Operation and Planning of Power Distribution Systems
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Lecture - 16
Reliability evaluation of multiple units connected to series and/or parallel

So, in my last lecture I discussed the basic mathematical modeling of reliability ok. So, what is reliability function and how to model mathematically these things are discussed in my last lecture ok.

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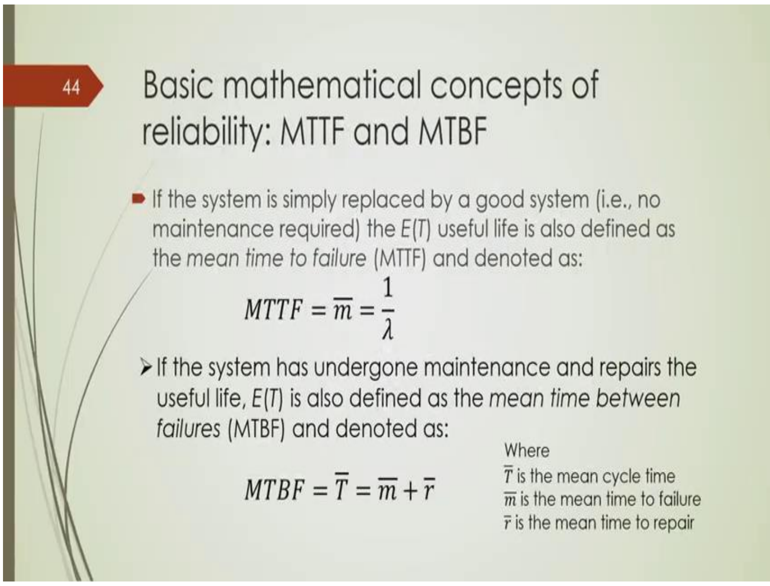

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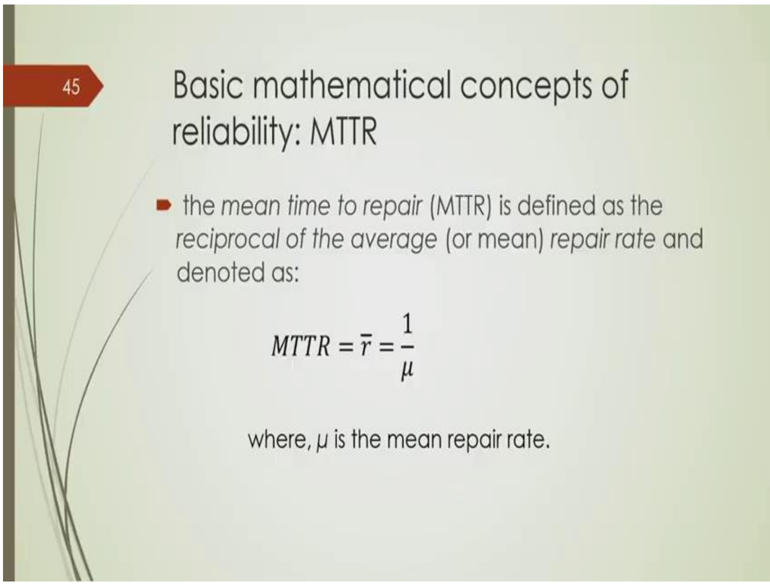

And finally, we got that for a system having constant failure rate this we got this expression that mean time to failure already defined is coming out to be 1 upon this lambda where lambda is the constant failure rate and this mean time to repair is 1 upon mu where mu is the constant repair rate ok.

And for any type of component which is repairable; there exist a number of cycle of failure and repair. So, basically there is a constant period of time by which this system will operate at healthy condition; then after that it will suffer from some fault and it needs maintenance or repair; so there is some period of time where it will undergo maintenance or repair ok.

And then after that it will again operate at healthy condition for a considerable time before it suffers from another fault or another failure. So, there is a cycle exist which is basically sum of this mean time to failure and mean time to repair. So, what is mean time to failure? Meantime to failure is basically if you sum up and take the average of all these durations for all the cycles for a particular component during which the component operates satisfactorily or components was in healthy condition ok.

So, this is called MTTF or mean time to failure and mean time to repair is basically; If you sum up all the repair duration and take the average of mean of that will represent this mean time of repair; mean time of repair and mean time of failure, if you add these two arithmetically then what you will get? You will get a cycle that is represented by this capital T which is called mean cycle time ok.

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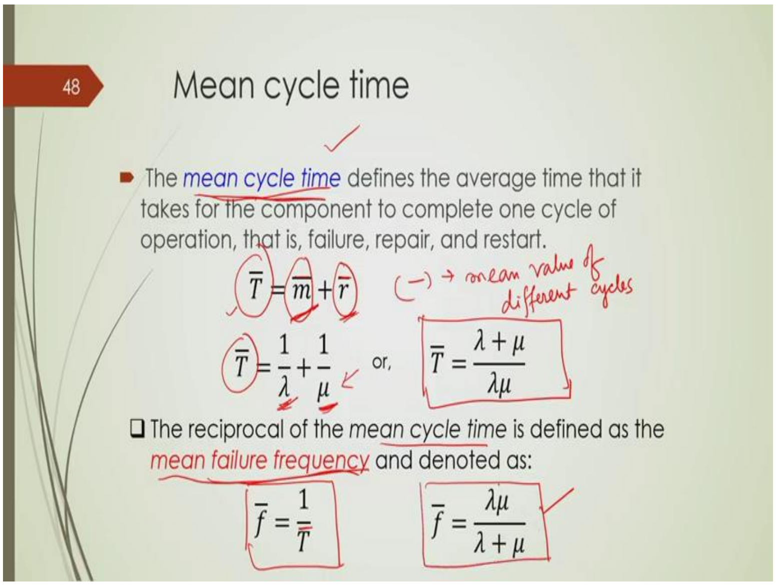

These things I discussed in my last lecture. So, mean cycle time is basically equal to summation of mean time to failure and mean time to repair ok. Now, if we represent this mean time to failure with a constant failure rate that is lambda. So, we already got this expression for mean time to failure would be equal to 1 upon lambda.

Similarly, since this mu is representing this repair rate this not repair duration so it is repair rate; that means, that much of repair per a given time. So, then from that you can find out this mean time to repair that is r bar. So, here bar is representing mean value of different cycles.

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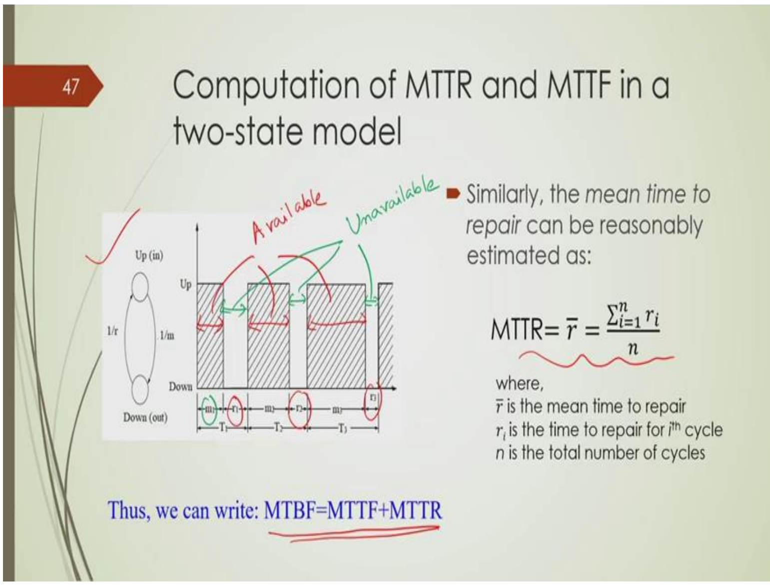

