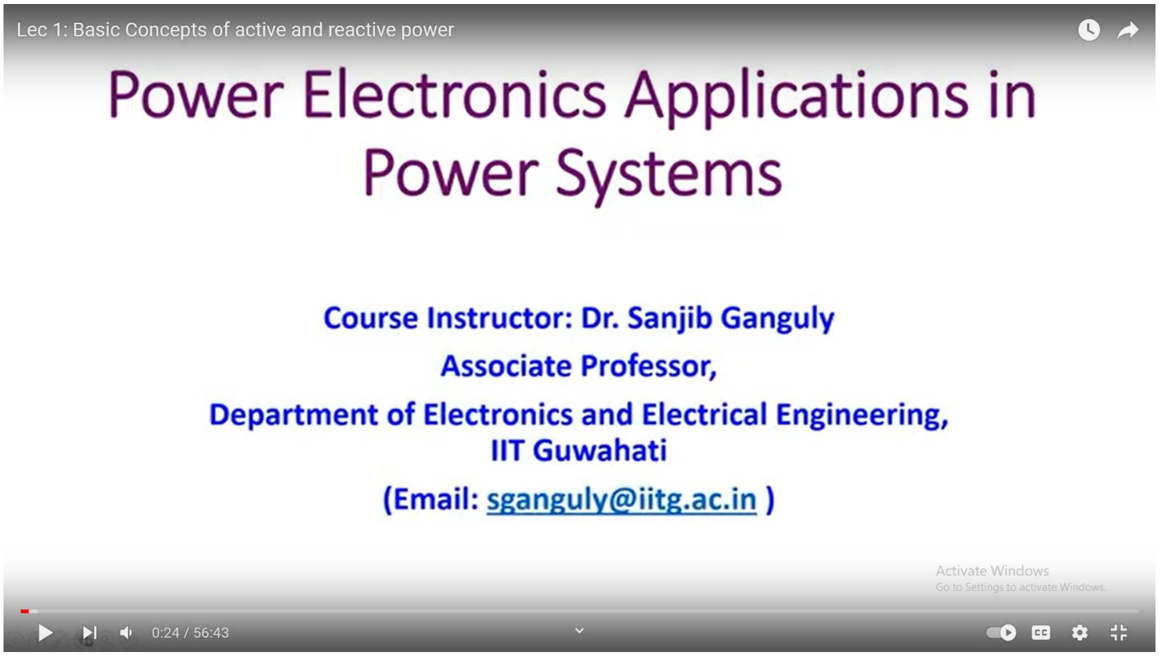
**Week: 01**

**Lecture: 01**

Lec 1: Basic Concepts of active and reactive power




So, welcome to the course Power Electronics Applications in Power Systems. As I mentioned in my introductory lecture, this power electronics applications refer to some power electronics-based devices, which are used to improve the energy efficiency of power systems, which are used to reduce the energy losses and thereby improve the energy efficiency. These devices are used also for regulating the voltages, thereby mitigating the over voltage and under voltages. These devices are also used to improve the dynamic performance, to improve the stability, to improve the damping of power systems. And, these devices are often called as compensators. And, reactive power compensator to be more precise.

And to understand what do you mean by reactive power compensators, one needs to have a fair idea what is reactive power. So in this particular lecture, I will discuss the basic concept of active and reactive power in a power system. okay. And then slowly I will discuss what do you mean by reactive power compensator and then I will move on this different modeling of power system devices and also modeling of power transmission lines.

So, let us move. So, in this lecture, I will start with basic concepts of active and reactive power. So, the goal of this particular lecture is to define active and reactive power. So, to understand what is active and what is reactive power, let us take an example. Let us start with a single-phase AC circuit.

So, we will start with single-phase AC circuit. So, let us consider we have a purely sinusoidal voltage source and it is connected to a load. Now, this represents purely sinusoid voltage source and this represents an inductive load. So, this box rectangular box is representing an inductive load. Now, in this purely sinusoidal voltage source, we can represent the instantaneous voltage as V t is equal to V m sin omega t.

This is the basics of electrical, this is the basic of electrical engineering. You might have seen this type of representation before. As you know in electrical engineering we have, we have representation of these different quantities in different way. For example, this representation is a time-domain representation. This representation is a time-domain representation and that is why this is instantaneous power, so instantaneous voltage.

So, this voltage representation means, at any instant of time say at t is equal to t, the voltage would be V m sin omega t, where this V_m is representing the maximum or peak value, peak value of voltage. And this omega represents the frequency of the sinusoidal voltage. Now if we plot this voltage, the plot would be something like that. So this is V t, this is omega t, whereas this peak or maximum value is represented by this V m. So, this is somewhat known to all of us.

