

Operation and Planning of Power Distribution Systems
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Lecture - 15
Mathematical concept of reliability

So, in my last lecture I have shown you how to solve numerical problems related to the reliability indices and how to determine the numerical values of those indices and what sort of information is required to determine the values of those indices ok. So basically, if you follow my last lecture, you will understand that in order to find those reliability indices we need the documentation or we need the tabulation of the data related to a particular fault event.

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31 Exercise:

Load data

Load point	Number of customers	Average load connected (kW)
1	1800	8400
2	1300	6000
3	900	4600

$N_T = 4000$
 Given that, number of customers interrupted/affected = 2200
 $CN = 2200$

Annual interruption event data

Load point affected	Number of customers interrupted	Load interrupted (kW)	Duration of interruption (hr)
2	800	3600	3
3	600	2800	3
3	300	1800	2
3	600	2800	1
2	500	2400	1.5
3	300	1800	1.5

$\sum N_i = 3100$
 $\sum N_i r_i = (800 \times 3) + (600 \times 3) + (300 \times 2) + (600 \times 1) + (500 \times 1.5) + (300 \times 1.5)$
 $ENS = (3600 \times 3) + (2800 \times 3) + (1800 \times 2) + (2800 \times 1) + (2400 \times 1.5) + (1800 \times 1.5) = \dots$

Determine SAIFI, CAIFI, SAIDI, CAIDI, ASAI, ASIDI, ENS, AENS, ACCI

In fact, if I go back and show you that, here this load data is anyway known to us, but this annual interruption event data, this we need to tabulate. And this corresponds to different load point, different types of fault events and any particular load point. And correspondingly how many numbers of customers affected and how much load was interrupted and what was the duration of the event.

Particularly, these three important data we require in order to compute all these indices. Apart from that, we need to know that how many customers are there in, under this particular feeder service area. And also, how many customers are really affected that is

CN, due to this kind of interruption ok, but this, in order to compute these indices, we need this data otherwise we cannot compute that. So, this is one you know, one of the ways of reliability assessment, which practical utilities follow ok.

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Basic mathematical concepts of reliability: General Reliability function

- The probability of failure of a given component (or system) is defined as a function of time as:

$$P(T \leq t) = F(t) \quad t \geq 0$$

where,
 T is a random variable representing the failure time
 $F(t)$ is the probability that component will fail by time t

Handwritten notes:
[Reliability Engineering]
 $P(\cdot)$ → Probability function
 t : time
[$F(t)$: Failure function / unreliability function]

But apart from that we have some probabilistic approach or some theoretical approach to predict some predictive reliability approach to predict the reliability level of a particular distribution line ok, and this way I am going to discuss in next two-three lectures ok. So, this concept is applicable to the well-known area of engineering, that is called Reliability Engineering.

So, this concept, this basic mathematical concept of reliability, general reliability function, this concept comes from reliability engineering ok. So, this is taught in reliability engineering which is a separate branch of engineering many of you might be knowing. Now, first of all we have some definition, the first definition is the probability of failure; probability of failure ok and this probability function is defined as P function ok.

So, P stands for probability function ok. Now, how do we define this probability of failure for a given component? Now, this component might be anything this component might be any component connected to a power distribution network. It might be a distribution line, it might be a feeder line section, it might be a power transformer, it

might be the circuit breaker connected to a particular feeder or it might be any component connected to a distribution network line.

Now, the probability of failure of that component means how much probability is there to fail that component and that probability of failure is mathematically defined as a failure function that is called $F(t)$, where $F(t)$ is the probability that the component will fail by this time t ok.

So, we call $F(t)$ as failure function or we also call this, later on I will show you that is also called as unreliability function ok. Now, what is that P ? In bracket capital T is less than equal to small t , what does it signify? It means that; here capital T is the random variable representing the failure time, capital T is a random variable that represents the failure time ok.

So, the probability of this capital T is of less than equal to small t . So, if we consider this t stands for time and it is continuous ok, it is a continuous function. And the probability of this failure time at this instant of time t is basically represented by this failure function ok that is capital $F(t)$ ok alright.

So, where t is greater than equal to 0 so; that means, this expression ultimately represents the probability of failure, that is also known as failure function which is basically representing a component may fail at time, before this time T which is represented by small t at any particular instant of time and the probability of that failure is represented by this failure function ok. Now, how do we represent this failure function?

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Basic mathematical concepts of reliability: General Reliability function

Here, $F(t)$ is the failure distribution function, which can be called as the unreliability function. Therefore, the probability that the component will not fail at a given time t is defined as the reliability of the component. Thus, the reliability function can be expressed as:

$$R(t) = 1 - F(t)$$

where,
✓ $R(t)$ is the reliability function
 $F(t)$ is the unreliability function

Reliability function
+
Unreliability function
= 1

So, before I go for this failure function or failure distribution function, we also call this as an unreliability function which I described in my last slide and this unreliability function is related to somewhat a reliability function. So, whenever you have an unreliability function so there exist a reliability function which complement the unreliability function ok.

So, therefore, the component will not fail at a given time t is defined as reliability of the component and that is represented by this reliability function $R(t)$ and that is basically equal to complement of this failure distribution function or failure function. So, here it means that if we call this $F(t)$ as unreliability function, then basically you know that reliability function plus unreliability function is equal to 1 ok.