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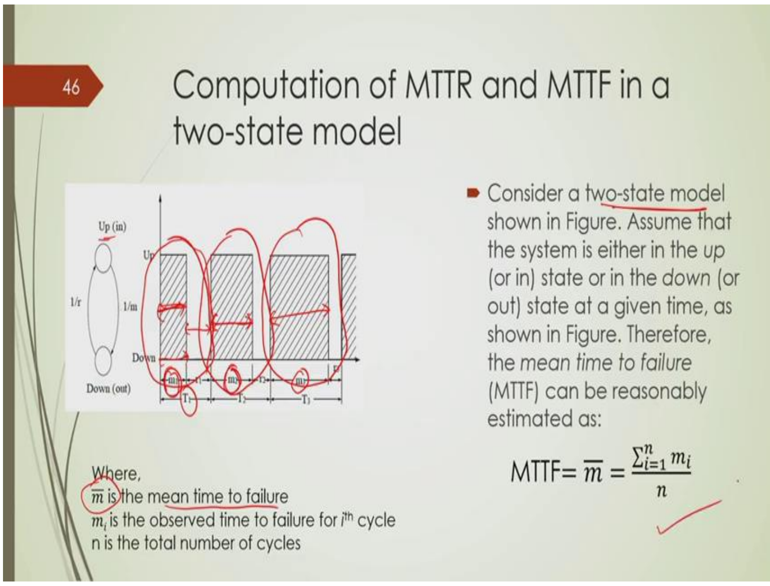

So, these cycles already I had explained in this particular slide. So, this is a typical cycle so during which this component was up and down similarly this is another cycle ok. So, this is another cycle which consists of some duration during which it was in up during this duration and some duration during which it was under maintenance one particular component was under maintenance. This is applicable for a two-state model already I discussed in my last lecture ok.

Now, if you sum up all these duration. So, this T 1 stands for basically the summation of the time for which the component is up plus component was down ok. So, this m 1 is basically representing the duration which this system component was operated satisfactorily or it was in up condition ok.

And similarly we have in different cycle different values of m; this duration may change for example, from here to here you can see duration has changed and then further it has change that it has increased and so on ok. Now, if you take this m values and if you take the average of that basically representing this mean time to failure that is MTTF which is explained here.

Similarly this mean time to repair if you take this r values and take the average of this then whatever you will get that will represent this mean time to repair and this summation of this mean time to failure and mean time to repair will give you mean time between failures that is MTBF and also it gives an average time or mean cycle time ok.

So, mean cycle time is basically represented as here capital T bar means it is a mean value. Similarly, here this m bar means it is the mean value of all m. Similarly, this r bar represents a mean value of all r ok, r 1, r 2, r 3 and, whatever r is there for a overall these cycles ok. Now, we can represent this mean cycle time as a function of this failure rate and repair duration similar to this and we get this expression ok.

Now, as you know if we take the reciprocal of this mean cycle time then it will give you mean failure frequency. Frequency as you know it is 1 upon this time period. So, it is 1 upon this T bar this should be T bar and, this expression for f bar is in terms of this or as a function of lambda and mu is given at this expression; it is simply the reciprocal of 1 upon T bar.

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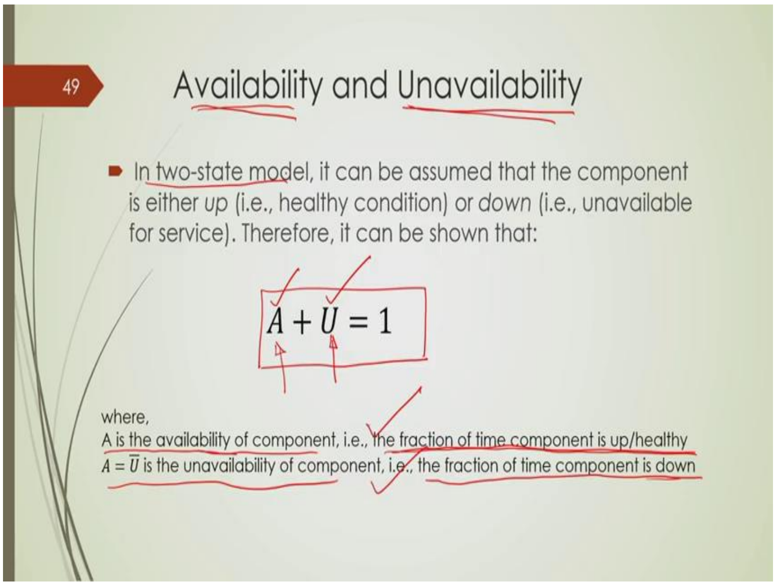

Now, we will also discuss about two important aspects; one is called availability another is unavailability of a particular component ok. Now, which type of component will have some period of availability and some period of unavailability? Of course, those components which are repairable; which are repairable because as you know in power system or power distribution systems we use several components some of them are repairable some of them are non repairable ok.

Now, for repairable components we need maintenance or repair; for non repairable components we need replacement alright. Now, how do you model this availability and unavailability? So, here availability is defined as the fraction of time the component is up or healthy ok. So, availability is defined as the fraction of time any component is up or healthy ok.

And unavailability which is simply that complement of this availability which; obviously, represents the fraction of time the component is in down or under repair ok. So, here availability is fraction of time the component is in healthy condition or operating satisfactorily and unavailability is the fraction of time the component is down.

Since both are fraction; that means their values will lie in between 0 to 1; so if you sum up these two component; one is fraction of time this component was available and fraction of time the component was unavailable this will represent 1 ok. So, here capital A is representing this availability of any component and capital U is representing unavailability of a particular component ok. And as you know component either would be available or would be unavailable ok.

So, they have this type of binary relationship. So, either any component can be operational can be at operational mode or under repair ok, there is nothing in between that. So, this is called two state model where we have two states of a particular component one is up state another is down state.

Up state is also called as healthy state and down state is also called as repair state ok. Alright, now as I said it can be in two-state model or an assumption is that a component is either up or down ok. So, up means it is at healthy condition or it is available for operation; down means it is under repair or it is unavailable for operation ok. So, you can understand this availability and unavailability both are complement to each other; that means, one system or one component would be either available or unavailable ok.

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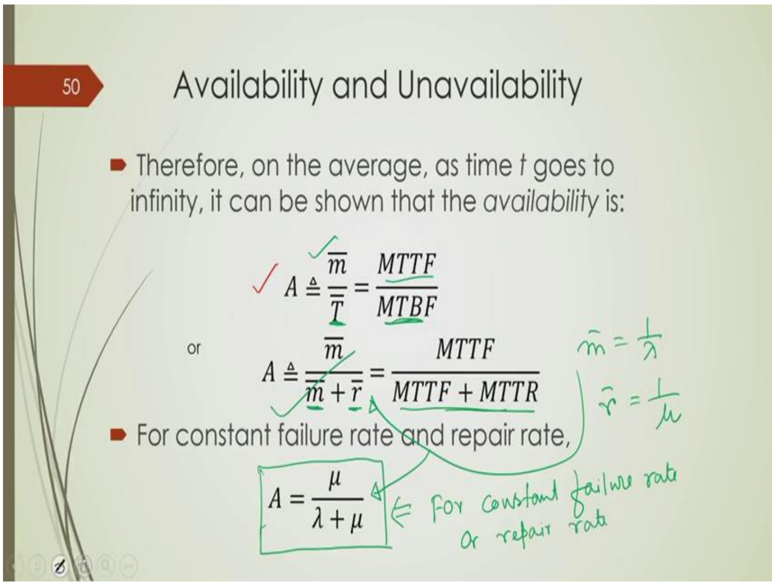

Now, we can represent this availability function as a function of this mean time to failure and mean time to repair ok. So, for a two-state model you can see for example, here. So, during this period this system was available. So, these are the time periods for which the systems are available. Similarly, these are the periods for which the systems were unavailable.

So, availability period is function of this m and unavailability period is function of r ok. So, we can represent this availability function which is equivalent to mean value of m or mean time to fault or mean time to failure divided by this mean time cycle which is called as MTBF ok. Similarly, this mean time cycle we know it is summation of mean time to failure plus mean time to repair like this and we can represent this availability in this form.

Now, we can replace this m bar by 1 upon lambda that is failure rate and r bar is 1 upon mu that is repair rarte. If you replace this in this equation this will give you this expression that is availability is equal to mu divided by lambda plus mu. So, this equation is applicable when a system will have constant failure rate or repair rate.

