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Module IV: Temporal Logic

Lecture III: Syntax and Semantics of CTL

So in our last class I have introduced the notion of temporal logic and I have talked about the temporal operators. Basically we can categorize the temporal logic in two define, one is your fast temporal logic and second one is your future temporal logic, and in future temporal logic we can have or we have discussed about the notion of four temporal operators one is your NEXT, second one is GLOBAL, third one is FUTURE and fourth one is your UNTIL operator which is a binary operator.

In this class today I am going to introduce special class of temporal logic which is known as your CTL, computational tree logic. So I will say what is the general notion of the temporal logic and what is CTL and after that we will go for syntax and semantics of CTL, and after that I will give some example how to represent those things with CTL. So in temporal logic what happens we have seen that we are going to give the meaning of a temporal logic operator with respect to a model okay.

So you can have a model which is nothing but a sequence of sets and we are going to give the meaning of a temporal operator in a particular set.

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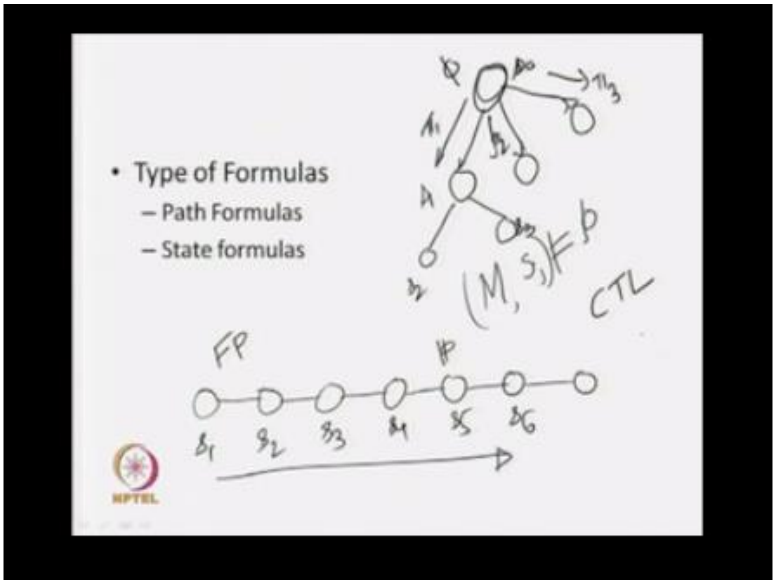

So we say that if I am having a model M and I am having set say S these particular combination model a temporal operator part, temporal formula part. Now we are having two notion one is called path formula and second one is called state formula. Now just that last class what we have discussed that we are having some state this is the sequence of state like that states are marked S1, S2, S3, S4, S5, S6 like that.

And so in S5 we are having that mechanic proposition P is 2, then what happens I cancel in state S1 the formula FP is stood in PUSA P is to. So though we are talking about the step S1, but it is related to this particular execution sequence. So in case of linear temporal logic what happens we are having one part and we are going to rejoin about that particular part.

But already I have mentioned that we are having the notion of your branching time where time branch out in several direction. So in that case we are going to root for all possible parts or there exist a part. So in this case what will happen we are going to talk about the state formula, here in notes general which are in this particular part for state S1 that happy holes.

Now in case of branching them what will happen we will just see that we are having a state S0 depending on several condition and time may branch out hence but at different position. So if I coming to S1 depending on the input combination or state of the system it can proceed either S2 or S3. Now when we are going to talk about in state S0 we have to see all possible combinations this is part π1, this is part π2 and this is part π3.

So in that particular case we have to see the measure of this particular temporal formula in all those possible part and depending on that we are going to say the behavior of that particular formula π in this particular step as 0. So in this case we are going to say that it becomes a state formula. So now we are going to talk about a particular class of temporal logic which is known as your CTL computational tree logic.

So in computational tree logic it is a branching then logic we have to root for all possible parts and when in upon a particular part we are going to look into all possible parts then we are going to setup that particular formula is to in step. So we are going to talk about state formula in case of CTL.

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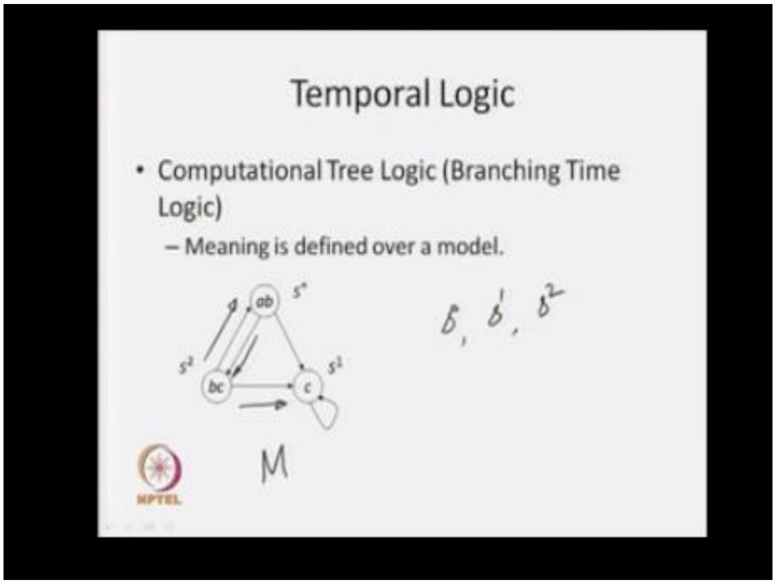

So now users computational tree logic it is a branching time logic and we are going to define the meaning of this particular CTL or computational tree logic with respect to a model. Now just see in this particular example, say I am having this particular model M it is having three states S), S1 and S2. And we are having some conditions or to consider this transition say S0 it is going to S2 by this particular transition.

When we are having in your S2 we can go to S3 or we can go back to S0 again. So in this part you can have a transition like from S0 to S2, S0 to S2 like that. Now in this case what happens this is a graph paper presentation, we can unfold this particular transition system and we can get some sort of tree.