Now, this voltage source is representing this purely sinusoidal source and it is connected to an inductive load. Now, what do you mean by inductive load? As we know that we have three basic elements in a circuit, three linear basic elements in electrical engineering circuit theory. One is resistive, another is inductive, another is capacitive. So, usual loads what we use in our day to day life represent an inductive load which has certain components of resistivity and certain components of inductivity as well. So, basically this inductive load we will represent as a series connected resistance and an inductance.

So, it is a representation; it is connected resistance and an inductance. So, and most of the practical loads with what we use in our day to day life are of inductive load. For example, an electric motor, which represents a bulk volume in an electrical load, either used in industries or even used in the domestic customers; even used by the domestic customers. So, these are highly inductive loads. Now when we have this representation, you know that this voltage source is changing its sign from positive to negative.

So, at any instant of time, let us say, the polarity of the source is like this, and due to this circuit connection, some current which is flowing through this, which is represented by the instantaneous current I_t. Here, this instantaneous current I of t is represented by, similar to this voltage representation, instantaneous voltage representation, it is represented by some peak value of this current multiplied by sin omega t minus phi, where I_m is basically representing peak value of the instantaneous current, peak value of the current and the phi is representing the phase difference. I will come to that, what do you mean by phase difference. So, now if we plot this voltage instantaneous voltage and this instantaneous current together in a same plot then we will understand that what is the phase difference. But apart from that this is the usual representation of instantaneous current in a time-domain representation of electric circuit.

Now, if I plot this current over the same plot here with this voltage and suppose this is I_t, then this representation of I_t will be something like that. It will also have the same frequency as you can see this omega and that omega are the same. So, it will have the same frequency and it will also be like a sinusoid, but its representation is something like that. Now the question is if you compare this voltage and this current then you will see that there is a phase difference and that phase difference is basically phi. Now what is phase difference? Phase difference is, you can see, if you consider this is the instant where voltage goes to zero and increasing in a positive half cycle as a reference, then with respect to that reference, the current starts increasing, this current reaches at 0 and increasing in a positive half cycle, after a high degree apart.

So, the difference of this is the phase difference. So, this difference phi will remain the same if you consider the peak value of these two quantities. So, whatever when this voltage goes to the positive peak after a phi angle the current will also go for its peak value and this will remain same and this will continue for all other instances. So, where the instant for if you compare the same instant of voltage and same instant of current, they will have a phase difference of phi degree. This is the normal concept that we have.

Coming back to our goal that we will try to define what is active and reactive power. In DC circuit what we have seen the power is simple the multiplication of voltage and current. But as you know in DC circuit the voltage and currents are constant in nature. They are not time varying. That is what the difference of AC circuit and DC circuit.

Now, in AC circuit both voltage and current are time varying. Now, the question is then how do you know that what is your power? So, here we will define a term that is called instantaneous power. This instantaneous power we represent it as a small p of t, it is basically the multiplication of this instantaneous voltage and instantaneous current. So, it is equal to V of t multiplied with I of t. Now, this instantaneous power is neither active power nor reactive power, it is a something different.

So, it is at any instant of time, if you just multiply the instantaneous voltage and instantaneous currents, whatever value we will be getting, that is instantaneous power. Now, we know that v(t) is equal to Vm sine omega t. and i(t) is equal to I_m sin omega t minus phi. So, if we put both the expressions over here, so what we will get is, this is V m sin omega t, this is, this multiplied with I m sin omega t minus phi. Now, we will do some simplification of this expression.

Let us keep this V_m I_m outside and then what we will get it sin omega t sin omega t minus phi. Now, this is an expression which is similar to sin a multiplied by sin b. So, when you have this expression, so what I can do is that I will multiply and divide 2 with this expression. So, what we will get, we know that 2 sin a sin b is basically equal to cos a minus b minus cos a plus b. So, what we will get is V m  I m divided by 2 cos A minus B is cos phi minus cos A plus B is cos 2 omega t minus phi.

So, this is what we get. Now, we will keep on simplifying this. so we know that again let me write it again Pt is equal to Vm Im divided by 2 cos phi minus cos 2 omega t minus phi. Now, we can expand this cos 2 omega t minus phi considering that it is cos a minus cos b. So, what we will get? Let us see, So, it is equal to V m I m divided by 2 cos phi minus. So, if we expand this, then what we will get is cos 2 omega t cos phi minus sin 2 omega t sin phi.