So, these things, one needs to understand. So, this gives that reliability is complements with unreliability function. Now, since reliability function is represented by capital $R(t)$. So, unreliability function is represented by capital $F(t)$ reliability, function that is capital $R(t)$ is equal to 1 minus capital $F(t)$ ok. Where, capital $R(t)$ is representing reliability function, capital $F(t)$ is representing unreliability function ok, this is the general concept.

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35 Basic mathematical concepts of reliability: General Reliability function

■ If the time-to-failure random variable T has a density function $f(t)$,

$$R(t) = 1 - F(t)$$

$$= 1 - \int_0^t f(t) dt = \int_t^{\infty} f(t) dt$$

$$= \int_0^{\infty} f(t) dt - \int_0^t f(t) dt$$

$$= \int_0^{\infty} f(t) dt + \int_t^{\infty} f(t) dt = \int_t^{\infty} f(t) dt$$

where $f(t)$: failure frequency

failure probability distribution function

$F(t) = \int_0^t f(t) dt$

$R(t) = \int_t^{\infty} f(t) dt$

Now, how do you represent this capital F t ? Capital F t is represented as capital F t it is not mentioned here, capital F t is represented by integration of 0 to t small f t dt ok small f t dt . Where, small f t is representing failure frequency ok. So, as we have seen this reliability function which is equal to 1 minus this failure distribution function or failure function.

So, since F t is defined as integration of 0 to t small f dt , where small f t is representing failure frequency or failure probability distribution probability or failure probability distribution function, failure frequency or failure probability distribution function.

So, if you integrate this with 0 to t with a limit of 0 to t , whatever you will get that is representing failure distribution function and that represents the probability of failure of any particular component on or before this time instant small t , ok. Then we can also find out this reliability function capital R t as a function of this failure probability distribution function that is small f t .

How to determine? So, simply we know that reliability function is equal to 1 minus this failure function that is failure distribution function ok, and failure distribution function is integration of 0 to t small f t dt where, small f t is a probability density function or probability distribution function.

Now, since we know the integration of any kind of probability a density function with a limit 0 to infinity will give you 1. So that means, we know, we can represent 1 as $\int_0^{\infty} f(t) dt$ and this is already we know that it is $\int_0^t f(t) dt$ ok. Now, if we change the limit of 0 to t. So, we can write it as $\int_0^{\infty} f(t) dt$ plus $\int_t^0 f(t) dt$ ok.

Now, this basically gives you, there is a mistake over here that $R(t)$ is equal to $\int_t^{\infty} f(t) dt$; $R(t)$ is equal to $\int_t^{\infty} f(t) dt$; that means, this is equal to $\int_t^{\infty} f(t) dt$ ok. So, $R(t)$ we got as integration of $\int_t^{\infty} f(t) dt$ ok and $F(t)$ we already know that it is equal to $\int_0^t f(t) dt$. Where, f is your probability distribution function or probability density function which is called as failure probability density function ok.

So, we got the relationship of failure probability density function as a function of reliability function as well as unreliability function or failure function.

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Basic mathematical concepts of reliability: General Reliability function

The probability of failure of a given system in a particular time interval (t_1, t_2) can be written in terms of the unreliability function, as:

$$\begin{aligned}
 \int_{t_1}^{t_2} f(t) dt &= \int_0^{t_2} f(t) dt - \int_0^{t_1} f(t) dt = F(t_2) - F(t_1) \\
 &= 1 - R(t_2) - [1 - R(t_1)] \\
 &= \underline{R(t_1) - R(t_2)}
 \end{aligned}$$

Now, we can also find out this probability of failure in a particular time interval, that is t_1 to t_2 ok. So, probability of failure at a particular time interval that is t_1 to t_2 we can write it as a limit t_1 to t_2 integration of $f(t) dt$ ok. So, this gives you the probability of failure of any particular system or any particular equipment within a time interval t_1 to t_2 ok and this can be written as by changing limit, this can be written as this.

And this one, you know, it is nothing but capital F t 2 that is probability of failure at time t 2 and this is nothing but probability of failure at time t 1.

And we can alternatively represent this probability of failure of any given system by using reliability function because reliability function, you know, F t 2 is basically equal to 1 minus R t 2 minus, you know that F t 1 is represented as 1 minus R t 1 ok. So, 1 and 1 gets cancelled out. So, you will get this is equal to R t 1 minus R t 2 which can be also shown by changing this limit.

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37 Basic mathematical concepts of reliability: General Reliability function

- The probability of failure of a given system in a particular time interval (t_1, t_2) can be written in terms of the **reliability function**, as:

$$\int_{t_1}^{t_2} f(t)dt = \int_{t_1}^{\infty} f(t)dt - \int_{t_2}^{\infty} f(t)dt = R(t_1) - R(t_2)$$

And it is represented by this, because as we know that, f t dt integration of f t dt at a time interval t 1 t 2 can be represented by integration of f t dt at a time interval of t to infinity and minus this integration of f t dt at a time interval of t 1 to infinity ok. Now, we know that; we already know that this reliability function it is equal to this R t is equal to f integration of f t dt where limit is t to infinity.

So here, this is basically representing this reliability function of t 1 and this is basically representing reliability as a function of t 2. So, we can get this and alternatively we got the same result in the last slide alright.