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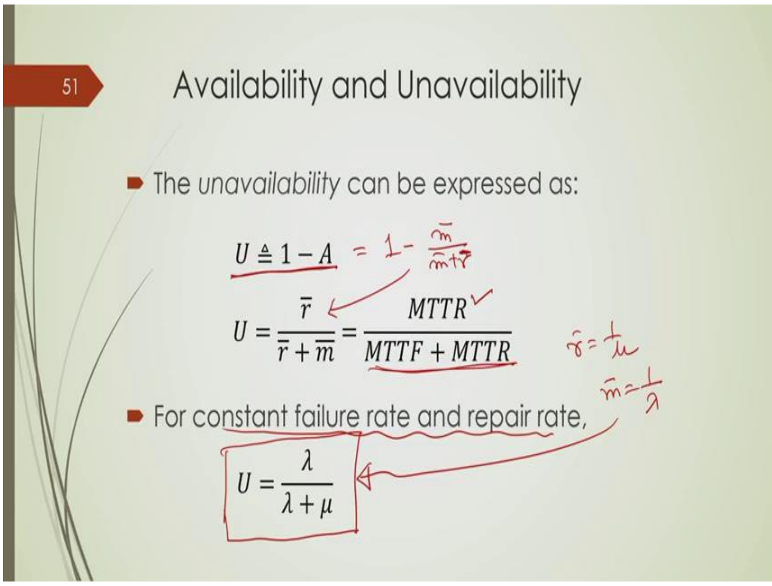

So, once you get this availability as we know the unavailability is 1 minus this availability function. So, unavailability represents the fraction of time the component will be unavailable so which is equal to 1 minus A. So, this is you can put simply that A value is m bar divided by m bar plus r bar. So, which will give you this value and this is nothing but mean time to repair divided by mean cycle time ok.

And similarly represent this r bar by 1 upon mu that is repair rate and m bar by 1 by upon lambda that is failure rate. So, which will lead to this equation that unavailability is equal to lambda divided by lambda plus mu, and that is also applicable for constant failure rate and repair rate; that means, a component is having with constant failure rate and repair rate ok.

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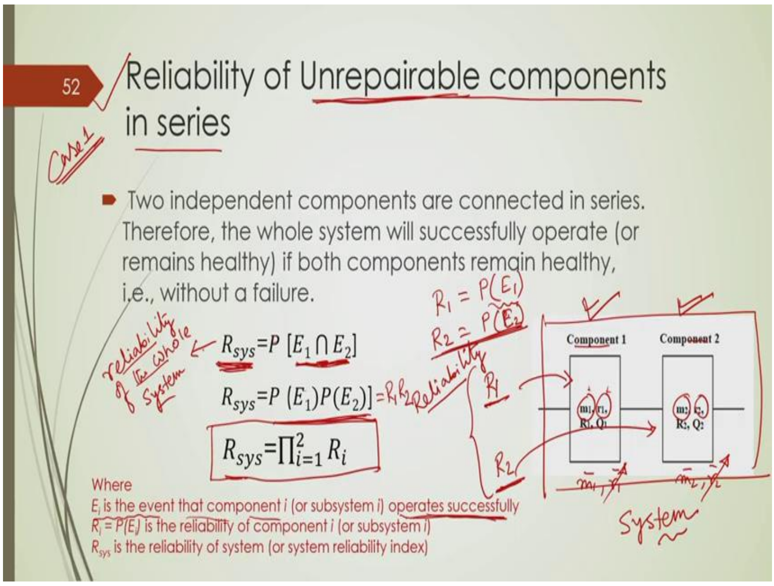

Now, here we will make some case studies first of all this is the first case 1. So, we will do some case studies here at this point number 1 is, suppose we have a number of unrepairable components in series. What do you mean by unrepairable components? Unrepairable components mean those components which are which cannot be repaired or which need to replaced if they suffer from failure ok.

Now, we will first analyze for a system having two components this is component 1 and this is component 2 ok. So, this m 1 and r 1 represent its mean time to failure and mean time to repair ok. Similarly, m 2 r 2 represents its mean time to failure and mean time to repair ok.

Now, since these components are not repairable so, r 1 will not exist so they should be replaced. So, there are many types of equipment for example, these fuses are not repairable items. So, if it is blown out then, obviously, we need to replace the particular fuse ok. Similarly, we have many components which are not repairable ok.

Now, for those equipment let us consider that this R sys represent that reliability of the system so, R sys basically represent the reliability of the whole system. Now, what is whole system? We have two components which creates a whole system. So, this makes a system; in the system we have two components which are connected in series ok.

Now, we need to determine that what would be the reliability of the whole system. In order to know that we need to know what is the reliability of the individual system. So, let us consider the reliability of this component 1 is basically R 1 and reliability of component 2 is basically R 2. So, these two are reliability of components 1 and 2.

Again, what is the definition of reliability that you need to recall my previous lecture. Reliability is basically a function or a probability which represents a component will not fail so it is the probability not to fail. So, you can go back and check my previous day lecture. So, this reliability is basically probability not to fail ok.

Now, if we have two components which are in series and if we consider that E is basically representing the event that the component i operate successfully; that means, that component i will be at healthy condition or will be up ok. Now, as you know for a series system overall system will not fail if both the systems are in healthy condition ok.

So, for a series system if we have two or multiple units or multiple components are in series then; obviously, you can understand from your general knowledge that if any of the components fail the overall system will fail ok. Now, what is the condition that the whole system will operate at healthy condition? The condition is that each and every component of the whole system should operate at healthy condition ok.

So, here E 1 and E 2 basically represent the event that the component 1 and 2 operates at healthy condition ok. And this P is representing this probability and this is basically intersection as you know. So, if E 1 intersects this E 2 then only you know that is basically the overall the probability of the overall system not to fail or it is the probability of the overall system to operate at healthy condition or at up condition ok.

So, in the probability theory the intersection of these two events can be replaced by this multiplication of individual probability that is P E 1 multiplied by P E 2. let us consider P E 1 basically represent R 1 that is probability of this component 1 to operate at healthy condition and P E 2 is basically represent by this function R 2 ok.

Where you know R 2 represents the probability of component 2 to operate at healthy condition. So, if we replace these two so you will get R sys is equal to nothing but R 1 multiplied by R 2 ok. So, we got a product rule in the determination of the reliability of the overall system ok.

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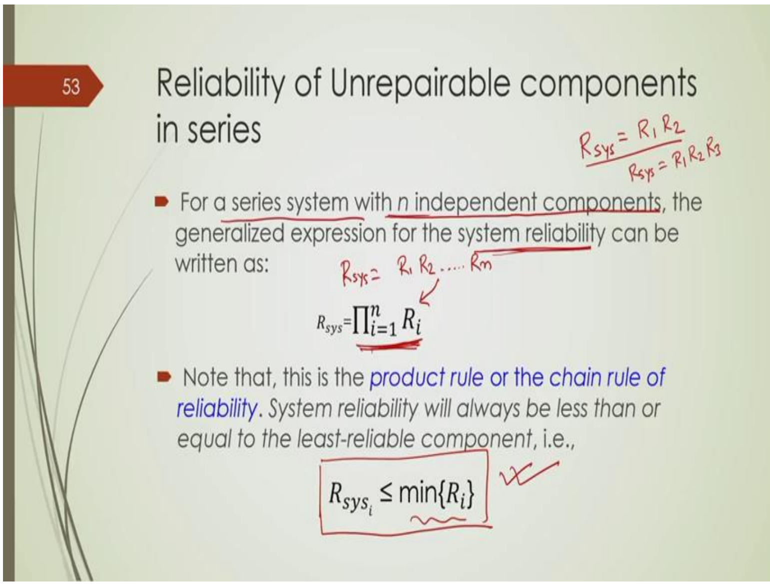

So, if we have similar kind of n independent components which are in series, how to determine the system reliability? So, in my last slide we got that for two components the overall reliability of the system; two components are in series of course. So, overall reliability of the system is of course, R 1 multiplied R 2. So, if we have 3 components it will be R sys is equal to R 1 R 2 multiplied by R 3 and so on.

