

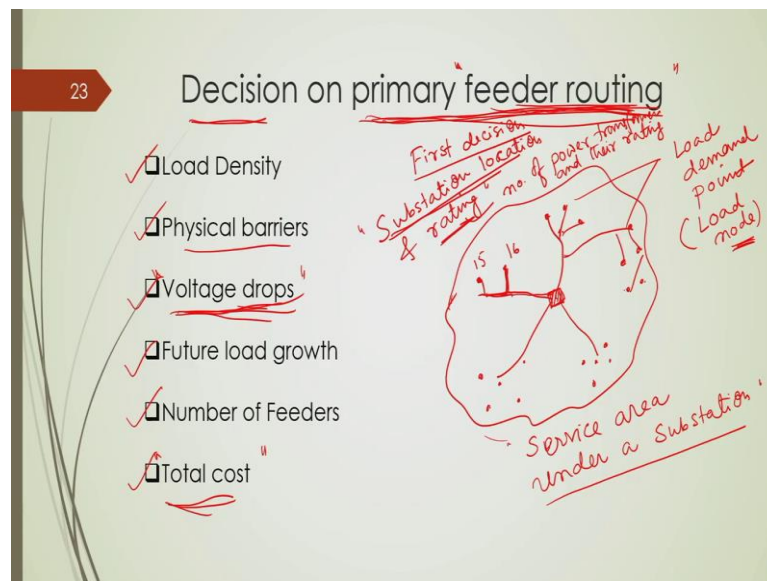
Operation and Planning of Power Distribution Systems
Dr. Sanjib Ganguly
Department of Electronics and Electrical Engineering
Indian Institute of Technology, Guwahati

Lecture - 10

Voltage drop and power loss computations for typical radial distribution feeders

So, in my last lecture, I discuss some design aspects of typical distribution substations, different bus bar schemes, and the typical design aspect of radial distribution network. Design in the sense how this radial distribution network is usually designed and it works ok. And, I also discuss something about sectionalizing switch and tie line ok. So, in this particular lecture I will start some analyzing aspect or some basic analysis of a typical radial distribution feeder ok.

(Refer Slide Time: 01:22)



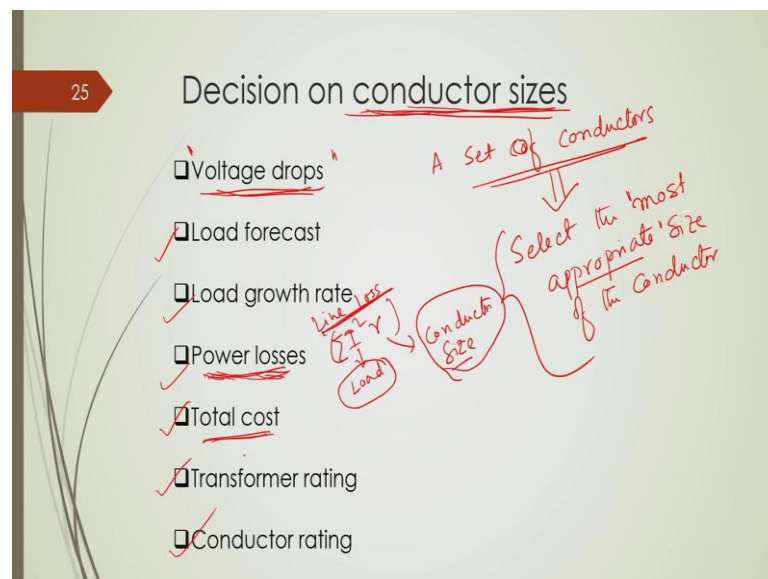
So, let us start. So, before I start this some analytical aspect of radial distribution network I will talk about three important aspects while designing a distribution network as a whole. There are three important decisions involved in designing this radial distribution network as a whole ok. So, these three decisions are: one is decisions on primary feeder routing.

(Refer Slide Time: 01:54)



One is decision on number of feeders.

(Refer Slide Time: 01:56)



Another is decision of conductor sizes. So, these three are important decisions that one distribution network planner should take or should decide based upon certain aspects. Now, what are the parameters? Which will directly influence these decisions? Those things I will just mention and in next few lectures; you will be able to understand that why they are so important; they are so important.

And, whenever I will talk about the different models, different planning models of power distribution system those decisions are very much crucial and those decisions sometimes are formulated as the decision variable of an optimization problem. And the optimal value of those decisions is determined through an optimization process.

So, but this we will not be going to discuss right now; we will be discussing this whenever we will discuss on power distribution planning models ok. At this point of time one should understand that what do you mean by feeder routing and off-course, number of feeders one can understand it nothing but how many feeders we should have and that also the decision on this conductor sizes.

These three important decision; apart from that there is another decision involves; one is that is basically the location of the substation. Now, in order to understand that what is feeder routing, let us consider that we have a service area. This one is basically the service area under a typical substation, service area under a substation ok.

Now, you can assume that whenever this IIT campus was in the developing phase to build; off-course, this area was the area under this campus ok. And we have different loading points which are scattered all over this service area which might be somewhere here, which might be somewhere here, which might be here, here and here. So, these are loading points or load demand points or wherever the customers need this power supply.

