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VLSI Design, Verification & Test

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Welcome to module 2 lecture 3 of the course VLSI design verification and test, we have been discussing resource constraint scheduling.

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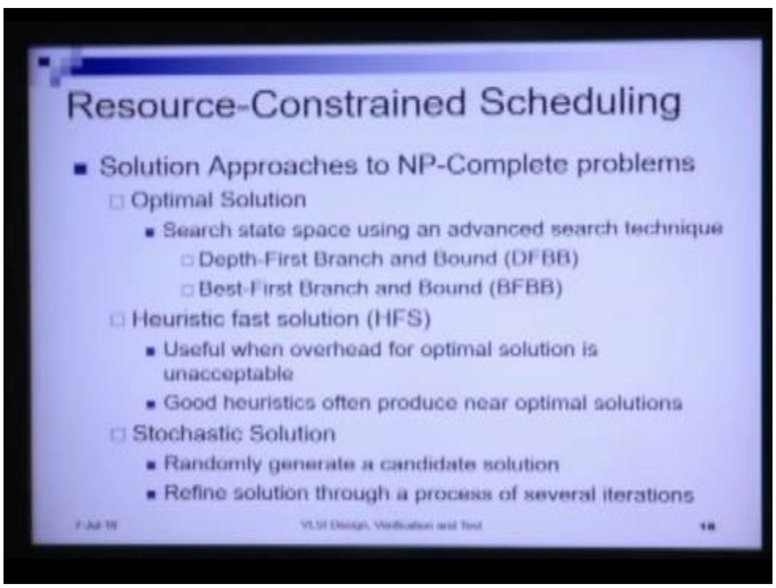

The general problem of resource constraint scheduling is NP-complete and therefore different strategies for solving the scheduling problem exists there are optimal solution strategies, heuristic for solution strategies, and stochastic solution strategies. So before going into other algorithms we will first try to characterize the problem by designing the ILP based problem formulation for the resource constraint scheduling problem.

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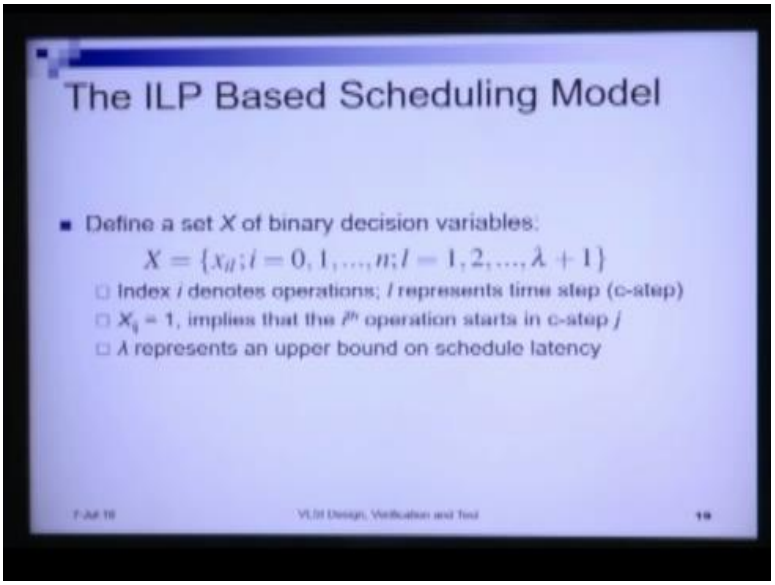

Now the ILP based scheduling model starts by defining a set of binary decision variables here we have the binary decision variables x and x =xil where i goes from 0 to N and L goes from 1 to dot up to λ+1. So here index i denotes operations and L denotes time step and xij the variable xij will be equal to 1 will be set to 1 only when the ith operation is scheduled to start in time step or C step J.

So xij = 1 implies that ith operation starts in C step j and λ represents an upper bound on the schedule latency. So for example in the operation constraints graph that we have been studying the ASAP schedule had given us a latency bound of 4 and we can have other latency bounds and on that latency bound we can perform a similar scheduling.