So, if we have n number of system the overall reliability of the system will be R sys is equal to R 1 R 2 dot dot R n. So, which is represented by this product rule that is product of R i ok. Now this product rule or chain rule of reliability is applicable only for when we have n number of components which are connected in series and they are not repairable ok.

And of course, since they represent this probability; they are fraction and that is why this overall system reliability is less than equal to minimum value of this R i ok. So, this is also explained with further example in one of my slides.

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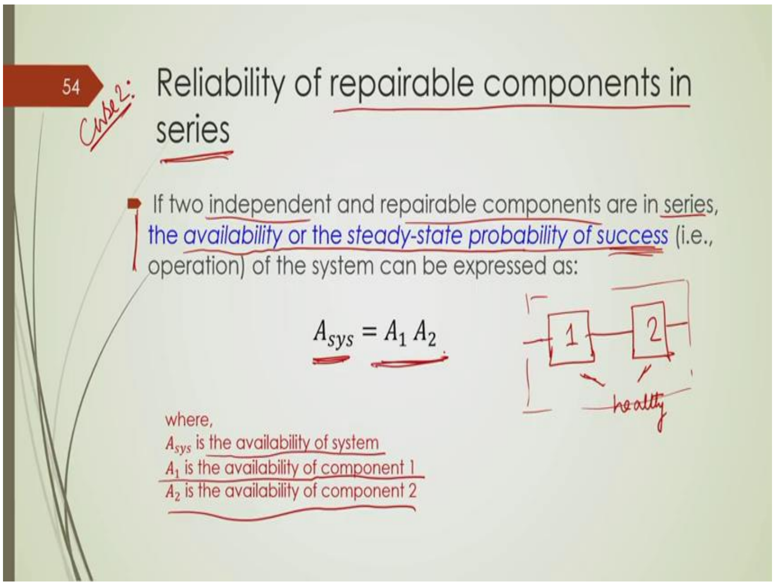

Now, we have case 2, 2nd case. When we have repairable components which are in series when we have 2 or more number of components which are repairable and they are connected in series. So, what would be the reliability or how do you analyze or assess the reliability of the system ok?

Now, for two independent we will of course, here study for two components and if you know that what should be this reliability function for two components system which are connected in series and they are repairable then you will be; obviously, understand how would be the generalized expression for reliability for such kind of system having n number of components ok.

Now, we will study with 2 independent and repairable components are in series. Now, here we will try to find out the availability of the system which is representing the steady state probability of success or the reliability or it is the probability of having the system in operation or in healthy condition ok.

So, here A sys is basically representing the availability of overall system, A 1 is basically representing this availability of this component 1 and A 2 is representing availability of the component 2 ok. Now, as you know for two component systems suppose this is component 1 and this is component 2. We will have the overall system available or in operational when both the components are in operational ok.

So, for more components are in series for series connection when a particular unit consisting of n number of components would be operated at healthy condition when all the components of the system are in healthy condition; so both the components are to be healthy then only the system would be healthy. So, the availability of the whole system is equal to A 1 multiplied by A 2 that is availability of the component 1 and availability of the component 2 alright.

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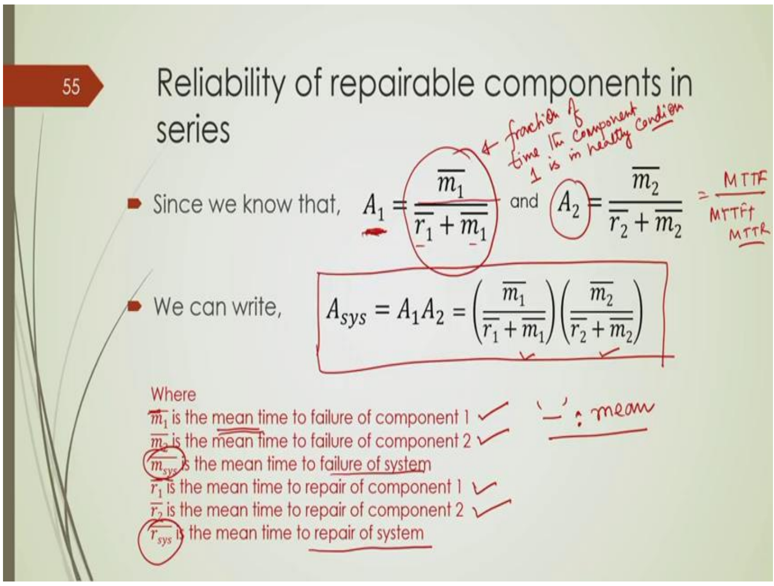

Now, let us consider that m 1 is basically mean time to failure of component 1 and m 2 is basically mean time to failure of component 2. Similarly, r 1 bar see this bar represent this mean time so this bar represents mean value ok. Similarly, r bar is representing that mean time to repair for component 1 and r 2 bar is representing mean time to repair for component 2.

And this m sys bar and r sys bar represent the mean time to failure for the overall system and mean time to repair for the overall system which consist of this component 1 and component 2 connected in series ok. Now, as we have seen the definition availability is the fraction of time this component 1 will be in healthy condition.

So, A 1 will be presenting this m 1 divided by r 1 bar plus m 1 bar. So, this will represent the fraction of time the component 1 is in healthy condition ok. Similarly A 2 is representing the fraction of time the component 2 is in healthy condition which is equal to m 2 bar divided by r 2 bar plus m 2 bar which is nothing but the ratio of MTTF divided by MTTF plus MTTR ok. So, which already we have seen.

So, the availability of the system which is product of this A 1 and A 2 we can get; we can get the expression for that is equal to this m 1 bar divided by r 1 plus m 1 bar multiplied by m 2 bar divided by r 2 bar plus m 2 bar ok.

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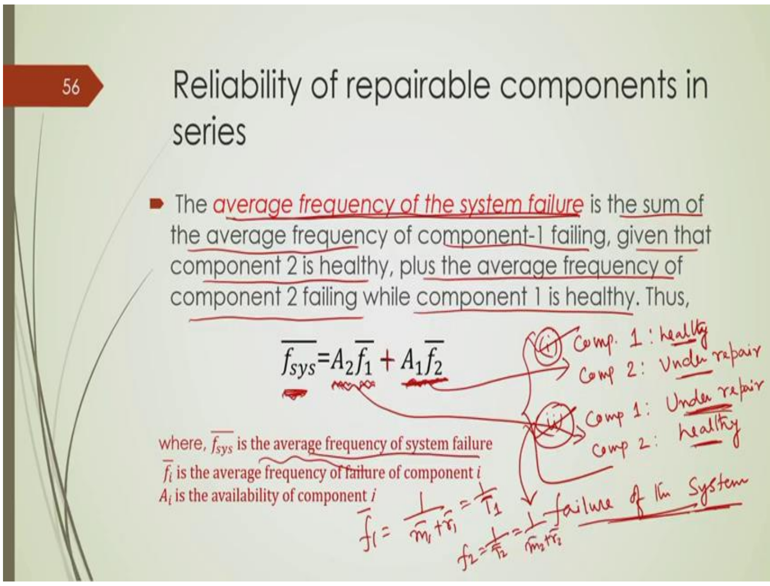

Now, we will try to determine the average frequency of the system failure. This is an important concept that one should understand; the average frequency of the system failure is the sum of the average frequency of the component 1 failing given that component 2 is in healthy condition, plus average frequency of component 2 failing while component 1 is healthy condition.

So, there are two conditions one is, component 1 is healthy and component 2 is under repair. And there is another condition that component 1 is under repair and component 2 is at healthy. So, both will drive to this failure of the system; failure of the system assuming that we ignore this simultaneous failure of both the components which probability would be of course, less ok.

So, this overall system will fail when this component was in component 1 is in healthy condition and component 2 is under repair or it is at faulty condition and second case is when component 1 is under repair and component 2 is operated at healthy condition ok.