So, this we got with by expanding this cos 2 omega t minus phi. Now, we can further simplify this as V m I m divided by 2. So, here cos phi and cos phi are common. So, I can take cos phi outside. So, that I can write this as a 1 minus cos 2 omega t.

Similarly, this side I will keep it as it is. So, I will multiply V m I m divided by 2 sin phi sin 2 omega t. So, I will get these two expressions. Now, what we will do here is we will consider that this is basically equal to p 1 t and this component is basically equal to p 2 t. So, we finally arrived at the expressions of this instantaneous power, remember this is the instantaneous power of the single-phase circuit.

$$p(t) = V_m I_m sin\omega t sin(\omega t - \phi)$$

$$= \frac{V_m I_m}{2}[cos\phi - \cos(2\omega t - \phi)]$$

$$= [VIcos\phi(1 - cos2\omega t)] - [VIsin\phi sin2\omega t]$$

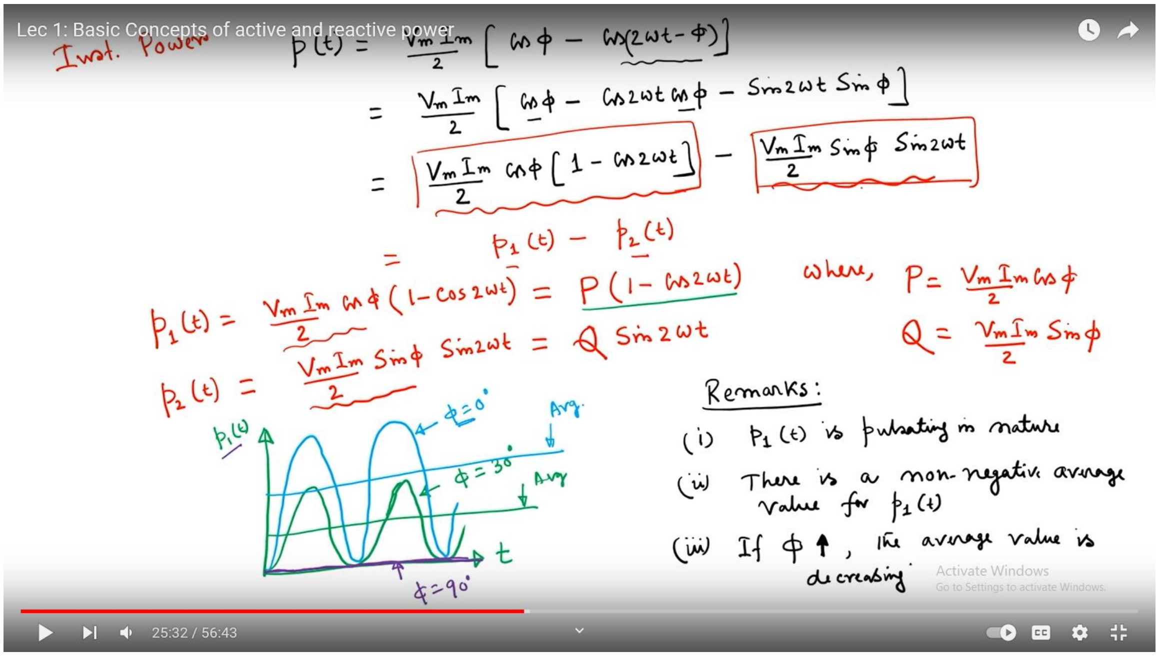
$$= [P(1 - cos2\omega t)] - [Qsin2\omega t]$$

$$= p_1(t) - p_2(t)$$

[Where, $VIcos\phi = P$ and $VIsin\phi = Q$]

As a resultant of two quantities, one is p1 t, another was p2 t, where p1 t is this and p2 t is this. Now, what we will do is, we can write p1 t is equal to  V m I m divided by 2 cos phi

1 minus cos 2 omega t. this V m I m by 2 cos phi this component is time invariant means that it does not change with time. So, I can write it as a constant this constant let us write as capital P. So, the whole expression becomes capital P multiplied by 1 minus cos 2 omega t where capital P is equal to  V m I m by 2 cos phi.




Similarly, we will represent p 2 p that is this component, this component. So, what we will write this is equal to V m I m by 2 sin phi sin 2 omega t. Now, again you look at this component is not time-varying. So, let us represent it as a constant that is capital Q. So, this represents capital Q sin 2 omega t, where capital Q is basically equal to V m I m by 2 So, if I summarize what I have done so far, then I can say that the instantaneous power of a single phase circuit is expressed as a resultant of two components, one is P1, another is P2 and P1, P2 expressions we have written over here.

