Design of Power Electronic Converters
Professor Doctor Shabari Nath
Department of Electronics and Electrical Engineering
Indian institute of Technology Guwahati
Module: Snubber Design
Lecture 33
RC Snubber Analysis - III - Overdamped and Critically Damped Case

Welcome to the course on Design of Power Electronics Converters. So, we were discussing snubbers and we will we were we had started deriving the RC snubber circuit. And in that we had seen that it ultimately looks like an RLC circuit, we have to analyse an RLC circuit in there we have three cases one is your underdamped then you over damped and critically damped.

So, underdamped condition the derivation we had seen and then with that derivation we had obtained the expressions for peak voltage across the device and also, we had obtained the rate of change dv by dt average dv by dt in the beginning that expression also we had obtained in terms of chi the initial carrying factor in zeta the damping ratio. So, now let us further continue with that and do the derivation for over damped case and critically damped case.

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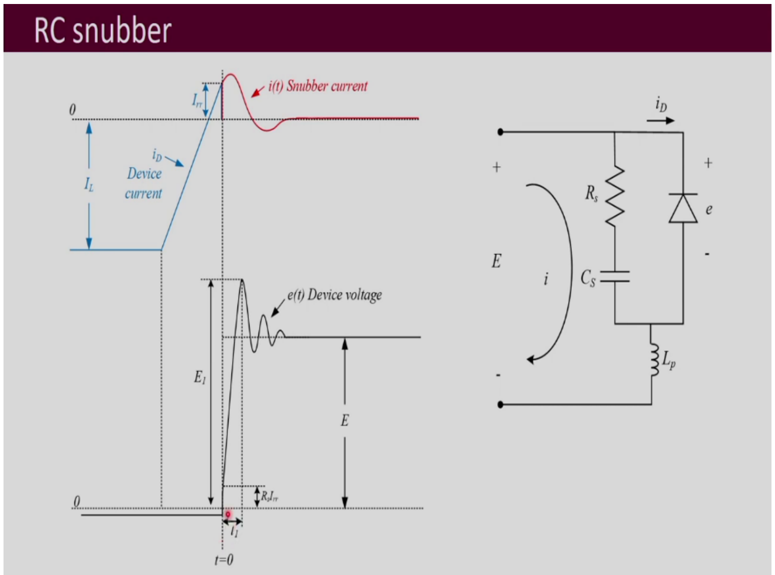

So, just for you to recall that this was a circuit with which we were doing the analysis. So, here this is your RC snubber and Lp is the parasitic inductance and this is your diode and across it as the voltage e and this voltage can reach up to a maximum of E1 and this is what we want to limit the spike. So, we have to derive an expression for this and then Irr is the reverse recovery current and this is the current at t equal to 0 which flows through this snubber the initial capacitor voltage is 0 that means, there is no charge in the capacitor at time t equal to 0.

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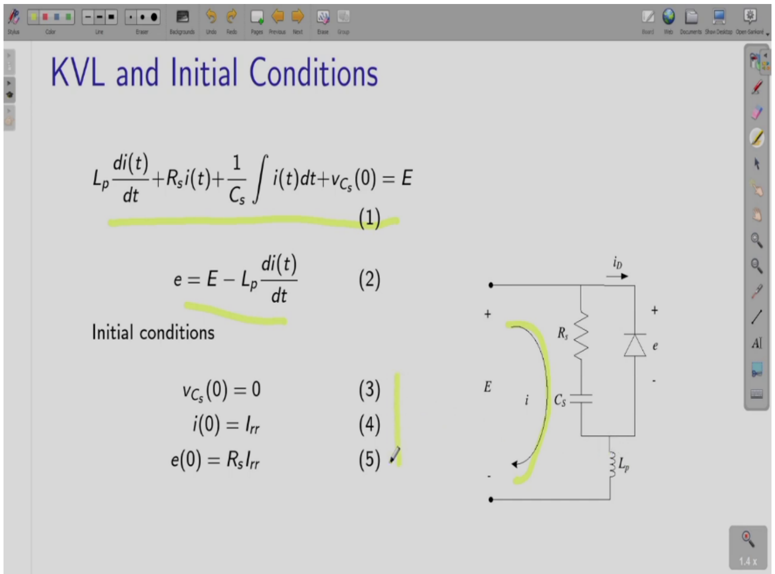

So, this was the KVL equation that we had written when we had applied KVL in this loop and then this is the second equation for e which is E minus of Lp di d by dt and these are the initial conditions.

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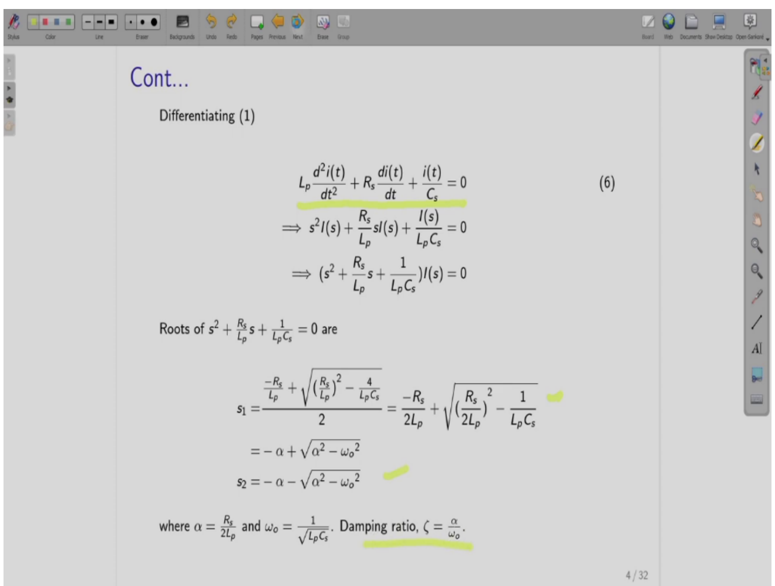

And then with that, we had obtained this differential equation for which we had this roots these s1 and s3 and this was the damping ratio zeta alpha by omega 0.