So, overall this frequency of the failure of the system is basically as you know representing that availability of this component 2 multiplied by the failure frequency of the component 1.

So, this corresponds to this case where your component 2 is in healthy condition and component 1 is under repair. Similarly, this is the case where component 1 is at healthy condition and component 2 is under repair; So, this basically gives you the frequency of this particular case.

And if you sum up these two then we will get the overall or average frequency of the system failure ok. Now, A 2 is basically the fraction of time this component 2 is available and during this period if you multiply this failure frequency of component 1, you will get failure frequency for this particular condition that this component 2.

This basically represents availability of this component 1 and this failure of this component 1. If you multiply these two this correspond to this condition when component 1 is in healthy and component 2 will be under repair and their summation is basically representing the total or average frequency of the system failure ok.

So, once we got that we know these expressions for A 2 from the last slide that is this and, we got the expression for A 1, similarly f 1 f 2 expressions we can also get because they are frequencies. So, f 1 bar will be equal to 1 upon m 1 bar plus r 1 bar. So, which is 1 upon this 1 upon T 1 bar which is representing the time cycle for component 1; so to this f 2 will be equal to 1 upon T 2 bar which is equal to 1 upon m 2 bar plus r 2 bar.

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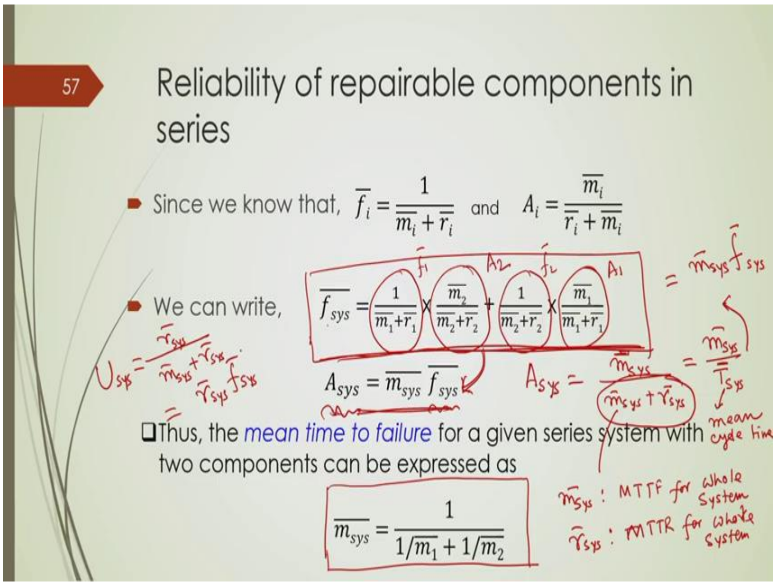

Now, once you know that you put these values to this expression and we will get this expression ok. We will get this expression where this is basically f 1 bar and this is basically A 2 that is availability of the component 2; this is basically f 2 bar and this is basically your A 1.

So, we know that sys is basically A 1 f 2 plus A 2 f 1 which we got from the last slide and we put these values ok. Now, we also know the availability of the system; availability of the system is basically equal to m sys bar divided by m sys bar plus r sys bar where m sys bar is representing the mean time to failure of the overall system and r sys bar is representing the mean time to repair for the overall system.

So, m sys bar is basically mean time to failure for whole system ok. Similarly, r sys bar is mean time to repair for whole system ok. So, this is nothing but equal to 1 upon T sys that is we can write m sys divided by capital T sys, where this T sys bar is representing mean cycle time; ok.

So, we can also write it as equal to m sys bar multiplied by f sys bar which is represented over there alright. Now, we have already seen the expression of A sys we got by multiplying this availability 1 and availability 2 that is this equation, and we get this f sys this equation; So, if we put this value of f sys at this expression, we will get this expression for availability of the overall system.

Now, we will equate both the equations one is this by putting this value of expression for f sys at this particular equation and left hand side we will equate with this equations ok. So, if you do this then you will get the expression for mean time to failure of the overall system as a function of m 1 bar and m 2 bar which are mean time to failure of component 1 and mean time to failure of component 2 alright.

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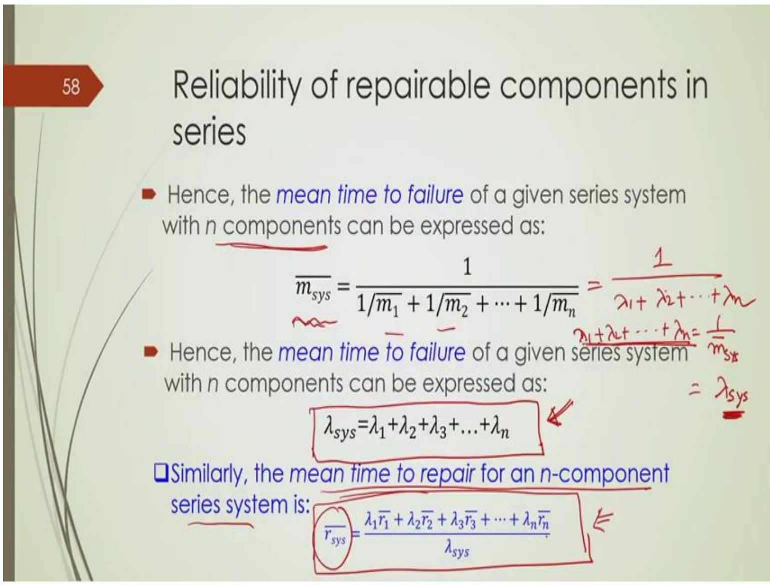

Now, similarly if we have n number of components in series which are also repairable then the mean time to failure that is MTTF of the overall system is equal to 1 by m 1 bar plus 1 by m 2 bar and so on ok. Now, as we know 1 upon m 1 bar is basically lambda 1 where we assume that the component 1 is having a constant failure rate. So, as the other components; that means, with the assumption that each of the components are having constant failure rate respectively.

So, we can replace this 1 upon m by lambda 1 and also lambda 2 lambda 3. So, this will lead to have this equation as equal to 1 by lambda 1 plus lambda 2 plus lambda n ok. And that is representing 1 by that is basically representing this mean time to failure. Now, we can alternatively write it as summation of this lambda 1 plus lambda 2 plus up to lambda n will be equal to 1 upon this m sys where this m sys bar represents the mean time to failure for the overall systems; so which is nothing but lambda sys ok.

So, here lambda sys is the failure rate of the overall system. Now, at this point we got this relationship that failure rate of the overall system is equal to sum of the individual failure rate of all components when a system is having n number of repairable components are in series ok.

So, if you know that there is a system which is having n number of components which are in series and all the components are repairable then you can find out the failure rate of the whole system simply by summing up the individual failure rate of the components ok. Similarly, this mean time to repair that is MTTR can be obtained as this ok. So, this you can get how to get this? I will give some hint.

So, this r sys is basically representing mean time to repair for a system having n number of components which are connected in series, and all the components are repairable ok. So, for that mean time to repair you can find out r sys with this expression. Now how to find this? So, in order to find this let us go back to this expression. So, this is basically availability of the system; now this is the expression of the availability of the system.

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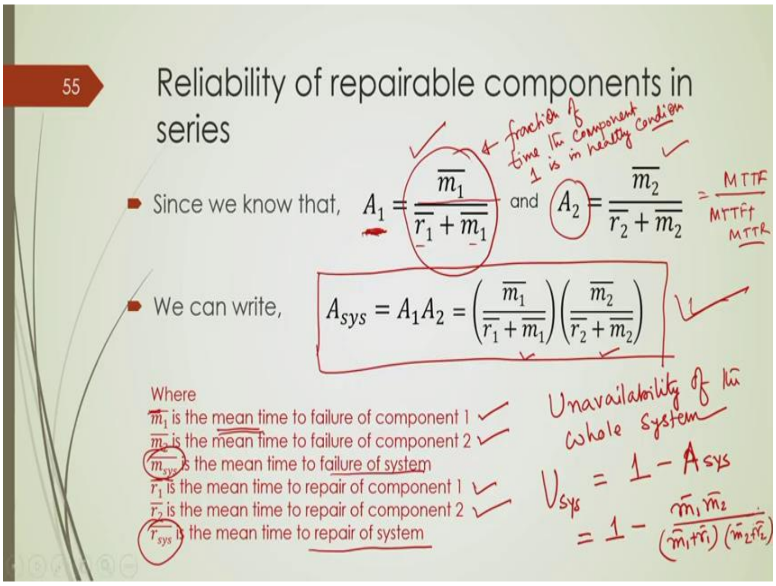

Now, for that system what would be the unavailability function? So, if unavailability of the whole system is represented by U sys then as we know U sys is equal to 1 minus A sys ok. So, as we know that A sys is equal to this so we can write it as U sys is equal to 1 minus m 1 bar m 2 bar divided by m 1 plus r 1 m 2 plus r 2 ok.

