INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

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VLSI design, verification & test

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Module VI: Binary Decision Diagram

Lecture IV: Ordered Binary Decision Diagram for State Transition Systems

Okay, we are discussing in the infrastructure is called OBDD or in particular ROBDD used binary decision diagram by which we can represent are Boolean function also. We got some algorithm by which you can manipulate the OBDDs like reduce, apply, restrict, exit like that and other hand we have saying that digital circuit can be represented by Boolean expression and the Boolean expression can be represented by ROBBD. And we have find the particular variable order in that bit the representation are when function is unique okay today. We will see how we are going to represented sequential circuit okay.

So how we are going to use module binary decision diagram for sequential circuit, when you look in to the sequential circuit then will find that how you can see that you can represented with the help of state transition diagram okay.

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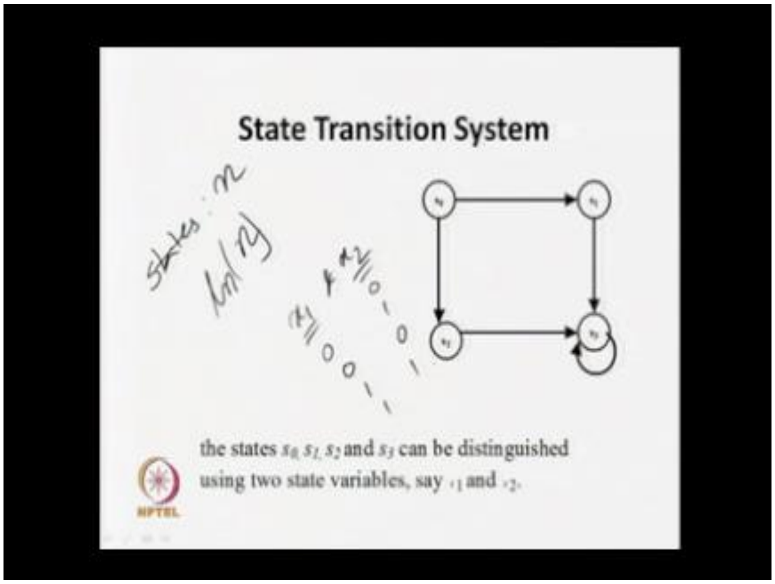

So basically it says that my system is in a particular state depending upon the present behavior and input sequences. It will make a transition to the next step okay. Now we have to represent those particular transition and we are going to talk about the sequential circuit. And basically sequential circuit it will be represented by the state transition system or the state transition diagram okay. Now how say this particular case that state transition system it is having four steps S0, S1, S2, S3. And we are having this particular transition we have transition from S0 to S1 when you are in the transition S1 we can make a transition to S3.

It mean as S3 then if will remain S3 on the other hand form S 0 is going to S2 and form S2 I can go for S3. So this is the state transition diagram and we know that having the state transition diagram then we can see the sequential circuit order in particular state transition diagram. So in this likes are we are going to see how we are going to represented those particular state transition system with Boolean function and eventually we are going to represent those particular Boolean function with the help of OBDDs are the binary decision diagram okay.

To represent this particular state transition system it is having four different states and represent this particular four states we need at least 2 step variables with the help of this particular states variable you can represent this particular states of this particular state transition system. So if in general if the particular state transition system is in number of states is equal to n, then the minimum number of state variables required to represent this particular state transition system is in n that means n. So this is the minimum required number of variable that we need to represent this particular system okay.

But we will take more state variables also. So depending on the state encoding with you follow me. Now see am going to said the at least in n to best two number variable to represents an states. Since we are having four such variables so at least we need two states variables and we are represent in this particular states variable is a x1 & x2. Which this particular state variable we can represent in particular position because we are having four different combinations 00,01,10,11 in this particular four unique combination. We can represent this particular four States okay. This is basically now we are going to say with the help of this particular states variable it consider the state.

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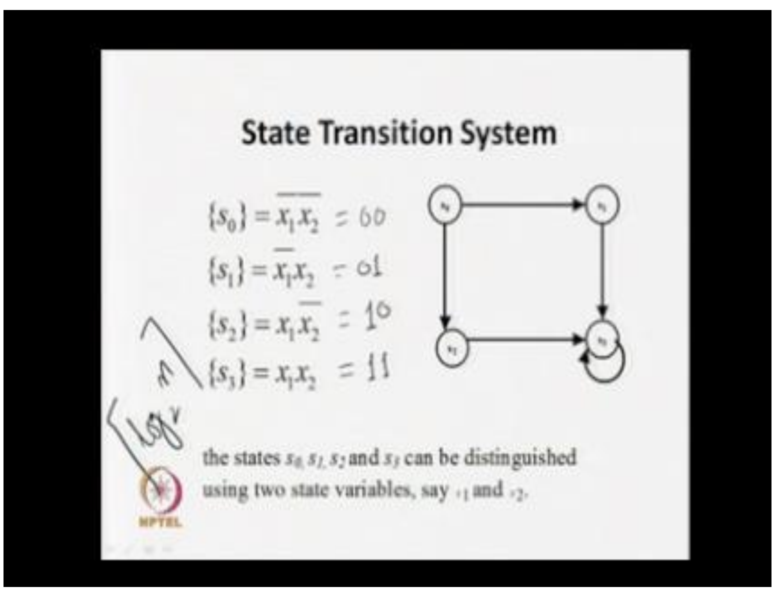

S0 will be represented by X1 bar X2 bar what happens when both are 0, 0 we are going to represent as 0 so S1 will be X1 and X2 that will be equal to and we can say that this is your 0 1 step and coding. S1 S2 bar that means state at calling is your 1 0. And S3 is your X1 and X2 that means that is that variable is 1, 1 and we can say these steps for this particular S3. So we need atleast two step variables to represent those particular 4 states. And already have mentioned that in general in finally an different state and we need in n the number of variables seeing of this particular that means log2n and this is the number of variables required minimum number of variables that means log to the n.

I am going to take a the particular arrow and this is the number of variable minimum number of variable we need to represented as and different states unique okay. So for this state's transition diagram and four different states and it said of state encoding of this particular four states.

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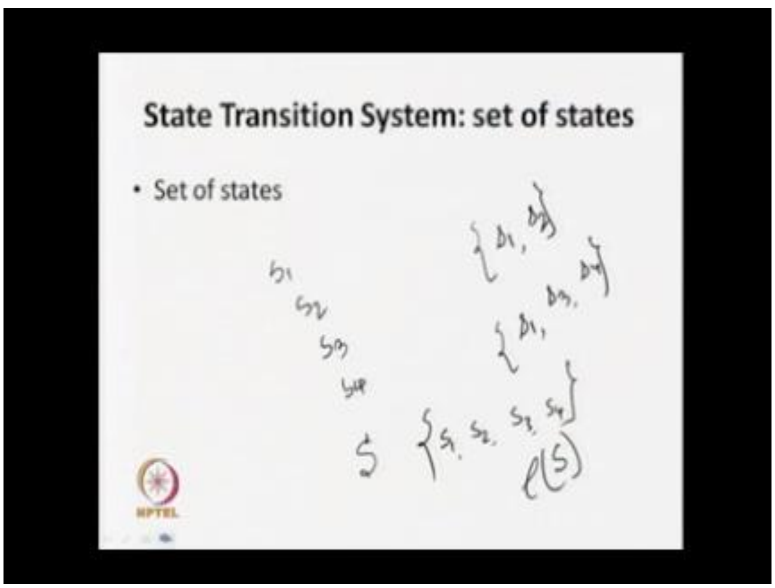

Now when we are going to talk about the state transition system we are four different states. S1, S2, S3, S4. Sometimes we insert the part of set of state and going to said the set of states is S1, S2 sometimes the insert to set of states S1, S3, S4 that means you can see the this order of set of this particular set of states okay. I am writing a set of states as S1, S2, S3, S4 this is the set of states. Then I could say this is a S. Then what are the set of states, what are the define of states we are going to get this is nothing but the states of S l s. That means we are going to get 16 different combinations. Now how to represent those particular set of states we are going to see this particular scenario,

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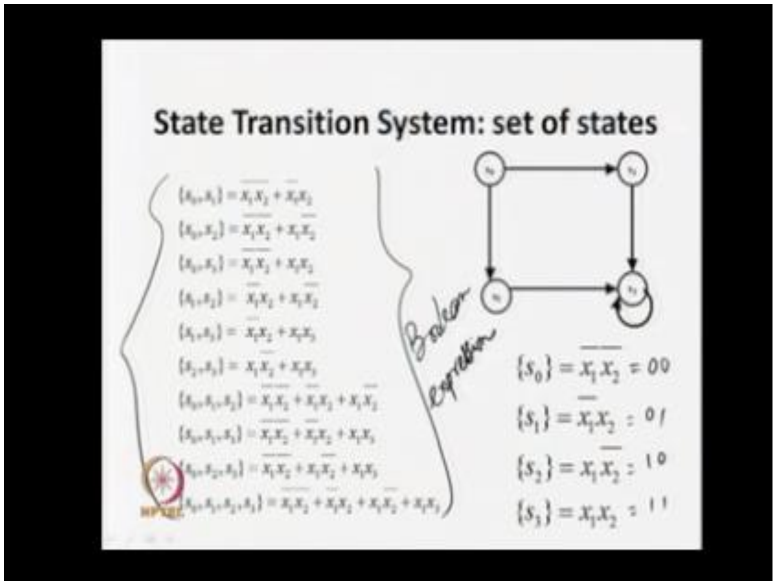

Now compact to this particular previous state transition diagram and having this state’s S0, S1, S2, S3 are the set of binary encoding and this particular four sets S0, S1, S2, and S3 that means this is your X1 bar X2 bar which I have mentioned. This is 00, 01, 10 and 11. So when are going to loop of set states I can talk about it S0 and S1 is the set of step like that S0, S2, S3 is the set of step. Now I can represent all those particular set of states with the help of Boolean expression, because S0 is represented by X1 bar X2 bar and S1 is represented by X1 bar X2. This is X1 bar X2 bar X1 bar X2. Similarly S0, S2 is a X1 bar X2 bar + X1 X2 bar.

