

Operation and Planning of Power Distribution Systems
Dr. Sanjib Ganguly
Department of Electronics and Electrical Engineering
Indian Institute of Technology, Guwahati

Lecture - 23
Capacitor placement at distribution feeder: Analytical approach

So, in my last lecture, I discussed the benefits of capacitor placement. I discussed two numerical problems just to illustrate you that what kind of benefits we can get with the capacitor placement ok. And at the end of my last lecture, I discussed that there are two types of planning processes for locating and sizing this capacitors or to determine the capacitor location and sizes.

One is you know that investment planning that is 1 step procedure that is basically to find out what are the optimal locations and sizes for capacitors by optimizing certain objectives; another is of course, that operational planning ok in which we are supposed to determine that how much compensation we require in a particular instant of a time in a day or particular duration of time in a day. We have seen that our load always varies and we can plot the variation of this load in a load curve ok.

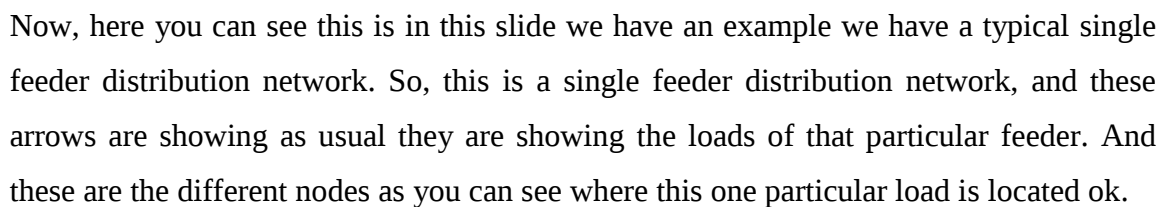
And that load curve can be 15 minutes load curve or 30 minutes load curve all these things I discussed. So, with this varying load demand with this time varying load demand our capacitor compensation needs to be varying ok. Otherwise, what could be the problem? Otherwise this capacitor compensation under very light load condition when load demand will be very low, but capacitive compensation is very high.

At this particular scenario this may cause harmful impacts to the network in a sense that this will cause over voltage in the network ok. How it will happen I will show you after a few minutes. Now, so in order to avoid that, we need to determine the time varying compensation that we require for throughout the day for a particular distribution network ok.

And this process is called operational planning process ok. Of course, this investment planning or the planning process in which we determine the sizes of the capacitor; this is usually done with the peak load demand data ok. That means we determine the capacitor

Now, this amount of capacitor size it is just for the sizing of the capacitor just to determine the optimal sizes of the capacitor. But that much of capacitor will not certainly kept throughout the day because this will lead to this; this procedure is called fixed compensation. So, if you fix the capacitor compensation throughout the day this will be a problematic when the load demand is low.

(Refer Slide Time: 04:53)



Now, without any capacitor how would be the voltage profile of that particular radial feeder. These things you have seen in my last module when I talk about this forward

backward sweep load flow and I had shown you the results of this forward backwards load flow forward backward sweep load flow.

And this result you can see I had shown the plots of the voltage magnitudes of individual test systems. There you can see that voltage will fall from starting from the substation to the end of this feeder ok. Now, this black the characteristics this characteristics is shown that this is the voltage profile of the network without any capacitor.

So, starting from this substation, voltage will gradually fall. And this how it will fall it may be linear or non-linear or whatever that depends upon this types of the loads and how they are located whether they are uniformly distributed or some amount is lumped or what.

So, but low; obviously, this voltage profile will be something like that it will fall starting from this substation voltage. So, this is our substation voltage and we keep usually this substation voltage as 1 per unit we call its 1 per unit ok. And if it is 1 per unit, you will see at the end of the feeder; that means, at this point of the network at this point of the network; that means, at the location where the distant load is located.

So, here you can see there is a; this is the distant node and at that particular node you will see the lowest voltage. Because in subsequent sections, you can see and subsequent lines you can see there will be voltage drops. So, here there will be voltage drop; here there will be voltage drop and so on. So, at the end of this feeder you will see the voltage profile is something like that and the voltage at the distant node is the lowest.

Now, there are two dotted lines which show this maximum allowable voltage and minimum allowable voltage. So, you can see this is supposing the minimum allowable voltage of the network. We call let us say it is, we call that 10 percent voltage drop is acceptable so minimum allowable voltage drop is here is 0.9 per unit ok. And this is the substation voltage which is 1 per unit.

So, you can see that at the distance node this voltage is lower than the minimum allowable voltage. So, this voltage is lower than the minimum allowable voltage ok. That might be any value lower than 0.9 that might be 0.85, 0.87 or whatever.

But at that node the customer who are connected at that node which is the distant node of that particular feeder they will experience under voltage they will experience under voltage ok. So, this is basically drawn, this voltage profile at rated load at or at peak load.

So, this is the voltage profile at peak load demand ok. Rated load or peak load whatever you can call it. So, this is the voltage profile this one is the voltage profile at the peak load. Now, suppose in order to alleviate the situation that some of the nodes may suffer from this under voltage. So, we can place a capacitor somewhere in this network, and it is arbitrarily chosen at let us say point A, we will put a capacitor ok.

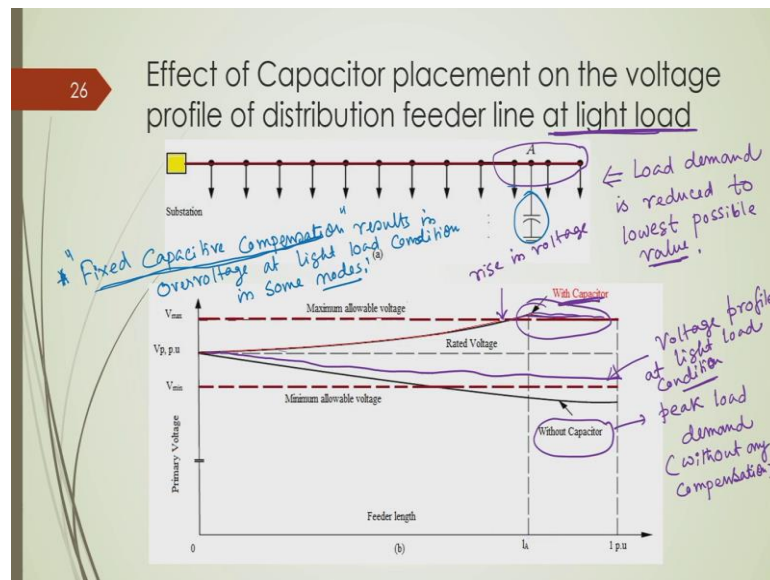
And we also select the size of the capacitor such that this voltage profile will be shifted at this point shifted like this and it will bring this node which was suffering from under voltage problem without any capacitive compensation. That means, this is shifted to this, the voltage of that particular node is improved or increased to that value which is within which is even higher than the allowable voltage.

So, I can suppose initially that voltage was let us say 0.85 per unit and with this capacitor placement it increased to 0.92 per unit. So, I can say that with this capacitor placement I improved this voltage of that particular node, which was suffering from under voltage problem. And that is what the purpose of the capacitor ok.

Now, what could happen if this load is reduced to some lower value than this peak load or load is reduced to the least value throughout the day? Because, as you know, load will vary throughout the day. So, this capacitor size and maybe its location also is decided, the size and location these are decided based upon this peak load demand ok.

Now, what could happen that if we keep this capacitive compensation same at any other loading point other than this peak load demand?

(Refer Slide Time: 12:19)



So, let us see another scenario. So, here everything is same; only thing is that the load demand is reduced to the light load. So, here as compared to the previous scenario everything is same but load demand is reduced to lowest possible value; lowest possible value.

So, under this scenario this you know. So, this was the voltage profile of this network at peak load; at peak load demand without capacitor or without any compensation; without any compensation. And with at this light load condition this voltage profile will be of course, shifted upward like this voltage will continuously declining and it will be like that let us say.

So, this is the voltage profile at light load condition ok. This is also without compensation. Now, with this capacitor; that means, by keeping this capacitor compensation fixed which we obtained from this previous scenario that is when this load demand to us at peak and at that time what could be the capacitive compensation.