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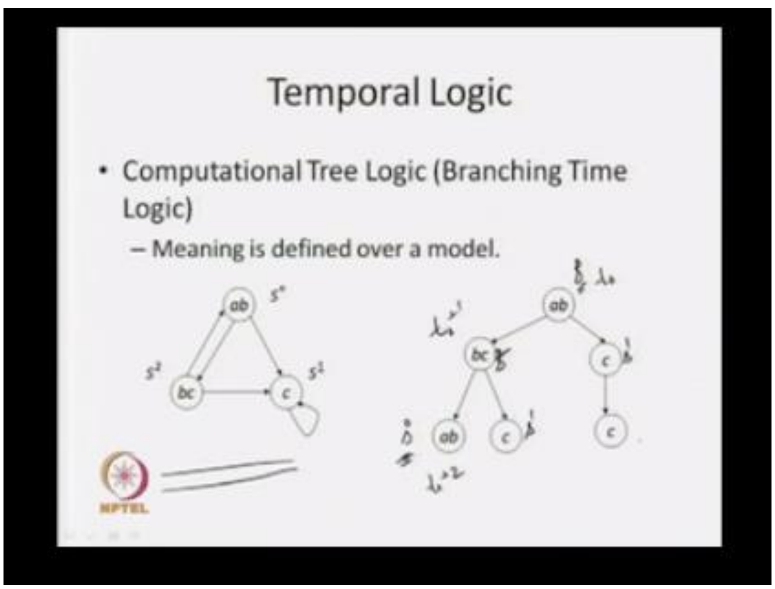

Now if unfolding this particular situation you see that in this particular state S0 depending on the scenario idea we can go to S2 or you can go to S1. So these are the next state behavior, so this is your in S0 we can come to S1 or we can come to S2. Now when we are in S2 again we are having two choices that we can go to your state S1 or we can go back to state S0 again.

So behavior of this particular step S0 and this particular step S0 is almost same or same basically apart from your timing instance if we can look into time instant, in some time instant P0 we are about here than in P1 I am going to be in this particular place P0+1 and next time instant in P0+2 we are in this particular. So apart from this particular time instant all the behavior of this particular state S0 and this particular state S0 is same.

Now in case of computational tree logic we are going to rejoin above this particular formula over a three step that is why the name coming as your computational free logic. So eventually if we are having a state condition system like that we can un pole it to a graph and we can and send the meaning of CTL with respect to this particular tree.

So we are going to say this is your computational tree logic. Now we are going to see, how we are going to define the CTL formulas.

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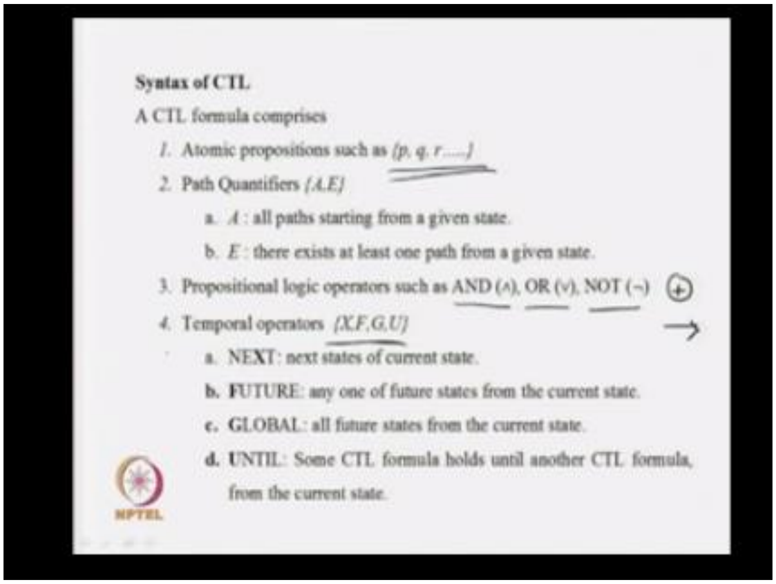

Now first when we are going to look for a language or look for logic and we have to look for the syntax of that particular logic. So what is the syntax of that particular logic CTL? So when we are going to write a formula in CTL then it involves several components, the first component is your atomic proposition. So we are having a set of atomic proposition these are represented as a say P, Q, R, S like that.

So these atomic proposition is going to take truth values either true of false like our proposition logic. So this is basically proposition that we have that truth values of this particular propositions are either true or false. Next we are having path quantifiers and the path quantifiers is specifically represented A and E. A means that in all possible part.

So in a particular state if you go for all possible paths then we were going to quantify these by this particular path quantifier A. Similarly E is another path quantifier which says that there exist a path if you are interested in a particular path, then we will look for this particular there exist quantifier. After that we are having all the proportional logic operators that we have in proposition logic can be used in our temporal logic also.

So like that AND conjunction, OR disjunction, NOT negation or maybe your explosive about like that your implication all those particular temporal proposition operators will be used in on our temporal logic also. So these are basically similar to your temporal proposition and logic apart from that we are having temporal operator we are going to use the temporal operator in our CTL.

So that temporal operators that we have discussed are used in this particular CTL also. So these are the four temporal operators that we are having X stands for your NEXT step, F stands for in FUTURE, G stands for GLOBAL operators and U stands for UNTIL operator. So P UNTIL Q so it is a binary operator P remains to UNTIL Q becomes Q. So these are the components that we are going to use while defining the CTL formulas.

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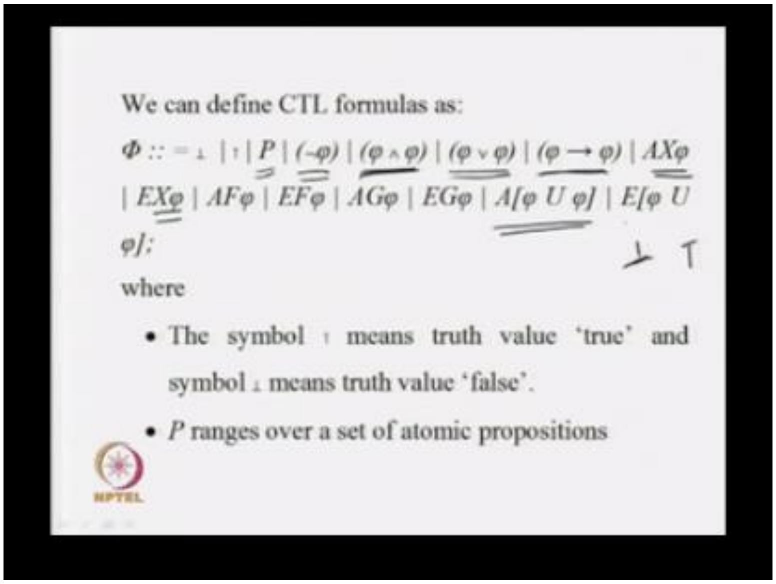

Now in BNF notation we can define the CTL formulas like that. So we are using two symbol one is your bottom and second one is your top. So this top and bottom are used as our CTL formulas, top stands for the truth value true and bottom stands for the truth value false. So you are having two truth values in our logic one is your true and second one is your false. So true is represented by the symbol top and false is represented by the symbol bottom.

So top and bottom are treated as your CTL formula the next we are having P all P will be treated as your CTL formula. What is P, P is nothing but these are the proposition that we are going to use in our system. So we are having some proposition those propositions can take value right, truth value like the true and false all these proposition will be treated as your CTL formula.