So if you see this these are the representation of the set state it is represent two state S0, S1 similarly this expression is represent in particular three states S0, S1, S2. And if you look at this particular left hand side you will find this is nothing but Boolean expression okay. Once we have this particular Boolean expression then what we can do we can construct within the for those particular Boolean expression. Now the state of steps can be represented with a help of BDDs or in particular we can say that ROBDs reduce order binary digital diagram.

And the representation we are going to get a unique representation if we seek the particular variable order, so now people having some states if we consider upsets of those particular set of states. Then how we can do we can represent particular upset or sub set of states with the help of Boolean expression and we can covert this particular Boolean expression are we can represent this particular Boolean expression with the help of ROBDDs, that means ROBDDs can be used to represent set of states okay.

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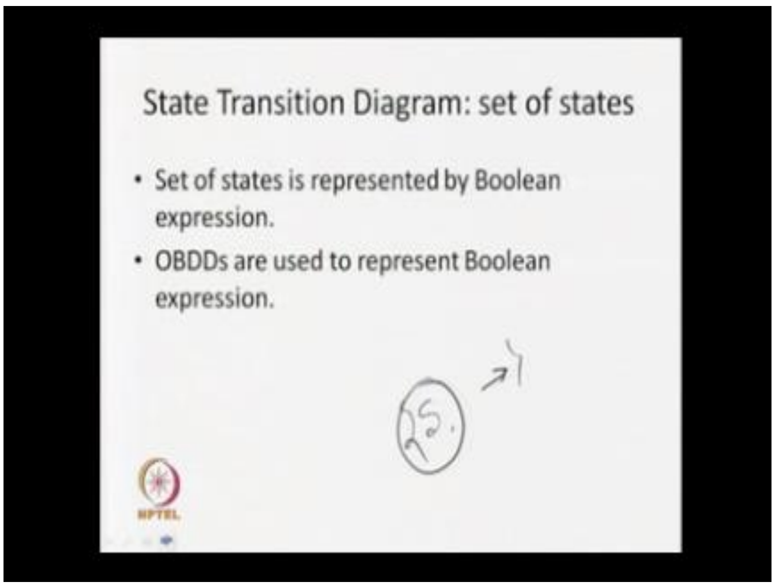

Now we can considered if we are going to look in the state transition system state transition system is nothing but the set of states and one transition relation okay. So basically you can consider this is basically state transition order to basic component set of state and transition relation. Now we have seen the set of states can be represented with the help of Boolean expression and once we have to the Boolean expression that Boolean expression can be represented with the help of OBDDs or it considered ROBDD if it is the particular variable ordered okay.

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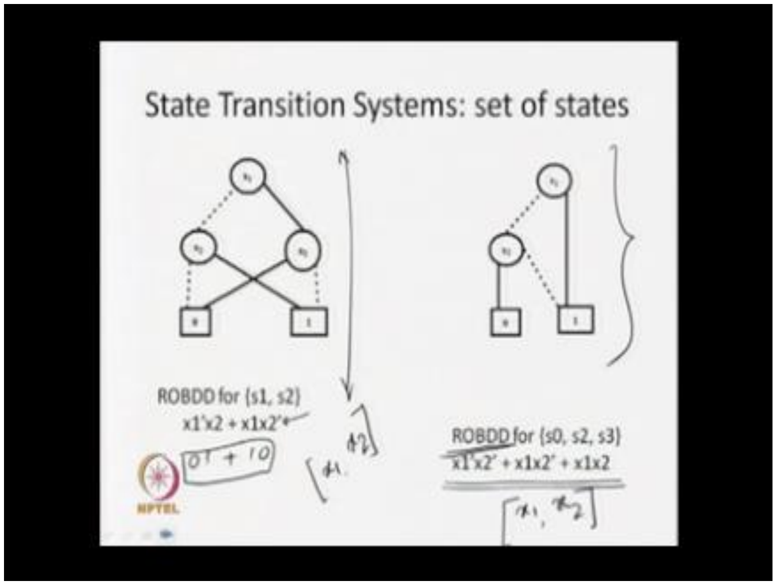

Next we are going to see how we are going to represent this particular states. So in this particular cases are simple example am showing about say this is the set of states am going to take S1,S2 so this is the up Boolean expression for this particular sub set S1,S2, X1' + X2+ X1 so this is basically is S1 is your 0 1 and S2 is your 1 0. Now once I have this particular Boolean expression we can construct the ROBDDs for the this particular Boolean expression.

And we are going to get this is our Boolean ROBDDs for this particular Boolean expression. That means these ROBDDs are going to represented set of states S1, S2 and what is the variable over here I can consider the variable order is your X1, X2 this the variable ordered in we are using this particular OBDDs. And if look into the particular OBDD we will find a no more detection can be apply to this particular OBDDs.

So we said that it is ROBDDs for S1 and S2 steps, similarly this is the angel of ROBDDs, this ROBDDs represent the sub set of S0, S2, S3. So with similar algorithm that we are having we are going to look for the sub set with consists of step S0, S2 and S3. Since we know the binary anchoring we can write down a Boolean expression and this particular sub set one. We have this particular Boolean expression, analogical constructer, ROBDDs for the this particular Boolean expression and eventually proved. Try to construct it you are going to get this particular ROBDDs for this particular Boolean expression.

And the variable order again we are switching for this particular order BDD is your X1, X2 okay. So with this particular variable ordering we are getting this particular BDD and this is basically ROBDDs for these particular subset because we cannot apply any more reduction rules to this particular BDD. So we are getting the ROBDDs like that if we are having step consisted. We are having several steps all those particular steps can be represented with higher power ROBDDs because those particular steps can be represented with the help of Boolean expression.

Secondly if we take any sub set of those particular states the how to happened again you can write the Boolean expressions are those particular Boolean subsets. And this Boolean expression can be represented with the help of ROBDDs. So we can now say that the set of states its can be represented with the help of ROBDDs. So we can now say the state of steps can be represented with the help of ROBDDs.

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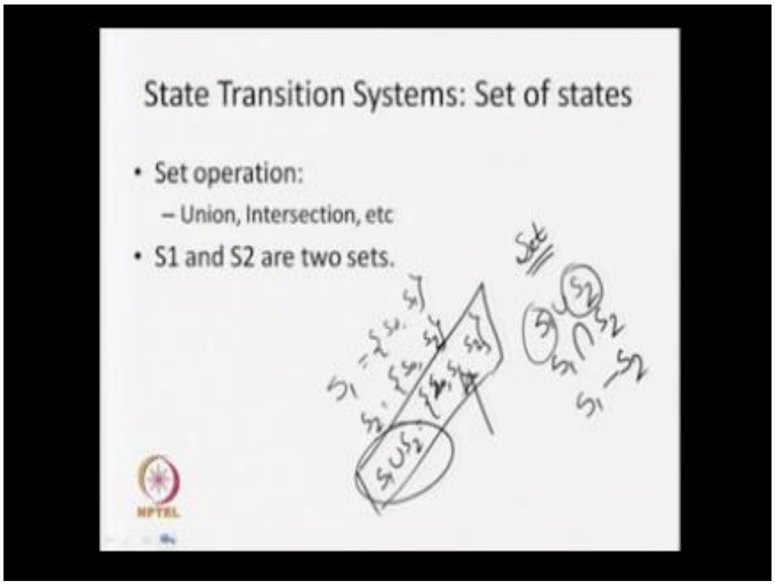

Now when we are going to have a set we are talking about set when we are having this particular set then generally you can apply some set priority operation on set. So if I going to considered two set S1 & S2 similarly it construct S1 union S2, S1 intersection S2, like that S1 minus S2 like that if we are working with a set then we can perform some set preparation on those particular step. Now this particular step S1 and S2 is representing with the help of ROBDDs. Now seem if I want to take the union of this two steps how can walk with this particular ROBDDs representation.

If I want to take the intersection can you use those particular ROBDDs to get the intersection of this two states or with the construct first have to calculate the union then construct the ROBDDs. This is also you can go we know S1 say it is your say S0, S1, we know S2 then we know, so this is S1, S2. So if you can say that S1 union S2 then this will consider S0, S1, S2. So I can calculate the particular union then I can construct the ROBDDs for this particular union. But if I am already having the ROBDDs are this two particular states whether can I used some algorithm or used some operation so that directly going to get S1 union S2 it is possible okay.

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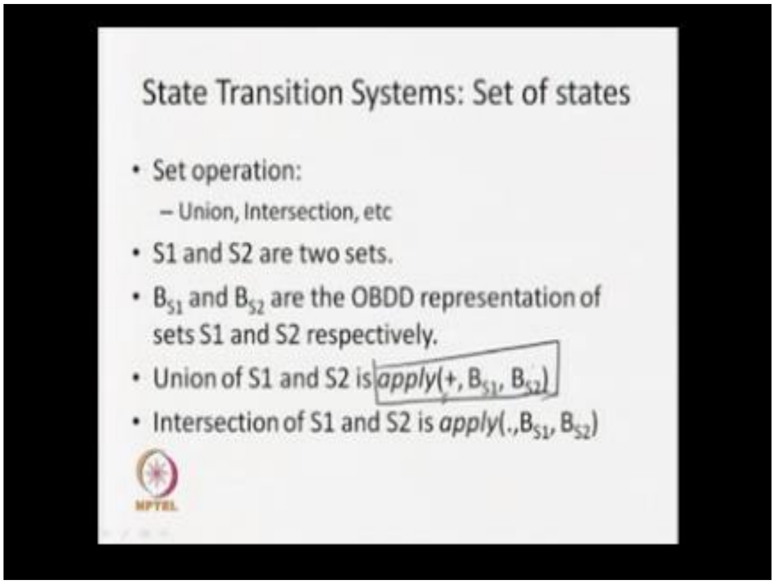

So how we are going to it now we are going to do S1 and S2 are two sets. Now we are going to say as BS1 and B under square S2 are the ROBDDs representation of Set S1 and S2 respectively. So BS1 is the ROBDDs representation of S1 and BS2 is the ROBDDs representation of S2. Now if we are going to take union of this S1 and S2 then its construct it with the help of this operation already we know the apply algorithm apply with some operator can be applied with the help of this apply operation.