So, that we can bring out those nodes from the problem of those nodes were suffering from this under voltage problem from the under voltage problem ok. Now, if we do so then and if we keep that same amount of compensation even at this light load condition the characteristic or voltage profile would be something like that.

So, here you can see there is a rise in voltage. So, this shows rise in voltage above the substation voltage for some of the nodes. So, you can see here some of the voltages, I mean some of the nodes who are nearby this capacitor who will experience this over voltage problem ok, due to this fixed capacitor compensation; that is why capacitive compensation cannot be fixed throughout the day.

So, this gives a lesson that fixed capacitive compensation results in over voltage at light load condition light load condition in some nodes.

So, some of the customers who are located near to the capacitor who are experiencing in fact, under voltage previously during peak load condition, we will see that they are suffering from an over voltage condition due to the fixed capacitor compensation. And therefore, this fixed capacitor compensation cannot be used throughout the day.

And we need a varying capacitive compensation; we need a varying capacitive compensation.

(Refer Slide Time: 16:57)

27

Effect of Capacitor placement on the voltage profile of distribution feeder line

- If only fixed-type capacitors are installed, there will be voltage rise problem during light load conditions.
- Therefore, some of the capacitors are installed as switched capacitor banks so they can be switched off during light-load conditions.
- Thus, the fixed capacitors are sized so that their presence would not cause voltage rise at light loading.

So, this is what I exactly trying to make you understand that only fixed capacitive compensation or only fixed type capacitor installment will lead to the voltage rise problem during light load condition ok. So, therefore, what the utilities used to follow that they keep some of the capacitors as switched capacitor. Because you know in you

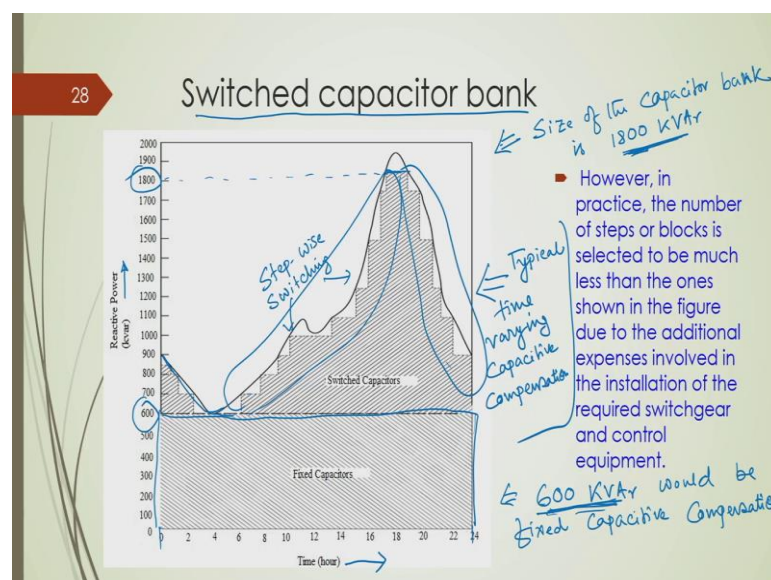
know capacitor banks there are many capacitors which are in series parallel combination which are connected in a series parallel combination.

And some of the capacitors are kept as switched capacitor, it means that they will be used only if these are really required or whenever there is a peak load condition ok. So, they will be switched during that and they will be switched off during light load condition. So, that at light load condition this degree of capacitive compensation is capacitive compensation becomes low ok.

And there would be of course, some fixed capacitors who will be present throughout the day ok. Because as you know capacitor switching is itself a problematic because it will cause transients in the network, it may cause over voltage ok. So, switching of the capacitor is something which the utilities try to avoid, but even if we keep that all capacitor fixed that is also a problematic in the sense that during light load condition some of the consumers or some of the loads will experience over voltage.

So, we have to have a trade off in these two situations that we have to keep some capacitors fixed capacitor switch able, but we need to minimize the switching as much as possible ok.

(Refer Slide Time: 19:15)



And this is a typical you know switching or typical you know time varying capacitive compensation. So, this gives a typical time varying capacitive compensation and this

characteristics is drawn by a in this axis this reactive power the capacitor is injected to the network and this is with respect to time. And this time starts from 12 o'clock at the midnight and ends at the 12 o'clock at the next day midnight.

So, you see that you know that during this period when load is very light up to this period, you can see that only some fixed capacitive compensation will be there throughout the day and; that means, up to 600 kVAr. So, this shows that up to 600 kVAr there would be fixed capacitive compensation. Then, this capacitor size is something like 1800 kVAr. So, the size of the capacitor bank size of the capacitor bank is 1800 kVAr, this is an indicative. This does not mean that it should be similar for every network or this is the common practice for all the utilities.

So, actually utilities follow this practice that some of the capacitors are kept fixed and some of the capacitors are switchable. So, you can see that even though that capacitor bank size is 1800 kVAr, out of which only 30 percent i.e., one-third of that capacitor are kept fixed ok.

And this is to compensate this base load condition and rest are switchable ok. And you can see that switching is done in step wise. So, this is step wise switching of the variable capacitor or switched capacitor. And during this peak load time, all the capacitors are brought into this service, and that time this compensation rises to 1800 kVAr.

But when this load is low or low then accordingly utility will decide that how much capacitive compensation will require. So, this type of step wise switching characteristics is usually determined through an optimization process and this is called operational planning ok.

(Refer Slide Time: 23:06)

29

Capacitor sizing

- Many utilities apply the following rule of thumb to determine the size of the switched capacitors:

$$\frac{\text{kVAr from (switched + fixed) capacitors}}{\text{kVAr of peak reactive feeder load}} \geq 0.70$$

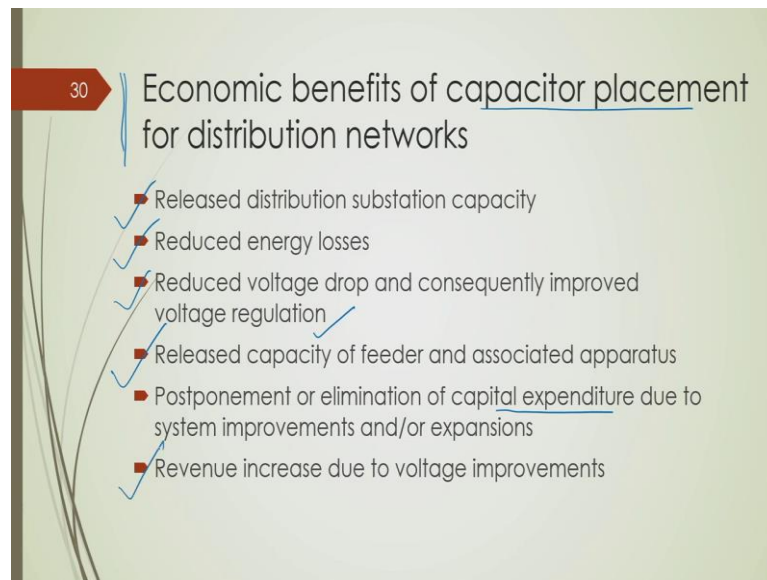
Some utilities use the following rule of thumb: *The total amount of fixed and switched capacitors for a feeder is the amount necessary to raise the receiving-end feeder voltage to maximum at 50% of the peak feeder load.*

Ref. T. Gonen. Electric Power Distribution System Engineering

So, generally many utilities follow a thumb rule that kVAr that they are in order to decide the size of the capacitors and here they used that up to 70 percent of this total feeder reactive load or reactive power demand would be compensated by a capacitor bank ok.

And that in that capacitor bank, of course, some percentage will be switched capacitor and some percentage will be fixed capacitor ok. And some utilities this is a thumb rule that does not mean that every time it should be 70 percent only, but it could be any value depending upon the network structure, depending upon the economy of the system and depending upon many factors ok.