Now the next thing that we will do is we will plot these two quantities that is p1(t) and p2(t). So if I plot this p1t with respect to this time, then what sort of plot I will get? We will look at it. It is equal to p 1 minus cos 2 omega t. So, if we plot 1 minus cos 2 omega t, what we will get is that it is pulsating above the abscissa. So, it will be something like this and so on.

Now, suppose  this corresponds to some value of phi that is equal to 30 degree. Now, if we consider phi is equal to 0, then how would be the plot? So, at phi is equal to 0, you can see this P will be equal to Vm Im by 2. So, that value will be eventually higher than that phi is equal to 30 degree. So, the plot would be something like this. So, this corresponds to phi is equal to 0 degree.

Now, similarly, if we consider phi is equal to 90 degree, then how would be the plot? If you consider that phi is equal to 90 degree, cos 90 degree would be 0, the plot will be exactly coincide with the x axis or abscissa. So, this corresponds to phi is equal to 90 degree. So, I will write this comment later on, but this is what we get. Now if you look at the in general you know characteristics of this P1 t then you will see it is pulsating above the abscissa. and it always has some non-negative average values.

So, if you take the average value corresponding to the plot phi is equal to 0 degree, this average value will come over here. Similarly, when you phi is equal to 30 degree, so let us take the average value here. So, this is, suppose, the average value when corresponds to phi is equal to 0 degree. So, this is suppose the average value. Then, corresponds to this phi is equal to 30 degree, the average value will be somewhere here.

And corresponding to this phi is equal to 90 degree, the average value would be 0. So, what you can comment over here is, this p 1 t is, I can write it as a remark. So, the first remark will be p1 is pulsating in nature. The second comment is that there is a non-negative average value for p1(t) and third is if we increase phi, if phi is increasing then the average value is decreasing. This arrow representing upward arrow means that if we increase phi, what we can see over here, if I increase phi, then average value is decreasing.

That is what the remark that we can get from this plot. Now, let us plot p2(t) as well, which is representing this. So, as we know p2(t) is equal to Q sin 2 omega t. So, if we plot P2t over time, then what it will also behave like a sinusoidal, it will behave like a sinusoid rather. So if your Q, suppose this is the plot of this p22 corresponds to phi is equal to 30 degrees, then how would be the plot when phi is equal to 90 degree? So, as we know Q is equal to Vm Im by 2 sin phi.

So, what you can see is here is that if phi is increasing q will also increase. So, here this Q, what Q represents? Q is basically the peak value. So, this is nothing but Q in this particular plot. So, when if you increase phi is equal to 60 degree, then this P2t plot will be something like this. Its frequency will remain same, only thing is that its peak value will change.

So this corresponds to phi is equal to 60 degree. If you further increase phi, let us say if phi is equal to 90 degree, then the characteristics will be something like this. This plot will be something like this. So this corresponds to phi is equal to 90 degree. Now what will happen when phi is equal to 0? When phi is equal to 0, this will, when phi is equal to 0, this Q will be equal to 0.

So, the plot will be just superimposed here. So, if I summarize what we got so far, then it will be something like that number 1 is p 2 varies like a sinusoid function. So, the Q represents the peak value of P2. Now, the third remark will be If we take the average of each of these characteristics, what would be the average? Since it represents a sinusoid, the average would be 0. So, the average of P2 will always be 0. why I wrote always be 0 that means irrespective of irrespective of this phi.