So we are going to apply now the operator is your plus Bs1 & Bs2. Now these operations after constructing this after applying this particular operation what will happen we are going to get the OBDDs. And that OBDDs are going to get set of states .basically and going to said union of S1 & S2. Okay, because it is going to take the common potion of Bs1 & Bs2 we can construct.

Similarly if I am going to perform the intersection of S1 & S2 then we can apply this particular operator apply. Bs1, Bs2, so by application of these things we are going to get the OBDDs or S1 intersection S2. So you just see that if I am in the OBDDs representation two states then I can use this particular apply operation on that to get the union and intersection of this two states. Already I have mentioned that when I used apply operation in to get in OBDDs for particular operations S1 & S2 and variable ordering of these particular result in operation same mutual Bs1 & Bs2.

Again you should remember one think when we are going to use particular apply algorithm both the OBDDs mass step combatable variable ordering that means that should have the same variable ordering and the variable ordering our resulting OBDDs is also same with those particular input OBDDs but whatever OBDDs we are getting is cannot be reduce one. So after that what happens I can use this   particular reduce algorithm to get ROBDDs are this particular intersection operation.

So if we are having OBDD representation of also in particular I considered all the OBDD representation of two set and I can use this particular apply operation apply algorithm to construct this union of these two states or intersection of these two states like that we can construct the operation also, so this is one advantage you are getting okay. Now what will happen?

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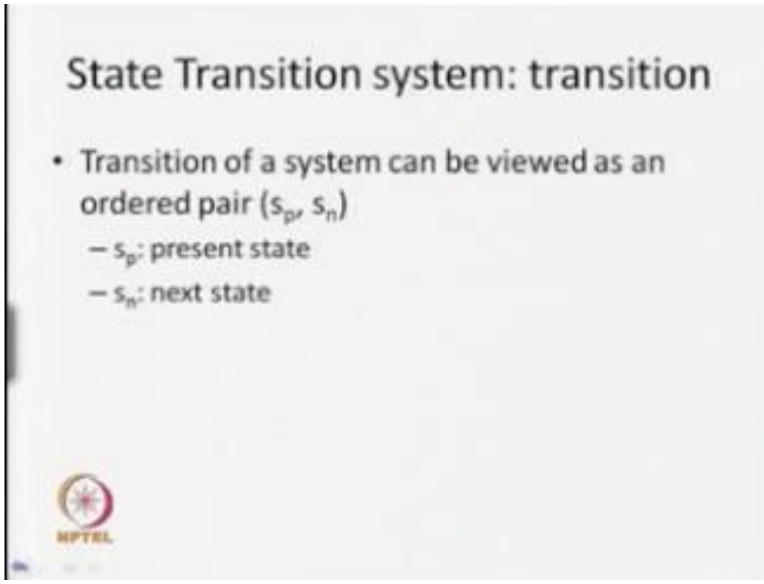

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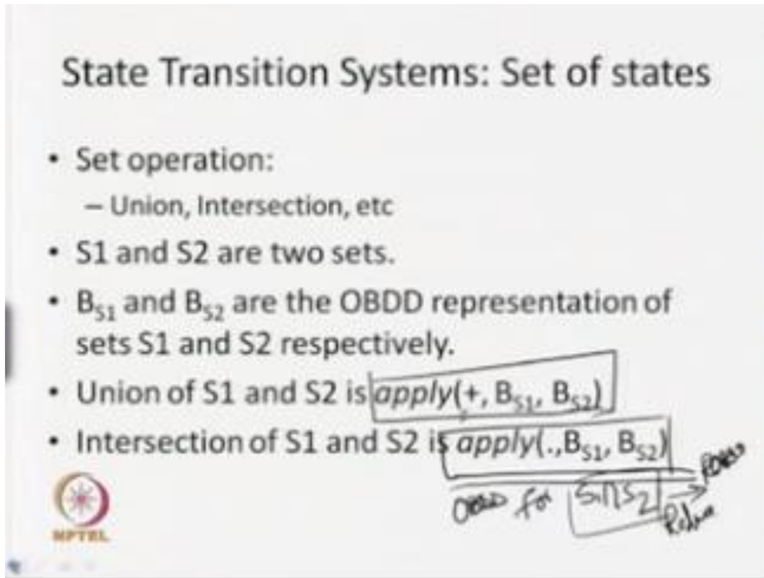

We have talked about as state transition system basically we have seen that in state transition system it is the collection of some states. And we have discussed how to represent those particular states with ROBDDs are too represented of set of states with the help of ROBDDs. Now when we talk about states transition system, we having another component which is the transition basically this is a state S0, and this is your state S1 now we having this particular transition and having this transition from S0 to S1 is state's transition graph. Now our aim is too how we should look for how to represent this particular transition okay, then if we can represent this particular transition then what we can do we can represent the entire states transition diagram.

Because that the states transition diagram is nothing but the collection of those particular transition. Now when we are going to look in to this particular transition then it can be put is an add up your S0, S1 okay we can comes here this particular transition is add up your which is the collection of the two states and here S0 it is having a transition from s0 to s1 so we can say that current state of presence there of my system once it make a transition then if you go to the s1 then I will say that this is your next step of my transition.

So that is why I am saying that this transition can be represented with the order for Sp and sn where sp is the present state and sn is the next state so with the help of this particular things we can represent the transition and once you collect all the transition then we are going to get unpaired state transition diagram or behavior of state condition diagram, now we are going to see how this particular transitions are going to represented or how we can capture this particular transition.

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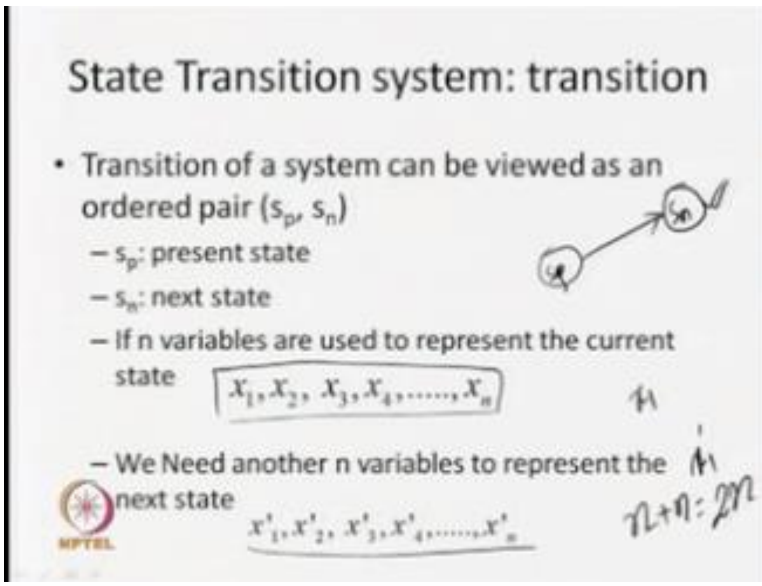

So in this particular case already I have said that if we are having n numbers states that we need total or minimum number of states variables as your log n to diverse 2, now is that states we are having the state transition system where n variable are used to represent the states. So you can said the these are the states variable x1 to xn these are the state variable required to represent the state and I am say that this particular state variable is represent the current state operation states.

My system is in a particular states in a particular instant of time and am going to said that in which steps I am are in which steps states transition system is I am going to represent this with the help of those particular state variables by the evaluation of this particular state variables it is going to the current states, now when am having a transition from one state S0 or said Sp to Sn present state then we have to represented this particular next states also where we are going to after making this particular transition.

I said the this particular next state behavior will be again represented next state will be again represent with the help of n state variable because this is n state variable is required to represent this particular present state. I need another n state n variable to represent this particular next state and I will said that the prime comparison of the original variables are used to represent this particular next state variable.

So if I giving an x1 variable which is related to your present state Sp then cross per the S1 and am going to one variable and I will say this is the x1 time, which will represent are which least to represent this particular next state. So that means we need another set state of variable to represent this particular next states, so we are having the representation of the present state we need n variables.

We are having the representation of the next states which involved another n variable so that means we need total 2n variable, so that means if we are going to use n variable to encode our states to represent this particular transition we need 2n variables because this is nothing but the combination of present states and next states n variables are used to represent the present states another n variable will be used to present the next states. Okay so we need two n variables so if we are having x1, x2, x3, x4 like that Xn is a present states variable then the prime comparison will be used to represent the next state.

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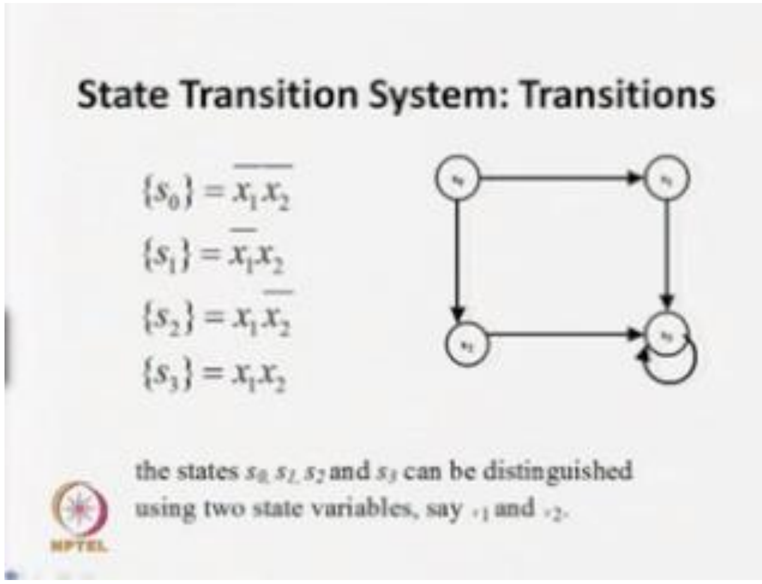

And combination of this particular variables are going to use or going to represent our that particular transition okay now we are going to see this particular representation with the help of our earlier examples what you have seen that this is the state transition diagram it is an four state S_0, S_1, S_2, S_3 and this particular states are represented by with the help of this particular state variable and this is the binary encoding of this particular states.