So, this black these dots, are basically representing load demand point or alternatively you can write as load node. Now, we design a distribution network basically in order to provide this supply power supply to those load points, ok. Now, here the first decision that one distribution planner needs to take is that first decision is substation, location and rating this is typically the first decision that. Where should I place the substation and what should be the rating?

Off course, substation rating depends upon how many power transformers are there ok so, and what are the ratings of those power transformers. So, if we have let us say two 10 MVA transformers so; obviously, the substation rating will be 2 multiplied by this 10 MVA that is 20 MVA ok. So, this rating depends upon number of power transformer, number of power transformer and their rating ok.

And substation location is another decision, which is not mentioned over here, but it is an important decision that where should I place the substation, should I place the substation at the centre of this service area or should I place somewhere in at the periphery of the service area or where I should place somewhere here or where?

So, substation location is itself an important decision for planning of power distribution network ok. And in many of the planning models, I will show you this decision is determined through an optimization approach so that the location of the substation is optimal ok.

But, there are many other factors like sometimes even though if you geometrically analyze and find a location as an optimal location, but that location may not be a feasible location. Because there are many factors that you one needs to consider before selecting or before choosing the site for the substation ok.

Now, this I will discuss in whenever in one of the modules whenever I will talk about different planning models. Now, at present, let us consider that at the centre I will place the substation, which is the optimal location and it is also feasible to place there ok. Why I am talking about the feasibility of placing a substation? Because sometimes if you find a location as the optimal location for the substation, but in that location if we have some hospital, some school or some this garden or park, you may not be able to place a substation there. So, that is why many of the distribution network planners they exclude this decision as one of the decisions, which is to be obtained through some planning or optimization approach. So, with their experience they can find out this location, which is the feasible location and accordingly they will construct the network ok.

But again, coming back to the main question that where should we? means what is primary feeder routing ok? Now, these are the typical nodes or load demand points, which already I mentioned. Now the question is how I will design this distribution network? Should I design a feeder, such that they will feed this node first or then they one another lateral will feed these nodes or vice-versa?

So, how do we construct? How do we construct this network? Based upon that this feeder routing will vary or feeder routes will vary. For example, suppose this is a typical node number let us say 15 and this is node number 16. So, I already have some feeder

here. Now, should we consider this both the nodes in a same lateral or should we create two lateral feeders to connect these two nodes?

So, those decisions will come under this feeder routing ok or this basically determine that routes of the feeder geographical, routes of the feeder and this depends upon many factors. These are the factors which directly influence of the determination of the routes of the feeder; one is load density; another is physical barrier sometimes you will see that there is a water body in between two dim note points.

So, even though they are closer, we cannot construct a particular distribution line, overhead line and this is one kind of barrier that one needs to consider. Then, also this voltage drop as I was talking about in one of my lectures, this voltage drop is an important measure that particularly designing this distribution network. Because, we should have some rules that that much of voltage drop would be allowable.

Let us say 10 percent voltage drop would be allowable, so that means, the path the customer, which is located for this point of the network he should not experience more than 10 percent of the drop of voltage. So, that one should understand. So, this voltage drop is another important measure and that is why we will use some analytical approach to determine this expression for voltage drop for different types of distribution network or distribution feeder; in next few slides I will explain this.

Then of course, future load growth number of feeders and also this cost because this is the ultimate factor which is somewhat related to the economy of the system. So, we can design something, but that should be economically feasible if it is not economically feasible, nobody will take the design that you are providing to that, ok.

So, feeder routing is nothing but to decide how we will choose the routes of the feeder. Should we choose the route of the feeder from one point to another or they should be chosen as a lateral feeders or what? So, this decision is an important decision ok. Next, we will go for an another decision, which is called number of feeders that how many typical feeders one should have under a particular substation.

So, if we go back to the previous slide and let us see that I can choose the typically four feeders to connect this all these nodes for this particular service area or I should connect them by using two feeders or even I can consider that I should go for three feeders. So,

that number that how many feeders I should use to supply all the loading points or all the load nodes that is also another important decision.

And there are some factors which will influence this decision; these are load density, feeder length. Feeder length is off-course, another factor because too much lengthy feeder will off-course, create excessive voltage drop along with it will be difficult for the person who are in charge of maintenance to look after them.

So, that is why this feeder length is another important factor. So, feeder limitations, primary voltage level substation capacity, conductor size, voltage drop are the most important you know factors for the such kind of decisions that; how many feeders we should have, what should be the routes of the feeder; all these things will be limited by this voltage drop.

And also we should take care about this future load growth. Then, another important decision is choice of the conductor size ok. Now, usually we have a set of conductors; usually we have a set of conductors, set of conductors from, which we should select a particular size for a particular feeder route.

For example, in between this few you know 15 and 16, this main feeder what type of conductor should I use? What should be the size of the conductor? This is an important decision. And that size would be off-course, different than the size of the conductor that we will use to create the laterals ok.

So, for designing the main feeder, a set of conductors should be used for particular sizes and for designing the lateral another set of conductors will be used and which particular conductor we will use from that set this is the important decision ok. So, out of this set of this conductor, we should select the most appropriate conductor, appropriate size of the conductor.