Now we can user the proportional connectives to form CTL formula. So if I is the CTL formula then not of ϕ will also be CTL formula, ϕ and ϕ will also be a CTL formula, ϕ or ϕ will also be a CTL formula, ϕ and plus ϕ will also be a CTL formula. So all propositions connective will be used to define CTL formulas from CTL formula. Now apart from that we are having temporal operators, these temporal operators are going to form some CTL formula.

So the temporal operator NEXT state, so we are having Xϕ is our temporal formula if this temporal formula is preceded by path quantifier A or path quantifier E then we are going to say that these are CTL formula. Similarly, for FUTURE operator we are going to have AFϕ and EFϕ that is path quantifier A and E we are going to get two CTL formula.

Similarly for GLOBAL also Gϕ so we are going to have two CTL formula AGϕ and EGϕ. Next we are going to have UNTIL operator which is a binary operator so if we are having two CTL formula ϕ and ϕ then we can say that Aϕ U ϕ is also be a CTL formula for Eϕ U ϕ will also be a CTL formula. So we can construct CTL formula using those particular rules if the formula conform to these particular rules then we are going to consider those as our CTL formulas others will not be declared as CPL formulas.

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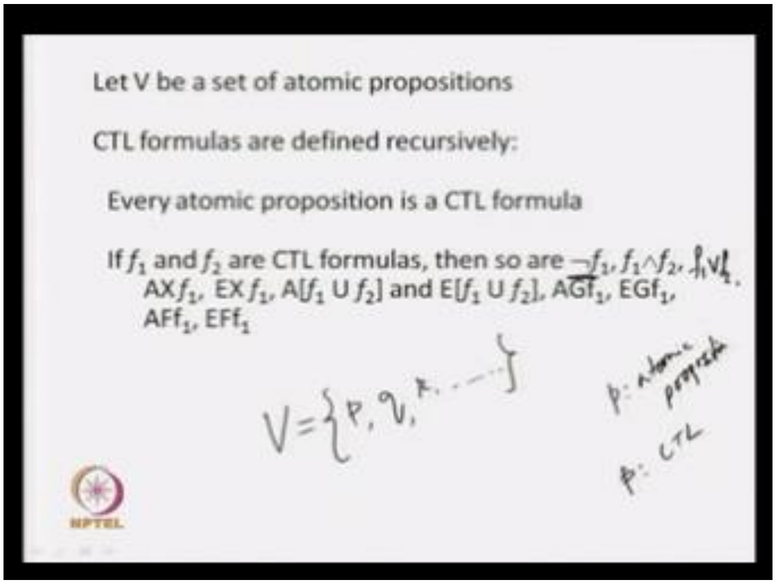

Now by looking into this particular BNF notation now we are going to see what are the CPL formulas that we are having so first we are going to set up we are having a set of atomic proposition so P is a set of atomic proposition you consider p, q, r etc are atomic proposition so they can take through values depth truth or false now we can defined a CTL formula with the help of this atomic proposition now every atomic proposition will be treated as a CTL formula.

Because in BNF notation as we have seen that all atomic proposition will be treated as the CTL formula so if p is an atomic proposition so in this case we will set p is also a CTL formula no for all atomic proposition we are going to get CTL formula now you just consider a reap we have two CTL formula f1 and f2 because recursively we are going to define it so if we are have two CTL formula f1 and f2 then we are going to set up not of f1 is also CTL formula f1 and f2 is also CTL formula.

Similarly we can say that f1 or f2 is also CTL formula can Axf1 is also CTL formula EXf1 is CTL formula Af1 until f2 is also CTL formula Ef1 untilf2 you will also CTL formula SO similarly AGf1 EGf1 AFf1 and EFf1 will be CTL formula so this way we can construct our CTL formula.

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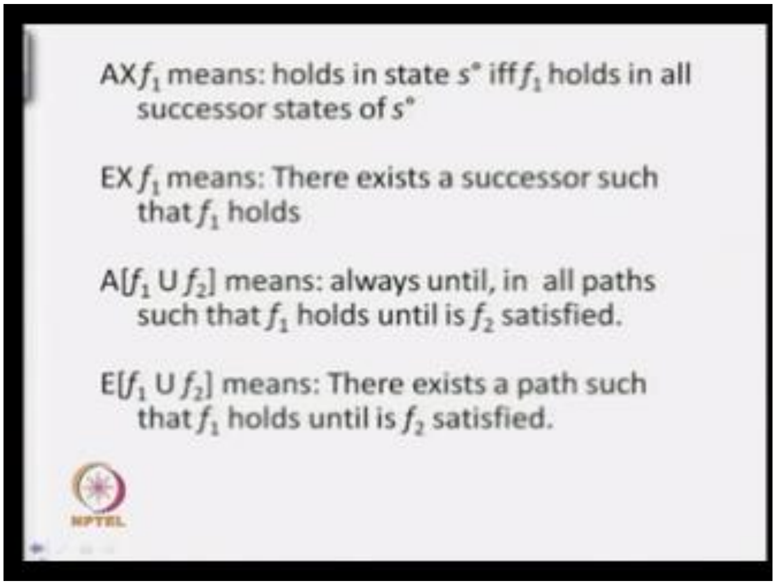

So what are the meanings now you just see that if you look into this particular formulas.

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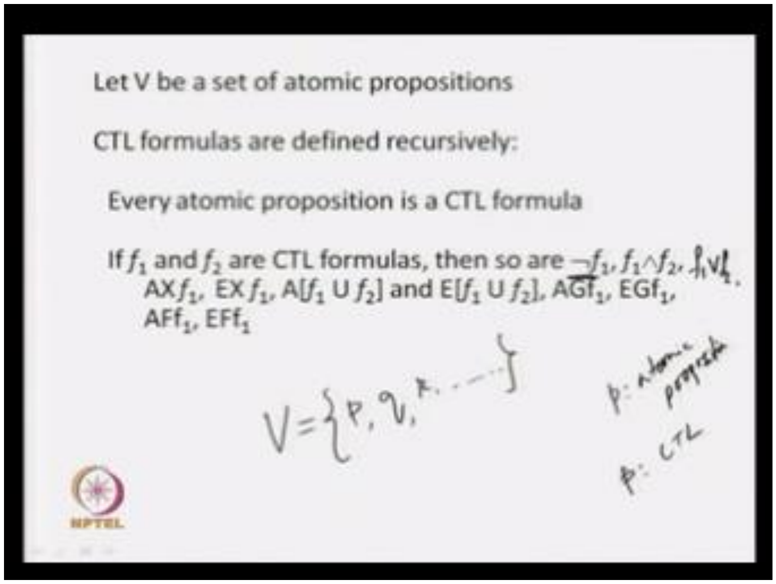

We will find that the meaning of those particular formulas with propositional connect if of very straight forward now we will see what is the meaning of the those particular low pointers.