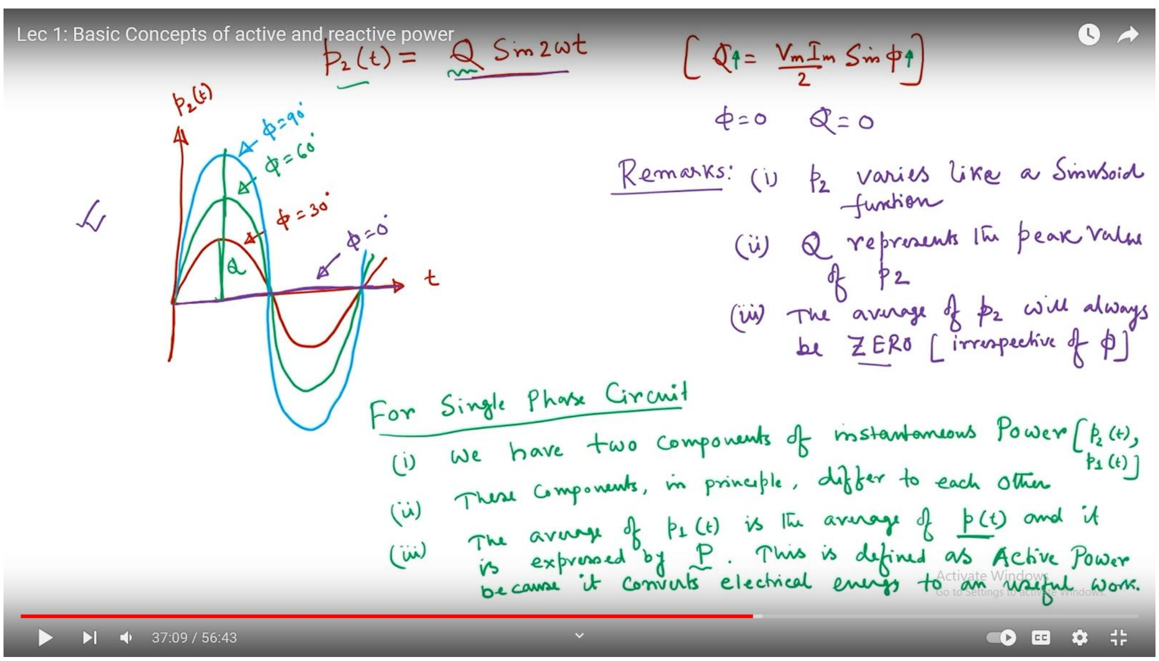
So, whatever the value of phi might be the average of this P 2 will be always 0. So, if we just compare these two characteristics one is p1(t) another is p2(t). then what we can comment that they are of different in nature. P 1 t is a component of this instantaneous power which is always pulsating in nature, but it pulsates above the abscissa, above the x-axis, above the horizontal axis. And it always has some non-negative or either 0, that means either 0 or positive average value.

Whereas if you look at this p 2 t, It also varies, but it varies, it also varies with time, but it varies like a sinusoid. It means that it has some positive value in some instant of time as well as it has some negative value. So when we have, so then as we know that for a sinusoid, this average value is 0. So, the average value of P2 will always be 0 irrespective of this angle phi. And as we know that from these characteristics, this value of Q is changing.

So, for example, whatever this value of this q was when phi is equal to 30 degree, the value of this q changes to this when phi is equal to 60 degree, further it changes to this when phi is equal to 90 degree. So, the value of q is getting changed if we change the phase angle. Now, if we summarize what we get in this particular you know study that we get the instantaneous power of two components; one is p 1 t another is p 2 t; one is pulsating in nature another is pulsating like a sinusoid; one is pulsating always above the abscissa; another is pulsating along this abscissa; one is having always a non-negative average, another is 0 average. So, what we will define is that we have two components of this power in a single-phase circuit. So, what we will write is that for single-phase we have two components of instantaneous power.

Now that means we have two components of instantaneous power one is p2t another is p1t and they differ by their characteristics. So how they are different with the characteristics you can see here you can  see if we take the average value of P1, what this average value is basically representing? This average value is basically representing nothing but P. So, this average value is changing when phi is changing. Now, when this phase angle will change, when will change different, when will change the  When we add this load or when we change the load from one particular type to another particular type, then phi might be changed. So when phi is changed, this average value of this will change.

And this average value will basically depend upon this, average value is basically nothing but this P. So, this P is of a important, P is basically is of important. So, this P, what it is basically doing? It is converting this electrical energy to a useful work. And that is why this P is defined as an active power. So, what we can write is that, these components in principle differ to each other.



Number 3 or third observation that we can write is or third remark we can write is this the average of this P 1 t is the average of P t itself. So, this you know that this average of this p2 t is 0 always. So, average of this p1 t is the average of the instantaneous power itself and it is expressed by capital P. This capital P, this is defined as active power because it converts electrical to an useful work. For example, if it is a, if this load is of a motor, then you know that motor consumes some amount of electric power, electrical power to convert its to mechanical energy, so that we can connect it with a pump or some other devices to do the same work like in lift we have, in the elevator we have, all this are run by motor.

Similarly, when we will pump water from downstairs to upstairs, we use motor. So, there are multiple purposes. In industry, if you go visit, you will see there are many motors for different applications. But all motors consume certain amount of electrical power just to convert it to some amount of mechanical power. And that is primarily done by this average of this instantaneous power and that is known as your active power since it is converting this energy to some useful work.