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38 Basic mathematical concepts of reliability: General Reliability function

The rate of failures in a given time interval (t_1, t_2) is defined as the hazard rate, or failure rate, during that interval. It is the probability that a failure per unit time happens in the interval, provided that a failure has not happened before the time t_1 , i.e., at the beginning of the time interval. Therefore,

Instantaneous failure rate $\rightarrow h(t) = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)}$

Now, we will define another term which is called hazard rate, or failure rate which is very important to understand. What is hazard rate and failure rate? This is basically the rate of failures in a given time interval t_1 to t_2 . So, we already have seen in the last slide how to represent the reliability function or the failure distribution function at a time interval t_1 to t_2 .

Now, this hazard rate or failure rate it is representing the rate of failures in a given time interval t_1 to t_2 ok. So, it is the probability, it is defined as the probability that a failure per unit time happens in that interval ok, provided that there is a condition; provided that a failure has not happened before this time interval t_1 .

So, this is something like you can understand, suppose this is continuous time t ok and this is the interval t_1 to t_2 . So, this is the time interval t_1 to t_2 . So, this failure rate is basically representing the probability of failure per unit time, probability of failure per unit time which happens within this interval t_1 to t_2 , provided that, this is a, there is a conditional probability, this is also a conditional probability provided that the failure has not happened before this time t_1 .

So, before this interval we assume that there is no failure, the component was fully reliable ok and that is why it is represented as mathematically you know that difference of reliability $R(t_1)$ minus $R(t_2)$ with a unit time interval that is t_2 minus t_1 divided by $R(t_1)$

1. Where, $R(t)$ is equal to 1 if this condition is satisfied that provided that a failure has not happened before the time interval, time t ok.

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Basic mathematical concepts of reliability: General Reliability function

Let, the time interval be defined as: $t_1 = t, t_2 = t + \Delta t$

$\Delta t \rightarrow$ duration of the interval

$$h(t) = \lim_{\Delta t \rightarrow 0} \frac{R(t) - R(t + \Delta t)}{\Delta t \cdot R(t)}$$

Δt is a very very small duration

$$= \frac{1}{R(t)} \left[-\frac{dR(t)}{dt} \right]$$

$$\boxed{h(t) = \frac{f(t)}{R(t)}}$$

Where, $f(t)$ is the probability density function

$$\frac{dR(t)}{dt} = -f(t)$$

$$R(t) = 1 - \int_0^t f(t) dt$$

$$-\frac{dR(t)}{dt}$$

Now, let us reorient this interval, let us consider this t_1 as t and t_2 is t plus Δt ; that means, this duration of the interval is represented by Δt . So, this is the duration of the interval ok. Now, according to this definition of this hazard rate which is an instantaneous failure rate. Since, it is a time varying function. So, we call it also instantaneous failure rate.

We call it also instantaneous failure rate, it is represented by $R(t_1) - R(t_2)$ divided by this time duration that is $t_2 - t_1$ divided by this $R(t_1)$ ok. Now, since we reorient this t_1 and t_2 . So, we replace this t_1 by small t and t_2 by small t plus Δt , where this Δt is very very small infinitely small and that is why this limit exist; limit Δt stands to 0, it represents this Δt is very very small duration, very very small duration, is of very very small duration or infinitely small duration ok.

Now, with this limit you know this $R(t) - R(t + \Delta t)$ divided by Δt it is representing the rate of change of $R(t)$. So, the rate of change of $R(t)$ $\frac{dR(t)}{dt}$ is equal to $R(t + \Delta t) - R(t)$ divided by Δt where limit Δt tends to 0, this is the definition of the differentiation that we know ok.

Now, since why this negative term comes? Because it is difference of $R(t)$ minus $R(t + \Delta t)$. So, if you take the difference $R(t + \Delta t) - R(t)$, there is a negative term coming. So, we get this instantaneous failure rate or hazard rate is a function of this rate of change of reliability function $R(t)$ divided by $R(t)$, there is a negative figure.

Now, what is that rate of change of reliability function? So, this we can find out because we know that relationship of $R(t)$ with $F(t)$, we know that; let us go back and see we know that $R(t)$ is equal to $1 - F(t)$. So, from this we can find out the relationship of $R(t)$ with $F(t)$.

So, if we rewrite here again, that $R(t)$ is equal to $1 - \int_0^t f(t) dt$. Now, if we take the $dR(t)/dt$, if we take the differentiation first order differentiation. So, we get this is equal to $-f(t)$ and we replace this with this minus $dR(t)/dt$ and thereby you get this instantaneous failure rate or hazard rate is coming out to be $f(t)$, $dF(t)/dt$ divided by $R(t)$.

So, it is coming out to be the ratio of $f(t)$ and $R(t)$. So, instantaneous failure rate is found to be the ratio of this failure probability density function $f(t)$ to this reliability function $R(t)$, alright. Now, we also know $R(t)$ is equal to $1 - F(t)$.