Now, again which one will consider as most appropriate size? This decision may come through an optimization process. Later on I will show you off-course, the economy is one of the factors if I go for higher size of the conductor then its cost will be also higher. So, we should judiciously select what should be the conductor size at a specific route of a feeder.

So, this basically depends upon different factors; again you see voltage drop factor is a common to all these decisions because this is an important factor which influence this length of the feeder which influence that number of the feeders, feeders root as well as what type of conductor we will use.

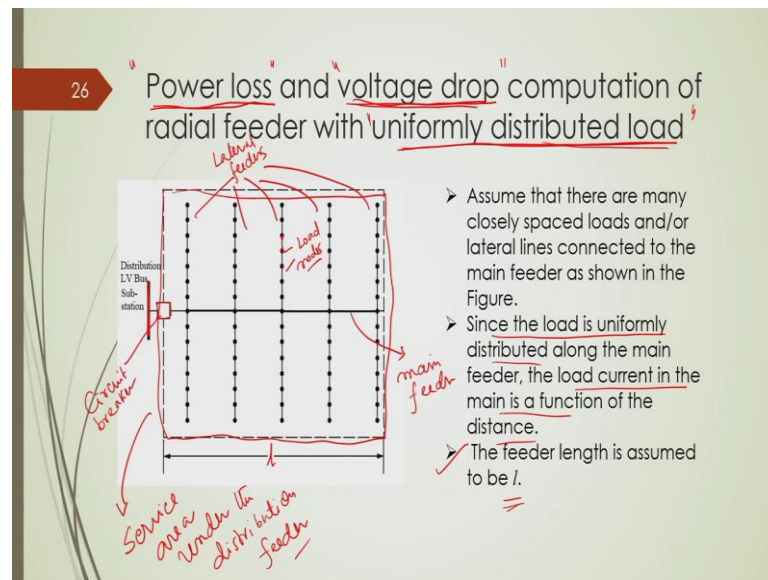
Because voltage drop means it is basically Iz drop ok; and I is you know this load current and z is the impedance of the feeder and this z is again dependent upon the conductor specification and the conductor spacing ok. And I depends upon this how many loads you are connecting and how you are connecting these loads ok. So, those things I will discuss in detail whenever I will talk about this parts distribution system planning ok.

So, this is an important factor; apart from that we have many other factors like load forecast, load growth rate, power losses because you know depending upon the conductor size power losses will vary because power losses means it is $I^2 r$ loss. So, I depends upon loading and r depends upon conductor size.

So, off-course, this power loss is another criteria that distribution system operator or owners will try to keep it under a specified value. Otherwise it will hamper their economy; it will be that loss for this particular distribution network owner. So, they should try to minimize it as much as possible.

So, this depends upon this load; you know suitably selecting the loading of the line and this also depends upon by suitable choice of the conductor size. So, this ' $I^2 r$ ' I am talking about this is line losses, distribution line losses. And this distribution line loss as a whole is summation of ' $I^2 r$ ' of all these feeder branches ok. So, this I will come later. Then, cost is the most important factor because economy depends upon the cost and also transformer rating and conductor ratings are important.

(Refer Slide Time: 19:16)



Now, I will go to some analytical approach to determine these two important factors; two important factors of a typical distribution feeder; one is power loss; another is voltage drop ok. These are also the limiting factors, these are also the limiting factors through, which one should decide that how much loading we should allow in a particular feeder or how many numbers of feeders, should we use and should we go for this capacity additions of the substation and all, ok.

So, these are the two important limiting factors ok. So, whatever I will talk about today these are some typical analysis analytical approach to find out these mathematical expressions for power loss and voltage drop ok, but these are under certain conditions ok. First of all, for this case we will consider that this is the service area.

So, this is the dotted line, it is the service area; this dotted line is the service area under this substation or under this distribution feeder. So, this one is service area under the distribution feeder and this distribution feeder is of a uniformly distributed load; this distribution feeder is having a uniformly distributed load and its service area is a rectangular area having this length of the feeder is l having this length of the feeder is l ; l meter or l kilometer whatever or l mile whatever you can consider ok.

And this is the main feeder you know, this is the main feeder, this is you know that circuit breaker of the feeder ok. And this is basically distribution system LV bus that 11 kV bus for example, in India ok. And these are the lateral feeders. So, these all are lateral

feeders; these all are lateral feeders. And this black dots they are representing this load node load nodes ok.

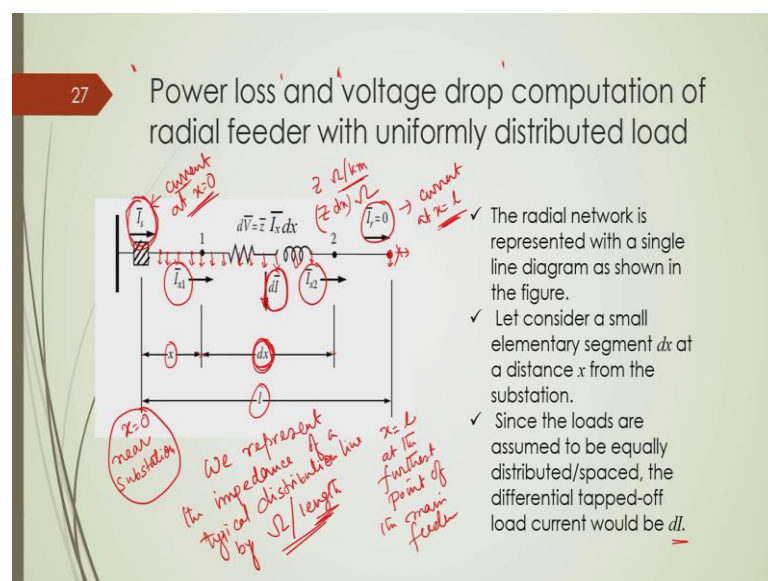
They are basically, you can consider an aggregated load demand and it is connected to a distribution transformer and that distribution transformer basically converts this distribution voltage level to the utilization voltage level and directly feeds it feed to the customer ok, alright.