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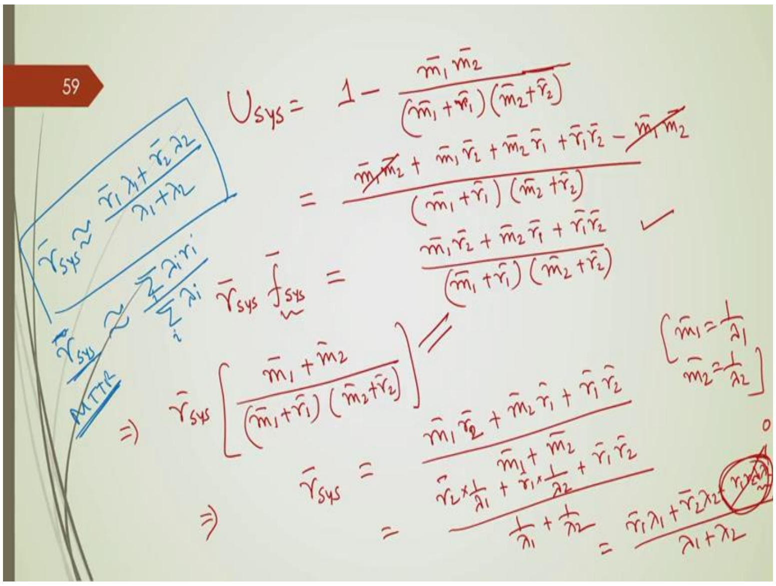

So, we got this, now we will write it here that unavailability of the system is equal to 1 minus availability that is m 1 bar m 2 bar divided by m 1 bar plus m 2 r 1 bar multiplied by m 2 bar plus r 2 bar alright. So, this we can further analyze so if we simplify this then we will get m 1 bar plus r 1 bar m 2 bar plus r 2 bar and here we will get m 1 bar r 2 and m 2 bar r 1 plus r 1 r 2 minus m 1 bar m 2.

So, this will get cancelled out ok. So, what we will get? We will get m 1 bar r 2 bar plus m 2 bar r 1 plus r 1 r 2 divided by m 1 bar plus r 1 bar m 2 bar plus r 2 alright. So, this is the expression for unavailability of the system. Now, similar to this previous one, you know that, this is the failure frequency and availability period basically m sys by this.

So, unavailability of the system will be equal to U sys is equal to r sys bar divided by m sys plus r sys. So, which will give you r sys bar multiplied by f sys bar ok. So, we can write here U sys is equal to r sys bar multiplied by f sys bar ok where we know the expression for f sys bar; so this is given here.

So, we just replace this expression as r sys bar multiplied by m 2 divided by this and m 1 divided by this which will lead to m 1 plus m 2 divided by these two unit. So, you can write it as m 1 plus m 2 divided by m 1 plus r 1 multiplied by m 2 plus r 2 ok. So, this equates with this equations you can see the denominator of both the sides are same.

So, one can replace it. So, we can write it as r sys is equal to m 1 bar r 2 plus m 2 bar r 1 plus r 1 r 2 divided by m 1 plus m 2 ok. Now, we know that m 1 bar is equal to 1 upon lambda 1 which is representing the constant failure rate of this component 1 and m 2 bar is equal to 1 by lambda 2.

So, if you put this here then what you will get, r 2 bar multiplied by 1 by lambda 1 plus r 1 bar multiplied by 1 by lambda 2 plus r 1 r 2 divided by 1 by lambda 1 plus 1 by lambda 2 which will give you r 1 bar lambda 1 plus r 2 bar lambda 2 plus r 1 r 2 lambda 1 lambda 2 divided by lambda 1 plus lambda 2. Now, this component which is multiplication of r 1 r 2 lambda 1 lambda 2; when you have this multiplication of lambda with r, since all these are fraction, and if you multiply these four fractions it will lead to almost negligible values. So, you can ignore it.

If we ignore we get this r sys approximately equal to r 1 bar lambda 1 plus r 2 bar lambda 2 divided by lambda 1 lambda 2. So, this gives you this last equation that is this, and for n number of components of course. So, this gives you approximately equal to summation of lambda i r i divided by summation of lambda i where i is varying from 1 to n if you have n number of components.

So, this gives you r sys which is value of mean time to repair MTTR. So, this basically gives you MTTR of the overall system. So, we can find out how would be the MTTR of the overall system.

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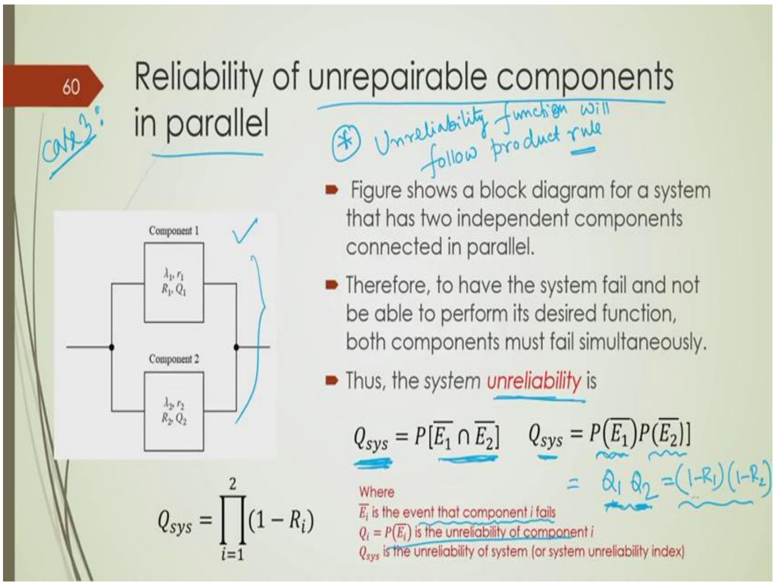

Now, this is our case 3 where we have unrepairable components are in parallel ok. So, previously we got two cases we studied two cases; case 1 was when we have n number of components are in series, and they are not repairable. Case 2 is when we have n number of components they are in series and they are repairable ok.

Now, here we have in case 3 we have n number of components they are in parallel and they are not repairable ok. So, two components which are in parallel and they are not repairable. So, similar to this the first case we need to determine the reliability of the system.

Now, when we have a parallel system with two or more number of components then the overall system will fail when all the components simultaneously fail ok. So, that is the general knowledge that we have if we have you know parallel system having n number of components then the overall system will fail when all the components simultaneously fail ok. So, here the failure function or unreliability function will follow the product rule ok.

So, the system unreliability is basically represented by probability of this event had E 1 bar; E 1 complement basically represents the event that component 1 fails and E 2 bar basically represent an events when events for component 2 fails and their intersection will give you the unreliability function of the system ok. So, Q sys is equal to P E 1 compliment P E 2 compliment ok.

So, P E 1 compliment we can represent it as Q 1 and P E 2 represent by Q 2 where, Q 1 is basically unreliability function of component 1 and Q 2 is unreliability function of component 2, and we can understand that relationship of this unreliability and reliability. So, Q 1 can be replaced by 1 multiplied by R 1, and Q 2 can be replaced by 1 multiplied by R 2 alright.