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40 Basic mathematical concepts of reliability: General Reliability function

By substituting $R(t)$ by $1 - F(t)$, we get,

Since we know $R(t) = 1 - F(t)$,
$$h(t) = \frac{f(t)}{1 - F(t)}$$

Hence,
$$\int_0^t h(t) dt = \int_0^t \frac{dF(t)}{1 - F(t)}$$

Hence,
$$\int_0^t h(t) dt = -\ln[1 - F(t)] \Big|_0^t = -\ln[1 - F(t)] + \ln[1 - F(0)]$$

Since $F(0) = 0$, $1 - F(0) = 1$, $\ln(1) = 0$,
$$1 - F(t) = \exp\left[-\int_0^t h(t) dt\right]$$

Taking derivatives of this Equation, we get,
$$f(t) = h(t) \exp\left[-\int_0^t h(t) dt\right]$$

$$f(t) = h(t) e^{-\int_0^t h(t) dt}$$

$$R(t) = e^{-\int_0^t h(t) dt}$$

Handwritten notes on the right side of the slide:

- $F(t) = \int_0^t f(t) dt$
- $\frac{dF(t)}{dt} = f(t)$
- $\Rightarrow \frac{dF(t)}{dt} = f(t)$
- $F(0) = 0$
- $1 - F(t) = e^{-\int_0^t h(t) dt}$
- $R(t) = e^{-\int_0^t h(t) dt}$

So, we replace it here because since we know $R(t)$ is equal to $1 - F(t)$. So, we replace this $R(t)$ by $1 - F(t)$ and we get the relationship of hazard instantaneous failure rate or hazard rate as this failure distribution function or failure density function to divided by 1

minus $F(t)$ ok, alright. And we also know the relationship of this $f(t)$ with this capital $F(t)$. Since, we know that $F(t)$ is equal to $\int_0^t f(t) dt$.

So, $dF(t)$, if we differentiate with respect to t , this will be equal to $f(t)$ or $dF(t)$ is equal to $f(t) dt$ ok, alright. Now, we replace this small $f(t)$ with capital $F(t)$ and this dt is basically it is supposed to be equal to small $f(t)$ is basically equal to $dF(t)$. So, this dt is placed in the left-hand side. So, that is representing $\int_0^t h(t) dt$ and right-hand side is $dF(t)$ divided by $1 - F(t)$ ok.

Now, we will integrate this both sides, both left-hand side and right-hand side with an interval of 0 to t . Now, if we integrate $h(t)$ that is instantaneous failure rate with a limit of 0 to t , what actually we represent? We will basically represent the integration of this all, instantaneous failure rate at a time span 0 to t . So, this is represented in the left-hand side ok.

And right-hand side it will be logarithm of $1 - F(t)$ with a time interval of 0 to t , because if we integrate this with respect to $dF(t)$ with a limit of 0 to 2 ; that means, we are integrating 0 to t both sides ok. So, this side will be, I will come to that what it would be, but right side would be equal to $-\ln(1 - F(t))$ from 0 to t ok, alright.

Now; that means, it is, it will be equal to $-\ln(1 - F(t))$ divided by $1 - F(0)$ ok, $1 - F(t)$ divided by $1 - F(0)$, ok. Now, how to find out what is $F(0)$? $F(0)$, as we know that $F(t)$ is equal to $\int_0^t f(t) dt$. Now at this F is equal to 0 both the limits will be equal. So, this will give you 0 .

So; that means, this will give you minus this denominator will be equal to 1 because $F(0)$ is equal to 0 . So, this will be equal to 1 . So, this is basically representing $-\ln(1 - F(t))$ ok. Now, this is what in the left-hand side, it is integration of $\int_0^t h(t) dt$, Now, we keep this left-hand side as it is and we will let you know the significance of that integration of $h(t)$ within a interval 0 to t later on ok.

Now, if we replace this logarithm and this will be $1 - F(t)$ will be equal to e to the power this negative will come to this other side and this will be exponential of $-\int_0^t h(t) dt$. So, this is the right-hand side of this, that this function and we kept it as it is. So ultimately, we get $1 - F(t)$ is equal to $\exp(-\int_0^t h(t) dt)$, alternatively

we can represent it as $1 - F(t)$ is equal to $e^{-\int_0^t h(t) dt}$, this exponential means it is equal to $e^{-\int_0^t h(t) dt}$.

So, this is the equation we got. And from this we can also get the relationship of $R(t)$ with respect to this $e^{-\int_0^t h(t) dt}$ term. So, since $1 - F(t)$ is nothing but $R(t)$. So, $R(t)$ we can write it as $e^{-\int_0^t h(t) dt}$ directly we can write it, $e^{-\int_0^t h(t) dt}$ alright ok.

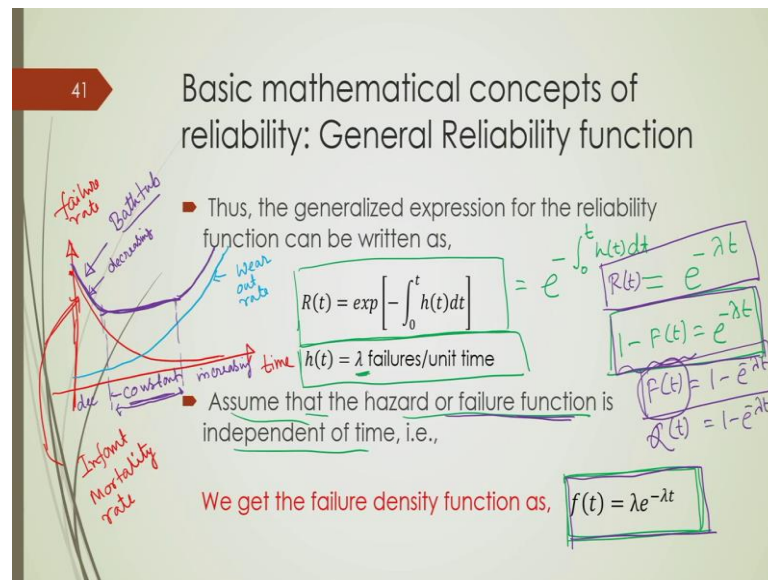
So, we get the failure function as a function of this hazard rate, we also get the reliability function as a function of this hazard rate ok. And we can we also know that, we can also find out what would be the relationship of this integration of $\int_0^t h(t) dt$ as a function of this failure distribution function.