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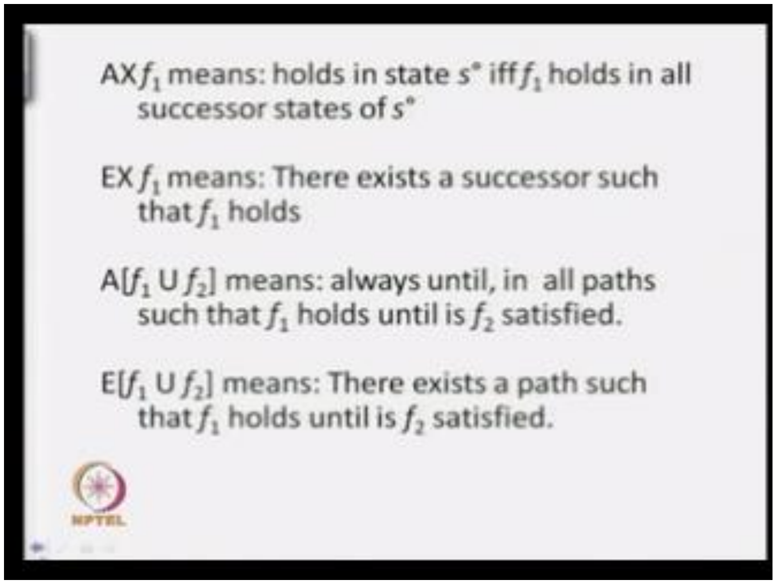

So one is your AXf1 what does it means that means if we are going to look for a particular step it is having several possibilities in further in a all those possibilities in next stage if up on holds then we will set up AXf1 holds at the pre most point or particular set that means what you say that in a particular set AXf1 holds if in all possible next step f1 holds so that is this is the this holds in a set as 0 if f1 holds in all successor state of your s0 similarly EXf1 it means that they are excess a successor state that f1 holds.

So it is having several possibilities in further but we are concern about a particular direction or in a particular further so if in a next state it holds then you can set up in that particular set EXf1 holds similarly Af1 until f2 so f1 that until is a binary operator so we need to formulas f1 and f2 and that is quantified by part 25 A in this it means that always until that means in all possible direction in all possible part f1 must holds f2 holds so this is Af1 until f2 similarly Ef1 until f2.

So we are concern about a particular part or we adjust the part not any part any part we are looking in to one particular path so if we get such a path that f1 holds until f2 holds then we say that in the particular set Ef1 until f2 holds.

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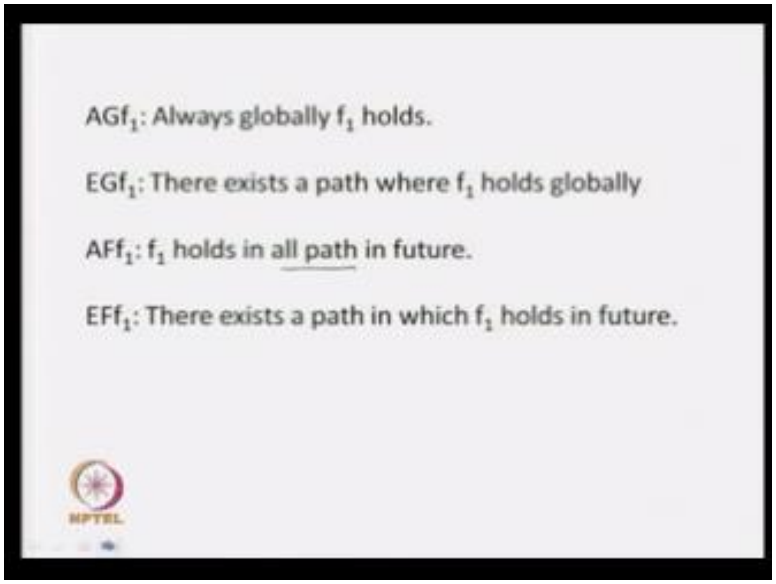

AGf1 this is always globally f1 holds so if we are in a particular state where ever we go in all direction in all path in apache that f1 must holds then we will set it AGf1 holds similarly EGf1 this is concerned to a particular path so we are have several possibilities but we look a particular path and in this particular path in all states f1 holds then we set depth that EGf1 holds similarity AFf1 this F stands for free server path or eventually aperture so that mean f1 holds in all path in further.

So we should get a further states where f1 holds then we say that AFf1 holds in that particular step so you have to loop for all path so similarly EFf1 looks for a particular path so if any part in further f1 holds then we say that EFf1 holds now you just see that we are having this particular 4 temporal operator and this particular 4 temporal operator will be proceed by part 24 A and E and that means we are going to get 8 different combinations and we are going to get different temporal formulas.

And the meaning of those temporal formulas will be defined over a modal formula we are going to defined a symmetric such I am given a brief idea about it how we are saying that these particular formula is slow.

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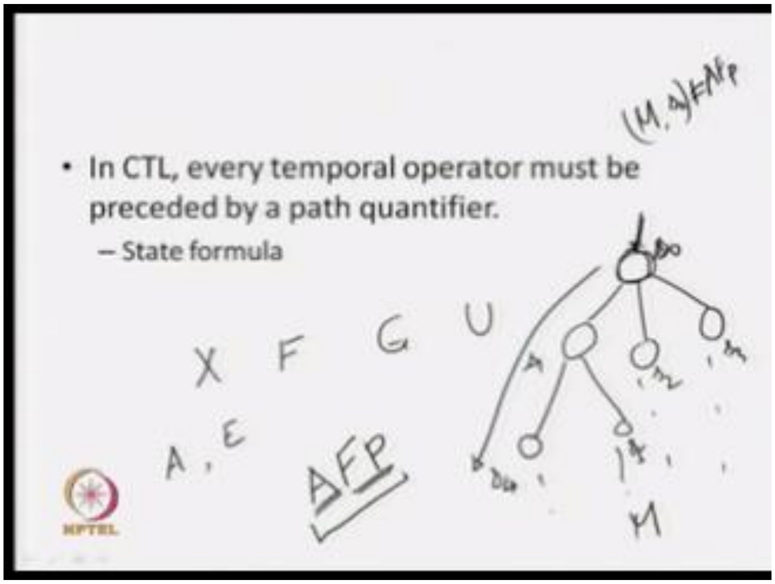

Now you just say if you look into this particular syntax then what will happen you will find that in CPL every temporal operator must be proceed by a path quantifiers so we are having say 4 temporal operator x fuser globally and until in CTL this path is of 4 temporal operator must be proceeded by path quantifier A or E then only we are going to set up these are CTL formula and due to this particular quantification by path quantifier that CTL formulas are basically step formulas.