(Refer Slide Time: 24:03)



Now, what are the economic benefits that we will get in a capacitor placement in distribution network ok? So, as I have explained in one of the problems in last lecture that due to the placement of this capacitor bank, if you can remember this numerical problem, then this generator can be loaded additionally 150 kilowatt ok.

So, this is something was shown in my last lecture ok. So, it means that due to this capacitor placement and due to this local reactive power compensation some amount of capacity of the generator is released ok.

So, same thing is applicable for other, if generator capacity is released; so as the capacity of the transmission line so as the capacity of the sub transmission line so as the capacitive of the substation. This is the hierarchical process and all are the sequential block of this power flow.

So, due to this capacitor placement we get released substation capacity, we get reduced energy loss, this is already explained because power loss is reduced. Also it reduces this voltage drop and consequently improve this voltage profile or voltage regulation.

Also it released the capacity of feeder and associated apparatus; and it needs deferral of this capital expenditure due to this up gradation of the system due to reinforcement of the system. And also this will increase the revenue because power loss is reduced; energy loss is also subsequently reduced ok.

(Refer Slide Time: 26:03)

31 Total financial benefits of capacitor placement

Therefore, the total benefits due to the installation of capacitor banks can be summarized as:

$$\sum \Delta\$ = (\text{Demand reduction}) + (\text{Energy reduction}) + (\text{Revenue increase})$$
$$= (\Delta\$_G + \Delta\$_T + \Delta\$_S + \Delta\$_F) + \Delta\$_{ACE} + \Delta\$_{BEC}$$

$\Delta\$_G$ is the released generation capacity beyond maximum generation capacity at original power factor, kVA
 $\Delta\$_T$ is the released transmission capacity beyond maximum transmission capacity at original power factor, kVA
 $\Delta\$_S$ is the released distribution substation capacity beyond maximum substation capacity at original power factor, kVA
 $\Delta\$_F$ is the annual benefits due to released feeder capacity, \$/year

Now, in many of the approaches optimization approaches as I mentioned in the last lecture, a suitable objective function formulation is done ok. In order to determine that what could be the possible size of the capacitor where we locate those capacitor ok. And in order to formulate this objective function one typical objective for some formulation is shown over here.

This objective shows that whatever benefits we are getting due to this capacitor placement one is demand reduction, one is energy reduction, another is increase in revenue they are aggregated and a composite objective function is formulated ok.

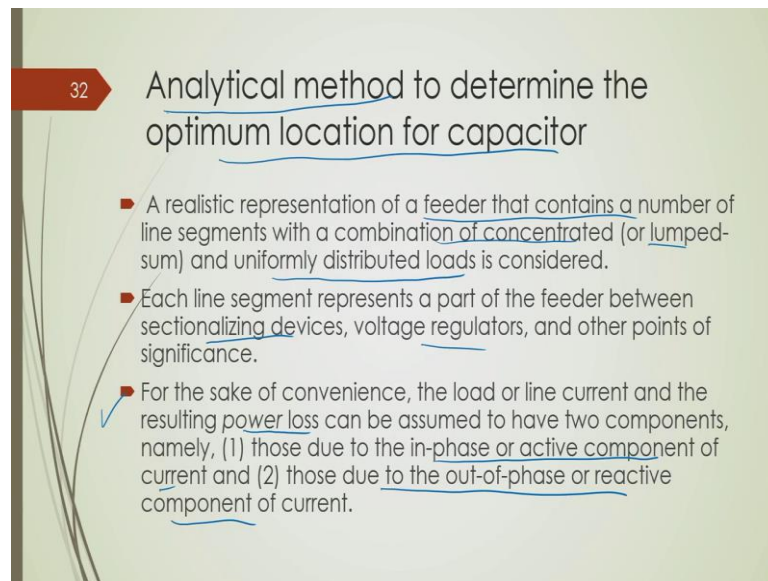
So, under this demand reduction you can see here we have this four components which are basically released capacities of the individual stakeholder hierarchical stakeholders which include generated or generation capacity, which include this transmission capacity distribution substation capacity and this feeder capacity ok. So, this is the release of the capacity of individual stakeholder.

Similarly this energy reduction is another component of this objective function and revenue increase could be another component. So, this is a composite objective function and accordingly one can optimize that objective so that it can decide that how much capacitive compensation it will provide or what should be the size of the capacitor; capacitor bank.

Again why I am calling it this is a capacitor bank; because as although in the figure it is shown a single capacitor although it is in the figure, but actually it is a capacitor bank where we have a multiple number of capacitors connected in series parallel fashion.

And they are connected to the switches for the switchable capacitor and for the fixed capacitor they are directly connected and they kept into service throughout the day throughout the year.

(Refer Slide Time: 28:45)



32 Analytical method to determine the optimum location for capacitor

- A realistic representation of a feeder that contains a number of line segments with a combination of concentrated (or lumped-sum) and uniformly distributed loads is considered.
- Each line segment represents a part of the feeder between sectionalizing devices, voltage regulators, and other points of significance.
- For the sake of convenience, the load or line current and the resulting power loss can be assumed to have two components, namely, (1) those due to the in-phase or active component of current and (2) those due to the out-of-phase or reactive component of current.

Now, in next few slides or the end part or the last part of this module I will discuss an analytical approach to determine this optimal location of the capacitor. I will discuss an analytical approach. As I said this determination of optimal location of the capacitor and the size of the capacitor, and also how many number of capacitor banks that I should install in a particular distribution feeder. Those are normally usually determined by formulating an optimization problem and by solving it ok.

But this approach analytical approach will guide you or will improve or will give you some more insights so that you can utilize this knowledge on further research and analysis ok. So, here this analytical approach is based upon certain assumption which I will mention time to time ok.

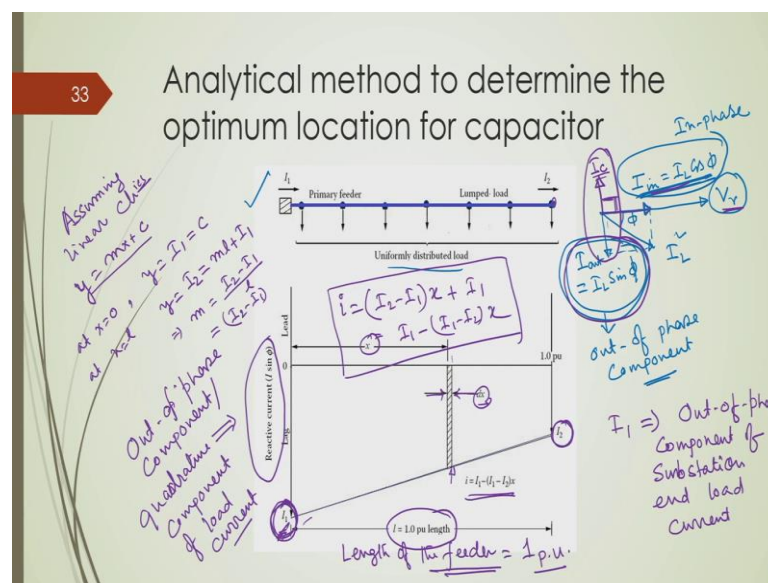
So, here we consider a distribution feeder that contains a number of line segments and with a combination of lumped load or concentrated load and uniformly distributed loads

ok. So, here we assume that there are some loads in a particular feeder which are lumped and there are some loads which are uniformly distributed ok. And also in that different line segment of the feeder we have sectionalizing switches we have many other compensation devices.

So, for the sake of the convenience we assume that this power loss can be represented with this in phase component of the current and quadrature or out of phase component of the current, which I already mentioned in my first lecture of this module. So, any current it has two components one component which is in phase of that voltage; that means, if your capacitor or any load is connected at the one particular side of a network.

So, across this load whatever the voltage will be there; so in phase component of the current with that voltage is called in phase component and quadrature component will be the out of phase component ok.

(Refer Slide Time: 31:24)



So, for example, so I can show you suppose this is that receiving end and its lumped load and although this feeder is not a lumped load completely; so we have some lumped load and some uniformly distributed load ok. Now, for a lumped load case which I explained at the very beginning how can we represent this voltage across this load and current drawn by this load.