So, this is typical type of distribution feeder which is having a rectangular service area, typically particular feeder I am talking about this is the main feeder or trunk feeder and these are the lateral feeders and each of the black dot is representing this you know load node; and we assume that this loads are of uniformly distributed, ok.

So, this analytical approach works under this condition. Other than that it will not work. So, we assume that loads are of uniformly distributed, service area is a rectangular and you know we have a one main feeder and few lateral feeders; each feeder, each lateral feeder are connected with some node i.e., load nodes ok.

So, you can understand, since load is a uniformly distributed along the main feeder and also in the lateral feeder, this load current in the main feeder will be function of the distance. And we consider that length of the feeder is , 1 meter or 1 kilometer or, not 1 meter it should be either or 1 kilometer or 1 mile ok.

(Refer Slide Time: 24:00)



Now, in order to determine this power loss and this voltage drop for typical that feeder, which is having uniformly distributed load and rectangular service area, what we will do? We will consider a typical single line diagram of this feeder; this gives you the single line diagram. This is the total length, this l is the total length and this x is basically providing way you the variable length.

So, x is 0 at near to the substation near substation; and x is basically l at the farthest point of the main feeder ok. So, x is equal to l at the farthest point. So, the remote test customer who is located at the end of this feeder at this point x would be equal to l ok. And we take at a distance x ; we take a small elementary length of dx . Although, in figure it is not showing that small, but you can assume dx is infinitely small length ok.

And, you all know that we represent the impedance of a typical distribution line by ohm per length; that means, if your length is represented in kilometer we typically say that this feeder resistance feeder impedance is let us say 1 ohm per kilometer. The typical value I am talking about may be 0.5 ohm per kilometer.

So, we represent this feeder impedance based upon the conductor size, which type of conductors we are using to construct that feeder and this feeder impedance is represented by impedance in ohm per unit length ok. So, since the length of the infinitely small section is dx , so, therefore, and if we have this impedance of this per unit length for this feeder is z ohm per kilometer or per meter whatever you can say.

So, then for this infinitely small section its impedance net impedance will be z multiplied by dx , z multiplied by dx . So, that much ohm would be the impedance of this infinitely small line section ok. Now again, do not consider this whole thing like a lumped feeder or lumped line transmission line, which usually power system engineers have to analyze because each and every node, each and every point of this particular feeder we have some load demand we have some load demand like this; for each and every point of this feeder ok. Now, therefore, we assume that the current near to the substation at x is equal to 0 is I_s that is the sending node current. So, this I_s is basically representing the current at x is equal to 0 ok. And this beyond this point beyond this the remotest customer this current would be of course, 0 because current will be drawn by this remote test current customer therefore, no current will flow through beyond that.

So, this I_r is basically representing the current of the feeder at x is equal to l , x is equal to l ; and as you know current itself is varying throughout this length starting from this substation current to this remote test point current ok. And this of course, this current which is flowing through this substation on at x is equal to 0 , that will be having the highest magnitude because it will basically carry carrying the load of all these loads whoever connected to that particular feeder.

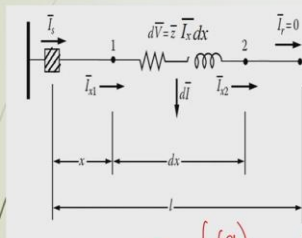
And this other side that at x is equal to l current would be automatically 0 ok. And we are assuming that current drawn by this elementary by this infinitely small section is dI ok. So, we have three components of current; one is current entering these two this infinitely small section, which is I_{x1} .

Another is current which is leaving this infinitely small section that is I_{x2} ; another is current which is drawn by this infinitely small section that is dI . So, we have some load at this you know infinitely small section which is drawing that dI amount of current.

(Refer Slide Time: 29:53)

28

Power loss and voltage drop computation of radial feeder with uniformly distributed load



- I_s is the sending-end current at the feeder breaker, and I_r is the receiving-end current.
- I_{x1} and I_{x2} are the currents in points 1 and 2, respectively.
- Assume that all loads connected to the feeder have the same power factor.
- Since the total load is uniformly distributed from $x = 0$ to $x = l$, we can write, $\frac{dI}{dx} = k$ (some constant)

$I_x = f(x)$
 Load current is function of length $\frac{dI}{dx} = k$

Now, what we can do? So, whatever I discuss and you also know that these are basically you know varying with the length. So, this current is basically I_x is function of x ok. So, this current is or I should say that load current is function of sorry function of length ok.