We are defined the parallels of the CTL formula over a step so thus simple like that I am going to set at the final min state as 0 so it is having three possible quantities in further this further also S1 S2 S3 people come to S1 say I am having two different user possibilities so this is your S4 and S5 like that we can have further possibilities in all these direction now when we talk about CTL formula that paroles of this CTL formula will be defined over this particular state.

So it is a step formula so any temporal formula say Fp in what it say that in further P holds so this is a temporal path temporal formula and basically this temporal formula defined over a path with respect to us staring step but while we are going to talk about CTL that means I can say that it will be AFP so this is the temporal formula FP is proceeded by this particular path quantifier so we are going to have this particular formula which is a facility formula and the two pale of this particular CTL formula will defined over this particular state or any step.

So if we talk about this as 0 say this is my model M so model M in state as 0 whether it models your AFP or not that means this CTL formula two pulse of this CTL formula is defined over this particular state so CTL have your step formula and what we can say which one is a CTL formula if every temporal operator is proceed by path quantifier then we say that this is your CTL formula.

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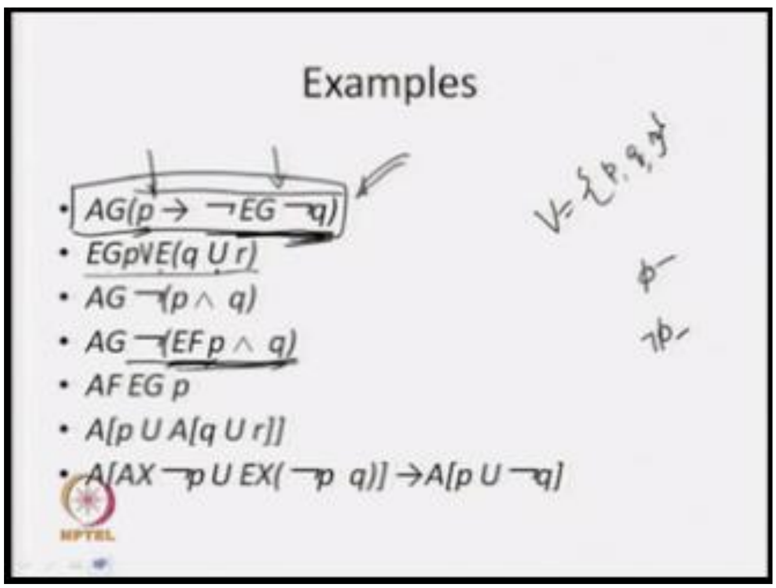

After looking for the syntax of the CTL formula let us say some formulas and let us whether these are CTL formulas are not so you consider about this particular pass formula now what did happen you can say that in this particular case you are having set of atomic proposition V say p, q w these are set of atomic preposition these are having a true value data true and false.

Now every atomic proposition will be treated as CTL formula so in this particular case I can say that q is CTL formula so lot of q is also CTL formula similar p is a atomic proposition so p is also CTL formula now G is a temporal operator it is proceed by E so E G0 q is also CTL formula so if we are having π as a CTL formula then what will 0 of π is also a CTL formula so in that particular test 0 (EG) π is also CTL formula.

So P is CTL formula and 0 (EG) 0q is also is CTL formula these two CTL formula are connected with the help of this particular implication of person so this wholes set is going to give you CTL formula now this CTL formula is again 2quantifier again having a TEMORAL operator called G so when a globally these holes are not again this particular globally temporal operator is proceed by this path quantifier s so that means this whole formula is CTL formula now similarly we can analyses each and every formula as you will find that these are valued CTL formula.

So here in the second example you just see that this is your q until r this until operator q and r are CTL formula so Eq and p r is also a CTL formula similar EGp so P is CTL formula so EGp is CTL formula now if we look into it EGp Eqr then you will find that this particular operator E is not is a CTL formula but it is not directly connected to it because of trophy we are having E so in the particular case it is not a CTL formula but you can make it CTL formula by putting some binary connectives are say positional connected if I say that this is r then it becomes a CTL formula.

So EGp it is CTL formula and Eq until another CTL formula because this E until your temorla operator is proceed by this particular E quantifier path quantifier similar G is proceed by your E path quantifier so this hole formula becomes a CTL formula like that you can analyses othetr one also so the 4^{th} one I am going to say p is atomic proposition so it I a CTL formula f is temporal operator and it is proceed by path quantifier.

So EFp is also CTL formula is also CTL formula because it a atomic proposition so they are connected by this particular conjoined so this whole thing also CTL formula ands already I have say that the π is a CTL formula then 0 π so 0 Efp and q is also CTL formula and this CTL formulas is again having a temporal operator G which is proceed by A that means in all globally whether these holds or not so D sol formula is also CTL formula, so like that we can set each and every formula and we will find that these are failure CTL formula.

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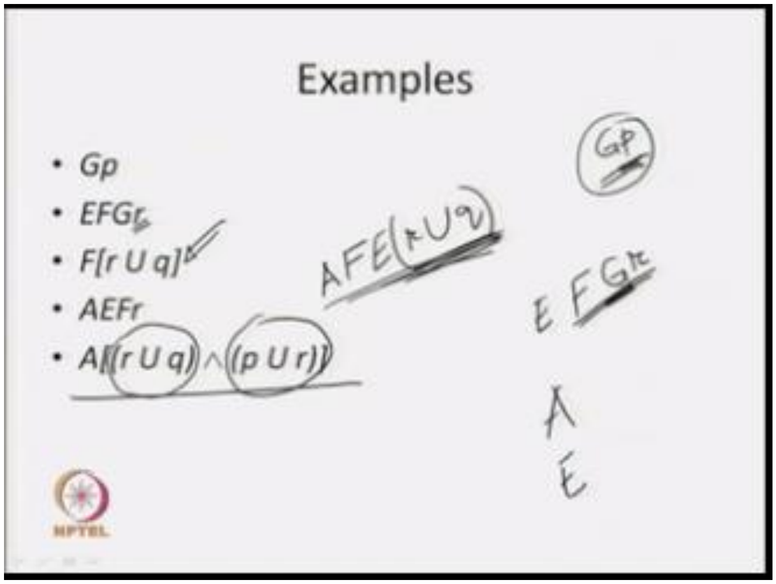

Now look for this another set of formulas the for formula is Gp so in case of your Gp is consider Pj atomic formula so it is a CTL formula G is the temporal operators so Gp is a temporal formula but this temporal formula is not presided by any part quantifiers A and E since it is presided by any part quantifiers so this temporal formula is not a failure CTL formula, so we are not going to said this is a CTL formula.