So, in order to find this as we know, if we simply take the derivative of this reliability function, if you multiply it with minus 1 then we will get the failure distribution function and that is how we got this failure distribution function, by simply deriving, by simply taking the derivative and multiplying with minus 1.

So, we will get it as $f(t) = h(t) e^{-\int_0^t h(t) dt}$. So, this exp is basically representing $e^{-\int_0^t h(t) dt}$ term. So, this is basically equal to $e^{-\int_0^t h(t) dt}$ ok. So, these three equations are important, one is this equation which relates this failure distribution function with hazard rate, instantaneous hazard rate and this is the second important relationship which gives the reliability function $R(t)$.

Of course, it is also an instantaneous reliability function as a function of instantaneous hazard rate and this gives the relationship of a failure probability distribution function or failure probability density function as a function of instantaneous hazard rate. Now, we will be doing some further analysis on these three equations ok.

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First of all, these equations we got already in the last slide that this reliability function is nothing but it is equal to e to the power minus integration of 0 to t integration of instantaneous hazard rate ok. Now, if we consider a case where that hazard rate is assumed to be constant, hazard rate is assumed to be constant with a value of λ ok.

So, we assume that this hazard or failure function is independent of time that is constant. So, if we consider so; so, what we will get? We will get this failure density function is $f(t)$ is equal to λe to the power minus λt , because if we consider $h(t)$ is constant that is λ .

So, this integration will result in minus λt . So, this will be equal to e to the power minus λt . So, $R(t)$ will be equal to e to the power minus λt and $R(t)$ is 1 minus $F(t)$. So, 1 minus $F(t)$ will be equal to e to the power minus λt ok and this failure distribution function which was equal to $h(t) e$ to the power minus 0 to t integration of $h(t) dt$ which results in minus λt .

So, this failure, this probability density function will come out to be small $f(t)$ as λ multiplied by e to the power λt . Now, what actually this analysis results into? Which is very important? Which actually this analysis result into? Now, if we plot this failure rate with respect to time, with respect to time, then we will get a characteristic that is called bathtub characteristics, this looks like a bathtub ok.

We will see there are two components which basically influence the failure of any particular equipment ok. One component is fast decreasing like this, one component is fast decreasing like this with time and it has initial higher failure rate and with t is equal to infinity or t very close to higher value of time it comes out to be 0.

That is basically called as, this one is called as infant mortality rate. What actually it does? It means that the initially when equipment is manufactured, there is a high probability that this equipment may fail if there is an error in the design or if there is an error in manufacturing ok. And if it is sort out at a very early stage and it would be of course discarded ok.

And that is how it is having high failure rate, probability of high failure rate during this initial phase of time and then it slowly in decrease in trend and there is another component of this failure, that is called as wear out rate and that probability rate is increasing with respect to time.

So, this is called wear out rate and this will basically be influenced by the aging of any component. So, with time all the components are aging and there exist a certain point beyond which this aging effect is more predominant and it will lead to the failure of a particular equipment.

So, wear out rate increasing with this time ok and actual probability of failure is summation of these two. So, if we sum up this, what sort of characteristics we will get? We will get a characteristic like this. So, these characteristic is called bath tub characteristic that is called bath tub characteristics ok.

So, if you look at this characteristic there are three regions, one region is you are having this decreasing failure rate, this is this region where this is decreasing failure rate. There is a region where you have this constant failure rate, this is constant failure rate, this is decreasing failure rate and there is a up to a certain time there is an increasing failure rate ok.

So, there are three regions and this period where this failure rate more or less constant is called the useful life of that particular equipment. So, this is the concept of reliability engineering ok. If you study reliability engineering this concept will be taught to you.

Now, this region where this failure rate is almost constant is called the useful lifetime, useful life of a particular equipment; useful life of a particular equipment.

So, during this useful life we may assume that this failure rate is constant as per our assumption and therefore, all our assumptions are would be valid during that specific period of time. And if we consider so, that is almost constant failure rate we can also determine the reliability function with the constant failure rate, we can also find out unreliability function or failure distribution function as a function of this constant failure rate.

And we can also find out how would be the failure probability density function as a function of constant failure rate ok.