So, as we know, I is varying with this current and since we have uniformly distributed load, this rate of change of this current of this over this section is a constant that is k ; this small k is representing a constant, because we have uniformly distributed load.

(Refer Slide Time: 30:56)

29

Power loss and voltage drop computation of radial feeder with uniformly distributed load

- By applying KCL at node 1, we can write,

$$I_{x1} = I_{x2} + dI$$
Or,

$$I_{x2} = I_{x1} - dI$$
Or,

$$I_{x2} = I_{x1} - kdx$$
- If $x = l$, the equation becomes

$$I_r = I_s - kl = 0$$
- This yields $k = \frac{I_s}{l}$
- The current at a distance x is

$$I_x = I_s(1 - \frac{x}{l})$$

Now, we apply KCL; we apply KCL at these two points; one is point 1 and point 2. So, point 1 is basically you know point 1 is the starting point of this, start point of this infinitely small section and point 2 is the end point; and you know distance between point 1 and point 2 is dx ok.

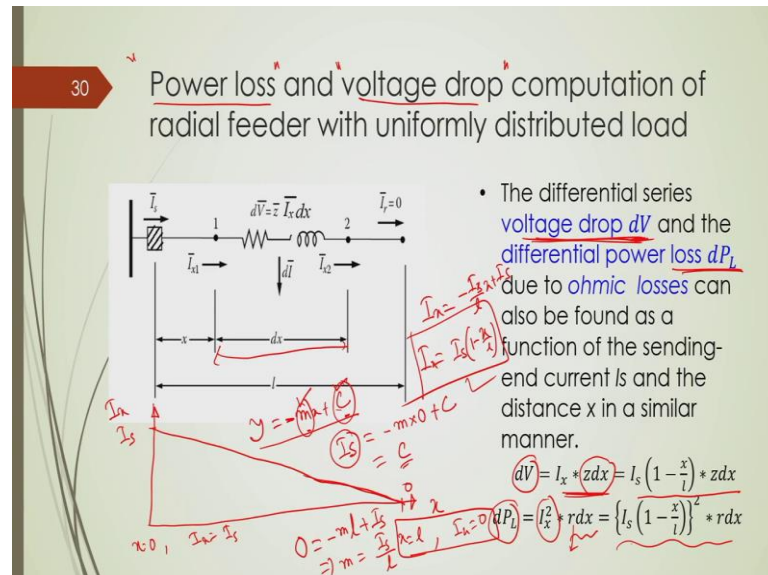
So, if we apply KCL at these two nodes KCL, I will get two equations or I simply apply this KCL at this point in order to have a expression of in order to have a relationship of I_{x1} , dI and I_{x2} which is like this. So, I_{x1} will be equal to dI plus I_{x2} because you know I_{x1} will be higher because it is nearest to the substation and I_{x2} will be off-course lower because it is distant away from the substation as compared to I point 1 ok.

And as you know that it is a radial feeder. So, power flow will be gradually reducing if you go from substation to the end customers ok. So, with this by replacing this dI with $k dx$ we will get this relationship. Now, we have two boundary conditions one is at x is equal to 0, I_x is equal to I_s and at x is equal to l , I_x is equal to 0.

So, if you put these two boundary conditions over here, you will get at x is equal to l ; this I_x is equal to 0, that is $I_s - kl$ is equal to 0, which will give the expression for k

and if you replace it with you know you will get the expression of this current that is I_x is equal to I_s minus I_s multiplied by $1 - x$ upon l ok.

(Refer Slide Time: 33:06)



So, this is how we can determine this. But, alternatively, we can also determine if you plot this characteristic simply if we plot this I_x versus x ; I am talking about magnitude here. Now, this bar symbolize that it is a phasor quantity, but here we are basically if you remove this bar then whatever the quantity will of the current would be there that would be the magnitude of that current ok. Now, if you plot this I_x with respect to x then as you know at x is equal to 0, I_x will be equal to I_s magnitude and x is equal to l , it will be 0.

So, if it and since this variation of the current or rate of change of current with respect to distance is constant for uniformly distributed load so, the characteristic will be a nothing but straight line characteristic, straight line characteristic ok. Now, you can assume that this characteristic is equal to y is equal to mx plus C where C is the intercept; m is the slope; and this slope you can see it is a negative slope. So, you can write minus on this.

Now, if you put these two boundary conditions at x is equal to 0, I_x is equal to you know I_s and at x is equal to l , I_x is equal to 0. Then, we will get the expression of these two unknown constant parameters; one is slope another is intersect. So, let us put it. So, at x is equal to 0, I_s is equal to I_x is equal to I_s . So, y is equal to I_s this will be minus

$m \times$ is equal to 0 means multiplied by 0 plus C. So, so constant C is coming out to be I s and at s equal to l this I x is equal to 0.

So, this will be 0, left hand side and this will be minus $m \times$ plus C, C is already determined as I s. So, we will get m as I s by x is off-course, at l. So, you have to replace it by l. So, this would be I s by l. So, once you get that if you put it over here. So, you will get I x is equal to y is here nothing but I x.