Now we are having some transition so 1, 2, 3, 4, 5 we are having five transition so transition t_1, t_2, t_3, t_4, t_5 well am going to represent this particular state transition system we have to represent those particular transition, now we are going to represented it.

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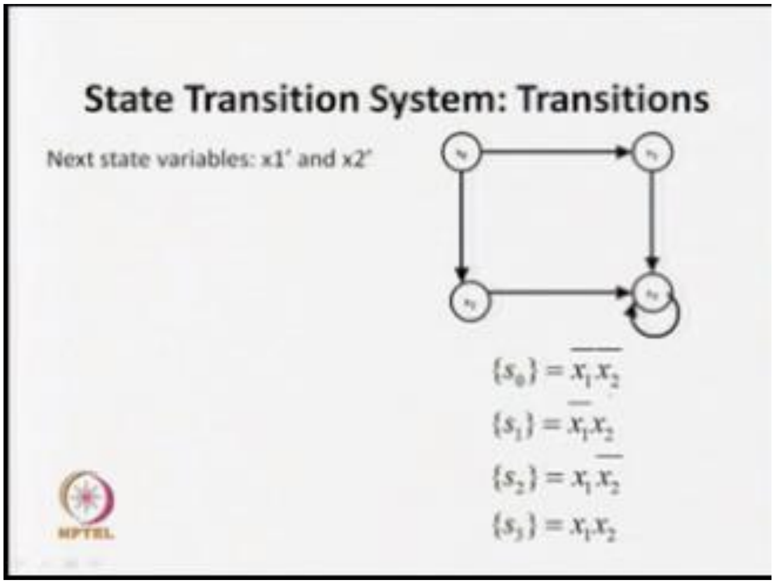

So you just see that this is are the are the states an x1 and x2 are used to represent those states and we can say that these are use to represent the present state then we are going to use next state variables x1’ and x2’ x1’ is correspond to variable x1 and x2’ is correspond to the variable x2. Now with the help of these particular four variables we are going to represent those particular transitions.

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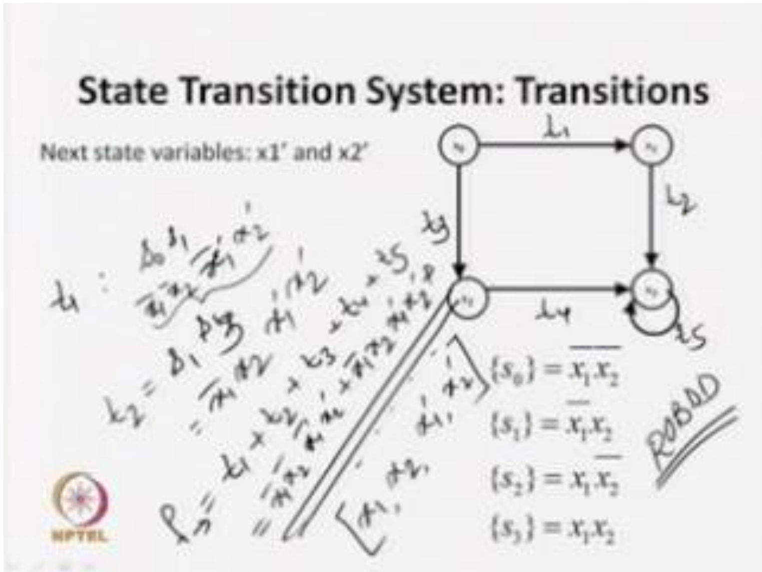

So how we are going to see this things now just say that now already I have said that this is your transition t1. So this is transition t1 is nothing but from S0 to S1, okay now what is your S0, I am saying that S0 is x1', x1bar x2 bar and it is going to next state as your s1, s1 is represented by x1 bar x2 okay so that means we can say that this is your transition I am going to represent x1' bar x2' okay.

So you just see that with the help of combination am going to represent this particular transition t1, so this is transition t2 now I considered transition t2 is nothing but is going from S1 to S4. So what is your S1 this is your x1 bar x2 and x4 I am representing sorry this is your x1 and x3x3 is represented by x1 and x2 that means this is the next state variables will be represented by prime bar so and x1' and x2'.

So like that I can have a transition like t3, t4 and t5 now all those particular transition will be represent that with the help of summation of those particular thing that means t1+t1+t3+t4+t5 these are the prime transition that we are having in the particular transition so this expression is going to represent this particular state transition behavior. And already we having seen that now t1is represent the condition t1 will be represented by x1 bar x2 bar x1’bar x2’ + x1bar x2 x1’x2’ like that I can write the expression.

Now you just see that collection of those particular transition is going to we need a behavior of this particular transition system that means now see what we have see in that again those all those particular transition can be represented with the help of an Bullion expression, we are getting an Bullion expression, once we have this particular Bullion expression then what we can do we can construct the BDDs for this particular Bullion expression and once we have this particular bullion expression we can use the reduce the algorithm to get the reduce binary decision diagram.

And if we stick to your particular variable ordering then we are going to get a ordered binary decision diagram, so ultimately we are going to get the reduce ordered binary decision diagram of this particular state transition system. So in this particular case I can say I can use the variable ordering as your x1, x2 x1’, x2’ I can take this particular variable ordering also.

If am going to take this particular variable ordering then what will get will get an ordered binary decision diagram of this particular Bullion expression and once we are getting the ordered binary decision of this particular Bullion expression. The how to considered a representing the state transition behavior of my system with the help of this particular ordered binary decision diagram.

After the reply apply reduce the algorithm to that particular BDDs ordered BDDs we are going to get ROBDDs that particular transition system. That means what we are considered ROBDD are can be used to represent state transition diagram okay.

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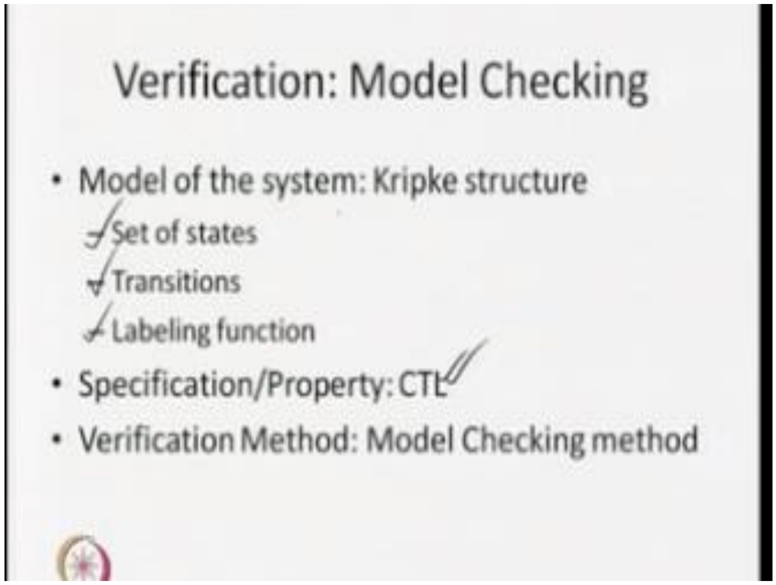

So that is why we are saying that state transition system can be represented by Bullion expression already we seem how we are going to represent this particular state transition system with the help of Bullion expression this is nothing but the combination of all those particular transition and once we are having in the particular Bullion expression then OBDDs can be used to represent those particular Bullion expression that means eventually you can say that OBDDs can be used to represent the state transition diagram.

That means the behavior of our state transition system has been represented by our ordered binary encoding decision diagram and once we reduce it then we can say that ROBDDs is used to represent this particular state transition diagram. And if this t1 particular variable ordering we are going to get a unique representation or that particular state transition diagram. Okay we have seen or we have discuss a particular states are called BDDs binary decision diagram and eventually you have come up to reduce ordered binary decision diagram.

With the help of this particular BDDs what we can do we can represent the Bullion expression and eventually you have seem this particular ROBDDs is can be used to represent any states transition diagram because if we are having a sequential circuit that sequential circuit can be represented with the help of state transition diagram and the behavior of the state transition diagram can be represent with the help of ROBDDs, now before that start the discussing about we are talking about verification technique and the name of this particular verification technique is your module checking.

We have seen this particular verification technique it is nothing but a property verification technique where we are going to represent particular state transition diagram or we say that the specific structure the property will be represent with the help of CTL because we talk about the CTL module checking only we are having other module checker also, so in module checking system we are going to use the kripke structure to represent the state or represent the transition system or model of our system and the kripke structure is having three components one is your set of states, second one is your transition, and third one is your leveling okay.

So these are the three components that we are having, now what we are having the specification an property we are going to represent with the help of CTL combinational pre logic because we discussing about CTL model checking only and after that we having a verification method in that verification method what we are going to see basically what we are doing it, we are check whether these particular given CTL property is true in this particular model of the system or not.

Okay now when we talk about this particular model checking algorithm what we have seen that this is nothing but s graph traversal algorithm okay because we are having the entire state space kripke structure and kripke structure can be put as a graph it is simple graph only and we are having a graph traversal algorithm to find out the state of steps where a particular given property is true.

But we have seen one problem with this particular model checking which is your clap base model checker it is having he problem with the state expression problem because already we have seen that the number of states are related to the number of states variable we have so if we have n variables or n state variable then we are going to get 2^n difference states okay. These are different possible combination that we may have and these are the different steps that we may have, but in particular all steps may not be the same but in my design say it reports another 1 variable. That means I can say that I cannot do it n variable I need n plus one variable.