P2t, if you can see that, since its average is 0, it means that it sometimes is positive and sometimes is negative. When it is positive, if you consider that it consumes certain amount of power, when it is negative, it returns back the same amount of power. So it is ultimately whatever it is consuming, it is returning back as well. So it is not consuming any power to do some useful work. So that is why we will call it, it is not a power for energy conversion, which is used for energy conversion, but it has some significance.

So that is why we call this as a reactive power. actually this P2 is not a reactive power but you can see that this peak value of P2 which is changing over this phi, this phase angle is called this reactive power because this will have some significance which we will study in the further lecture. So what we can write is that the peak value of this P2 that is Q is called or defined as reactive power. okay, It is not this P2, but its peak value. Why it is so? Because you can see that when this phi is changing, only peak is changing.

Other shape, etc., other properties remain same. So, that is why peak is basically changing with respect to phi and this is an important parameter. Later on, we will see in the performance of a power system and that is why we defined this as a separate quantity that is reactive power, ok. Now, this study is all about the single phase circuit. Now, what will happen if we have a three phase circuit because as you know that normal power system is of three phase.

So, let us do the same analysis for the three-phase circuit. So, suppose we have a balanced 3-page supply. So, here our assumption is that we have a balanced 3-page voltage source and we have a balanced prefetch load. Now what do you mean by balanced three-phase source? It is taught in a basic electrical engineering course that when you have a three-phase source which are having same RMS or this peak value and they are exactly separated with a phase difference of 120 degree. or 2 pi by 3 to each other.

So, when we have so, then it is a balanced 3-phase source. Similarly, what do you mean by balanced three-phase load? Balanced three-phase load means three identical loads in every aspect. If it is an inductive load, then all these loads, and inductive loads are identical. Similarly, if it is a capacitive load or resistive loads, then they are identical. So, three identical loads constitute a three-phase balanced load and three-phase voltage source having the same RMS value of individual phases and they are apart from each other 120 degree, that is 2 by 3, then it is a balance source. So, what will be the expression of the balance source and balance load? So, if I consider that three-phase sources are like this, one is VAT and it is represented by Vm sin omega t; then for balance source, then V b t will be equal to V m sin omega t, let us say minus 2 pi by 3 and V c t is represented by V m sin omega t minus 4 pi by 3.

$$v_a(t) = V_m sin\omega t$$

$$v_b(t) = V_m \sin(\omega t - 120^o)$$

$$v_c(t) = V_m \sin(\omega t - 240^o)$$

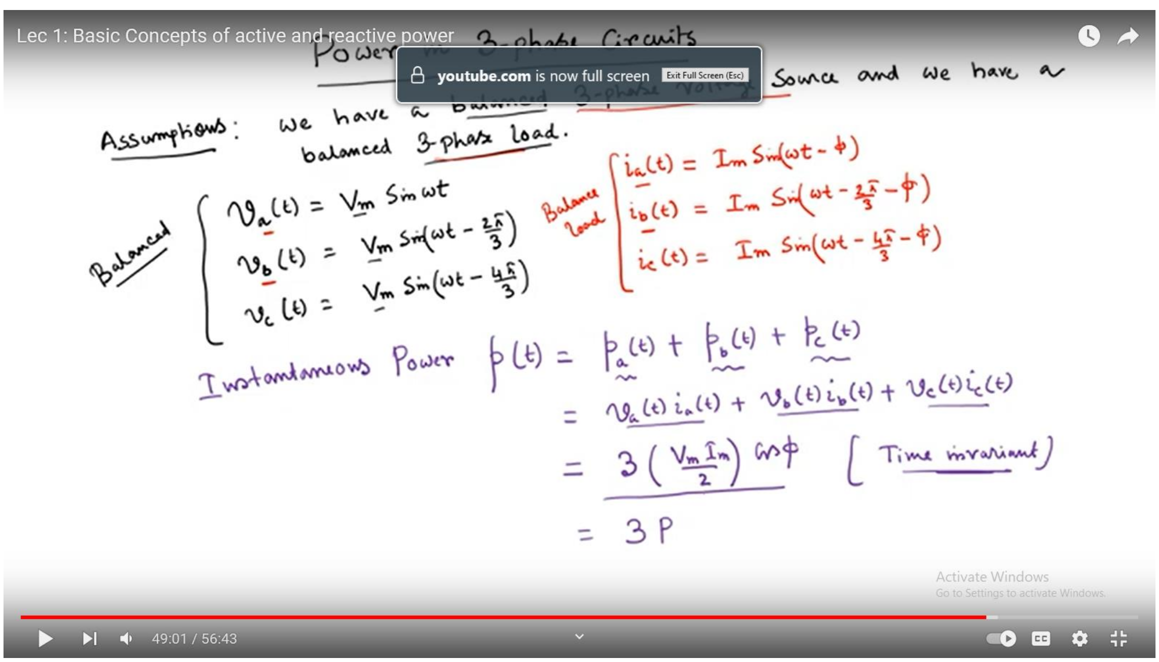
So, you look at these three sources, they are having same rms value because they are having same peak values, they are having identical frequencies and their phase difference between this to this is 2 pi by 3, here also between this to this is 2 pi by 3 and this to this also is 2 pi by 3. So, when you have such a kind of source then it is a called balance source. Now, when you have a balance source and three identical load connected to this source, then what we will do is, they will draw the similar current, what we have seen in case of single phase circuit. So, let us write it as a $i_a(t)$; $i_a(t)$ is the current drawn by this phase A, the load connected to phase A. So, this is suppose I m sin omega t minus phi, I b t is equal to I m sin omega t minus 2 pi by 3 minus phi.