So, here as you know this unreliability function they will follow this product rule. So, I can note down here so in this particular case unreliability function will follow product rule; we will follow the product rule ok. So, that is why this overall unreliability of the overall system will be equal to product of the unreliability function of each and every component ok, which is equal to 1 minus this reliability function of each and every component ok.

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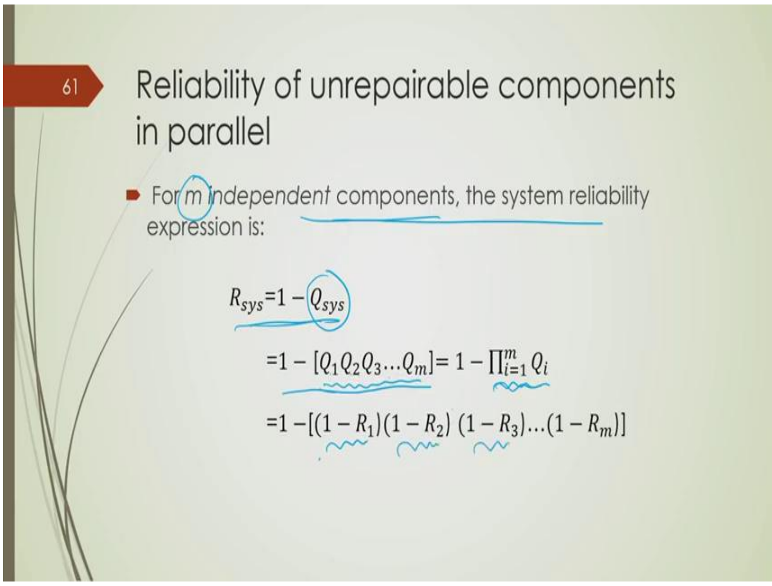

So, overall system reliability will be equal to 1 minus this Q sys where Q sys is the unreliability function of the system. So, which is equal to 1 minus Q sys which will follow this product rule that is Q 1, Q 2, Q 3 up to Q m. Where we assume that we have m number of independent components are in parallel so this will give you this product of this Q i where i value vary from 1 to m ok.

Now, in terms of reliability function also we can find out this is equal to 1 minus this Q 1 is replaced by 1 minus R 1 Q 2 is replaced by 1 minus R 2 Q 3 is replaced by 1 minus R 3 and so on ok. So, we can find out the overall reliability function like this ok.

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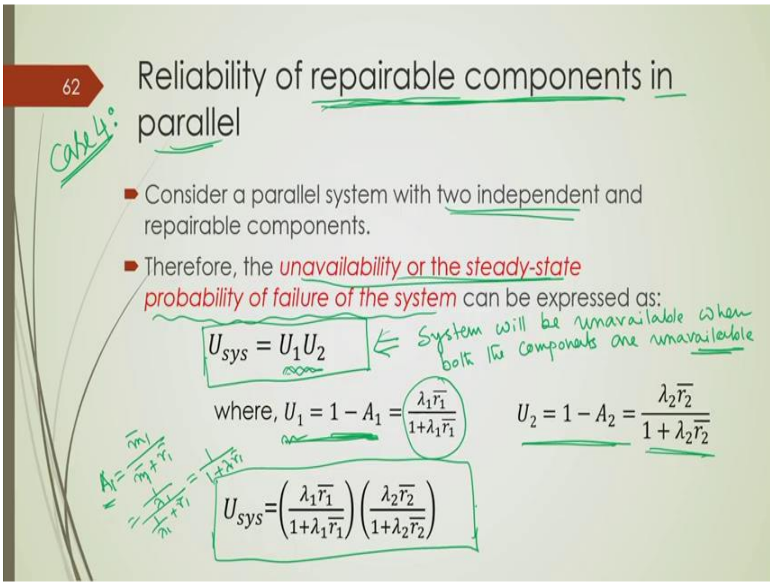

Now, we will have this case study that is case 4, where we have m number of components or we have a number of components which are repairable individual components, all components are repairable and they are in parallel ok.

And we need to find out or need to assess the reliability of the overall system. Now, here the difference is that we have repairable components are in parallel. Now, previously we have seen that when we had n number of components which are connected in series and they are repairable, the availability of the overall system follows the product rule.

Now, in the last case we have seen that when we have m number of components which are in parallel and they follow this product rule of the unreliability function. So, similarly here the unavailability of the overall system which also represents the probability of the failure of the system will follow this product rule ok.

That means, the system will be unavailable when both the components, when we have two independent components are in parallel when both the components are unavailable ok. So, this basically represents that system will be unavailable when both the components are unavailable ok.

Now, we know that U 1 is basically representing that 1 minus A 1 that is availability of the one and we know this A 1 is basically equal to m 1 divide m 1 bar plus divided by m 1 bar plus r 1 bar. Now, m 1 bar, we can replace by lambda 1 so this is equal to lambda 1 plus r 1 bar so which is basically representing this equal to 1 divided by 1 plus lambda 1 r 1 bar.

So, if we take you know 1 minus A 1 which will give you lambda 1 multiplied by r 1 bar divided by 1 plus lambda 1 r 1 which is the unavailability of the component 1. Similarly, unavailability of the component 2 we can get like this and the unavailability of the system will be the product of these two unavailability of component 1 and unavailability of the component 2.

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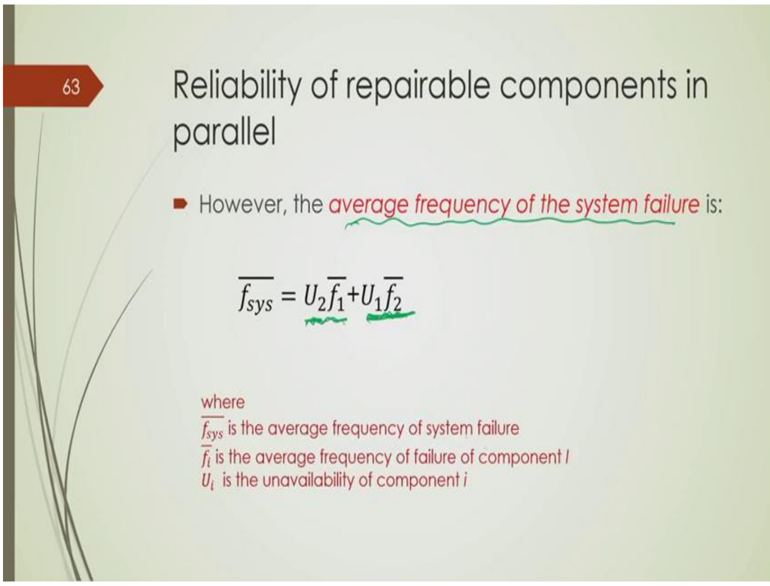

Now, similar to previous case 2, we can find out this average frequency of the system failure. So, when the system will fail; when one is component 2 is unavailable and the frequency failure of this component 1, and when the component 1 was unavailable and the frequency of failure of component 2. So, previously it was the availability and here we have unavailable multiplied by the frequency of the failure of individual component.

So, this will lead to the case when component 2 was unavailable and failure frequency of component 1 and this will lead to the second component this will lead to the case where component 1 was unavailable and multiplied by the frequency of the failure of the component 2. Now, we got this U 1 and U 2 expressions from here and simply we will put it, and we know expressions for f 1 and f 2 as well.