So, this is suppose voltage across this load and this is suppose this current which is drawn by this load I_L . So, this I_L will have two components one is in phase component that is this I or I in another is out of phase component or quadrature component that is I out ok.

So, this I is basically representing in phase component of the current and I out is basically representing the quadrature component of this load current which is connected at a point where this voltage is V r ok. Now, here this in phase component, since this current is you know lagging with respect to voltage because all loads currents are lagging only because loads are usually inductive in nature.

So, if we consider this phase angle of this current is ϕ so I is basically equal to $I_L \cos \phi$. So, this is the in phase component of the current and I out is equal to $I_L \sin \phi$. So, this is quadrature component of the current. So, this is quadrature component and this is in phase component and this is out of phase component.

Now, here we are concerned about this out of phase component only. Why it is so? I will come to that next slide I will explain ok. We will not consider this in phase component of the current because we are basically compensating this out of phase component of current through this capacitor installation.

Now, if you install a capacitor at this particular loading end suppose this capacitor is drawing a current which is leading and this is I_C then you can see it does not have any influence of this in phase component of the current; rather it will compensate this out of phase component of the current.

Because this out of phase and this they are capacitive current they are cancelling each other ok. And this I_C will be capacitor current which will be quadrature with this supply voltage assuming that I capacitor bank is fully ideal. And therefore, it does not have any influence on this reduction of this in phase current ok.

So, this part is the plot of this reactive current which is also called quadrature current or out of phase current. So, this is out of phase component or quadrature component of load current alright. Now, we are concerned about only this out of phase or quadrature component of current which can be compensated with this capacitor placement or with the capacitive current.

So, this I_1 stands for out of phase component or quadrature component of the current at the substation end; so this I_1 is substation at quadrature component of substation end load current.

And this I_2 is basically this quadrature or out of phase component of the load current at the distant load of this particular feeder ok. And we are assuming that this I as you know this current will vary throughout this length and we are assuming that this length of this distribution feeder is 1 per unit. So, we are assuming the length of the feeder is equal to 1 per unit alright.

Now, you can see the substation end; obviously, this load current will be higher so its quadrature component would be higher, and at the distant end load current will be at the least value, and the so as the quadrature component ok. And we can understand this how we can find out this characteristics which shows that this variation of the current; we assume that it is a linear characteristics.

So, assuming linear characteristics, we consider that this I is following this y is equal to $m x$ plus c ok. So, at x is equal to 0, this y is equal to you know that I_1 and at x is equal to 1 this y is equal to this I_2 . So, at x is equal to 0, y is equal to I_1 . So, as this c because if you put this x is equal to 0 you will see y is equal to c which is equal to I_1 .

So, c is equal to I_1 and at x is equal to 1, y is equal to I_2 . So, this is equal to your $m l$ plus c ; c is our I_1 . So, m is equal to I_2 minus I_1 by l , here we consider l is equal to 1 per unit so it is equal to I_2 minus I_1 that is; so I can put it here. So, this current I is equal to m is equal to your, I_2 minus I_1 multiplied by x , x is written over here and plus this I_1 . Or alternatively we can write it as this is equal to I_1 minus I_2 multiplied by x ok.

So, this is how this quadrature component of the load current will vary from this substation end to the distant node ok.

(Refer Slide Time: 39:07)

34

Analytical method to determine the optimum location for capacitor

- Assume that the power losses are caused by a lagging line current I flowing through the circuit resistance R . Therefore, it can be shown that:

$$I^2 R = \underbrace{(I \cos \phi)^2 R}_{I_{in}} + \underbrace{(I \sin \phi)^2 R}_{I_{out}}$$

Power loss without capacitor bank
- After adding a shunt capacitor with current I_c , the resultants are a new line current I_1 and a new power loss. Hence,

$$I_1^2 R = \underbrace{(I \cos \phi)^2 R}_{I_{in}} + \underbrace{(I \sin \phi - I_c)^2 R}_{I_{out}}$$

Power loss with capacitor bank

Now, here you can understand that why we are taking only the quadrature component of the current. Because you can see that, I can represent this line losses as $I^2 R$ losses. So, $I^2 R$ can be resolved into two components. So, this is in phase component $I \cos \phi$ and this is quadrature component $I \sin \phi$ ok. Similarly with this adding capacitor we can have a current I_c ok.

So, what would be the power line losses? So, this is, suppose, represented by $I^2 R$ ok. So, where this is your again this I in that is in phase component of the load current and this is out of phase component of the load current, but with this will be subtracted this I_c ; because I_c and this out of phase component or quadrature component they will cancel out each other, and the net amount of this quadrature component would be $I \sin \phi$ minus I_c ok. So, this is something like that.

Now, if we consider that this is the power loss without a capacitor and this is power loss with capacitor. Capacitor means capacitor bank you can understand, capacitor bank ok. Now, if we take the difference of these two, one is power loss with when there was no capacitor bank and this is the power loss when we have a capacitor bank installed.

(Refer Slide Time: 41:09)

35

Analytical method to determine the optimum location for capacitor

- Therefore, the loss reduction as a result of the capacitor addition can be found as:
$$\Delta P_{LS} = I^2 R - I_1^2 R$$
$$\Delta P_{LS} = 2(I \sin \phi) I_C R - I_C^2 R$$
- Thus, only the out-of-phase or reactive component of line current, that is, $I \sin \theta$, should be taken into account for power loss reduction as a result of a capacitor addition.
$$\Delta P_{LS} = f(I_{out})$$

If we take the difference of this and suppose this difference is represented by ΔP_{LS} where ΔP_{LS} is basically representing this loss reduction due to this capacitor placement. So, if you take the difference of these two, then this I in phase component of this power loss would be cancelled out. Because there will be no improvement of this in phase of this power loss which occurs due to this in phase component of the load current ok.

But only in this difference of this power loss term, we will have this quadrature component; that means, this loss reduction, suppose it is represented by ΔP_{LS} it is only function of the quadrature component. So, we can write over here, only out of phase or reactive component of the line current that is $I \sin \phi$ should be taken account for the power loss reduction.

Because power loss reduction that is ΔP_{LS} is only function of this I_{out} or quadrature component of the current and that is why only quadrature component of the current is considered in the further studies.

(Refer Slide Time: 42:29)

36

Analytical method to determine the optimum location for capacitor

- Assume that the length of a feeder segment is 1.0 pu length, as shown in Figure. The current profile of the line current at any given point on the feeder is a function of the distance of that point from the beginning of the feeder. Therefore, the differential power loss of a dx differential segment located at a distance x can be expressed as:

$$dP_{LS} = 3[I_1 - (I_1 - I_2)x]^2 R dx$$

Now, since we have a 3 phase system so, this we represent this ΔP_{LS} that is differential power loss at a distance x having a differential segment with the length of dx located at the x distance away from the substation is represented by this equation ok. So, here we take a small different infinitely small length of dx at a distance x away from the substation ok. And we are trying to determine that what should be the power loss at this infinitely small line segment ok.