$$i_a(t) = I_m \sin(\omega t - \phi)$$

$$i_b(t) = I_m \sin(\omega t - \phi - 120^o)$$

$$i_c(t) = I_m \sin(\omega t - \phi - 240^o)$$

Here phi is the phase angle difference between this current and that voltage. Similarly, this phi is the phase angle difference between this current and that voltage. Similarly, this will be equal to I c t is equal to I m sin omega t minus 4 phi by 3 minus phi. Now, this represents balance load. So, when we have a three-page balance source and a three-page identical load, an identical load stands for three identical loads connected to three different phases. So, then what would be the expression for instantaneous power? So, here the instantaneous power p_t will be equal to P a t plus P b t plus P c t, where P a t is basically the instantaneous power in phase a, P b t is representing the instantaneous power in phase b, P c t is representing the instantaneous power in phase c.




So, P a t is nothing but the multiplication of v_a(t) with i_a(t), p_b(t) is representing the multiplication of v_b(t) with i_b(t), p_c(t) is basically representing multiplication of v_c(t) I_c(t). So, if you put the expression of v a t and i a t and multiply and add with what you get with v b t i b t and v c t i c t, then you will get a result which is substantially different to the single phase circuit. What we will get? The result would be, I am not deriving all this thing, this is I am leaving to you, if you just do this mathematical derivation, then what we will get is, this is equal to V m I m by 2 cos phi. So, that is what is substantially different to what we get in a single phase circuit. So, what we get over here is that if we sum up the instantaneous power then we will get a expression which is time invariant.

$$p(t) = p_a(t) + p_b(t) + p_c(t)$$

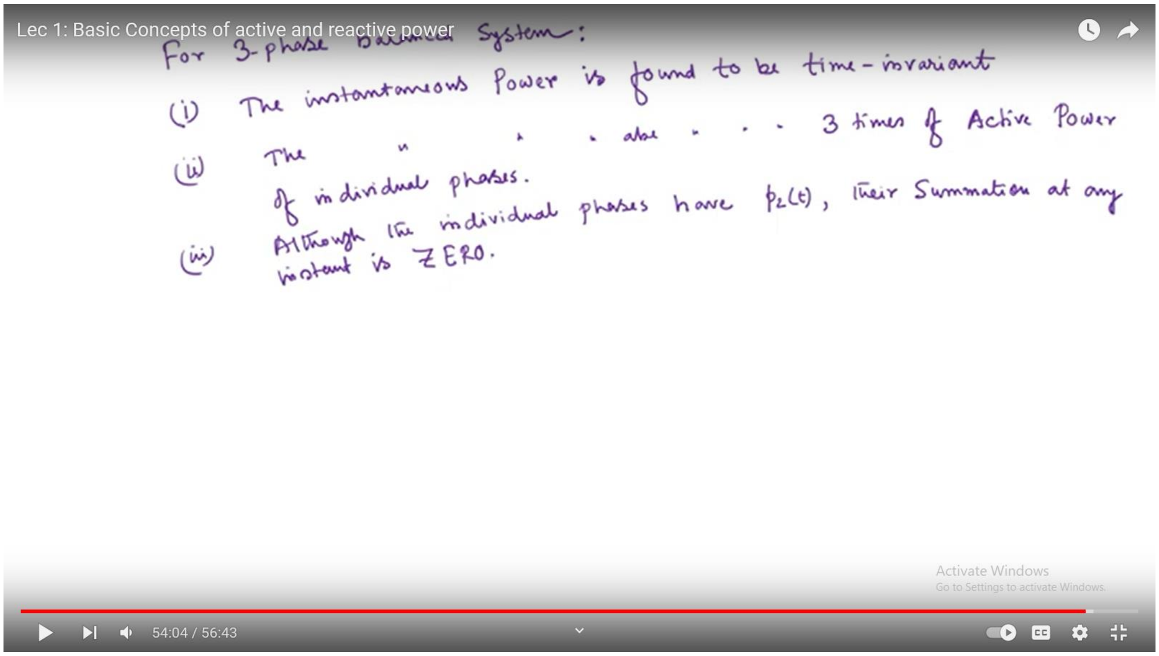
$$= v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t)$$