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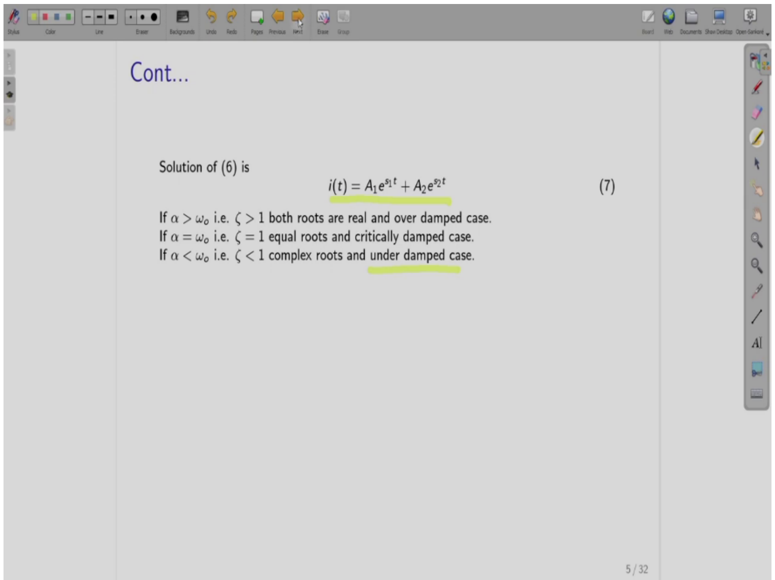

And this is the solution of the differential equation for it and we had discussed this case of underdamped now, we are going to discuss these 2 over damped case and critically damped case zeta greater than 1 and zeta equal to 1.

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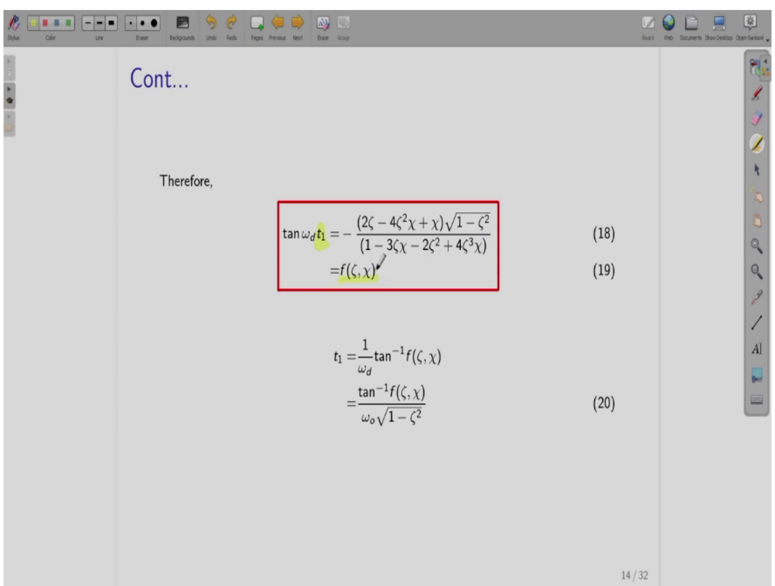

So, for underdamped case, this is the one important result that we had obtained that is we had to find out this time t1. Now, this time t1 came in terms of tan function tan omega d t1 and this is in terms of zeta and chi and then this was denoted as f of zeta comma chi.

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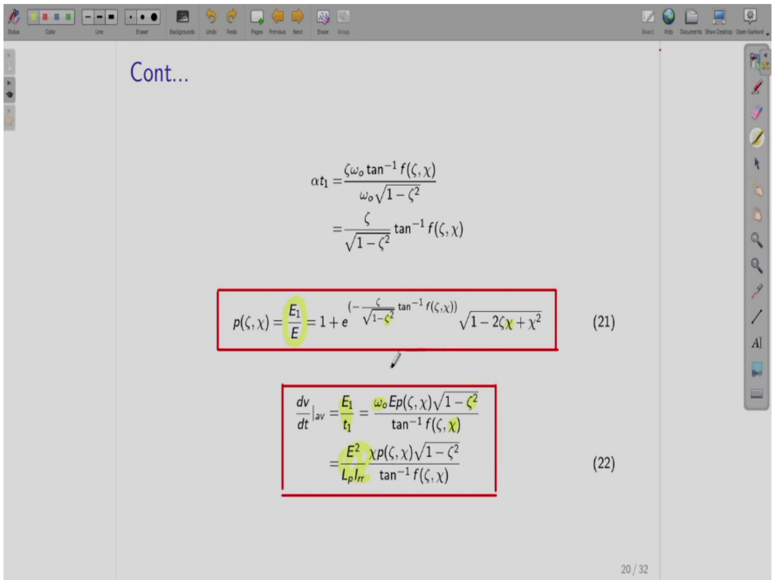

And then these are other two important results that is your E1 by E your peak voltage by the blocking voltage E in terms of your zeta and chi and this dv by dt average which is your E1 by t1 and this also was written similarly as a function of this zeta and chi and then here also this natural frequency also came here and this the same could also be expressed in terms of this E square by Lp I by Lp into Irr multiplied by chi. Now, for overdamped case and critically damped case we are going to do the derivation in the similar manner and reach out to these results.