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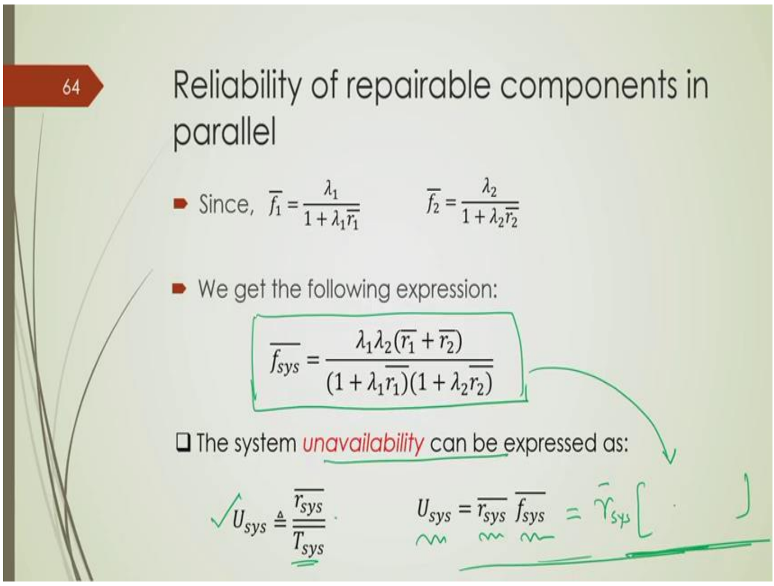

So, we get this relationship as f sys is equal to this alright. Now as we know this unavailability of the overall system is equal to r sys bar divided by overall mean time cycle that is T sys bar which is equal to r sys bar multiplied by f sys bar ok. Now we already know the expression of U sys that is this expression; and if we put this expression over here just by replacing this f sys bar, then we will get this equation.

So, this f sys bar we get from here; and we will put it to here; and this will give you one expression for U sys that is unavailability of the system; and that equation will equate with this equation ok.

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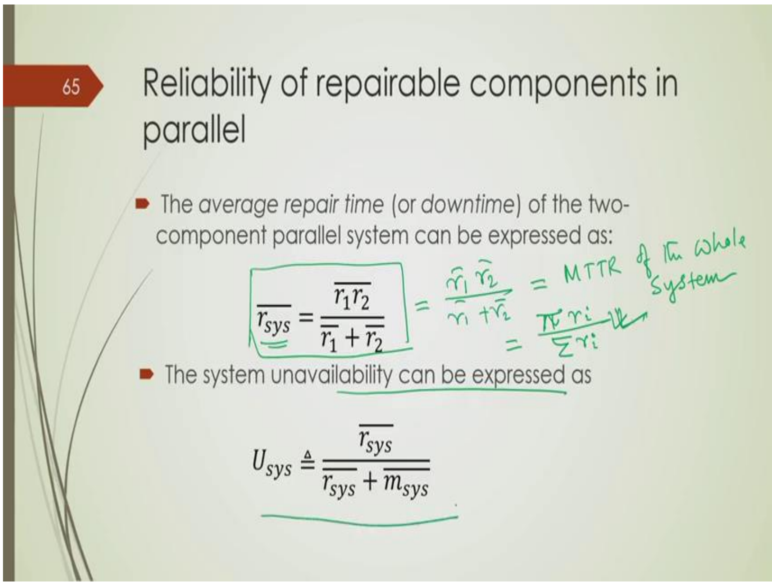

And if you equate then you will get the relationship of R sys as a function of r 1 r 2 bar this is actually r 1 bar r 2 bar divided by r 1 bar plus r 2 bar ok. So, here you get this mean time, so this is basically MTTR of the overall system which is equal to you can see and the numerator, it will follow this pi or product rule.

So, which will be equal to r i and the denominator will be this summation alright. Now, this needs to be verified whether it works for m number of system, you should verify it and unavailability of the system which can be found out from this.

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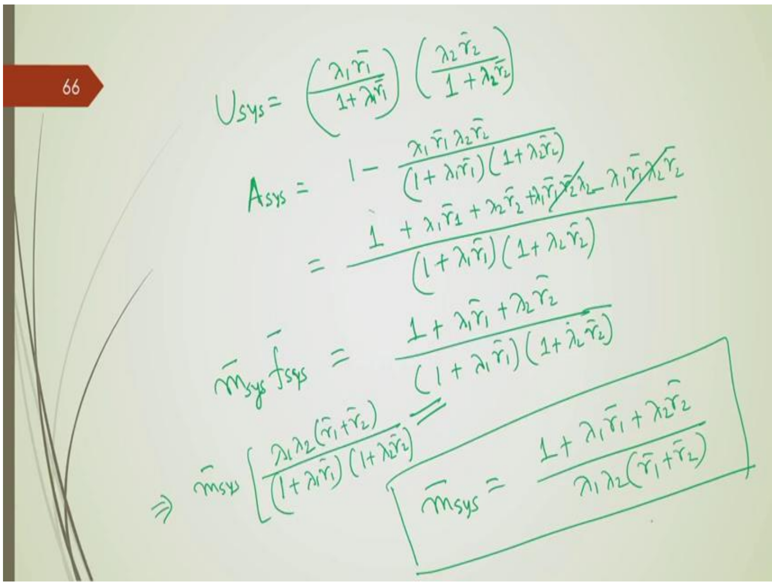

And this average time to failure, once you get similarly m sys bar also you can find out ok. So, how to find out this m sys bar? You can see that unavailability function is given over here that is this. So, if you write it here, unavailability of the system is equal to lambda 1 r 1 bar divided by 1 plus lambda 1 r 1 multiplied by lambda 2 r 2 bar divided by 1 plus lambda 2 r 2 bar, so this we got from here ok.

Now, this is basically representing this overall unavailability. So, availability will be equal to that is A sys will be equal to 1 minus this. So, this will be lambda 1 r 1 bar lambda 2 r 2 bar divided by 1 plus lambda 1 r 1 bar 1 plus lambda 2 r 2 bar ok. Now, we can simplify it like this which will give you 1 plus lambda 1 r 1 bar 1 plus lambda 2 r 2 bar.

So, this and this will cancel out so this will give you 1 plus lambda 1 r 1 bar plus lambda 2 r 2 bar, in the denominator it will be 1 plus lambda 1 r 1 bar 1 plus lambda 2 r 2 bar ok. Now this is what the availability of the system and we know that availability of the system is basically m sys bar multiplied by f sys bar ok.

And we know this expression of f sys from here that it is equal to lambda 1 lambda 2 multiplied by this. So, once you put it here, this will give you m sys bar, let me check again this f sys bar is equal to lambda 1 multiplied by lambda 2, r 1 plus r 2. And the denominator was 1 plus lambda 1 r 1 bar and 1 plus lambda 2 r 2 bar.

So, this if you equate with this expression that this denominators are identical. So, you get this value of m sys bar as 1 plus lambda 1 r 1 bar plus lambda 2 r 2 bar divided by lambda 1 lambda 2 r 1 bar plus r 2 ok. So, this is the expression that I have shown in the last slide, this is how to derive this alright.

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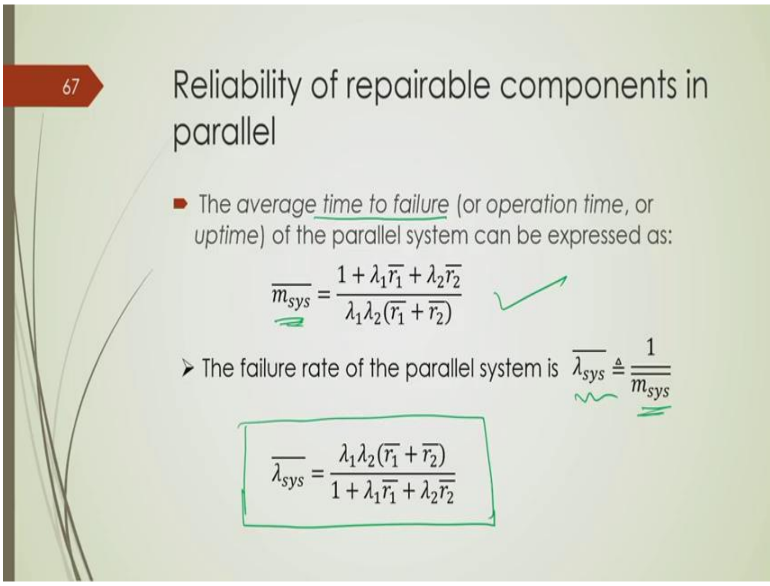

And as we know one upon m sys bar which gives this failure rate of the overall system. So, you can find out this is the failure rate the overall system ok.

So, I will stop today at this point we will continue next lecture.