So in the particular case the number of states we are going to get about 1 to $2^n +1$ that means the number of states is exponential in that n with respect to those variables. So this is a measure problem if you go for the bigger system then in this case it will be very higher, now how to contain this particular state space, so this BDD can be used to contain this particular state space extension problem because we have already seen that the state contingent system can be represented with the help of the ROBDDs and in most of the cases, we have seen that most of the Bullion expression can be represented by help of the compact ROBDDs.

Also I have mentioned that size of these ROBDDs depends on the variable ordering, for the particular variable ordering we may get a very compact representation but for the same function, we define variable ordering may not get a compact representation it may a bigger one also to find out the appropriate variable ordering is a hard problem. We do not have any algorithm which will say that this particular variable ordering will give you the best representation best ROBDDs representation for this particular Bullion function.

So it a hard problem, you use here some realistic to get some variable ordering which will give us a reasonable size of ROBDDs. So that OBDDs can be used to state transition system but lien or kripke structure can be another component which is your labeling function. Now we are how we are going to represent the particular labeling function okay.

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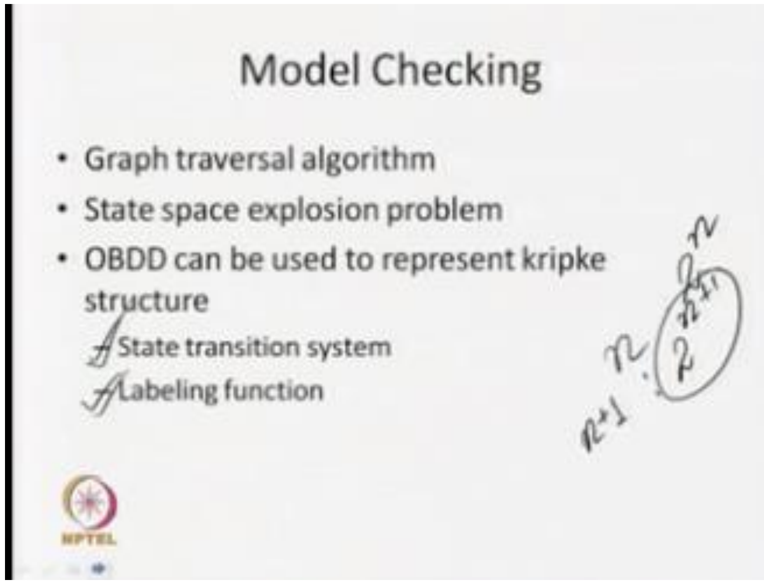

So in this particular case what will happen you just see how I am going to give this particular labeling function?

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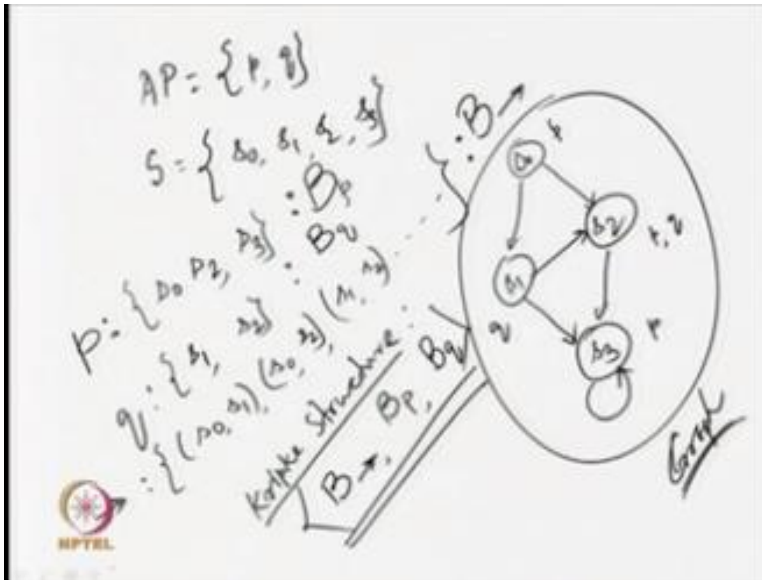

Now consider one particular state it become say this is s0, s2, s2, s3 so. So this is a kripke p, q. Now you just see, in this particular state transition system or say kripke structure I am having two this things, atomic proportion and that say state of atomic proportion is your p and q and it is having four state as s0, s1, s2 and s3 okay. On the other hand it is the level with this particular atomic proportion. So when we talk about kripke structure we need the state transition system and along with this particular labeling function.

Now when we are using OBDDs to represent the state transition system along with that we mark with the information of this particular labeling function. We are on the model checking or you can say that ROBDDs can be used for model checking algorithm. In this particular case now you just see that, which side of the state, where this particular atomic proportion p is stored. So if you see these things, then this is your s0, s2 and s3 and where which are the state where this particular atomic proportion q is 2.

Then in this particular case you will find that this is your s1 and s2. You know that state transition system, here we are having here is a transition and what are the transition, I can say this is s0 to s1, s0 to s2 like that s1 to s2 like that you can list all those particular transition 1, 2, 3 to 4, 5, 6. So these are the incomes we have. Now this is a sub sets, this sub set can be represented with the help of BDDs and I can say this is a BDD representation of those particular states, where this particular atomic proportion p is true.

Now I can represent this particular sets okay with the help of another BDDs and say this is a BDD bq which represent the set of state where this particular set of state atomic proportion q is true and these are the state transition where we have in our system and already we have seen how to represent this particular transition behavioral of the system with the help of ROBDDs. So I can say that this is your b arrow is the ROBDDs representation of this particular transition relation.

And you just think that if I am talking about order BDDs all this BDDs are having compatible variable order, now what I can say that for kripke structure? We should have the state transition behavior and along with that labeling function. So that means we can say that b arrow bp and bq, this three BDDs is going to represent kripke structure okay. Now if I am having particular kripke structure, that state transition diagram along with this particular labeling function. So this information can be captured with the help of these three BDDs. Now you just see that, that means ROBDDs can be used to represent the kripke structure also.

So once we represent the kripke structure with the help of ROBDD now we see whether that model checking algorithm can be perform with the help of this ROBDD s or not because already we know that I am representing this kripke structure with the help of a graph then I can use some graph afforded algorithm to implement of model checking algorithm to find out which are the states that a particular property is true so this is the graph you have to gather we are using.

So since we are suing this particular keep guest BDDs to represent this kripke structure now whether can I use this particular ROBDD presentation to come up with some model checking algorithm.

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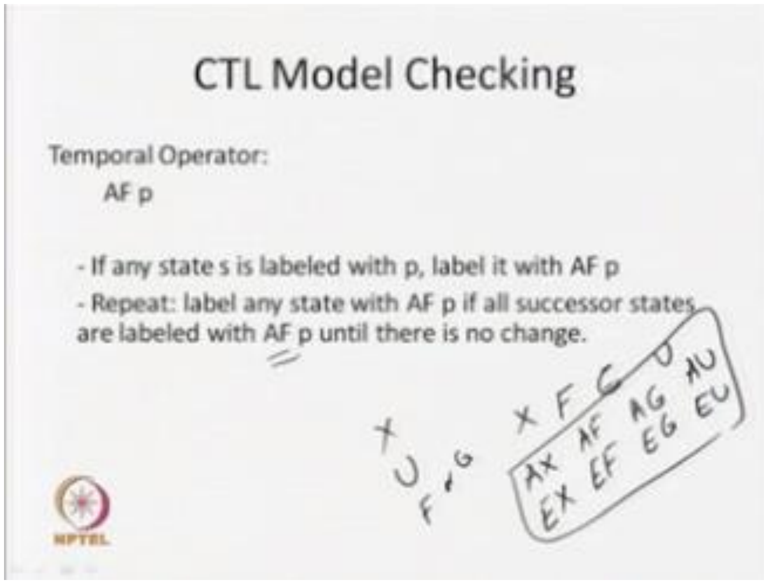

So in this particular case it is possible and we are going to say this is your symbolic models. So if OBDDs are use to implement our model checking algorithm then we say this is the symbolic model checking algorithm and we are giving the terms symbolic because here the state transition diagram has been represent the symbolic alive with the help of ROBDDs and set of states say I am going to have a combination of set of state say s0 s1s2 like that this particular set of states again represented with the help of an ROBDD.

You can say this is the symbolic representation of this particular subset let show people are using this particular terms symbolic, symbolic model checking so symbolic model checking is nothing but your model checking algorithm only, but here we are using the structure BDDs to represent our entire stats transition diagram that means we are using that ROBDDs to represent the kripke structure.

Now we are going to see how we are going to implement our model checking algorithm with the help of those particular ROBDDs, now you just see that I am coming up this particular already we have discussed about this particular temporal operator AFp because already we know that we are having some operators say next state future globally and until and along with that we are having two part quantifiers so in all part AX Af Ag and AU until operator is a binary operator what xX and GRO in your operator and another part quantify you are having EX EF EG EU now we need the procedure to evaluate this particular operator but already we have seen that we need three operators which are the adiquature of operator at least it should contains one next state operator one until operator and we can chose one of the operator like F or G.

So already we have discuss this particular operator AFP and we have see that how we are going to evaluate this particular AFP we are having the algorithm to evaluate this particular temporal operator AFP, so ion a nature how we are going to do? If any state s is leveled with p then leveled with it AFP then we are going to repeat this procedure level n is state with AFP if all successor states are leveled with AFP until there is no sense.

Now you just see that we are starting from some state if it is leveled with p I am going to say that afp is true over here then we have going to find out all the predecessor of this particular state okay, if this all the predecessor state also we say that AFP is true because in future we are going to get some state if all the successors are having leveled with AFP okay, so this is the way that we have going to do find out this particular operator AFPO which is graph preferably algorithm.