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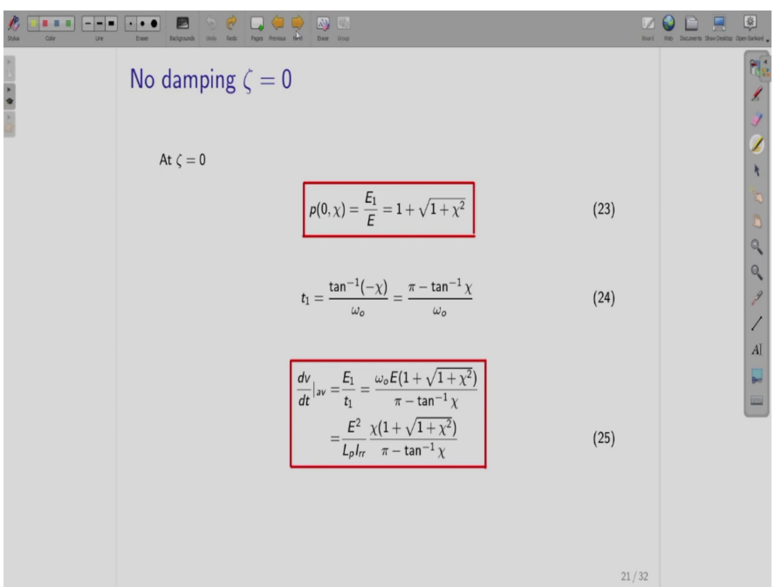

And we had done the same thing for your no damping case zeta is equal to 0 as well.

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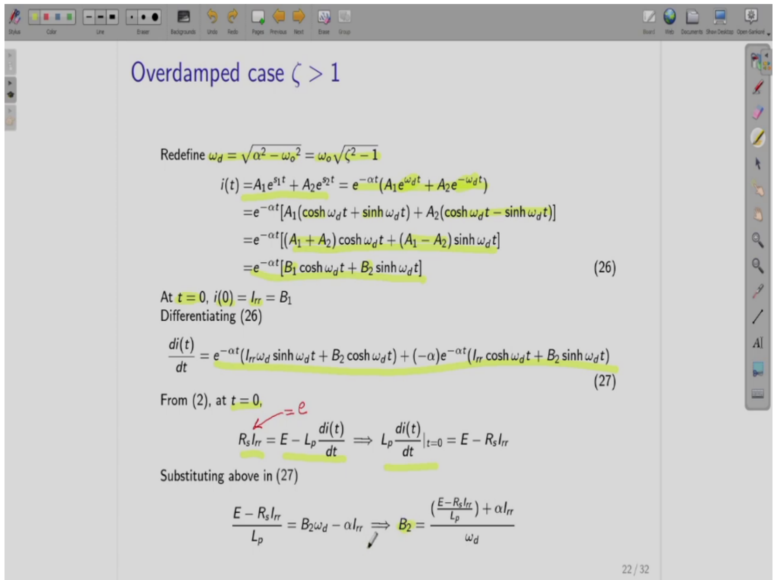

So, now having recalled what we had done last lecture, now, let us look here for overdamped case zeta greater than 1. So, here this is the solution of the equation the general solution and now, when we have this zeta greater than 1, we define this omega d as equal to root over of alpha squared minus omega 0 square, which can be written as omega 0 root over zeta square minus 1.

Note that when we were discussing underdamped case this square root was a square root over 1 minus zeta square. Since zeta there was less than 1. Now, we have reversed this now, the definition for omega d is this is equal to omega 0 root over of zeta square minus 1 as zeta is greater than 1. So, now both the roots are real and so we can write these like this e power minus alpha t A1 e power omega dt plus A to the power minus omega dt.

Now, this e power omega dt and e power of minus omega dt, this can be written in term of (())(06:01) cos h and sin h, your cos hyperbolic and sin hyperbolic functions, we can write it. So that is how we have written you have expanded and then further we try to reduce it. So, when you try to reduce this is what you will be obtaining, and let us say A1 plus A2 equal to B1 and A1 minus A2 equal to B2. So, this is the solution that we will be obtaining in terms of cos h and sin h.

$$i(t) = e^{-\alpha t}[B_1\cosh\omega_d t + B_2\sinh\omega_d t] \quad (26)$$

Now, as we did before, we do the same procedure we go for initial condition. So, at t equal to 0, your i0 equal to Irr. So, this is 0. So, cosh omega dt is 1 so Irr equal to B1. Then further we differentiate this it so d it by dt you differentiate you will be obtaining

$$\frac{di}{dt} = e^{-\alpha t}\left(I_{rr}\omega_d \sinh \omega_d t + B_2\omega_d \cosh \omega_d t\right) + (-\alpha)e^{-\alpha t}\left(I_{rr} \cosh \omega_d t + B_2 \sinh \omega_d t\right) \quad (27)$$

this equation and then against substitute for t equal to 0 and then we had this equation E minus Lp di by dt, which your small e was equal to Rs Irr, this one is this is equal to e and this is equal to Rs into Irr. So, then, we obtained from here Lp d it by dt at t equal to 0 this is equal to E minus Rs Irr. So, substitute all that same here in this d it by dt this equation. So, then we obtain this your E minus Rs Irr by Lp. So, this is what you are going to get the B2 as.