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42 Basic mathematical concepts of reliability: basic single-component concepts

*Expectation Concept of Probability Theory

Theoretically, the expected life, that is, the expected time during which a component will survive and perform successfully, can be expressed as:

$$E(T) = \int_0^{\infty} t f(t) dt$$

$$E(T) = - \int_0^{\infty} t \frac{dR(t)}{dt} dt$$

$$E(T) = -tR(t) \Big|_0^{\infty} + \int_0^{\infty} R(t) dt$$

$$= -[\infty \cdot R(\infty) - 0 \cdot R(0)] + \int_0^{\infty} R(t) dt$$

$$= [0 - 0] + \int_0^{\infty} R(t) dt$$

$$E(T) = \int_0^{\infty} R(t) dt$$

since, $R(t=0) = 1$

$f(t) = -\frac{dR(t)}{dt}$

$R(t) = \int_t^{\infty} f(t) dt$

$R(0) = 1$

$R(\infty) = 0$

$E(T) = \int_0^{\infty} R(t) dt$

And we can also find out the expected life period of that particular equipment. What do you mean by that expected life period? This concept comes with the concept of expectation concept of probability theory. This concept comes from expectation concept or of this probability theory which is well known and this is represented by capital E or sometimes one symbol is used, this type of symbol is used to represent expected outcome of a particular event in a probability theory ok.

Now, how to represent this expected time or expected live period or the definition of, expected live period is basically the expected time during which a component will

survive and perform successfully, which is very important to us. This should be capital T; this capital T is basically representing as we have shown you here that is basically representing a random variable representing the failure time ok.

So, this expected value of this capital T can be obtained as $\int_0^{\infty} t f(t) dt$ with a limit of 0 to infinity. Now, we know already $f(t)$ is equal to $-\frac{dR}{dt}$, already we got $f(t)$ is equal to $-\frac{dR}{dt}$. So, this we got from this particular relation. And if we put over here so what we will get? E capital T is equal to $\int_0^{\infty} t dR(t)$ ok.

Now, if you integrate it, then you will get E capital T is equal to $-\int_0^{\infty} t R'(t) dt$ with a limit of 0 to infinity plus integration of $R(t) dt$ with a limit of 0 to infinity ok. Now, we can find out what would be that $R(0)$ and what would be $R(\infty)$ ok, what would be the value of $R(0)$ and what would be the value of $R(\infty)$.

Now, we already know this $R(t)$ is equal to $\int_t^{\infty} f(t) dt$ ok, this is something that we know. Now, if we put t is equal to 0, then this limit would be 0 to infinity. Now, if you integrate any probability density function with a limit of 0 to infinity this will gives you 1.

Now, what would be this when R reliability function at t is equal to infinity? If you put t is equal to infinity both the limits will be equal. So, this will be equal to 0 ok. Now, if you look at this part; so, this part gives you $-\int_0^{\infty} t R'(t) dt$ multiplied by $R(\infty)$ and 0 multiplied by $R(0)$.

Now, since we know that this $R(\infty)$ is equal to 0. So, this part will be equal to 0 multiplied by infinity and this part will be equal to 0 multiplied by 1 which will result in 0 ok. So, this part will be 0. So, ultimately we get E capital T; that means, expected value of this capital T is equal to $\int_0^{\infty} t R'(t) dt$ this we get ok.

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43 Basic mathematical concepts of reliability: basic single-component concepts

Therefore, the expected life can be expressed as:

$$E(T) = \int_0^{\infty} R(t) dt \quad \text{or} \quad E(T) = \int_0^{\infty} \exp \left[- \int_0^t \lambda(t) dt \right] dt$$

$$= \int_0^{\infty} e^{-\lambda t} dt$$

When there is a constant failure rate,

$$E(T) = \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda}$$

Now, this is the right one. So, E capital T is equal to 0 to infinity $R(t) dt$ or expected value of this capital T is equal to 0 to infinity. Now, we know this $R(t)$ is equal to in terms of this constant failure rate, $R(t)$ we obtained as $e^{-\lambda t}$. Now, if we put it that $R(t)$ is equal to $e^{-\lambda t}$, that is, what we will get 0 to infinity $e^{-\lambda t} dt$ and this gives you $1/\lambda$ ok.

So, ultimately and this for a constant failure rate, the expected time of this you know expected life period will be equal to $1/\lambda$ which gives the time of that particular equipment to fail, which gives the expected lifetime, expected period in which this component will survive and operate satisfactorily.

Now, before that I will also show you the relationship of this reliability function and unreliability function. You can see that reliability function we got as $R(t)$ is equal to $1 - \lambda t$ and unreliability function which is represented by $F(t)$ is equal to $1 - R(t)$.

So, if we represent $F(t)$. So, this will be equal to $1 - e^{-\lambda t}$ ok. Now, this is also called unreliability function, sometimes it is represented by capital $Q(t)$. So, capital $Q(t)$ is equal to $1 - e^{-\lambda t}$ ok.

Now, if we have an exponentially decaying probability density function, suppose this is your failure probability and it is, of an exponentially decaying function this axis is time,

this is where t is equal to 0 and this is where t is equal to small t ok alright. Now, you know that the definition of this unreliability function what actually it gives, it gives your unreliability function that is $F(t)$ is equal to integration of 0 to t of $f(t) dt$ where this one is basically nothing but failure probability density function ok.

So, if you integrate this particular function with a interval of 0 to t whatever value you will get that will represent the unreliability or that will represent the probability of the failure. So, this area under this particular curve will represent your unreliability function, unreliability probability ok.