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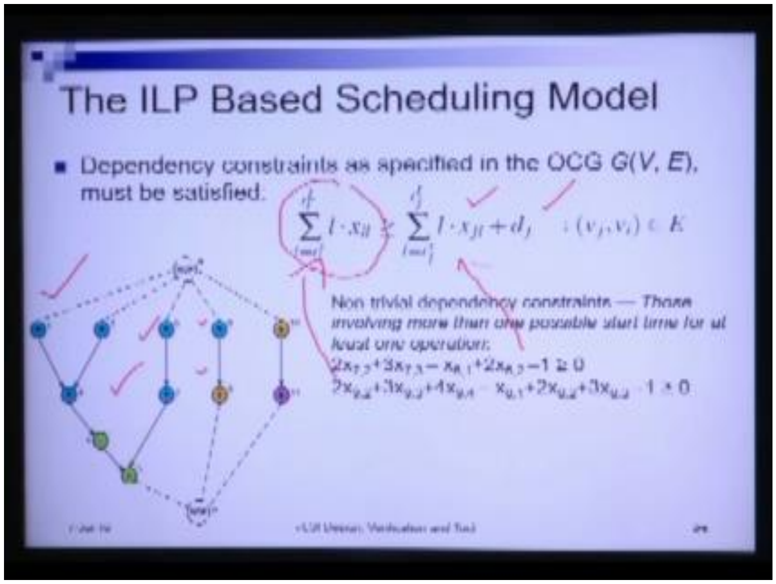

We are saying that an operation will be one when that operation is scheduled in a given time step. Now therefore if we have the ASAP and the ALAP times for the ith operation say we have the ASAP as soon as possible time to be TIL and given any latency bound let us say that the ALAP time of the same operation is TIL.

So therefore in my schedule in the solution this ith operation can start at any time between TIS and TIL between the ASAP and ALAP time. However, an operation must have a unique start time the same operation in the same solution cannot have more than one start times right. So therefore to denote this through a constraint in the ILP model in the ILP model we have an objective function and a set of constraints.

Now to model this as us as a constraint that an operation can start at a single one and only one time step we apply the following that is shown here in the left. So summation N=TIS to TIL XIL=1. So if xil is greater than 1 because xil can only take values 0 or 1X IL=1 when operation I in schedule time step n and xil = 0 otherwise.

So there can be only one value of XIL which will be one between its ASAP and ALAP times. And hence summation XIL=1. Now given this ASAP and ALAP times what will be the start time of that operation, the start time of that operation can easily be denoted as shown on the right of this slide TI=L=TIS to TIL summation L into XIL. So for example, let us say that I have an operation that has actually started in time step 3.

It is possible to schedule it in say time 1, 2 or 3 and in a given solution my ILP has found that I should be scheduled in time step 3. So xi3 the ith operation xi3 will be equal to1. So TI will have the value 3. So this therefore correctly models the start time of a given operation. Now we will take an example and see how this constraint can be applied on the example operation constraint graph that we have been using.

So in the operation constraints graph we see that for all operations having zero mobility can only have one start time the zeroth operation can only start in time step 1, 1 and 2 can only start in time step 1 this can only start in time step 2, this can only start in time step 3 and the fifth operation can only start in time step 4. We are saying that X01 which tells me that the zeroth operation in time step 1 should be 1, X11 tells me that that the first operation in time step 1 should only be one for any other times that it should not be 1.

The second operation should also be scheduled in time step 1 and hence x21=1 x32 = 1y because the third operation should only be scheduled in time step 2 and nowhere else y is x43 = 1 because the 4th operation similarly should only be scheduled in time step 3 similarly x54 = 1 and xn5 which is the last operation should be 1 because the start time of the sink node should be fifth time step because the schedule should complete in4.