So, I s is equal to minus I x I s by l, x plus I s. So, which is nothing but I s 1 minus x by l. So, this we will get alternatively ok. Now, what we will determine? Because our goal is again to determine the expressions for this power loss and voltage drop, power loss and voltage drop that is why we are doing all these things ok.

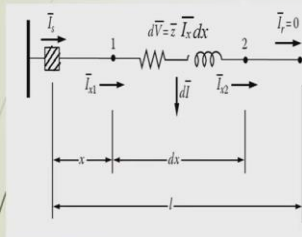
So, to find out this voltage drop of this infinitely small section, let us consider this voltage drop is dV and this power loss is dP L ok. L stands for this loss P stands for power ok. Now, this dV that is voltage drop at this infinitely small section would be equal to this I x multiplied by this zdx, which this zdx is basically representing the impedance of the infinitely small section.

So, this is equal to I s; one up I x is replaced with this expression, this expression and zdx will remain. Similarly, this dP L expression we will get I square I x square multiplied by rdx where r is basically per unit length resistance of the feeder. So, if you multiply it with this length of the infinitely small section that is dx, rdx will represent the resistance of the infinitely small section. So, if you replace with this expression of I x you will get this ok. So, this two are the expressions for this voltage drop and power loss of the infinitely small section dx of dx length, which is located x distance away from the substation ok, alright.

(Refer Slide Time: 37:56)

31

Power loss and voltage drop computation of radial feeder with uniformly distributed load



- The differential series voltage drop dV and the differential power loss dP_L due to ohmic losses can also be found as a function of the sending-end current I_s and the distance x in a similar manner.

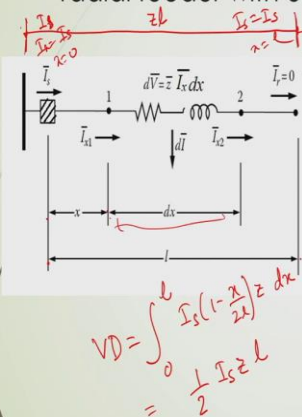
$$dV = I_x * z dx = I_s \left(1 - \frac{x}{l}\right) * z dx$$

$$dP_L = I_x^2 * r dx = \left\{ I_s \left(1 - \frac{x}{l}\right) \right\}^2 * r dx$$

(Refer Slide Time: 38:00)

32

Power loss and voltage drop computation of radial feeder with uniformly distributed load



- The series voltage drop VD_x due to I_x current at any point x on the feeder is:

$$VD_x = \int_0^x dV = I_s \left(1 - \frac{x}{2l}\right) * z x$$
- Hence, the total series voltage drop along the full length is:

$$VD = I_s \left(1 - \frac{l}{2l}\right) * z l = \frac{1}{2} I_s z l$$
- Similarly, the total power loss would be:

$$TP_L = \int_0^l dP_L = \frac{1}{3} I_s^2 r l$$

Now, what we will do next? We will integrate this from 0 to x so that you will get voltage drop up to this section x . So, if you integrate this expression whatever you will get dV from 0 to x with this limit; that means, up from here to here. Then, whatever this value of this voltage drop that we will be getting that will be voltage drop up to this point x ok, because this section is infinitely small. So, up to this point we are getting.

So, if you do this integration here then what we will get? You will get this voltage drop will come out to be this is for voltage drop up to x . Now, if you take the full length then

this limit will be changed to 0 to l. So, actually this VD will or this full length or full voltage drop of this feeder or the voltage drop of the feeder from the substation to the furthest customer will be integration 0 to l this $I_s (1 - x) \cdot 2 \cdot l \cdot dx$, ok.

Now, if you do this integration you will get this value as this. So, there is a typo over here; this would be dx this would be dx . So, you will get this expression as half of $I_s \cdot z \cdot l$ ok. You can verify it; you will get ok. So, this is the basically the expression of voltage drop of whole feeder, starting from this substation to the furthest point of the feeder. Similarly, this total power loss of the feeder would be this if you integrate the whole expression for power loss with the whole range starting from 0 to l.

So, we will get this the expression of total power loss as one third of $I^2 r \cdot l$ ok. Now, look at these two expressions, look at these two expression why this factor half or why this factor one-third these are coming? Why this factor this half and why this factor one third they are coming in the expression of voltage drop and power loss? Because of the uniform distributed nature of the load.

Now, if we have a lumped load; that means, we have a here we have substation and we have a feeder like this and at the end all loads are concentrated; this is my load. So, this form of the load is called lumped representation. We do not have any load points in between this substation and this furthest point.

Now, what could have been this voltage drop you can try? In that case voltage drop would have been I_s multiplied by $z \cdot l$, because the current which is flowing through this line starting from the substation is I_s and current at this point that will be also I_s . So, here for this lumped load for lumped load this I_x is equal to I_s at x is equal to 0.

Similarly, I_x is equal to I_s at x is equal to l ok. Now, if this length is of length of the feeder is l and the impedance is small z per unit length. So, total impedance will be z multiplied by l ok. So, total voltage drop will be nothing but this $I_s \cdot z \cdot l$. So, there is no half factor there now. This half factor is coming because of this uniformly distributed load, as if that effective length of the feeder for a uniformly distributed load of a typical or a feeder with uniformly distributed load the effective length of the feeder become half, in order to compute this voltage drop. That means, also this is an advantageous because if you compare this voltage drop of a lump feeder or lump load, then it will come as I_s

multiplies by $z l$. If you compare this with this then you this is 50 percent of the voltage drop as compared to this lumped load ok.