(Refer Slide Time: 43:25)

37

Analytical method to determine the optimum location for capacitor

- Therefore, the total power loss of the feeder can be found as:

$$P_{LS} = \int_{x=0}^{1.0} dP_{LS} = 3 \int_{x=0}^{1.0} [I_1 - (I_1 - I_2)x]^2 R dx$$

$$= (I_1^2 + I_1 I_2 + I_2^2) R$$

where,

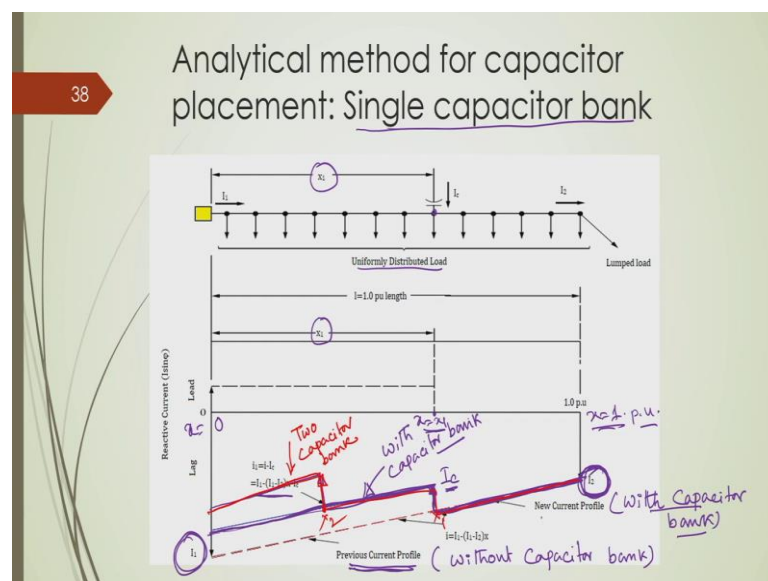
- P_{LS} is the total PR loss of the feeder before adding the capacitor
- I_1 is the reactive current at the beginning of the feeder segment (at Substation)
- I_2 is the reactive current at the end of the feeder segment
- R is the total resistance of the feeder segment
- x is the per unit distance from the beginning of the feeder segment

Then we will integrate this power loss and we integrate it with this varying x is equal to 0 to 1 where x is equal to 0 corresponds to the sending end side and x is equal to 1 correspond to the end of the feeder. So, if you integrate that then it will give the overall power loss of the feeder or power loss of the feeder.

And that is if you do this integration with by and then if you put this limit x is equal to 0 to 1 then you will get this is coming out to be that ok. Where I_1 is basically reactive current or out of phase current or quadrature current which is out of phase or quadrature component of the load current at the beginning of the feeder segment; that means, at substation or near to the substation.

Similarly, I_2 is out of phase component or reactive component or you can call it as quadrature component of the load current at the end of the feeder segment. And R is the resistance of the feeder segment and X is the reactance, X is not applicable of course here ok.

(Refer Slide Time: 44:43)



Now, we will see that if we put a single capacitor bank at some point which is X_1 distant away from the substation; which is X_1 distant away from the substation. How our reactive current component will change? So, this red dotted line was this previous current profile; that means, current profile without capacitor bank.

So, what would be the change if you put a capacitor bank here it will draw a capacitive current I_c and this I_c will be opposite to this out of phase component of current. So, it will lead I_c current here ok. So, this is I_c current and due to that this previously up from this capacitive component to this you know up to this point this characteristic of the new current profile which is with capacitor bank.

So, this characteristic is with capacitor bank, it will follow this whole profile ok. But with this I_c current, the new current characteristics will be this. So, here this is the new current profile which is following a new current profile with this capacitor placement.

So, due to this capacitive compensation, reactive current is reduced to an I_c amount from here to the substation end and that is what the advantage of this capacitor placement you have seen, I explained at the very beginning that with this capacitor placement at some point that much of reactive current or that much of reactive power we need not to bring from the substation ok. And that is why from here to here this reactive power flow will also be reduced ok.

How much it will be reduced that much the capacitor is providing or injecting reactive power to the network ok? And this will be our new profile, so this will be with capacitor bank characteristics; with capacitor bank.

(Refer Slide Time: 47:22)

39

Analytical method for capacitor placement: Single capacitor bank

- The insertion of one capacitor bank on the primary feeder causes a break in the continuity of the reactive load profile, modifies the reactive current profile, and consequently reduces the loss, as shown in Figure.
- Therefore, the loss equation after adding one capacitor bank can be found as before:

$$P'_{LS} = 3 \int_{x=0}^{x_1} [I_1 - (I_1 - I_2)x - I_c]^2 R dx + 3 \int_{x=x_1}^{1.0} [I_1 - (I_1 - I_2)x]^2 R dx$$

with capacitor bank
without capacitor bank

$$P'_{LS} = (I_1^2 + I_1 I_2 + I_2^2) + 3x_1[(x_1 - 2)I_1 I_c - x_1 I_2 I_c + I_c^2] R \quad \Leftarrow$$

Now, what we will do this is our characteristics of current reactive power characteristics which is from 0 to x, x is equal to x 1; and this is our characteristics which is from x is equal to x 1 to x is equal to 1. That is the length of the feeder that is we considered as 1 per unit ok.

So, we will have two current characteristics; one is here to here another is here to here. So, in order to determine this power loss, we need to consider these two characteristics separately so which is done over this here. So, from x is equal to 0 to x 1, we will follow this characteristics which is with capacitor this is with capacitor bank, and from x is equal to x 1 to x where x is equal to 1, we will follow with old characteristics that is without capacitor bank.

And we integrate this as we did previously we will get P LA dash is equal to that expression, if you do this integration you will get that characteristics. So, this part will be as usual previous expression that is this part and this part is the additional expression.

(Refer Slide Time: 48:53)

40 Analytical method for capacitor placement: Single capacitor bank

Thus, the per unit power loss reduction as a result of adding one capacitor bank can be found from:

$$\Delta P_{LS} = \frac{P_{LS} - P'_{LS}}{P_{LS}} = \frac{\text{Power loss reduction due to capacitor placement}}{\text{Power loss without capacitor}}$$

$$\Delta P_{LS} = \frac{-3x_1[(x_1 - 2)I_1 I_c - x_1 I_2 I_c + I_c^2]R}{(I_1^2 + I_1 I_2 + I_2^2)R}$$

$$\Delta P_{LS} = \frac{3x_1}{1 + (I_2/I_1) + (I_2/I_1)^2} \left[(2 - x_1) \left(\frac{I_c}{I_1} \right) + x_1 \left(\frac{I_2}{I_1} \right) \left(\frac{I_c}{I_1} \right) - \left(\frac{I_c}{I_1} \right)^2 \right]$$

$$= f \left(\left(\frac{I_2}{I_1} \right), \left(\frac{I_c}{I_1} \right), x_1 \right)$$

Now, what we will do we will take this difference of this P LS dash which is that power loss due to the capacitor bank placement at distance x 1 from the substation, another is power loss without this capacitor bank without this capacitor bank.

And we will take this ratio of this power loss reduction with respect to this power loss without this capacitor bank. So, this is basically you can write it as a ratio of power loss

reduction due to capacitor placement with respect to power loss without capacitor so this is the ratio.

And you can see that this part is basically eliminated, because this is common and this part will remain ok. And this is divided by this power loss without capacitor. So, if you simplify this, then you will get ΔP_{LS} is coming out to be this expression ok. So, you can verify whether it is true or not. So, here we purposefully make two ratios one is I_2 by I_1 another is I_C by I_1 .

So, we get two ratios here and this ΔP_{LS} is brought out to be function of these two ratios. And of course, this will be the function of x_1 as well where x_1 is the location where we put this capacitor ok.

(Refer Slide Time: 50:56)

41

Analytical method for capacitor placement: Single capacitor bank

- ▶ If c is defined as the ratio of the capacitive kVA (kVA) of the capacitor bank to the total reactive load, that is,

$$c = \frac{\text{kVA of capacitor installed}}{\text{Total reactive load}}, \text{ then } c = \frac{I_C}{I_1}$$

c = degree of Capacitive Gouphard
- ▶ If λ is defined as the ratio of the reactive current at the end of the line segment to the reactive current at the beginning of the line segment, that is,

$$\lambda = \frac{\text{Reactive current at the end of line segment}}{\text{Reactive current at the beginning of line segment}}, \text{ then } \lambda = \frac{I_2}{I_1}$$

$\lambda = 0 \Rightarrow \frac{I_2 = 0}{I_1}$
 $\lambda = 1 \Rightarrow \frac{I_2 = I_1}{I_1}$

Now, why we put these two ratios what is the significance? Because you see that, this ratio I_C by I_1 is named, it is represented by a variable c small c and it is representing that kVA of the capacitor bank to the reactive load of the system, Where I_1 is the total reactive load of the system because it is the reactive component of the load current at near to the substation.

So, this small c is the ratio of the capacitive kVAr to the total reactive load ok. Of course, it is reactive load because it is only the out of phase component of the load current. And this ratio I_2 by I_1 where you can remember this I_2 is basically reactive current or out of

phase component of the current at the end node, and I_1 is the reactive current or out of phase component of the current or quadrature component of the current at the substation end ok.