Now we look at operations having mobility one which are the operations we see here operation 6 and operation 7 so we have to model that operation 6 can be scheduled either in time step 1 or in time step 2 but in any one of them not in both of them similarly operation seven can either be scheduled in time step 3 or time step 2 but not in both of them hence what do we have x61 + x62 =1 similarly x72 + x73=1 so x72 for example x72+ x73 = 1 tells me that the seventh operation can be scheduled either in time step 2 or in time step 23 but not both similarly for operations having mobility 2 we have this the eighth operation can be scheduled in time step 1,2 or 3 so x81 +x8 2 + x83 =1 similarly operation 9 can be scheduled in either the second third or fourth time step.

Hence x92 x + x93 + x94= 1 the tenth operation can be scheduled in the first second or third time step similarly the 11th operation can be scheduled in the second third or fourth time step.So after we have modeled the unique start times of each operation we come to the next constraint the next constraint is dependency constraint what does it tell me that operation for example here operation 3 must be scheduled only after both operation to and operation 1 have been scheduled only after both operation 1 and operation to has both been scheduled again for example operation 7 must be scheduled only after operation 6 have already been scheduled right so how do we say that as we said in the previous slide that this represents the start time of operation j so the so the left one this indicates the start time of operation I this represents the start time of operation j.

The start time of operation I should be after operation j starts and the whole propagation delay of of the j^{th} operation is also crossed so therefore the delay or propagation delay of operation j + the start time of operation j should be the earlier start time of operation I this is what it says so the start time of operation I is should be at least the start time of operation j+ the delay of operation j when is has this to happen when there is a dependency constraint between operation j and operation I going to show through the edge Vj Vj, vi if there is an H Vj , vi then this one should be true okay.

Now in the in our example we do not show all the dependency constraints here but show only the non prevail dependency constraints for example and what do we mean by non trivial dependency constraints those dependency constraints involving more than one possible start time for at least one operation so what did we say that all operations in the critical path operations 1, 2, 3, 4, 5 all of them have only one start time on one possible start time so there is a trivial constraint will always exist that means that operation 4 should only complete after operation 3.

Now in our operation constraints graph another assumption is that all operations have unique delay so therefore dj is always equal to one for all operations j in dj is always equal to 1 so our non-trivial operations constraints become this for example as we said there is a constraint between operation 7 and operation 6 so how are we representing that this part this one this part this one is this one here so summation l into x $_{il}$ here we are saying that the seventh operation I = 7 and time steps are 2 and 3 l = 2 and 3 so we as we know that the seventh operation can start either in the third time step or in the second time step.

So if it is if it is if it starts in the second time step its time is given by 2 into x7, 2 if it starts in the third time step its time is given by 2 into X 7, 2 if it starts in the third time step its time is given by 3x X(7, 3) because only one of X 7, 2 or X 7, 3 can be 1 so the value of this expression 2x 72 + 3 x73 can either be two or three right now for now continuing the second part here X 6 1 + 2 x 62 this part is therefore denoted this one.

So it says that the sixth operation can either start in the first time step or in the second time step and how do we denote its times, so it can be either X6 1x X 61 or 2 x X 62 so the start time is denoted by this expression, now what we are saying that the difference of the start times which is given by 2 x 72 + 3 X 73 - X 6 1 + 2 x 62 this is the difference of the start times of seven and six this, this has to be separated these start times has to be separated by at least one.

Because all operations are unit time operations this can this has to be separated by at least one and therefore we get this whole corresponding expression here this inequality shows represents this constraint. So similarly we get other constraints as what as there is a constraint between nine and eight the start time of operation nine is represented by what 2 x X92 + 3 x X93 + 4 x X94 the start time of operation 8 is represented as X 81 + 2 x X82 + 3 x X83.

Now again the difference between the start times of operation 9 and operation 8 has to be atleast one and is therefore represented by this constraint, hence progressing in this way we get the constraint between operation 11 and operation 10 as because operation 10 can be scheduled in either the second third or fourth time steps so it start time is represented by 2x 11 2 + 3 x 113 + 4x11 4 and the start time of operation.