Similarly, second example is consider R is the CTL formulas since it is atomic proposition so A at G is the temporal operators so GR is a temporal formula FG are now F is the temporal operators G is the temporal operator and R is a CTL formula because it is a atomic proposition so this particular temporal operator G is not presided by any part quantify directly we are getting F so this FGr is the temporal formula no doubt but this is not a CTL formula.

Since it is not the CTL formula so EFGr is also not a CTL formula, so to have or we say at it will be a CTL formula provided each temporal operator is presided by your part quantifier and with the help of this part quantifier we are making or we will said that the CTL formula the step formula we are going to talk about the truth fellows of the CTL formula over a step. Now similarly this next formula if you see that R and U q so until is your temporal operator r and Uq is a temporal formula.

Because either angular pole so this temporal operator is not presided by any part quantifier so it is not a CTL formula. But if you say that E r and U q then it becomes a CTL formula because this temporal operator is your presided by your part quantifier so Fr and U q is not the temporal formula but we are saying that is a temporal formula. But since it is a temporal formula so I can say that F and this temporal formula if I write this thing then it will become a temporal formula but if it is not a CTL formula now because this particular temporal operator F is not presided by any part quantifier.

But I can make it they temporal formula CTL formula by placing a part quantifier in front of it so this whole formula becomes a CTL formula acts as this particular third formula is not a CTL formula but it is temporal formula but if I would part quantifier in front of every temporal operators then it becomes a CTL formula.

So with similar argument you can say at this equation 4 or say expression 5 are also not a CTL formula you just consider, you can apart and then find that this may be a CTL formula because it is presided by your this particular part quantifier A. But if you look component wise first loop into this particular component p and U r this is a temporal formula but it is not CTL because this particular until it is not presided by ant temporal part quantifiers.

Similarly r and U q is also temporal formula but this is not a CTL formula because this anti operator is not presided by any part quantifiers. So that is why this since these two components are not a CTL formula so they are conjunction will not also be a CTL formula, so in that case we will say that this is not a CTL formula. So now we have seen the syntax of our CTL computational three logic we can use atomic proposition all atomic proposition will be treated as our CTL formulas.

CTL formulas can be connected by your propositional connective like AND, OR, XOR in precaution extra and those will become from CTL formula. Again if we are having any CTL formula they can be construct another CTL formula by temporal operators but each and every temporal operators must be presided by a part quantifier, then only we will say that this is a CTL formula, we do not presiding by a part quantifier we are not going to get CTL formula but we will may get temporal formulas.

So CTL is a subclass of my temporal formulas so with part quantifiers we are making CTL formulas secondly when we are going to say that part quantifier basically we are going to say that it part quantifier A say that in all possible part from a particular step S or E says that they are exists a part from a particular step S, so in that particular case this CTL is treated as our step formulas we are going to look for the close values of CTL formula with respect to steps.

But we should have either all possible computational part or they exist a computational part they exist the meaning of A and E so with the help of part quantifier A we are making the step formula.

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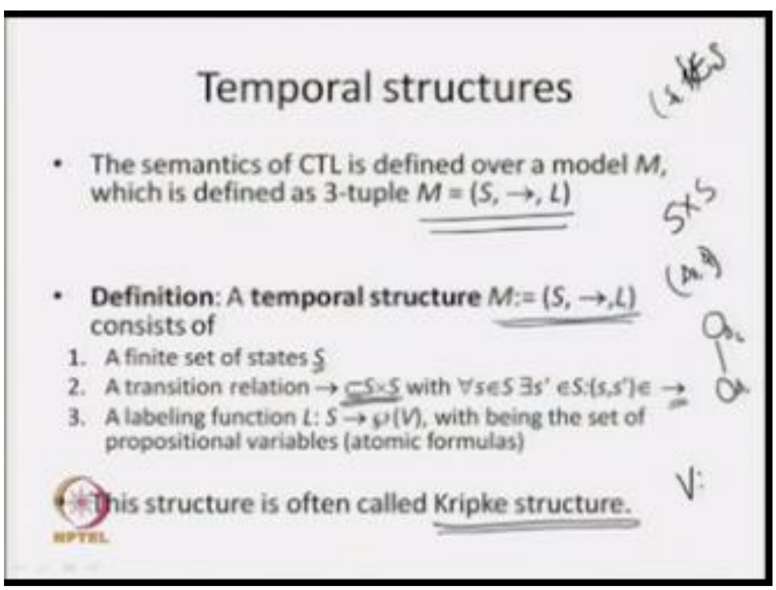

Now we have to look for the semantics on meaning of this particular CTL formula already I have mentioned that if we are going look for a true fellows of the temporal formula it is always define over a model. So for CTL also we have to define the meaning of CTL formula over a model. So in this particular case we are going to have a model M call M and this model is having three components, so this model M is defined as S this arrow and L.

So we are going to have a model which is a treat oppose basically we having three basic components S we are talking about one arrow and we are talking about L with the help of this we are going to define a model and in this particular model we are going to define a meaning of our CTL formula. Now what are the three history components just see the formal definition of this particular temporal structure, so we said at the eights temporal structure M is having a three topple where we are having three component S arrow and L.

Where S is a set of finite steps that means we are having steps in these steps is a finite step so we are having finite number of steps in out sets so we are representing it by S. The second one that arrow that we are representing it is a transition relation and transition relation is nothing but a subset of Cartesian product of S and S that means we are having a transition from one step to the other step so that is what we are saying that it is a subset of this Cartesian product S and S.

So Cartesian product S and S basically will give you the components like that S0 and S1 say if S0 and S1 so it belongs to your say S then S0 S1 will be a component in our this Cartesian product. What does it mean that means we are having a transition from S0 to S1 so now second component is the transition relation which is subset of the Cartesian product of S and S which are particular and sticks on it says that for all S belongs to S they are existent as just sorts that S as thus belong to this particular transition relation.

Now what that it means so, if we are having a set from that particular step we should have a transition to some other steps that to which says that transition relation is complex so this is one restriction the CTL structure that we are depending over here the transition relation must be complain that means if you keep up any random any step S it must have a successor step, if steps are have not having successor step then we are not going to treat these things as a CTL structure.

Along with that we are having a labeling function which is known as basically L define as L it is nothing but S arrow ρ(V) preside what is V here V is basically this is set of atomic proposition and ρ(V) basically says that power set of this particular set of atomic proposition so we will give an example so the step will be level by the state of atomic proposition and what it is in gets set the proposition variable and these are atomic formulas or atomic variable.

So what basically this leveling functions says it says that this particular atomic proposition is true in this particular steps so we are having a leveling function because we know since these are atomic proposition we know the true fellows of those particular atomic proposition either they will be true or they will false. So if a particular atomic proposition is true in a particular step we level this particular step by that particular atomic proposition.