So, if we have a uniformly distributed feeder or this a feeder with uniformly distributed load an effective voltage drop becomes half as compared to the voltage drop of a lump feeder. Same is applicable when we will have this power loss calculation; here this you know actual power loss will be one-third of this power loss of a lumped feeder ok.

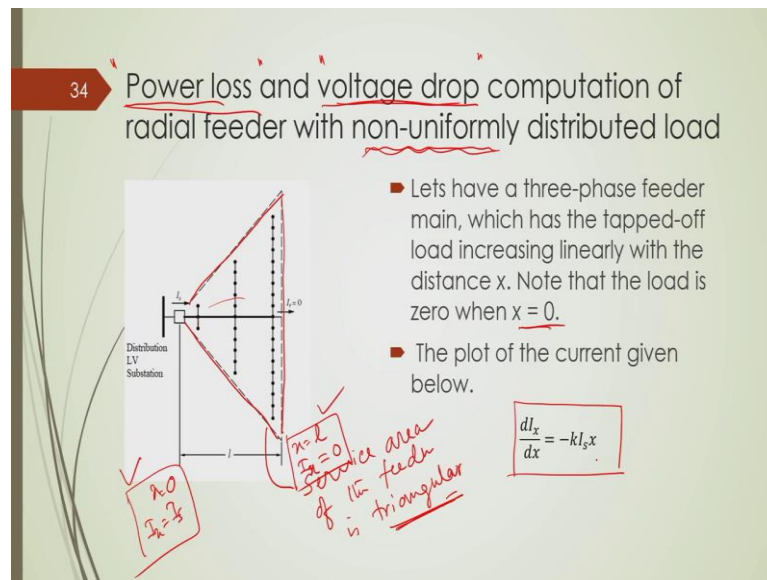
(Refer Slide Time: 43:07)

33 Power loss and voltage drop computation of radial feeder with uniformly distributed load

- Therefore, for a uniformly distributed feeder, the voltage drop will be similar to a lump load connected at the mid-point of the feeder.
$$\text{i.e., } x = \frac{l}{2} = 0.5l$$
- For a uniformly distributed feeder, the power loss will be similar to a lump load connected at the one-third length of the feeder.
$$\text{i.e., } x = \frac{l}{3}$$

So, this is alternatively represented like this for a uniformly distributed feeder the voltage drop will be similar to the lump load connected at the midpoint of the feeder ok. Now, for a uniformly distributed feeder the power loss will be similar to the lump load connected at the one-third length of the feeder ok. So, that is somewhat new for this type of analysis ok.

(Refer Slide Time: 43:38)



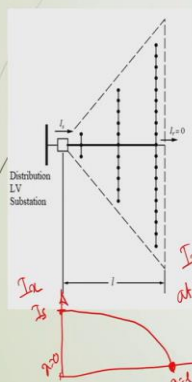
Now, I will also talk about power loss and the voltage drop expressions for a uniformly distributed load. This is not non-uniformly distributed load, but for a uniformly distributed load, but having the service area of triangular ok. So, here the service area, service area of the feeder is triangular and load basically is uniformly distributed.

But, since it is triangular, then throughout the line this load becomes non-uniformly distributed because near to the substation less loads are connected if you go away from the substation more loads are connected ok, alright. So, for that case we also need to determine what would be the expression for power loss and what would be the expression for voltage drop, ok.

(Refer Slide Time: 44:52)

35

Power loss and voltage drop computation of radial feeder with non-uniformly distributed load



- The constant k can be obtained as:

$$I_s = \int_0^l dI_x = \int_0^l k * I_s * x dx = k * I_s * \frac{l^2}{2}$$
- Hence, $k = \frac{2}{l^2}$
- Substituting k , we get

$$\frac{dI_x}{dx} = -2I_s \frac{x}{l^2}$$
- Integrating this, we get the current at x

$$\int_{I_s}^{I_x} dI_x = -\frac{2I_s}{l^2} \int_0^x x dx$$

$$I_x = I_s \left(1 - \frac{x^2}{l^2}\right)$$

Now, again this we can also determine if you plot this characteristic how this current or the load current will vary from the substation to the furthest point. So, here also we have two boundary conditions that, at x is equal to 0, I_x is equal to I_s ; x is equal to l I_x is equal to 0. So, these two are boundary conditions, say as similar to the previous example, but here the service area is triangular that is the only difference of that with respect to the previous example ok.

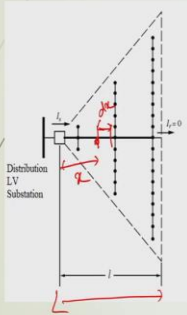
Now, in order to find out this expression for I_x here, if we know the characteristic of this I_s with respect to x then if you simply put these two boundary conditions you will get the exact expression of this I_x ok. Now, how to find that? We know here this the rate of change of current with respect to this distance; it will be some constant multiplied by this x ok.