$$= V_m I_m[sin\ \omega t \sin(\omega t - \phi) + \sin(\omega t - 120^o) \sin(\omega t - \phi - 120^o) + \sin(\omega t - 240^o)\sin(\omega t - \phi - 240^o) = 1.5 V_m I_m cos\ \phi$$

So, this expression is time invariant that is what a substantial difference to the single phase circuit. Then, does it mean that we do not have the, you know, a similar type of definition like active and reactive power? No, the answer is not. So, what is this 3 V m I m by 2 cos phi? So, if we go back and see in a single-phase circuit, then we know that V m I m by 2 cos phi is basically P. So, if we consider that this is P, then this is nothing but 3 times of P.

So, 3 times of P. So, the question is one may understand that it is a 3-phase system. So, we will get power since all these phase voltages are identical and balanced and these loads are identical. So, overall power consumption is 3 times of the individual phase power consumption. That is somewhat logical to understand. But the question is where does this Q goes? Here, does it mean that we do not have the concept of reactive power here? Actually, it is not. So, what we can write as a remark for this particular system? So, for three-phase balance system, what will be the remark? the instantaneous power is found to be time invariant.

The instantaneous power is also found to be three times of active power of individual phases. So this is a great advantage rather in a three-phase system that even though you have that, you know, individual phases, these powers, instantaneous powers are pulsating in nature. Individual phases, the instantaneous power will pulsate similar to what we discussed in a single phase circuit. So, in spite of the individual phases having pulsating instantaneous power, when you sum up, you will get a constant quantity, that is an advantages. And that is, you know, why, you know, the three-page induction motor is one of the kind of robust and most rugged and efficient motor used in last 140 years since its invention, okay.




So, this is what the benefit, advantage, okay. The another thing is that although the individual phases have this P2T that is the pulsed sinusoidal you know the component which varying as a sinusoidal  So, individual phases have p to t, their summation at any instant is 0. you can see, you can go back and, you know, this, you know, find out this P1, you can just resolve this individual instantaneous power into P1 a t and P2 a t. Similarly, here P1 b t and P2 b t, here also P1 c t and P2 c t, what is the similar way I did in case of single-phase circuit. If you do, if you do it, then you are, you will see that P2  a and p to b and p to c their summation at any instant of time is 0. So although we have reactive power in the system but it is not appearing in the instantaneous power and instantaneous power is a constant quantity and that is an advantage.

So this is altogether our finding in this lecture. So if we summarize our finding here  that I can say that in a single phase circuit, we get two components of instantaneous power. One is pulsating in nature with some average value, either it can be 0 or it can be a positive value, but it never be negative. So, since it is converting this power to some useful work, we call it active power or sometimes real power. And we also have seen that in a single phase circuit, there is another component of instantaneous power which is varying like a sinusoid.

So, its average is obviously 0. But, its peak value depends upon the phase difference of the voltage and current. And obviously, that depends upon the types of load that we will be using. And that is basically also an important to us. Why it is important to us that I will explain in the next lecture.

And that is defined as a reactive power. reactive power is not defined as the average because average of P2 is basically 0, but its peak value is defined as the reactive power. It has some role in power system, it has some significant role in power system, which I am going to discuss in the next lecture, but we will define it as a reactive power. Whereas, when we consider a three-phase balance source and three-phase balance load, If we sum up the individual phase instantaneous power, what we have seen is we will get a constant quantity or time invariant quantity, which is nothing but 3 multiplied by individual phase active power consumption. And in individual phase, although we have reactive power consumption, if you sum up at any instant of time, it becomes 0.

So that is what we found from this today's lecture. And we will proceed this with the next lecture to understand what is the role of this reactive power then and why do we need this reactive power compensation. This is very important. So, this is all about the first lecture. Thank you for your attention. Thank you.