So, if you take this ratio what it will signify? This ratio is represented with a variable lambda and it is representing the ratio of reactive current at the end of the line segment to the reactive current at the beginning of the line segment ok.

(Refer Slide Time: 52:33)

42

Analytical method for capacitor placement: Single capacitor bank

- Therefore, the per unit power loss reduction can be found as:

$$\Delta P_{LS} = \frac{3cx_1}{1+\lambda+\lambda^2} [(2-x_1) + \lambda x_1 - c]$$
- where x_1 is the per unit distance of the capacitor-bank location from the beginning of the feeder segment (between 0 and 1.0 p.u.).

If α is defined as the reciprocal of $1+\lambda+\lambda^2$, that is $\alpha = \frac{1}{1+\lambda+\lambda^2}$

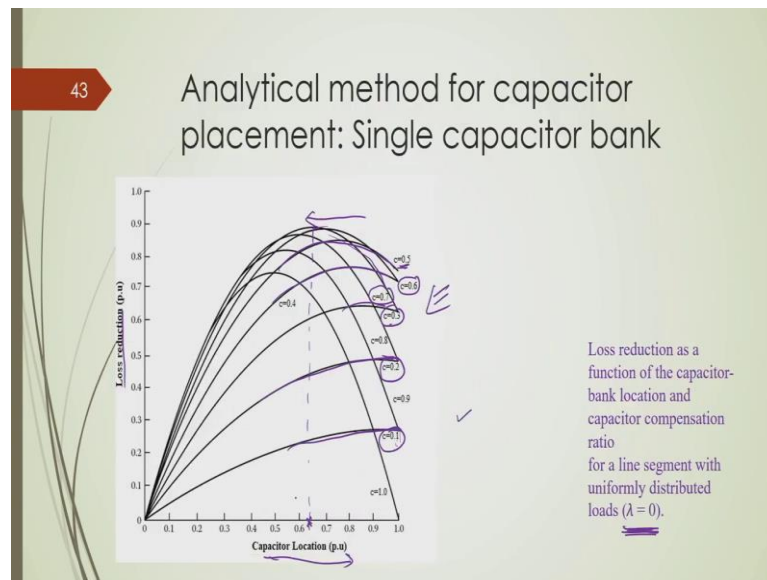
$$\Delta P_{LS} = 3\alpha cx_1 [(2-x_1) + \lambda x_1 - c]$$

$\lambda=0, \alpha=1$

$= f(\lambda, c, x_1)$

So, it has some significance I will come to that. Now, we represent this ΔP_{LS} that is per unit loss reduction as a function of these two ratios that is c and λ and we get this ok. And again we represent one upon this 1 upon 1 plus λ plus λ square is equal to variable α , and we further deduce this expression like this, which is representing per unit loss reduction ok.

(Refer Slide Time: 53:05)



Now, what we will be doing, we will plot this characteristics by varying this lambda and c, here we have as you can see this is function of lambda c as well as x 1. So, we will vary this lambda and c and plot these characteristics alright.

Because rest of this thing even alpha is also function of lambda and so this expression is of only function of a lambda and c. Now, by varying lambda and c we will vary the characteristics. So, here what we will do, we consider a single capacitor bank because this expression is applicable only for a single capacitor bank placement at a distance x 1 from the substation.

Now, if we consider lambda is equal to 0 and if we plot c then what could be the characteristics? So, that characteristic is plotted over here ok. Now, what this lambda is equal to, So lambda is equal to 0 means if you look at lambda is equal to 0 means correspond to I 2 is equal to 0.

That means, this reactive component of the current or out of phase component of the load current at the end of this feeder segment is 0. When it is possible? When this load is only uniformly distributed ok, so, this I explained in module 2, and you can understand. So that means, lambda is equal to 0 which corresponds to the fact that line loads are of uniformly distributed. Now, at that particular condition if we vary c ok, if we vary c then how would be the variation of per unit loss reduction ok.

So, here you can see, this correspond to c is equal to 0.1; this corresponds to c is equal to 0.2; and then c is equal to 0.3, c is equal to 0.4 here, c is equal to 0.5. So, if we increase c then this loss reduction is getting increased.

And now c is our degree of capacitive compensation, because c represent this ratio of capacitive kVAR to the load reactive kVAR; so c is basically representing degree of capacitive compensation ok. So, if c is equal to 0; that means, it is the case when there is no compensation, and if c is equal to 1; that means, you are providing 100 percent compensation; that means, hundred percent reactive power is compensated ok.

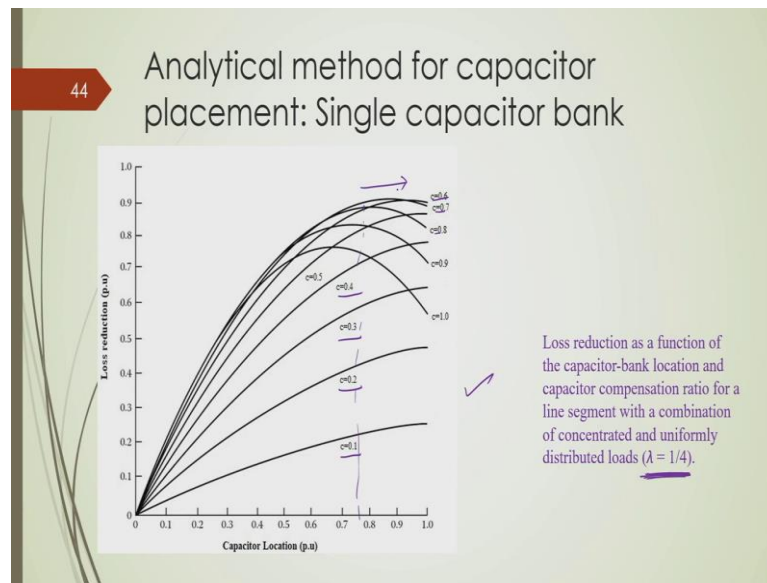
So, if c is increased that loss reduction will also increase up to a certain value when c is equal to 0.5 then you can see that peak loss reduction; after c is equal to 0.5 or c is equal to 0.6 will not change rather only this characteristics means the location where this peak loss reduction will take place is shifted to that side ok.

So, at c is equal to 0.1, so at the end of this feeder, if we put a capacitor then we will get highest amount of loss reduction. Similarly, at c is equal to 0.2 it is also near to the end of the feeder if we put a capacitor bank we will get maximum loss reduction.

But and this holds for c is equal to 0.3, c is equal to 0.4, c is equal to 0.5 and so but when c is equal to 0.6, c is equal to 0.7 then you can see this peak loss reduction is almost same, but where if you put this because this x is representing your capacitor location x 1;

So, that means, higher value of c means that this you should place this capacitor much before this end of the feeder; much before the end of the feeder. For example, at c is equal to 0.7, you can see this is the characteristics. So, that means if you put capacitor at a location which is 60 percent away from the substation or 65 percent away from the substation we will get the peak loss reduction ok.

(Refer Slide Time: 58:24)

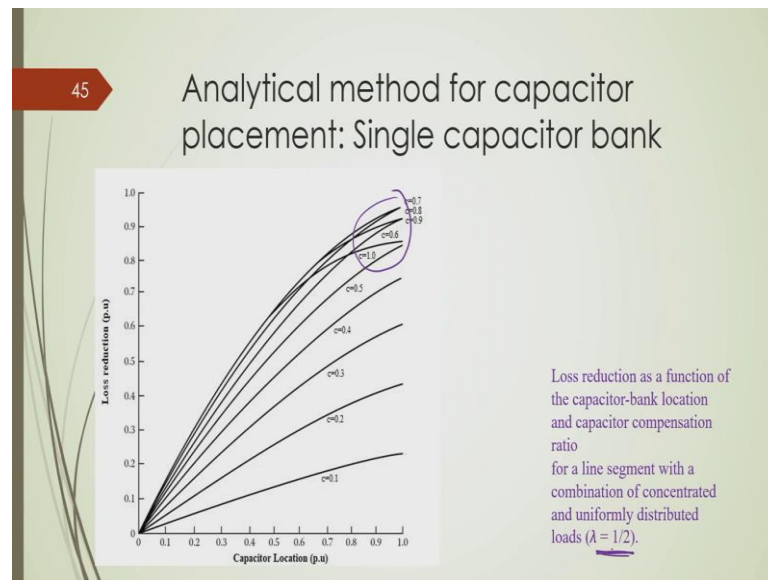


Now, next characteristics correspond to λ is equal to 1 by 4. So, when λ is increasing starting from 0 to any value; that means you have certain amount of loads which are lumped load and certain amount loads are uniformly distributed and that is why λ is non-zero and it means that out of all this, certain loads are of lumped and certain loads are uniformly distributed.