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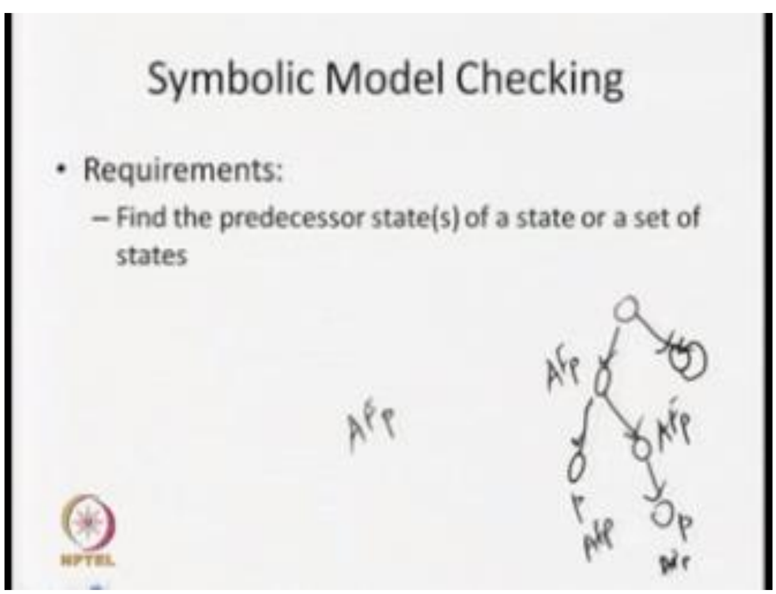

So basically what is our basic requirement if you look in to it and saw find a predecessor state or a state or a set of state so basically you just see that if I am going to have this particular transition system, okay now say if this is your P and p is true and in this true case I am going to say that AFP is true now what I am going to say I am going to see a predecessor so from this particular predecessor I will say that AFP is true over her AFP is true over here.

Now in this particular is this predecessor having only one successor AFP is true so I am going to leveled with it AFP okay, now for this particular state seen both the successor are having a piece why I am going leveled with AFP, now when I come to this thing I will find out one is leveled with AFP but he second one is not leveled with AFP so we are not going to level with AFP, so you just see that in this particular method what happens we have to find out he predecessor states of a particular set of states or a particular state and depending on our operator we are going to say that one particular CTL formula is true on this particular state or not.

So in our graphed say leveled algorithm so basic requirement is to find out the predecessor state or particular set of states.

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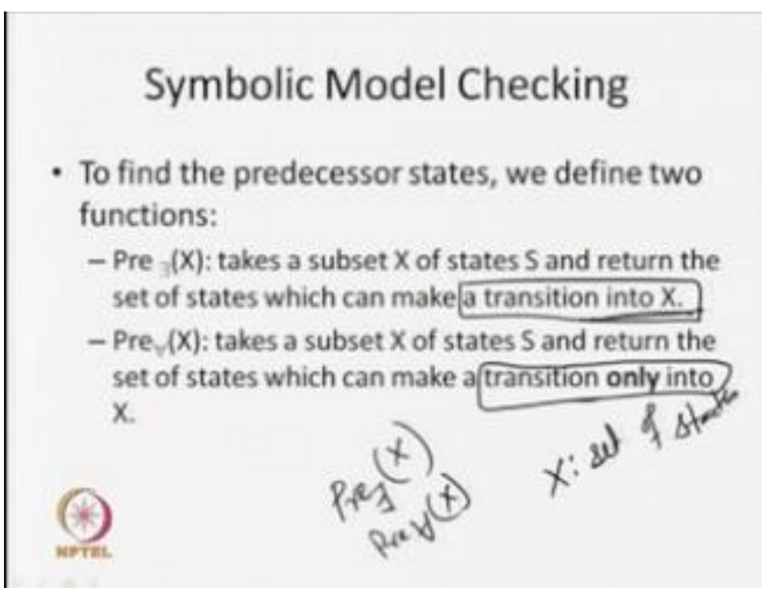

Okay now for that now when we are going to look for this particular symbolic model checking that means we need two operators one we are going to select pre they are exist X and p for all acts so you say that two operator will be p they are exist X and p or all X where x is a set of state, so what we are going to say that p they are exist x, now we take a subset x of state sand written the state of x which can make a transition into x, so we are having a subset x we are going to set of states which can make a transition to this particular subset.

And pre for x it is similar to pre they are exist x but in this particular case we are going to take those particular states which can make a transition only into this is the graph transition only into and here we can say that make a transition into x, so we are existing at least one transition is coming to acts I can say that they are exist the transition which is going to this particular subset x that means if I am giving subset x then we are going to find out the predecessor state of those particular sub set x.

Where at least one transition is coming to this particular states but when we talk about pre for all x that means you are going to consider those states which are going to make a transition to this particular subset x only that means all transitions should through mote this particular acts so that is why we are saying that make a transition only in to x it should not go to this. So in model checking algorithm our basic requirement is to find out the predecessor state and we can find out this particular predecessor state with the help of this two operator pre they are exist x and pre for all acts.

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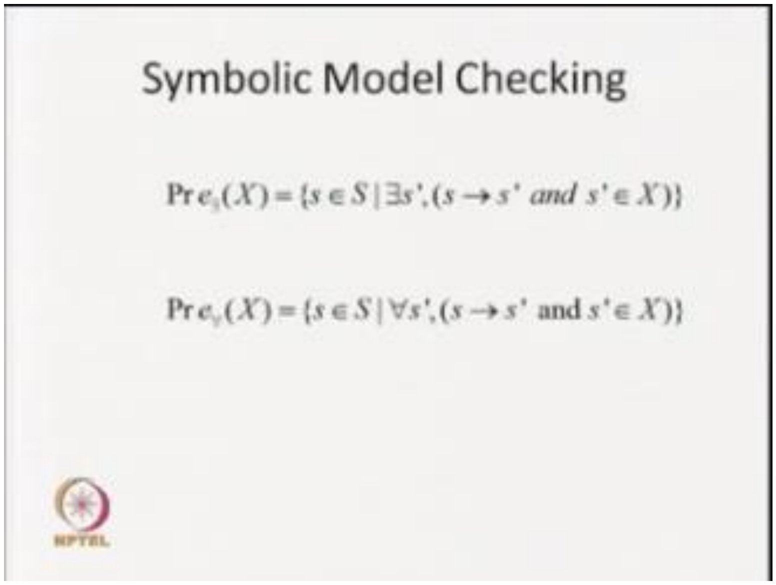

Now this is basically in our metrical analysis we can say that p they are exist x we are going to collect all those particular states belongs to x such that they are exist a state as that and we are having a transition from s2 to s- and s- belongs to x, and p for all x we are going to collect all states from s which are member of this given entire state space where for all s – if we are having a transition from s to s- then s- must belongs x so this is your for all transition must go to x they are pre they are exist at least one transition must go to x so these are the pre for all x and pre they are exist x. Basically these two properties are going to a predecessor step of a given state of step okay.

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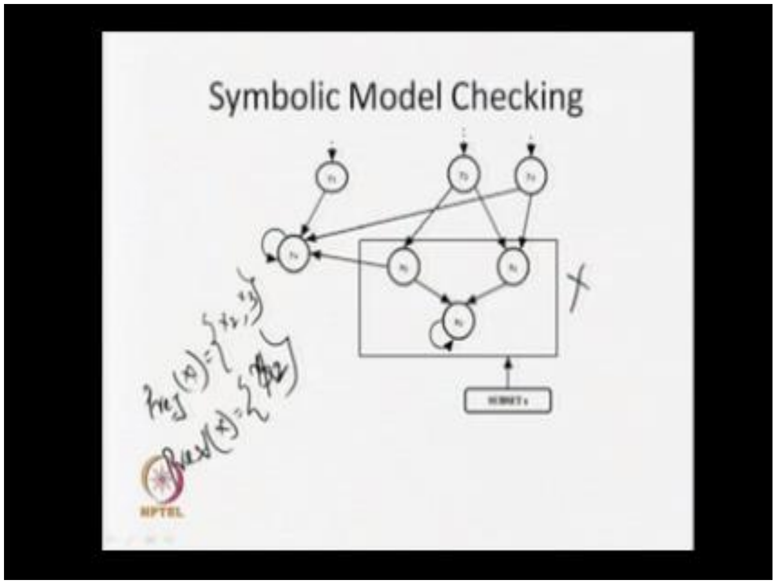

Now see this is a symbol example this whatever we are talking about P there exist X and pre for all X. Now in this particular case this is a subset we are considering this is X it is having particular three states. Now we are going to look for basically you are going to look for going to evaluate the predecessor state in successor atleast one transition will come to this particular X, subset X, or P for all X all the transition come to this particular state. So in this particular case if we said the pre for all X will find particular states S1, S2, S3. So for S2 having two transitions both the transition are coming to this particular subset X.

So basically it is going to say X2 is the states which are all the conditions are coming to this following. And pre X atleast one condition must come to these things. So in this particular case since all are coming pre X is also that X2 when I come to this particular X3 states, then you will that it is having two condition one is coming to particular subset X so X2, X3 is going to set of states. Where it is satisfied as a set of state pre X is X okay.

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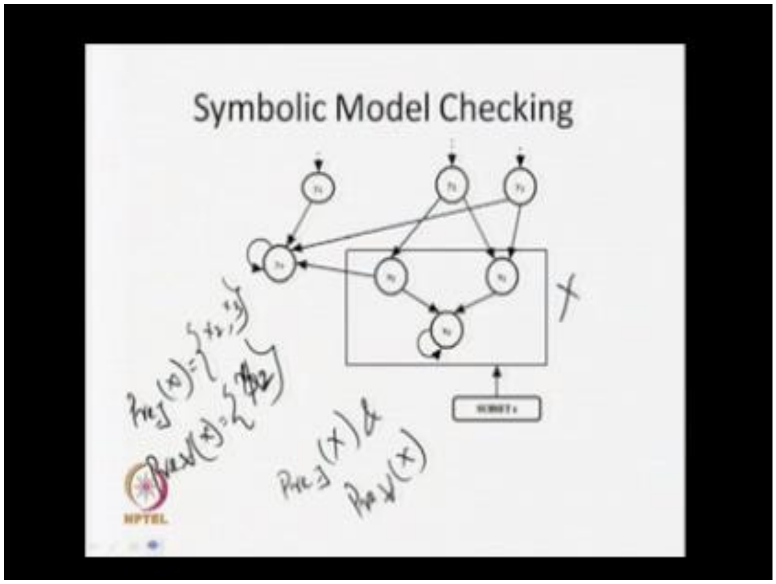

So these are the things will be need here. Now you considered to you having this two operator were there exist and pre for all our X with the help of these things and find out the predecessor steps okay. Now here when we are going to calculate this particular predecessor state and we know that these are required to evaluate or model checking algorithm. Because already seem this particular process when we are going to said up in types of scenario that at least calculate the predecessor states and we are going to collect the all type of particular states. Now we are seeing that pre for all X and pre X. You can evaluate those particular states. You know pre for X all the states are coming to this particular subset X. there exist that is one condition is coming to this particular set of steps.