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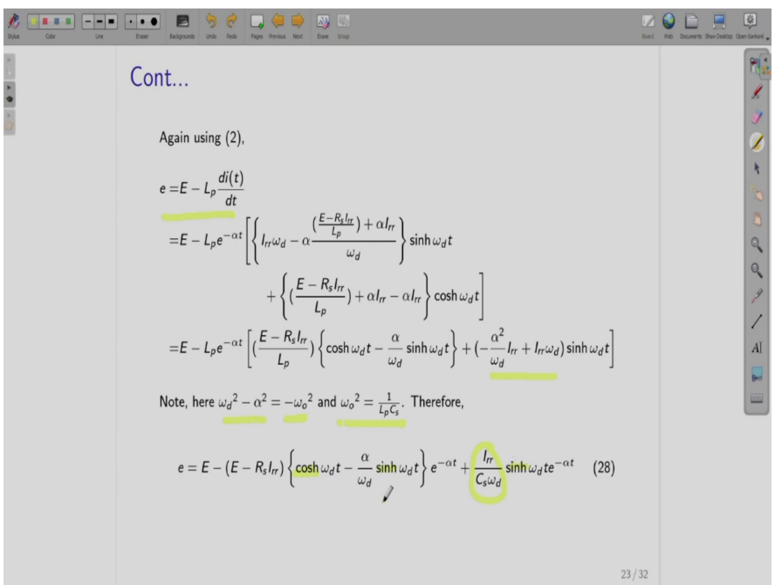

Further what we do is we again use this equation and substitute for d it by dt that we had just obtained in last slide. And then when you try to solve it, you try to reduce it and when you reduce it just substitute here for this one as omega d square minus alpha squared equal to minus omega 0 square now and omega 0 squared equal to 1 by LpCs. So, you will be reducing this to Irr by Cs omega d. So, what you will observe is that this equation

$$e = E - (E - R_s I_{rr})\left\{\cosh\omega_d t - \frac{\alpha}{\omega_d}\sinh\omega_d t\right\}e^{-\alpha t} + \frac{I_{rr}}{C_s\omega_d}\sinh\omega_d t e^{-\alpha t}$$

(28)

is similar to the one that we had obtained for underdamped case so only differences here this is cos hyperbolic and sine hyperbolic (())(08:45) instead of cos and sin.

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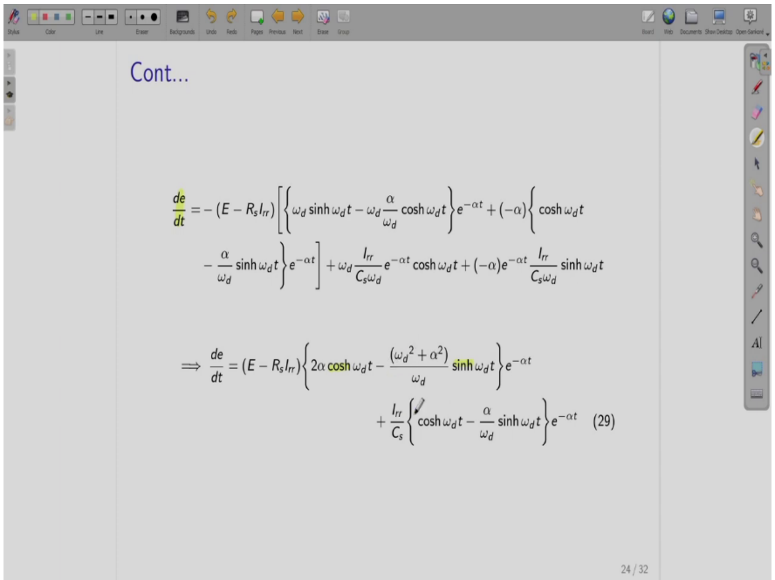

Then we differentiate de by dt. So, with differentiating e you will be obtaining this try to reduce it. So, when you reduce this is what you are going to obtain (29) shown in screenshot. This equation is also similar to the d by dt equation in case of underdamped condition, only thing is now, these are all cos h and sin h. And this term there is instead of omega d square minus alpha squared this is plus over here.

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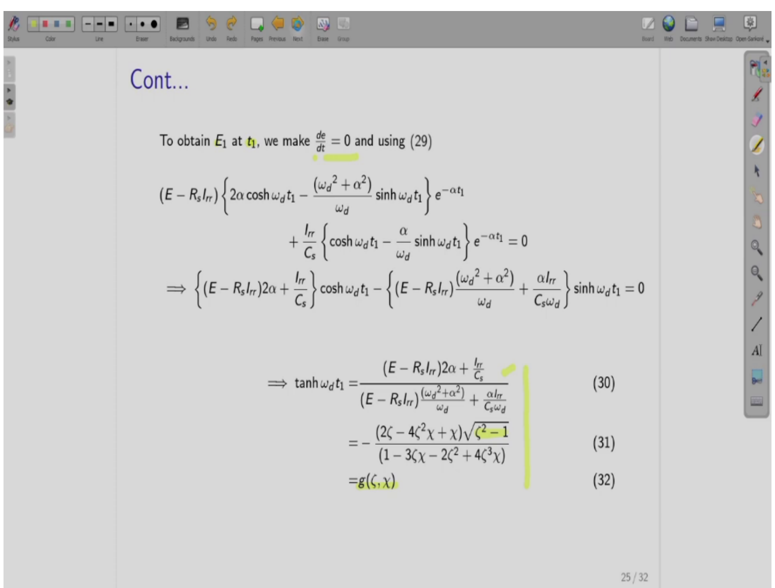