Under this how would be the change in the characteristics you can see if you increase c , this peak loss reduction is increasing and only thing is that at c is equal to 0.6 and 0.7 and 0.8, this location of the capacitor is not at the end of the feeder; it should be in between 70 percent length of the feeder or at 80 percent length of the feeder.

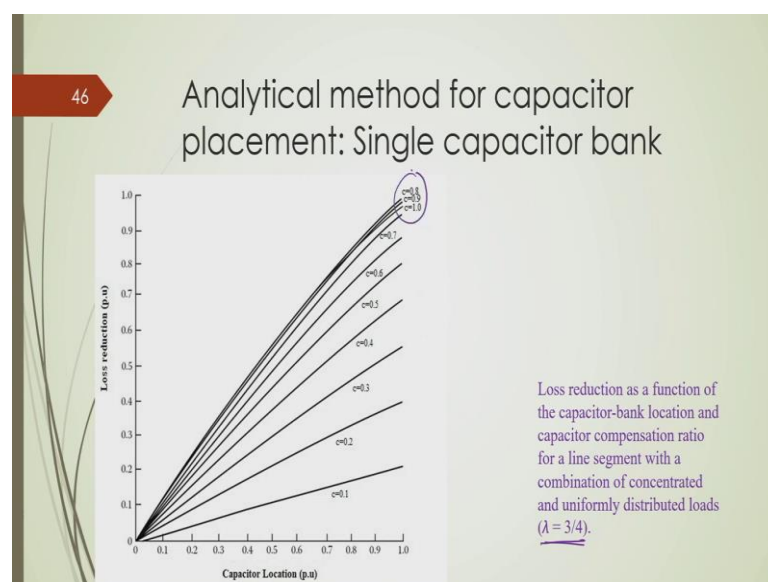
So, characteristics is similar to the previous thing only, thing is that you can see if you compare these characteristics with previous characteristics, the location of the capacitor is getting changed ok. So, location of capacitor for peak loss reduction is towards the end of the feeder ok.

(Refer Slide Time: 59:50)



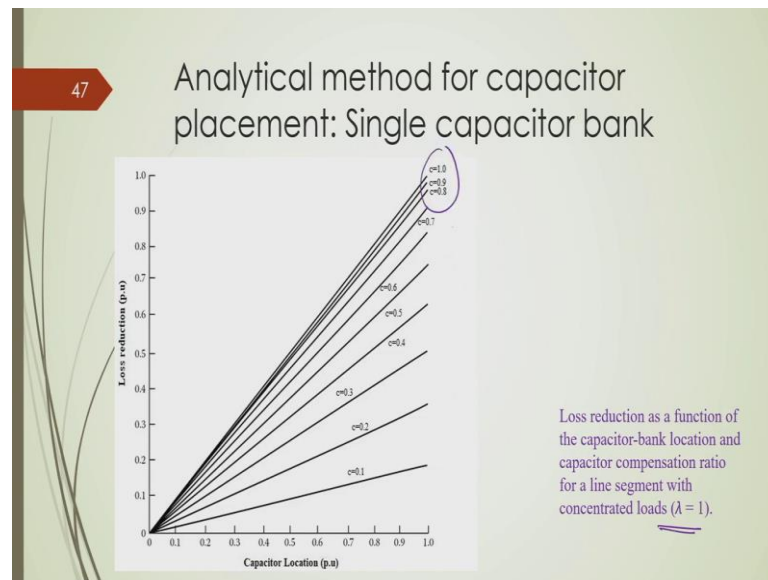
Now, if we further increase this lambda you can see again this location of this capacitor in order to have this peak loss reduction is at the end of the feeder ok. Because again increase in lambda means the amount of the load which is lumped they are increasing ok.

(Refer Slide Time: 60:14)



Then lambda is equal to 3 by 4, you can see almost linear characteristics and the highest value of loss reduction will take place if you put this capacitor at the end of the feeder.

(Refer Slide Time: 60:32)



Again lambda is equal to 1 means it corresponds to this lumped load; that means, current characteristic will be flat, if lambda is equal to 1 means I_2 is equal to I_1 , it means it is a completely lumped load, all loads are concentrated at the end of the feeder.

So, where will be the capacitor location so that we get highest value of power loss reduction? We get at the end of the feeder.

(Refer Slide Time: 61:08)

48 Analytical method for capacitor placement: Single capacitor bank

Optimum Location and Optimum Loss Reduction		
Capacitor Bank rating, p.u	Optimum Location, p.u	Optimum Loss reduction, %
0.0	1.0	0
0.1	0.95	27
0.2	0.90	49
0.3	0.85	65
0.4	0.80	77
0.5	0.75	84
0.6	0.70	88
0.7	0.65	89
0.8	0.60	86
0.9	0.55	82
1.0	0.50	75

So, if lambda is increasing; that means, if lambda is increasing; that means, we are slowly moving from this uniformly distributed load to the lumped load which is

concentrated at the end of the feeder. So, capacitor location would be at the end of the feeder so that we can get highest benefit in terms of power loss reduction ok.

So, here some numerical this table provides you some numerical value for determination of optimal location and optimum loss reduction. And this, I believe that this corresponds to lambda is equal to 0 ok. And you can see that you can get the maximum loss reduction if you place this capacitor at this 0.65 or that is almost two-third distance of that particular feeder from the substation.

(Refer Slide Time: 62:09)

49

Analytical method for capacitor placement: Two capacitor banks

■ Let us assume that, two capacitor banks of same size are located at distances of x_1 and x_2 from the substation. The power loss reduction will be,

$$= 3 \int_{x=0}^{x_1} [I_1 - (I_1 - I_2)x - 2I_c]^2 R dx + 3 \int_{x_1}^{x_2} [I_1 - (I_1 - I_2)x - I_c]^2 R dx + 3 \int_{x_2}^1 [I_1 - (I_1 - I_2)x - I_c]^2 R dx$$

$$= 3acx_1[(2 - x_1) + \lambda x_1 - 3c] + 3acx_2[(2 - x_2) + \lambda x_2 - c]$$

Now, we will talk about the case with two capacitor banks. So, if we have two capacitor banks which are located at X 1 and X 2 distance from the substation then how much loss reduction we will get?

Now, it means that again you can understand that then how would be current profile. So, you can see if I go back to this figure. So, here we have one capacitor bank placement so if we have another capacitor bank placement suppose at here then this characteristics will be something like this and so on.

So, this is basically for two capacitor banks two capacitor banks that one is located at X 1 another is located at X 2 let us say so this is X 1 and this is X 2; and this will be then the current profile for two capacitor bank placement; two capacitor bank placement and

so on. So, if we have more number of capacitor banks accordingly that many steps we will be having in this particular characteristic.

So, for this two capacitor banks so our expression for this we will be having 3 segments. we have seen one from x is equal to 0 to x_1 another is from x_1 to x_2 another is x_2 to 1. So, this corresponds to without capacitor; this corresponds to 1 capacitor and this corresponds to 2 capacitors that is why it is subtracted by 2 I c ok.