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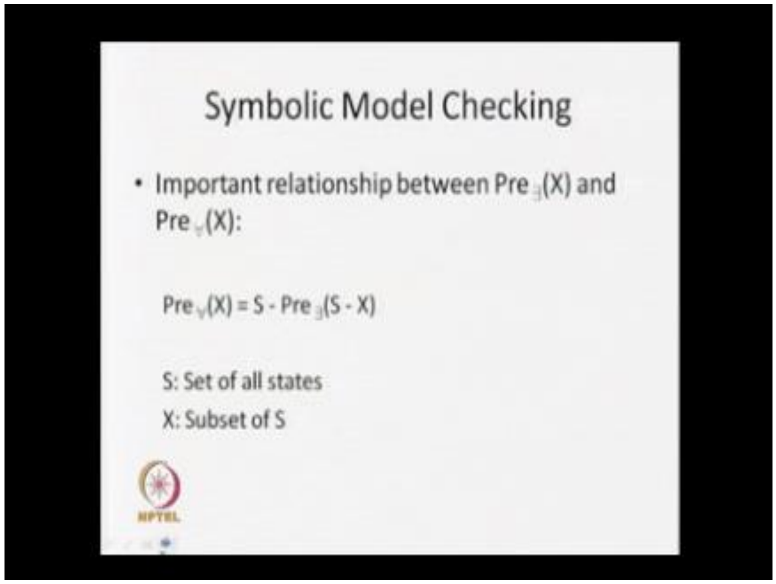

Now we are having a relationship between these two things and pre X and pre for all X. Now this the important relationship between pre there exist X and pre for all X. Now this for pre for all X can be represented with the help of pre there exist X this is the relation pre for all X=S- pre X-X. Where S is the set of all state and X is the subset of X okay.

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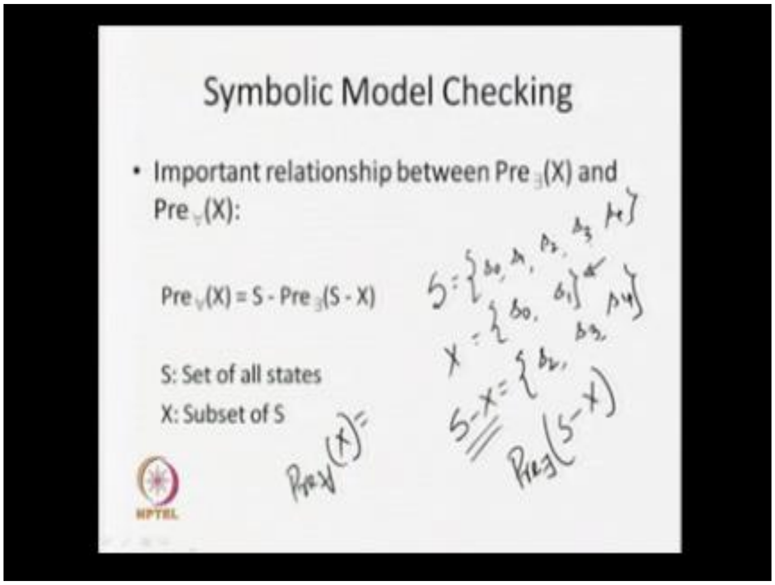

Now what we considered how we are going to have this things pre for all X= S – pre S- X. I am having a given set of state S equal to say I am having S0, S1, S2, S3, S4 okay. Now I am having a subset which having a X which is having say S0, and S1. Now in this particular case we are having S0, S1, S2 like this then what is my S-X. S – X is basically S2, S3, S4 okay. Now considered if am going evaluate of pre for all X then what will happen I am going to consider all the all those predecessor state when all those condition is coming to this particular two states S0, and S1.

This S is the set of state, S0 and S1. So pre foe all X in this particular case all condition must come to this particular steps. Now am going to take the set deference I am going to loop for S-X, so S2, S3, S4 is the set of state which represent X- S, now when I am going to considered a pre there exist S- X. So in this particular case what will happen form a particular state atleast one condition is coming to this particular S- X. So you just see that if for a particular states if one condition is going to S-X. That means all transition will gone cannot to this particular X so see that, so this is my,

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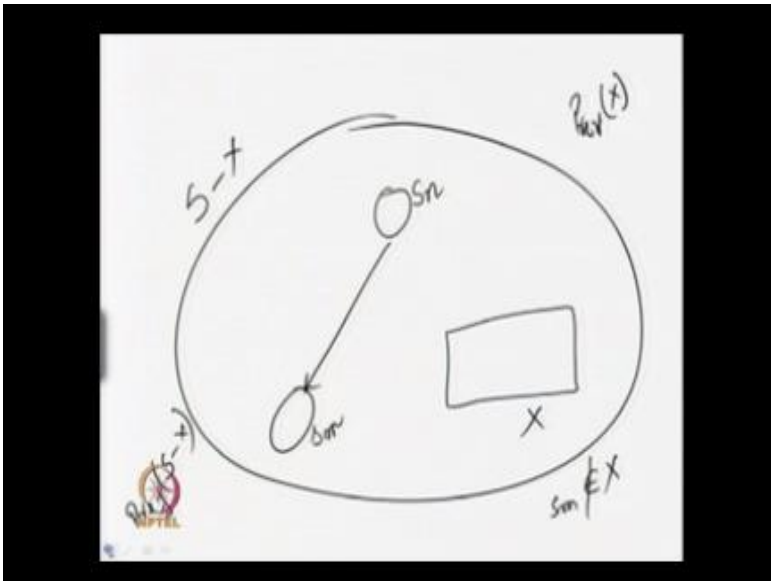

Set of state x and adopt potion is basically S- X. Now if I considered one particular set say S N okay. Now if one of the transition is coming to this say this is SN which is not belongs to your X that means SN is not belongs to your X. That means which we are having one condition which is not going to this particular X okay. So this is the things that am getting this condition I am going to look of pre there exist S- X. So this particular state will not come on the pre for all X okay. Because all transition are not going to X, so that is why what we are doing first pre there exist S- X.

So such types of things we are getting and that means type of state SN will be picture and it should not be included in pre for all X. So in this particular case I am evaluating in these particular things and eventually removing those particular states from that is. Where at least one condition is going outside of the particular X. So the remaining state will be or remaining states will go to give me pre for all X okay. So in this particular case what will happen you itself see that here we got the pre for all X with the help of pre there exist S-X okay.

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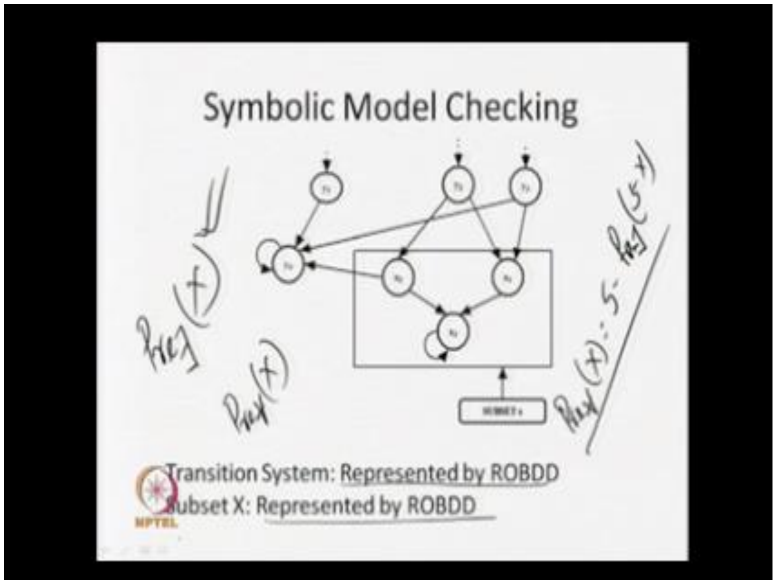

This is the way we are going to evaluate, that means if we are having a procedure for pre there exist S-X then from here you can pre for all X. That means you itself see that you need one procedure which is going to evaluate pre there exist X. Then very well you can calculate pre for all X okay. So that is why what happens now you itself see that the whole transition system we can represent with the help of ROBDDs. If we having a subset of X it can also be represent by ROBDDs. Now we have to see how we are going to evaluate this particular predecessor steps okay.

Already we have seen that we can if I can get a procedure pre there exist X okay. If this going to give me all the state form which at least one transition is coming to this particular subset X now once we have this particular procedure I can pre for all X, because we know the relationship pre for all X = S- pre there exist S-X. So if am having a procedure for pre there exist X we can develop pre for all X.