With this, you can find out that the value of this constant like this the way it is done. And then as you know if you integrate this current from substation to this point that x then whatever expression that we will get that will be the expression of this current. But I can show you an alternative way to find out this expression ok.

(Refer Slide Time: 46:36)

36

Power loss and voltage drop computation of radial feeder with non-uniformly distributed load



- The expressions for the voltage drop and power loss can be computed as:
$$dV = I_x * z dx = I_s \left(1 - \frac{x^2}{l^2}\right) * z dx$$

$$dP_L = I_x^2 * r dx = \left\{ I_s \left(1 - \frac{x^2}{l^2}\right) \right\}^2 * r dx$$
- The series voltage drop at the point x can be computed as:
$$VD_x = \int_0^x dV = I_s \left(1 - \frac{x^2}{3l^2}\right) * zx$$
- Hence, the total series voltage drop along the full length is:
$$VD = I_s \left(1 - \frac{l^2}{3l^2}\right) * zl = \frac{2}{3} I_s zl$$

So, if you plot, in fact, this plot is I hope it is there somewhere no, this if you plot this current with respect to this distance I x is basically current and x is the distance then as I know this bound two boundary condition at x is equal to 0, x is equal to 0 this current will be I s and at x is equal to l this current would be 0. So, these two boundary conditions we know.

Now, previously because it was a uniformly distributed loaded feede, the current was linearly varying. Now, here how it will vary? Here it will vary like a parabola ok and therefore, I can represent this I x with the expression of C 1 plus C 2 x square ok. Now, since this slope of this is basically negative, so, I can replace it by 0, I can replace with the negative slope.

Now, if we put these two boundary condition at x is equal to 0, I x is equal to I s. So, I can replace it. So, C 1 will be equal to I s ok and at x is equal to l I x will be equal to 0. So, this would be 0. So, this would be C 1 minus C 2 l square. So, C 2 will be equal to C 1 by l square. C 1 itself is I s. So, this is basically I s by l square. So, once you get this C 2 and this C 1, you put it here. So, you will get the expression of I x as C 1 is I s minus I s by l square x square.

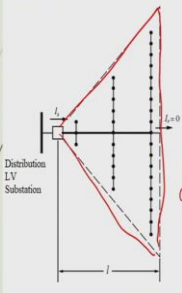
So, this is nothing but I s 1 minus x square by l square. So, this is the expression of I x. So, the same expression you can get alternatively if you plot the expression and plot this current I x with respect to x ok. So, once you get this expression of this current I x with

respect to x . Then in similar way we determine this voltage drop at the point, which is you know the x length the distant away from the substation.

And let us take an infinitely small elementary length of dx ; find out the voltage drop of that particular length by similar way we got earlier and also find out the power loss of the infinitely small section. So, once you get that. So, if you integrate over 0 to x , you will get the expression for voltage drop up to this point. Now, if you change this limit from 0 to l then we will get the voltage drop of the entire section. So, this will come out to be two-third of $I_s z l$, two-third of $I_s z l$ ok.

(Refer Slide Time: 50:00)

37 Power loss and voltage drop computation of radial feeder with non-uniformly distributed load



- Similarly, the total power loss would be:

$$TP_L = \int_0^l dP_L = \frac{8}{15} I_s^2 r l$$
- Therefore, to determine the voltage drop, the total load current is assumed to be concentrated at:

$$x = \frac{2l}{3} \approx 0.667 l$$
- To determine the power loss, the total load current is assumed to be concentrated at:

$$x = \frac{8l}{15}$$

And similarly power loss will come out to be 8 upon 15 $I_s^2 r l$ ok and; that means, for a triangular service area of a uniformly distributed feeder, for a uniformly distributed feeder with triangular service area, you can assume that low total load current concentrated at two-third distance of the length.

In order to find out this voltage drop and in order to find out this power loss, you can assume that load current is concentrated at 8 upon 15 multiplied by the whole length of the feeder, ok. So, or alternatively you can consider that the effective length of the feeder of a uniformly distributed feeder with triangular service area in order to find out this voltage drop becomes two-third of the actual length ok.

Previously for uniformly distributed feeder with triangular service area, we got half of the this effective length, we got as half of the actual length in order to calculate the voltage drop, but here we got it as two-third of the actual length in order to find out the voltage drop. In order to find out the power loss this factor is found out to be 8 by 15.

So, what you can find out if you compare? So, two by third is nothing but around 0.667 of this length this x . So, this would be the actual you know effective length of the feeder and for uniformly distributed load having a rectangular service area we found out is 0.5 of the l ok.

So, you can see that because of this triangular service area this effective length is somewhat increased; effective length is somewhat increased which means the voltage drop will also increase. If you compare with uniformly distributed load with a rectangular service area and triangular service area because of that particular shape of the service area your effective length is increased and thereby this voltage drop will also increase, ok.

So, similarly similar type of observation you will be getting in case of power loss ok. So, here in this particular lecture I focused on the analytical approach to find out two important limiting factors; the expression of two important limiting factors for a typical two cases.

Case 1 is feeder with a triangular service area with uniformly distributed load; another is feeder with a rectangular service area with uniformly distributed load. And we can understand that how this voltage drop and power loss expressions are different than a typical lumped load, which is concentrated at end point of the feeder ok.

Thank you.