(Refer Slide Time: 64:17)

50

Analytical method for capacitor placement: Three capacitor banks

- For two capacitor banks of same size are located at distances of x_1, x_2, x_3 from the substation. The power loss reduction will be.

$$= 3acx_1[(2-x_1)+\lambda x_1-5c] + 3acx_2[(2-x_2)+\lambda x_2-3c] + 3acx_3[(2-x_3)+\lambda x_3-c]$$

$$= f_1(x_1, \lambda, c) + f_2(x_2, \lambda, c) + f_3(x_3, \lambda, c)$$

So, if you solve this, you will be having this relation you will be having this relation ok. So, if we you have 3 capacitor banks, you can see which are located at X_1, X_2 , and X_3 , you will be having this relation ok.

(Refer Slide Time: 64:28)

51

Analytical method for capacitor placement: n capacitor banks

- For n capacitor banks of same size are located at distances of $x_1, x_2, \dots, x_n$ from the substation. The power loss reduction will be,

$$= 3\alpha c \sum_{i=1}^n x_i [(2 - x_i) + \lambda x_i - (2i - 1)c] \quad \text{--- } f(\lambda, c, x)$$

- The optimum locations for the capacitor banks can be obtained by taking first order derivative with respect to x_i and equates it with zero. This yields,

$$(x_i)_{\text{optimal}} = \frac{1}{1 - \lambda} - \frac{c}{2} \left(\frac{2i - 1}{1 - \lambda} \right)$$

Now, we can generalize this, if we have n capacitor banks from this previous two expressions, you can find out the expressions for loss reduction and that generalized expression is coming out to be this. Here x_i is the location of the capacitors; x_1 is the location of the first capacitor; x_2 is the location of the second capacitor; and this will be up to x_n where, x_n is representing the location of n th capacitor.

And other than that it is essentially same; that means, this is function of λ c and this x_i ok. Now, one thing that you can observe from this characteristics here that this expression has some components one is this component; another is this component; another is this component.

So, this component, if we write it as a component f and then if it is f_1 , f_2 and component f_3 ; then you can see this component is a function of x_1 only, apart from that it is usual function of λ and c . Similarly this component is only function of x_2 apart from that λ and c ; and this component is a function of x_3 , apart from λ and c ok. So, this is something you can understand.

And so we can easily determine that what should be that x_{optimal} by differentiating this expression with x_1 and equals to 0. so we get this is the value of x_1 optimal. So, at that you know if x_i is equal to 1.

(Refer Slide Time: 66:46)

52 Analytical method for capacitor placement: n capacitor banks

- The optimal loss reduction with n -capacitor banks will be,

$$= 3\alpha c \sum_{i=1}^n \left\{ \frac{1}{1-\lambda} - c \left(\frac{2i-1}{1-\lambda} \right) + \left(\frac{i^2 c^2}{1-\lambda} \right) - \frac{1}{4} \left(\frac{c^2}{1-\lambda} \right) - \left(\frac{ic^2}{1-\lambda} \right) \right\}$$
- With algebraic reduction, we get the expression for the optimal power loss reduction,

$$= \frac{3\alpha c}{1-\lambda} \left[n - cn^2 + \frac{c^2 n(4n^2-1)}{12} \right]$$

Then it will give you the location of the first capacitor bank or only capacitor bank which will be placed. If x_i is equal to 2, it corresponds to the second capacitor bank location and so on ok. Now, if you put this expression over here, then we will get this optimal loss reduction expression which is coming out to be this. And if you simplify with there are many in finite series, so if you simplify this then it is coming out to be this expression ok; which gives you the algebraic expression for loss reduction when we have n number of capacitor banks in a feeder ok; n number of capacitors in a feeder. So, it is as usual function of n it is function of λ and c as well.

(Refer Slide Time: 67:22)

53 Analytical method for capacitor placement: $n=1$

- For $n=1$, we get the power loss reduction expression as, $\Delta P_{LS} = \frac{3\alpha}{1-\lambda} [c - c^2 + \frac{c^3}{4}]$
- Differentiating with respect to c and by equating it to zero, we get the optimal value of $c = \frac{2}{3}$ and $x_1 = \frac{2}{3} \left(\frac{1}{1-\lambda} \right)$ Capacitor bank location
- The power loss reduction will be $\Delta P_{LS} = \frac{3\alpha}{1-\lambda} \left(\frac{8}{27} \right) = \frac{8}{9} \left(\frac{\alpha}{1-\lambda} \right)$
- For $\lambda = 0$, i.e., uniformly distributed load, the power loss reduction will be $\Delta P_{LS} = \frac{8}{9}$ p.u., for the capacitor placed at $x_1 = \frac{2}{3}$ p.u. distance with $c = \frac{2}{3}$ p.u. size. This is well used thumb-rule for capacitor placement.

Now, we will only study when this capacitor n is equal to 1; that means, when we will only use one capacitor bank. So, if you put n is equal to 1 in this expression as well then you will be getting the location and optimal power loss for that particular feeder.

So, if you put n is equal to 1 this gives to this ok. Now, differentiating this with c ; that means, this is a function of now only λ and c ; so differentiating with respect to c gives you that optimal value of c , that optimal value of compensation you require to get this least power or highest power loss reduction.

And this happens to be at x_1 is equal to this so x_1 gives you the capacitor location. So, this is capacitor bank location and this c gives that capacitor bank size ok. So, if you put these two over this expression, you will get ΔP_{LS} is coming out to be $\frac{8}{9}$ multiplied by α divided by $1 - \lambda$.

Now, if we consider λ is equal to 0; that means, it is a kind of uniformly distributed load throughout the feeder, then α which is equal to $1 + \lambda + \lambda^2$, you can see and α which is equal to 1 divided by $1 + \lambda + \lambda^2$. So, when λ is equal to 0, α will be equal to 1 ok.

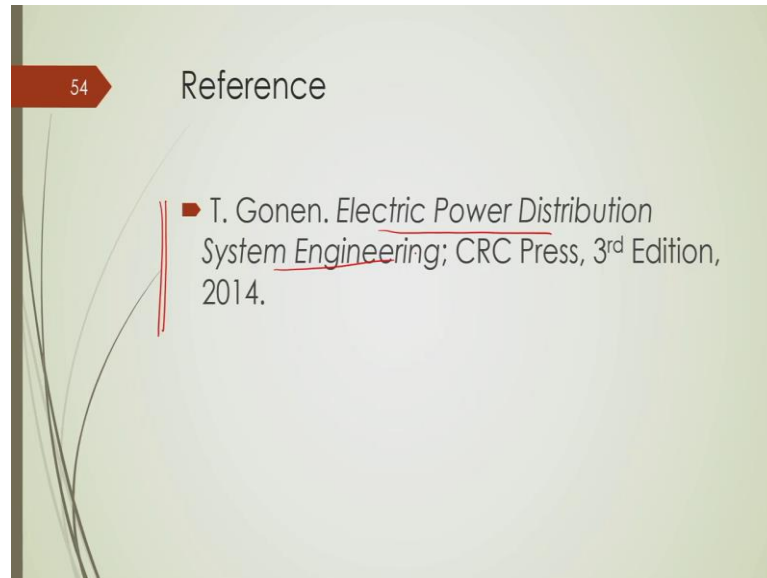
So, if you put this 2 values over here, so α is equal to 1, λ is equal to 0, so ΔP_{LS} will be equal to $\frac{8}{9}$ per unit. So, this gives that much of power loss reduction is possible which is almost equal to 88 percent; 88 point some percent ok. Loss reduction is possible if you put this capacitors, because this part will be equal to 1.

So, if you put this capacitor at two-third of the distance or two-third at the length of the feeder and if your capacitor size is two-third per unit of the total reactive power demand of the network, this is called the well-known two-third thumb rule of the capacitor placement and many utilities they follow this ok.

So, they used to place if one capacitor bank is to be placed, they used to place a capacitor at two-third distance of the length of the feeder and by keeping its size of the two-third of the total reactive power demand of the feeder. So, with this they get highest amount of loss reduction ok.

And without if someone is blind about where to place this capacitor and what should be the size of the capacitor they follow this thumb rule and this is well known two-third thumb rule for capacitor placement.

(Refer Slide Time: 70:52)



So, with this I will stop and I sincerely acknowledge this material is are taken from Turan Gonen's book on Electric Power Distribution System Engineering ok.

Thank you very much for attending this lecture.