Further, what we try to do is we try to obtain the peak voltage the spike that is E1 at time t1 and that maxima can be obtained by differentiating this with any equating it to 0. So, d by dt equated to 0 and then you try to solve it and you solve this is what you are going to get

$$\tanh \omega_d t_1 = \frac{(E - R_S I_{rr})2\alpha + \dfrac{I_{rr}}{C_s}}{(E - R_s I_{rr})\dfrac{(\omega_d^2 + \alpha^2)}{\omega_d} + \dfrac{\alpha I_{rr}}{C_s \omega_d}}$$

(30)

$$= -\frac{(2\zeta - 4\zeta^2\chi + \chi)\sqrt{\zeta^2 - 1}}{(1 - 3\zeta\chi - 2\zeta^2 + 4\zeta^3\chi)}$$

(31)

$$= g(\zeta, \chi)$$

(32)

Now, since this is similar to what we had done in case of underdamped condition, I have not shown here again all the steps just note down that you this is similar to what we got for underdamped condition the expression that we are obtaining. And so, you when you solve it for zeta and chi by substituting it, you will be getting similar expression only differences this is now, zeta square minus 1 instead of 1 minus zeta square. And so, we represent this function is g zeta comma chi. Instead of f zeta comma chi which was it was how it was presented for underdamped condition.

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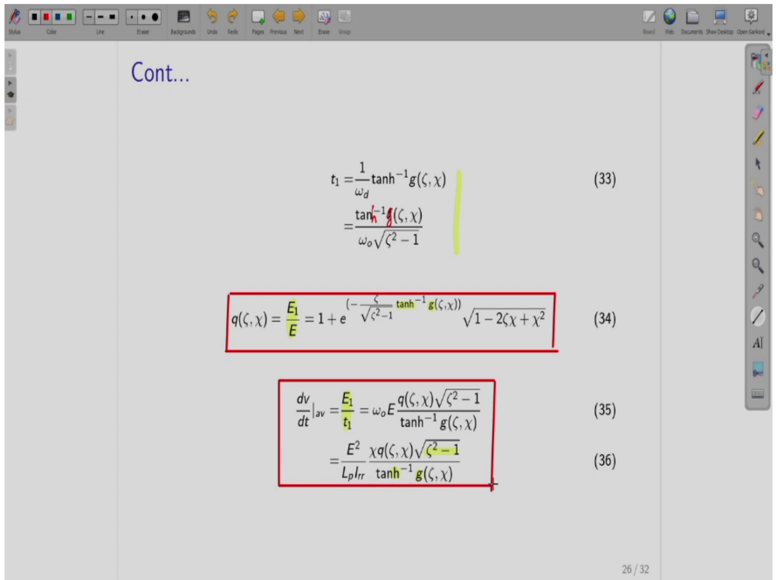

Then, next to the similar we tried to obtain t1 form here. So, this is your tan h. So, tan h inverse of this will be g zeta comma chi by, omega 0 root over of zeta square minus 1. And then further you write this one your q zeta comma chi which is your E1 by E and you substitute for everything and you are going to get.

$$q(\zeta,\chi)=\frac{E_1}{E}=1+e^{\left(-\frac{\zeta}{\sqrt{\zeta^2-1}}\tan^{-1}f(\zeta,\chi)\right)}\sqrt{1-2\zeta\chi+\chi^2}$$

(34)

Now, this equation again is similar to what we had obtained in your underdamped case only difference here is this is tan h inverse and this function is g instead of f and this is very important result that we obtain. Because this is what we are further going to use for your snubber design. Then, we are using it we can also obtain this E1 by t1,

$$\left.\frac{dv}{dt}\right|_{av}=\frac{E_1}{t_1}=\frac{\omega_0 Ep(\zeta,\chi)\sqrt{1-\zeta^2}}{\tan^{-1}f(\zeta,\chi)}$$

(35)

$$= \frac{E^2}{L_p I_{rr}} \frac{\chi p(\zeta,\chi)\sqrt{1-\zeta^2}}{\tan^{-1} f(\zeta,\chi)}$$

(36)

this is also similar to underdamped case only thing is this is tan h and this is g and this is now here zeta the square minus 1 instead of 1 minus zeta square. So, this is also another important result, which we will be using for your snubber design.

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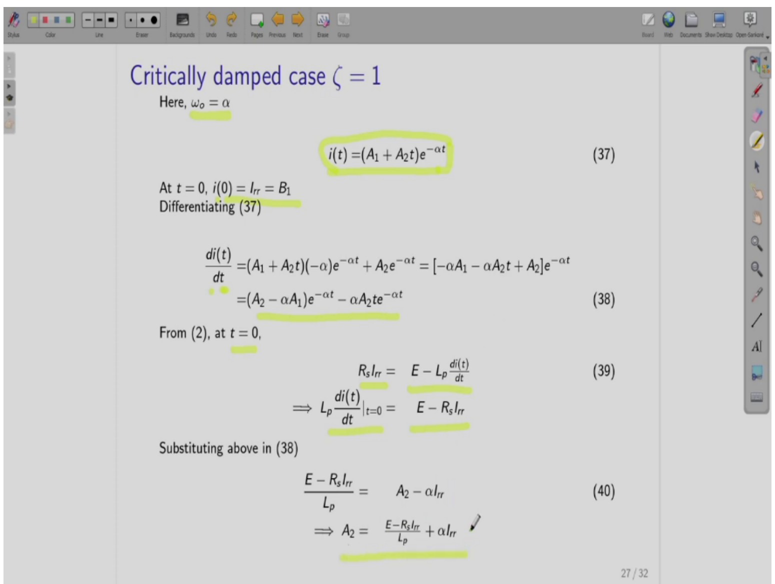

Now, let us go to the critically damped zeta equal to 1. So, critically damped case now, here omega has 0 equal to alpha and it equal to

$$i(t) = (A_1 + A_2 t)e^{-\alpha t}$$

(37)

this you must be knowing for your RLC circuit this is what is the solution that comes for critically damped case. Because both the roots are equal now.