And this structure is often called Kripke structure so basically the semantics of your CTL formulas define over Kripke structure we use the temporal Kripke structure so this is the Kripke structure which is having the history component S transition relation and L and what is the basic recommend one is your transition relation must be complete for every step there should be a successor step.

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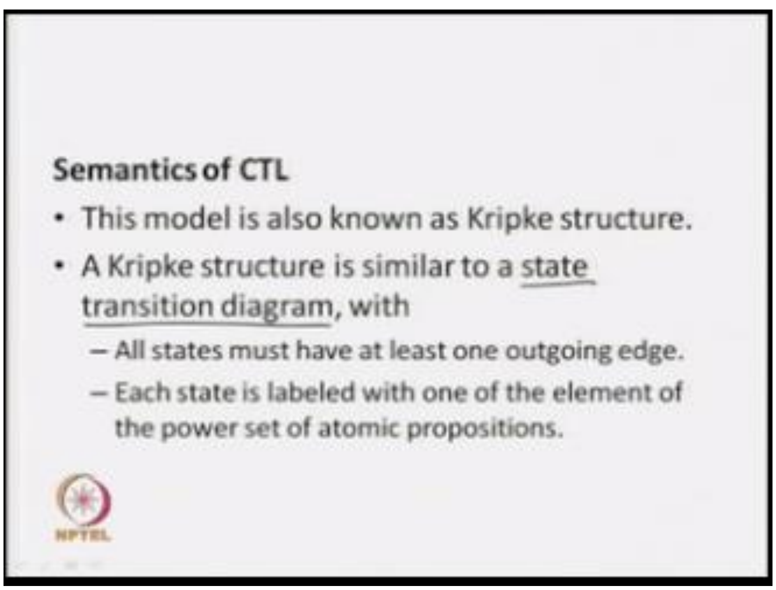

So we going to define it over a Kripke structure and if you look into behavior of the Kripke structure you will find that it is nothing but very much similar to your step transition diagram it is a state transition diagram and in this particular step transition diagram we are having two restrictions one is your all state must have at least one outgoing edge that means every state must have a successor state and secondly we are having a labeling function the state will be level by the atomic proposition why it is particular atomic proposition is true. Okay, we will just see one example then it will clear.

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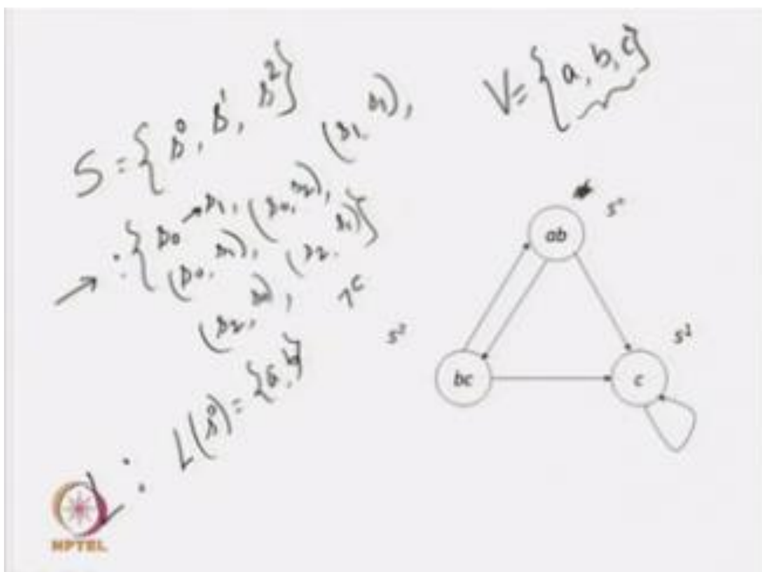

Now you just see the compare to it is particular transition diagram so it is a finite step transition diagram because we are having the state S what are these particular states we can say that this is your s0, s1 and s2 so we are having three states along with that we are having the state of atomic proposition V here we are having a state of atomic proposition or the atomic proposition that we have out a, b and c these atomic proposition get two fellows either true or false along with that we are having this particular transition function so here what is the transition function that we have, we are having a transition from s0 to s1.

So we are having s0 to s1 this is a transition on the notation I can say that since I am saying it is subset of partition but I can say s0 s1 also I can write like that, I get from s0 to s2 I am having another transition so I can say that, that transition state have s0 to s2. Now when I come to s1 you will find that we are having a transition from s1 to s1 so s1 to s1 we are having another transition.

Now only one condition we have and similarly when you have come to this s2 then we find that from s2 we are having that condition to s0 and form again condition in having form s2 to s1 so this set of condition relation basically contents this particular pipe condition 12345 now it should satisfy one condition that this particular condition relation must be complete that means the pre step mass have a such that.

Now if you said at the it will look in to state 0 will find that if it having two successful step if we look in to s2 again it is having w successor step if we look in to s2 1 it is having cell group that means we are having a s1 as it is whole success or that means it is complete. Now along with that we should having a leveling function L so what is this leveling function it says that we are going to level each and every step by the powers step of this particular set of automatic variable.

So in a particular state if that atomic variable is true then we are going to level it by this particular atomic position so here it says the l of your s 0 is basically nothing but a and b so what it says in when we are coming to this particular status you know then this atomic position a and b is true and then early it says that in this particular s0 c is not that two we can say that not of c that regard say that can again need by not of c also if says that things is not c is not to interpreter respect.

But in condoms what is that say if it is not leveled it any atomic proposition we say that these atomic proposition is falls in this particular state. Similarly these particular state s2 it is back with b and c that means the atomic portion b and c or 2 in this particular state 2 but atomic portion a falls over here, and in s1 it is level which sees so it say that the atomic portion c is to this particular step has s1 but the atomic proposition b and c of falls over here.

So if atomic proposition is falls regionally therefore marked it but in eternal meaning is that these atomic position is falls in this particular step because atomic proportion can have tow eternal either two and true and false if it is true your leveling if we are not leveling we says that these atomic proportion is fall in that particular step.