So if DI is the delay or propagation delay of the I th operation and if this I th operation is executing at the current time step it must have started at most DI time steps earlier it cannot start earlier than this so we represent that in this term so M = L- DI + 1 to n so this represents the time it goes back at most DI time steps from the current time step and finds out whether it has if whether the operation has started at anytime between the DI eight previous times that and the current time step if it has started this at most this time steps earlier then it is also currently executing.

So given this given this we obtain the constraint as given below here so for each type of resource that we have so for each type of resource k we said that k denotes the types of resources and AK denotes the number of resources of type k, so at each time step not more than a k operations that needs to be scheduled on resources of type K can be concurrently what do we represent as the constraint here speaking again if we have a resource of type K and we have AK resources of that type we cannot schedule more than AK operations that needs to be scheduled on this resource type at any given time step right.

And so here in this the term that we defined here becomes important because this term here tells me that the X that the i8operation is running in the current time step or not if this whole this summation m = L minus DI+ 1 to l x IM = 1 that means that in the current time step l my operation this operation the I th operation is still executing on some resource of type k now then what does this summation tells me that for each type of resource for each type of resource the total number of nodes or operations that can be concurrently operating at this current time step is limited by at most AK.

This is what this resource constraint tell me tells me again as we proceed to we now proceed to the example as we have been doing so let us assume that we have AL use and again two multipliers to L use and multipliers so the aliens are used for the operations plus minus and comparison and the multiplier is used for the star or multiplication operation now what we will do we will now therefore as we have said for each resource type at each time step the resource constraint has to be satisfied.

So hence for the first time step for the multiplication operation what do we have we say that X 11 what are the what are all the possibilities the first operation has a possibility of being scheduled in time step 1 the second operation also has a possibility of being scheduled in time step 1 the sixth operation the sixth operation has a possibility of being scheduled in time step one and the eighth operation also has a possibility of being scheduled in operation in time step one.

There are no other multiplication operations that can possibly be scheduled in time step one so among these four possible operations the constant tells me that it was two of them can be scheduled in time step one why because we have a resource constraint of two multipliers at time step 2 similarly x 32 that means the third operation can be scheduled in time step 2 the sixth operation can be scheduled in time step 2 it can also be scheduled in times the one but can possibly be scheduled in time step 2 as well. Similarly the seventh operation can be scheduled in time step to the eighth operation can also be scheduled in time step two and these are all multiplication operations it may be noted that operation one and operation two having mobility 0 cannot be scheduled on time step 2 and hence we have there source constraint for time step 2 as follows x 32 + x 6 2 plus x 72 plus x 82 less than equals to 2 in time step 3what do we have X 73 + x 83 less than equals to 2 because in time step 3.

What can be shaded either the 8th operation which has a mobility of two can be scheduled in time step 3 or the seventh operation which can be scheduled in either time steps two and three can be actually scheduled in time step 3 so the resource constraint at time step 3 for operation multiplication becomes X 73 + x 83 less than equals to two now we come to the resource constraint on the addition subtraction and comparison operations that has to be performed using the ALU.

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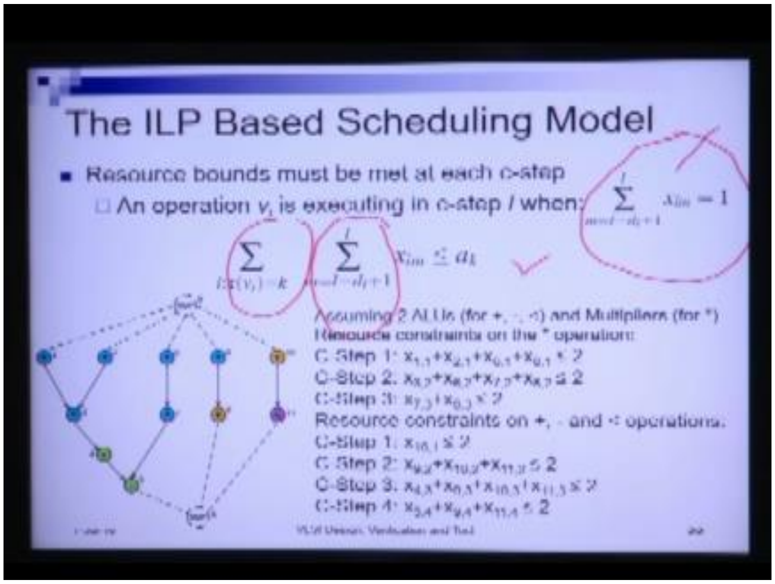