And so, again do the same thing trying to obtain this constants A1 and A2 by using the initial conditions. So, at t equal to 0, i0 equal to Irr are equal to B1 and then further we have this,you differentiated d it by dt.

$$\frac{di(t)}{dt} = (A_1 + A_2 t)(-\alpha)e^{-\alpha t} + A_2 e^{-\alpha t}$$

$$= (A_2 - \alpha A_1)e^{-\alpha t} - \alpha A_2 t e^{-\alpha t} \quad (38)$$

So, when you differentiate this is what you are going to get again substitute for t equal to 0 you know that are this

$$R_s I_{rr} = E - L_p \frac{di(t)}{dt} \quad (39)$$

So, at t equal to 0 this is

$$L_p \left. \frac{di(t)}{dt} \right|_{t=0} = E - R_s I_{rr}$$

substitute back in this equation. So, you will be obtaining for your A2.

$$A_2 = \frac{E - R_s I_{rr}}{L_p} + \alpha I_{rr}$$

(40)

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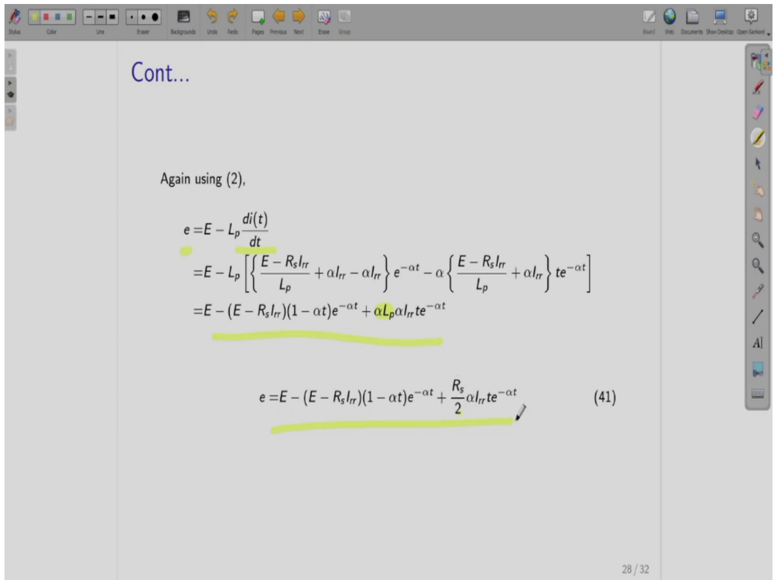

So, now using it, we have this expression for E, we substitute for Lp d it by dt just obtained the last slide and you solve it, this is what you are going to get

$$e = E - (E - R_S I_{rr})(1 - \alpha t)e^{-\alpha t} + \frac{R_S}{2} \alpha I_{rr} t e^{-\alpha t}$$

(41)

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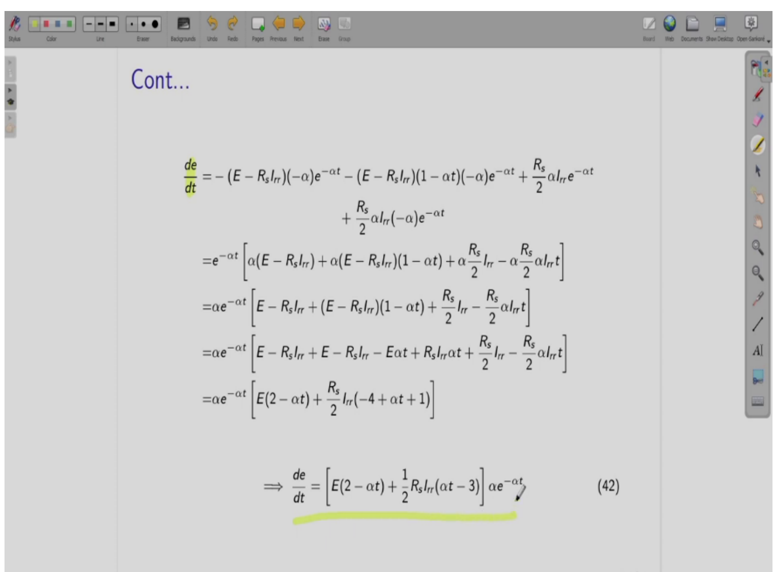

Now, next thing as I did before differentiate. So, de by dt you do it and try to reduce it once you reduce this is what you are going to get.

$$\frac{de}{dt} = \left[E - (2 - \alpha t) + \frac{R_S I_{rr}}{2} (\alpha t - 3) \right] \alpha e^{-\alpha t}$$

(42)