And we also know that reliability function $R(t)$ is equal to $\int_t^\infty f(t) dt$, this is also something which is known to us; that means, what do you mean by this integration of $f(t)$ from t to infinity? This is basically the area under this particular curve, area under this particular curve from starting from t to infinity. So, this gives you a reliability probability ok.

So, this is something you should know and summation of this reliability and unreliability will give you 1.

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Basic mathematical concepts of reliability: MTTF and MTBF

- ▶ If the system is simply replaced by a good system (i.e., no maintenance required) the $E(T)$ useful life is also defined as the mean time to failure (MTTF) and denoted as:

$$MTTF = \bar{m} = \frac{1}{\lambda}$$

$\lambda \rightarrow$ constant failure rate
- ▶ If the system has undergone maintenance and repairs the useful life, $E(T)$ is also defined as the mean time between failures (MTBF) and denoted as:

$$MTBF = \bar{T} = \bar{m} + \bar{r}$$

$(-) \sim$ mean value of successive cycles

Where

\bar{T} is the mean cycle time
 \bar{m} is the mean time to failure
 \bar{r} is the mean time to repair

} non-repairable
 } repairable

Now, we will talk about a single system, we will talk about a single system and it is E capital T , that is useful life of or expected life on which the system will survive, a system will work satisfactorily.

So, what would be that E capital T, for a single component already we determine that its useful time is equal to $1/\lambda$, where λ is our constant failure rate, this we derived in fact, last slide this ok and this is called Mean Time to Failure MTTF ok.

So, if we have any equipment or any system which is a good system and it would be periodically replaced, there will be no maintenance required then the useful life for that particular equipment is also defined as mean time to failure, mean time to failure. So, this is for that kind of equipment which are not repairable or which are not repaired ok.

So, they will be rather replaced with a new equipment. So, this definition mean time to failure is basically the useful life for that equipment which are not repaired, which are not to be repaired, but rather replaced with a new one ok, and that is equal to $1/\lambda$ divided by this λ , λ is failure rate.

Now, if we have some equipment which will undergo some maintenance and repairs ok, these equipments are repairable. So, this equipment is non repairable or unrepairable and this equipment are repairable ok, for that equipment the expected life time is also defined as mean time between failures ok.

Because I will show you for this type of equipment, we will undergo certain ups and downs, ups mean for some period of time they will be in operation, they will operate satisfactorily or they will not fail and for some period of time they will be under repair ok.

And during that time, they are not in operation, they are not operating. So, this type of system will undergo some cycles of satisfactory operation and repair followed by this repair, then again, some satisfactory operation followed by repair and so on.

So, this if that kind of system which is repairable, this mean time or useful life is also defined as the Mean Time Between Failure that is called MTBF in short, MTBF stands for Mean Time Between the Failure which is represented by \bar{T} , here this bar symbol means it is mean value. So, this bar symbol is representing mean value, mean value of successive cycles.

So, for this kind of system there is repetitive cycles of operation and repair. So, one particular time period is obviously, summation of this operating period plus repairing

period. And of course, this mean time of before this failure is this is represented by T bar which is equal to this m bar plus r bar. Where, m bar is mean time to failure and r bar is mean time to repair ok.

And capital T bar is representing mean cycle time, mean cycle time, I will illustratively explain what it is.

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Basic mathematical concepts of reliability: MTTR

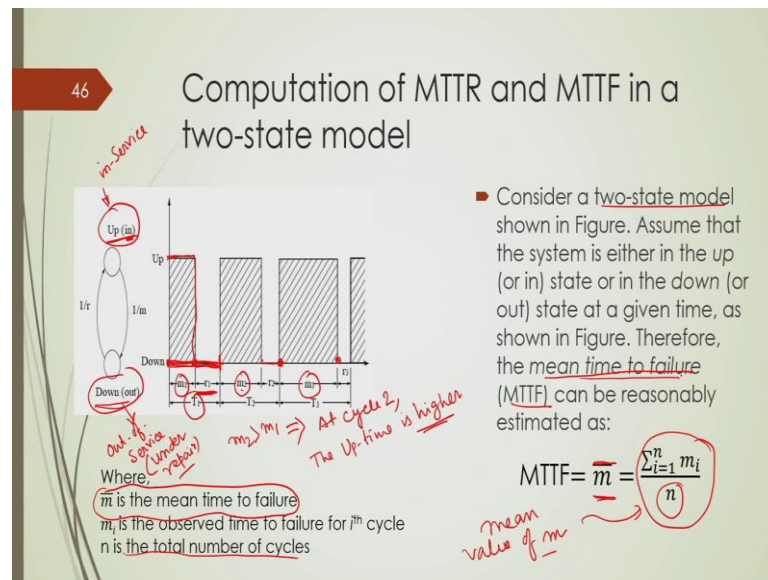
- the mean time to repair (MTTR) is defined as the reciprocal of the average (or mean) repair rate and denoted as:

$$MTTR = \bar{r} = \frac{1}{\mu}$$

where, μ is the mean repair rate.

Now, there is another definition that is called Mean Time To Repair, that is MTTR, which is defined as the reciprocal of average repair rate and it is denoted by r bar and it is equal to 1 upon mu where mu is the mean repair rate ok which is the mean repair rate.

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Now, we have an example of a two-state model, two-state model means we have one particular equipment which will be either up upstate or down state. So, there are two states for that particular equipment, either it will be in upstate or it will be in down state, upstate means it is analogous to in-service.