The two ALU use that we have so what do we have we have a single operation the tenth operation that can possibly be scheduled in time step one using the ALU operator ALU resource and therefore we have the constraint as X 10 one less than equals to two in time step two what do we have the tenth operation can also be scheduled in time step to the 11th operation can be shared will possibly scheduled in time step 2 and the ninth operation can we possibly scheduled in time step 2

So what do we have we have the constraint as X 92 plus X 10 2 plus X 11 2 less than equal to 2 so although we have we are assuming here that it is possible to schedule the tenth in the 11thoperations together this possibility will be discarded by the dependency constraints that we have seen in the last slide so in for the third time step what do we have X 4 3 plus X 9 3 plus X10 3 plus X 11 3 less than equals to 2and lastly at the fourth time step we have X 54 plus X 9 4 plus X 11 for less than equals to 2 these are all the resource constraints that we have for our example now given all these given all this constraints.

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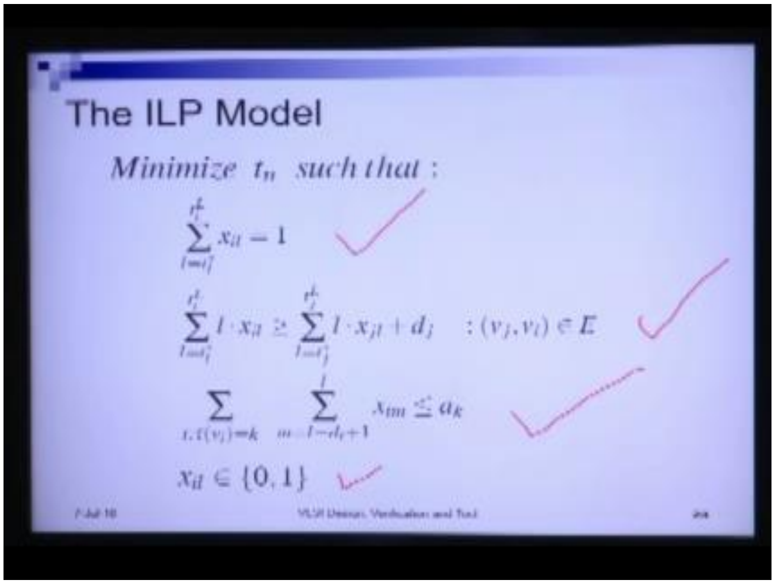

We can now obtain our final ILP formulation now the formulation will first have an objective now here we have put the objective as to minimize TN that is the start time of the sink node or the NH operation now if we minimize the start time or the sync operation that means we are basically minimizing the length of the schedule right so therefore the final formulation becomes minimize TN such that and TN can be obviously alternatively represented as summation l equals 2t is to TN s to T and L, L into X n L the time at which TN is scheduled the start time of the in it node.

Now therefore we can obtain the final formulation as minimize TN such that all operations have a unique start time or dependency constraints are satisfied or resource constraints are satisfied and the last constraint being that the our decision variables exile are binary right just to say that let us say if my objective function would be a bit different for example instead of trying to minimize the NH operation that means the start time of the sink node if you want to minimize the summation of the start times of all nodes just arbitrarily speaking if we had this one then our whole optimization problem would, would be a bit different.

As we can as we know what, what will then be the objective function because we now know how to represent the expressions for the start times of all these nodes therefore our objective function will therefore be an addition over the start times of the of all nodes in my operation constraints graph and each start timing represented by it is corresponding expression with this we come to the end of module 2 of lecture 3.