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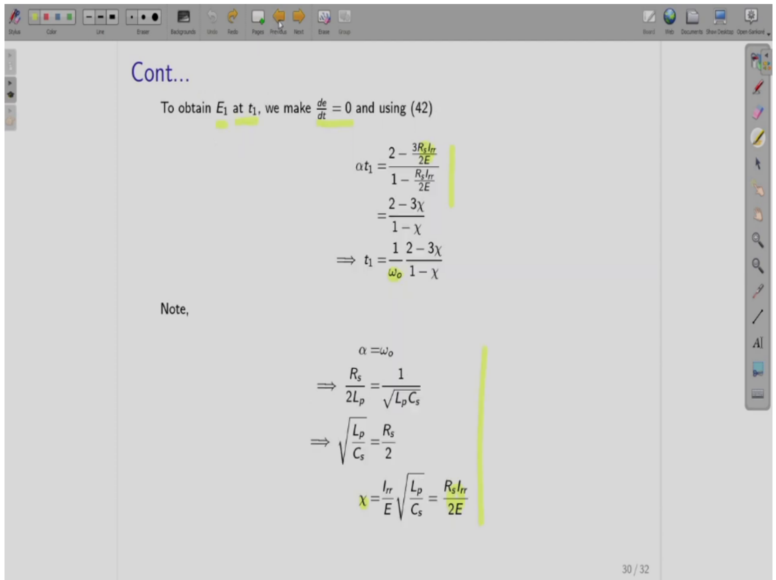

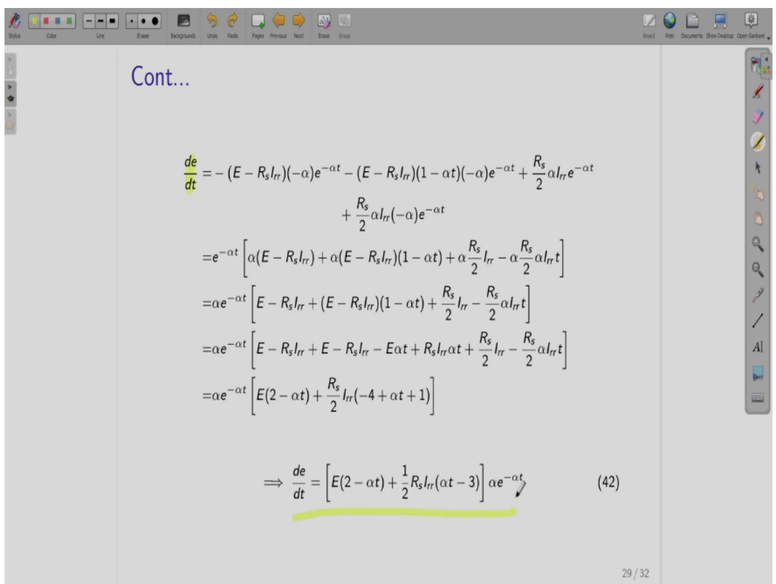

Then after that what we do is that we equate this to 0 to try to obtain the maximum that is your E1 and let us say that occurs at time t1. So, when you do that and then you equated this is what this alpha t1 is what you are going to get. So, this one you equate it to 0 and rearrange this is what you will be obtaining you can do it is simple.

Then, we try to replace this Rs Irr by 2E and you can do the substitution and by doing this rearrangement you will be obtaining chi is equal to Rs Irr by 2E, which here you can replace that. So, this alpha t1 comes out in terms of this as a function of chi and then from here t1 is equal to 1 by alpha now, alpha here is equal to omega 0. So, 1 by omega 0 2 minus 3 chi by 1 minus chi.

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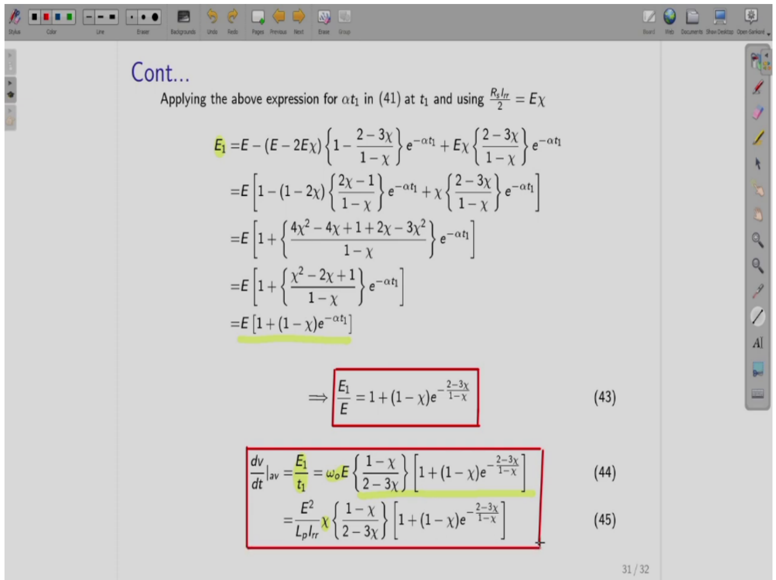