So, upstate means in-service state and downstate means out of service, out of service state ok and this is basically representation of this up and down. So, during this period of time it is operated successfully or satisfactorily, then it undergoes some fault or some requirement of maintenance and this is the period where this maintenance is done that is represented by small n ok.

And this starting from this point to that point it creates one particular cycle that is represented by capital T 1, capital T is, already I mentioned, it is representing this cycle time and T bar is representing the mean cycle time. So, this one is particular cycle. So, starting from this period that is T is equal to 0 to this period then, similar type of cycles repeated ok, multiple times.

It may so happen sometimes you know that part, same equipment can be operated in even more time satisfactorily, where you can see m_2 is higher than m , which is basically representing that at cycle 2 the up time is higher, it means that during this cycle two that particular equipment can be operated more time than the previous cycle and then you can see this repair time also is less, it is also possible.

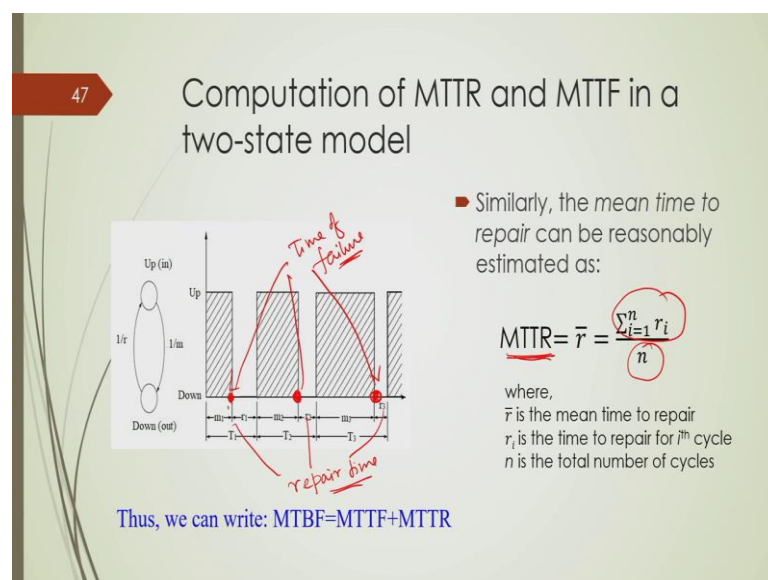
So, this m and r they are basically representing that the duration which that particular equipment will be in service and out of service or that means, out of service means it is under repair ok. Now, this duration may vary from one cycle to another cycle, sometimes that same equipment will be having more duration up time or in service time or sometimes it will be of less.

Similarly, sometimes a repair time would be higher, sometimes repair time would be lower. So, that is why we take mean value of this in service time, it is also called mean time to failure and that is represented by \bar{m} ok. So, it is nothing but if we have n number of cycles, your n is basically representing the total number of cycles ok.

Now, if n is 1, n would definitely have some finite value. So, if you sum up all this m value for different cycles and you divide it by n , then this basically representing mean value of m , mean value of m and this is represented by \bar{m} ok and this is only this mean time to failure because that much time it requires.

So, m means in third cycle that much duration is required before that particular equipment is failed or before that particular equipment is out of service and that is why it is called mean time to failure.

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Similarly, for the same equipment in two state-model only mean time to repair is basically equal to summation of all r , r here is representing this repair times, this r is

representing repair time. And that may vary cycle to cycle depending upon what sort of problems are there and what sort of maintenance is required. And, if you sum up all this duration, repair duration and divide it by this number of cycles that is n this will represent Mean Time To Repair, that is MTTR ok.

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Mean cycle time

- The mean cycle time defines the average time that it takes for the component to complete one cycle of operation, that is, failure, repair, and restart.

$$\bar{T} = \bar{m} + \bar{r}$$

$$\bar{T} = \frac{1}{\lambda} + \frac{1}{\mu} \quad \text{or,} \quad \bar{T} = \frac{\lambda + \mu}{\lambda\mu}$$

- The reciprocal of the mean cycle time is defined as the mean failure frequency and denoted as:

$$\bar{f} = \frac{1}{\bar{T}}$$

$$\bar{f} = \frac{\lambda\mu}{\lambda + \mu}$$

And as I said that mean time between this failure; that means, here you can see this is one particular point where there is a failure and then this is the duration of repair and then up to this time this system will operate satisfactorily. So, mean time between two successive. So, you can look at this dot where this failure will occur or we need this maintenance for this particular equipment.

So, these are representing the time of failure which basically necessitates the repair of that particular equipment ok. Now, this mean time of this between two failures; so, this is one failure this is failure 1, this is failure 2, this is failure 3. So, in between two failures like here to there, you see it is equal to m plus r ok.

So, mean time between two failure is basically equal to capital T which is summation of m bar plus r bar ok and we can represent this m bar by 1 upon λ where λ is constant failure rate and r bar by 1 by μ where μ is the repair rate and we can find out this.

If we take the reciprocal of this \bar{T} , $1/\bar{T}$ then what we will get that is called mean failure frequency, mean failure frequency and that can be also represented by in terms of λ and μ . If you simply take the reciprocal of the time period you will get the frequency and since it is the time period for between two falls. So, its reciprocal gives the failure frequency ok.

So, with this I will stop today.