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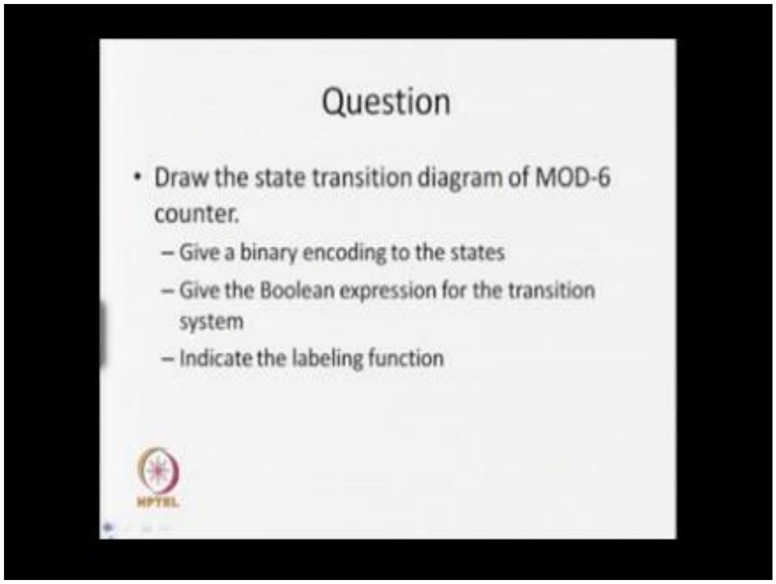

Now how to evaluate these things, now in this particular case now we are going today I am going to stop here the next class I am going to say evaluate pre there exist X. And once you can had the procedure for this particular pre there exist X, then what happens we can evaluate pre for all X also, and once we have this particular procedure then we will see how we are going to implement or model checking algorithm with the help of this particular ROBDDs.

That means we are going to represent state transition system with the help of ROBDDs okay. And now we are going to implement those particular models checking algorithm with the help of particular ROBDDs and in this particular case we are going to said this is the symbolic model checking model checking algorithm okay.

Now look in this question that I am saying that draw the state transition diagram of MOD-6 counter. So what we have to do give the binary encoding to the states, give the Boolean expression for the transition system and indicate the labeling function okay we have indicate the leveling function now how we are going to say I am talking about the MOD 6. So MOD6 counter is basically count from 0 to 5 again it is set to 0, that means what happens I can say that if this is your 0 then I am going to have 6 states we are having 1 and I am going to have 2, then I am going to have 3, 4 sorry 5 then I can move that to 0.

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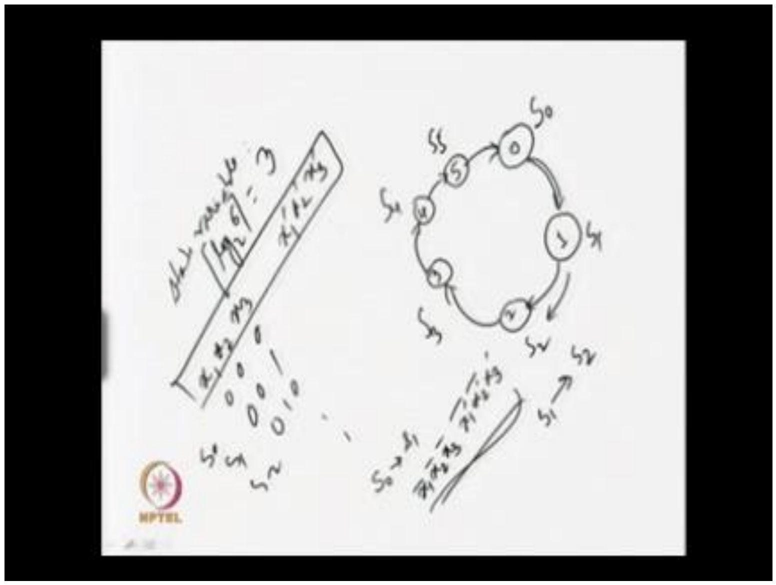

That means how many steps we are having 6 steps we are having that means how many step will be needed for it this is your log 6 to the 2 so that means 3 variable so I can considered X1, X2, X3 are the three variables. And what did I say if I say this is my state S0, S1, S2, S3, S4, S5. Then I can say that 0 step will be your 000 S1 will be your 001, S2 will be 010, like that others state encoding. Now what I am saying that first you have to say keep the binary encoding of the states give the Boolean expression for the state transition system.

Now what will happen, what are state transition you can say that we have transition from S0 to S1. Now since do represent this condition data I need another state of variables then I can say that X1', X2', X3'. So these transition prime X0', X1' can be represent with the help of X1 bar X2 bar X3 bar this is 00 to 01, X1' bar, X2' bar, X3' bar.

So this is the transition behavior for this particular transition similarly for this second transition also I can represent is going for S0 S1 to S2 like that I can represent all those particular six transition with the help of this particular six variable X1, X2, X3, X1', X2', X3'. And we can able now Boolean expression all those particular state transition diagram. Now indicate the level of function I am going to talk about these particular transition behavior that you will find that X1 X2 X3 all are 0 that means all are falls they are in S1.

I am going to setup this is evaluate X3, this is evaluate X2 because what happen I can say that this is nothing but X1', X2', X3 this is 001. That means X3 is 2 in this particular step statement that means what I can say that this is evaluate X3s similarly this is evaluate of X2 and I can say that this is evaluate of X1 X2 X3 like that we can have the level. And already I have say that if I have the level I will be knowing which are the states that X1 is true now you can collect these particular states and we can represent this particular states with the help of ROBDDs so that is why am saying that indicate the labeling function so this is the way we are going to do.

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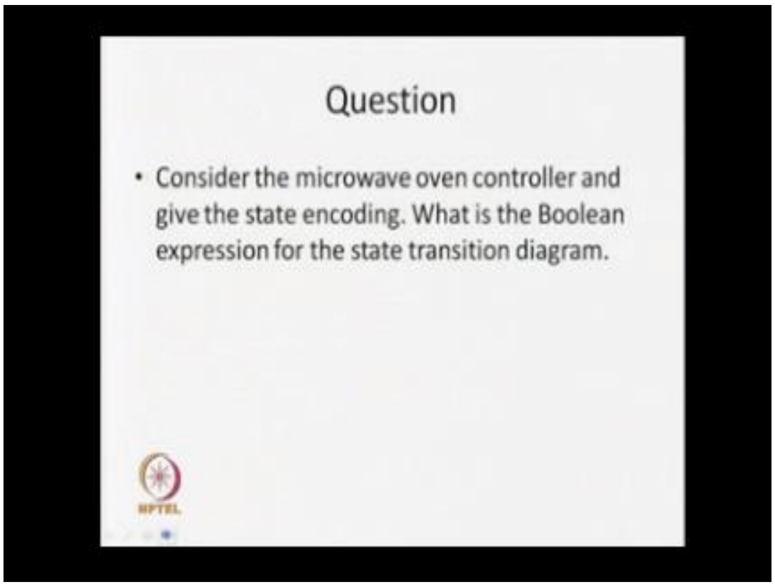

Secondly we are going to say that consider the microwave oven controller and give the state encoding. What is the Boolean expression for the state transition diagram? So this is another question that already we have discuss about these microwave oven controller and we are having the state transition diagram so this is the state transition diagram that we have,

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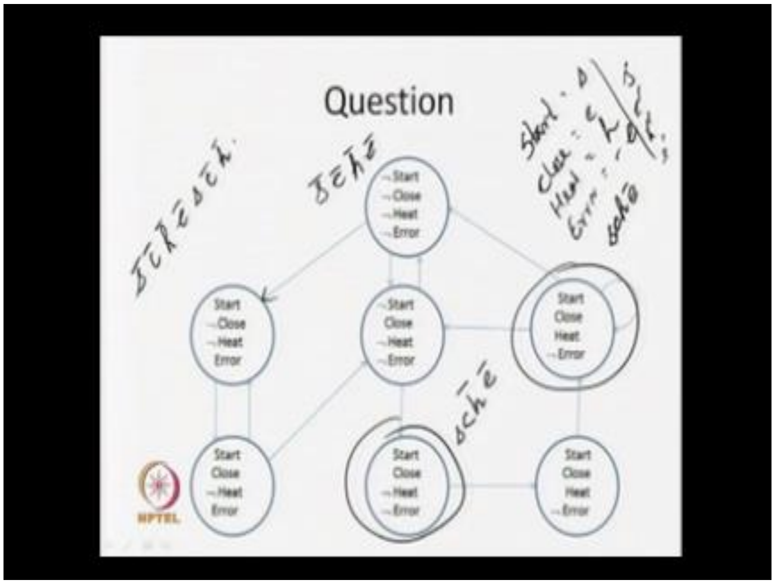

So this is the state transition diagram that we have. Now in this particular case I am having in 4 step variable can say that start so am going to say that represent start by S. Then I am having close and I am going to represent C this particular close, heat I can say that H is the mean Heat representing answer error E say I am going to Error the E. So this are the 4 state variables will be leading to represent this particular state. Now when is say that what is the binary encoding of particular states I can say that this is evaluate basically, S bar C bar H bar and E bar.

Similarly when is say that what is the state encoding of particular state say this is part is, so START is true CLOSE is true, HEAT is true, but ERROR is false so these are the stated encoding. So similarly when I come to particular step what is the state encoding. I can say that this is start is true, close is true, heat is false, error is false and state encoding. So what I am saying that consider a microwave oven controller and give the state encoding. So we can add the state encoding. What is the Boolean expression for the state transition diagram?

Now In this particular state transition we have to consider all those particular transition I am leaving this particular transition because this going from. Now we need another set of variables where I can say that variable will be whole S’, C’, H’, and E’. So these particular transition I can represents like that so S bar C bar H bar E bar to S Close bar H bar and E bar okay. So this is the present state this is the next state so this is particular transition with the help of Boolean transition represent with the help of particular Boolean expression like that I can write the expression for all those particular transition and eventually I can get an Boolean expression.

So that is why am saying what the Boolean expression for the state transition diagram? Is To get the Boolean expression for the state transition diagram, and once you get it and what we can do we can represent these particular Boolean expression with the help of ROBDDs okay. We stop here, in next class we are going to see how we are going to evaluate pre there exist X and with the help of that operator we are going to see how we are going to implement of model checking algorithms using BDDs okay. And that means we are going to get the algorithms of third parts symbolic model checking algorithms thank you and bye, bye.

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