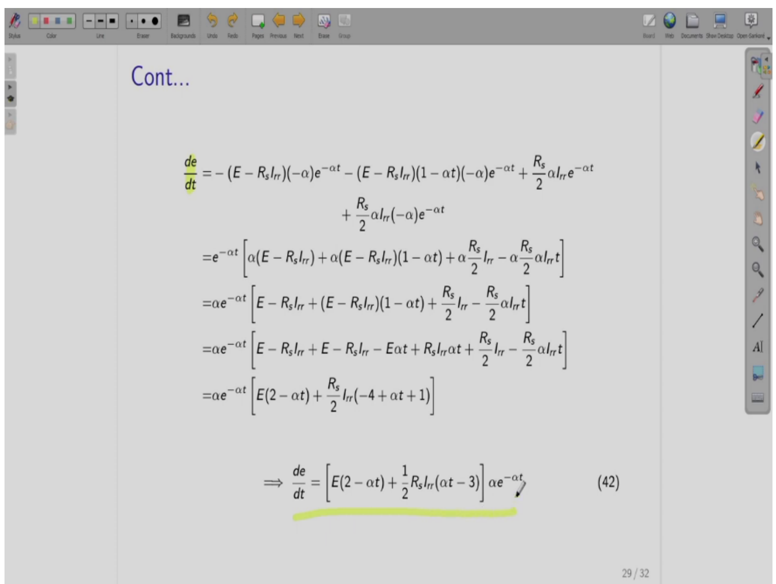

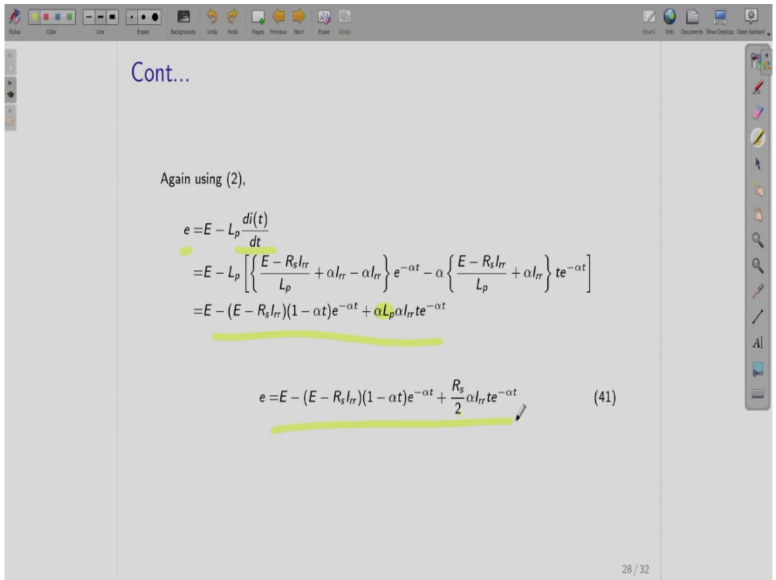

Next, what we do is that, we are going to apply for this alpha t1 in the expression of E. So, we have obtained this expression (41) for E now, we have to substitute for alpha t1 here and all these places in here also this is t1. So, when you substitute that you will be obtaining the expression for E1 the peak voltage. So, you substitute try to reduce when we reduce it, this is what we are going to get.

So, E1 by E finally turns out to be

$$\Rightarrow \frac{E_1}{E} = 1 + (1-\chi)e^{-\frac{2-3\chi}{1-\chi}} \tag{43}$$

is called 1 plus 1 minus chi e power of minus 2 minus 3 chi by 1 minus chi. So, this is the important result, which we will be using for your snubber design. Then next up what we do is, we again divide this by t1, so, E1 by t1 this is your average rate of rise in the beginning. So, that expression is also important. So, that also you substitute and then further you can write in terms of this either you can write it as omega 0 e multiplied by this or E square by Lp Irr into chi multiplied by the same. So, this one is also another important expression.

$$\left.\frac{dv}{dt}\right|_{av} = \frac{E_1}{t_1} = \omega_0 E \left\{ \frac{1-\chi}{2-3\chi} \right\} \left[1 + (1-\chi)e^{-\frac{2-3\chi}{1-\chi}} \right]$$

(44)

$$= \frac{E^2}{L_p I_{rr}} \chi \left\{ \frac{1-\chi}{2-3\chi} \right\} \left[1 + (1-\chi) e^{-\frac{2-3\chi}{1-\chi}} \right]$$

(45)

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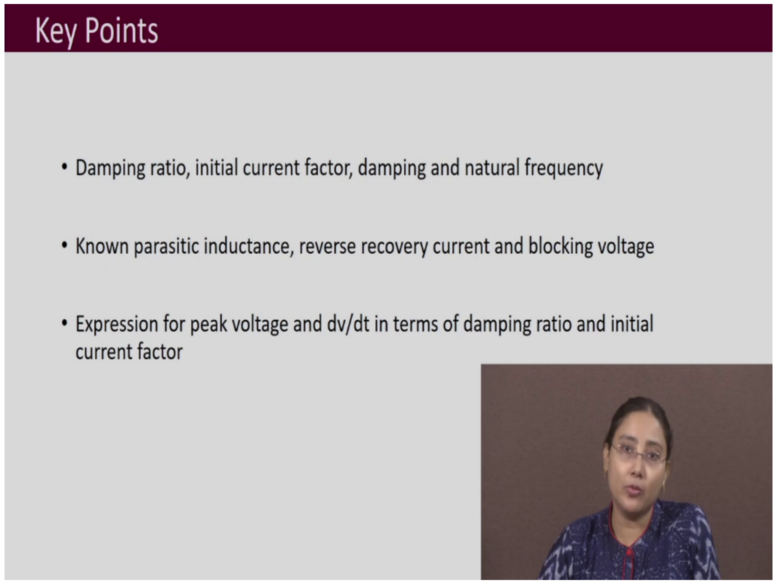

So, the key points of this lecture and the same as what we had discussed before that the important results are these normalized E1 by E, that is your peak voltage or the spike voltage divided by the blocking voltage and the dv by dt rate of change of the voltage across the device initially and that is obtained as the spike voltage divided by the time it takes to reach to that peak voltage.

So, E1 by t1 and these two are obtained in terms of your initial current factor chi and the damping ratio zeta and other important terms, which you have to remember is your natural frequency omega 0 and what is assumed to be known in this derivation is your parasitic inductance Lp and the reverse recovery current Irr and also the blocking voltage which is very easy to find out in whichever power electronic converter you are going to use. So, for all the cases, we had done the derivation and we obtained these expressions. Now, further we will be using these expression to do the snubber design numbers. Thank you.