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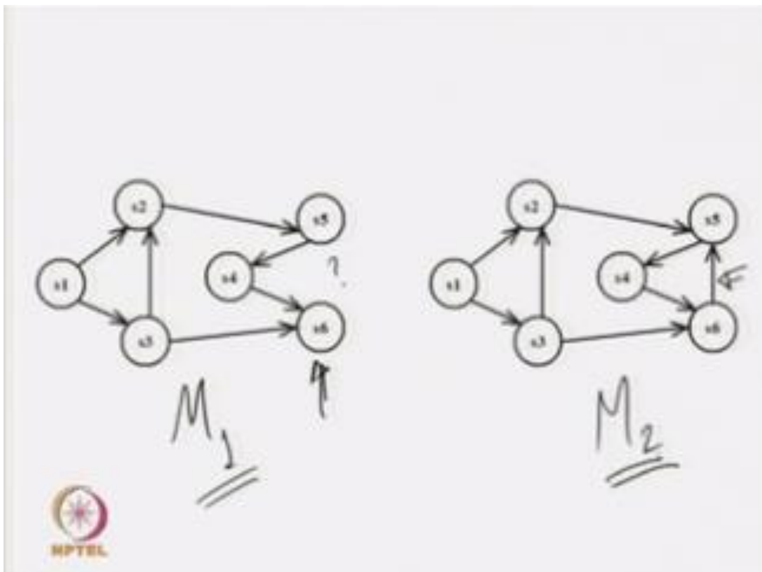

Now just look in to this particular model I am saying this is m1 and this is m2 what are these two models are keep the structure so both are having the similar structure it is a similar structure then we have to talking about the transition all way leveling function we are not showing it over here because we are not talking about the set of theorem proportion but if you look in to this two models m1 and m2 whether they are fix the structure if you observe with you will find that votra identical what in m2 we are having one extra aero accept transition this transition is not present in your m1.

Whether this is going to contribute something over here if you observe it then in m1 in find that this step s6 it is not having any success there you just see that there is no outgoing as formed is particular step S6 so since it is not having any success so that this transition may lesion is not complete so you are not going to treat this particular m is a kept the structure. So m1 is not a keep get structure similarly m2 nor if you look in to it in all the structure we are find that we are in transition or we are having success as that.

So m2 is a keep get structure so we have to careful about it it is not like that in find x that model we can define a meaning of the CTL formula it must be keep get structure and what is the basically common optic structure that transition relation must be complete for each and every steps we must have the success step. So here m2 is a keep get structure but m1 is not keep structure.

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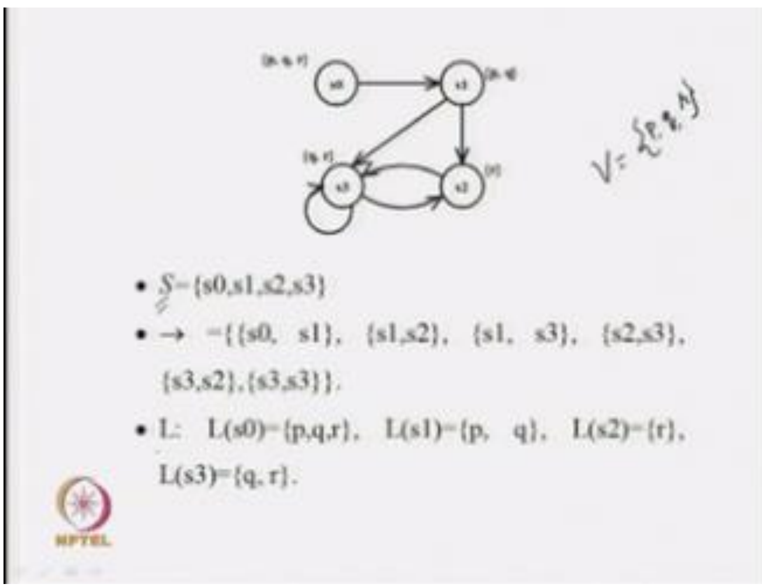

Now look in to another examples sy already I have explain this things now by observing it also you will find that this is a keep get structure because if to look in to all the steps all the steps are having pure a transition not in having a success all so now what are the feed basic component we are having set of states so this is the 0 s1s2s3 this is the state of step and the transition relation how many transition we are having 123456.

So we are having this six components ways of transition relation and this is the leveling function this as said that we having this level over here that means you are having a set of atomic proposition then the atomic proportion that we are having here is your pq and r.

So as 0 is leveled it pqr it says that all pqr is to over there so level of s0 is pqr in s1 p1 p1 qs1 so level of s1 is p and q level of s 2 is only r because r is2 and level of s still in q and r so I think with this particular explanation I think with the notes of keep just this clear about it because it is having three component as the set of states transition relays on and the leveling function, and what is the leveling function already I have aspect.

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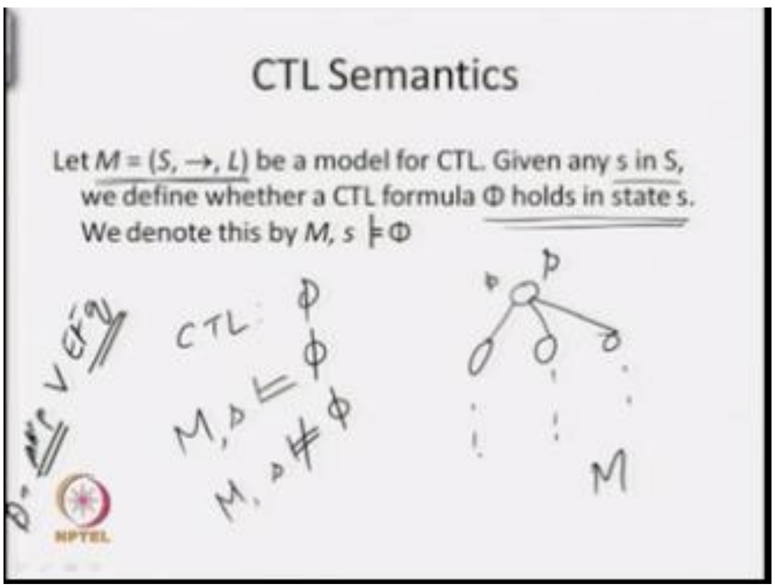

Now we are going to defining the semantics our CTL formula so it is like you already temporal formula I have said one in CTL formula also is said that we are in to consider one particular model m which is having this three components set of states transition relation l we have model of CTL so now keep an any step s of s if you are consider any step s of s we define whether the CTL formula φ holds in this particular state or not.

And we donate and we denote it by m s φ that when if I am having some sort of your model m and we are going to consider one particular step s you are having a CTL formula step φ will said that M S models φ it says that this particular CTL formula φ holds this particular step L. now and similarly we use this particular symbol also m s does model φ in this particular guess what is says that in state s your formula φ does not holds.

So basically we are going to set up φ holds in a particular step of φ does not hold in a particular step so in this particular case if φ holds in a particular φ sometime you can have a level of φ φ over here, and the meaning of those particular CTL formula will be defect on the structure of this particular CTL formula because if we are going to talk a particular CTL formula it may have struck formula also.

So basically in that table you are going to define the meaning on this particular structure of CTL formulas consider one particular formula say A F P or EFQ that means this is one CTL formula this is another CTL formula first we should have done level of this particular CTL formula in a step then these EFQ holds inside s can we can set up AFP or EFQ also holds in this particular circuits I delve of this non components is to in this fellow. So if we look for one particular formula we have prove loop for this particular component wise and after that we are going to look for the entire formula.

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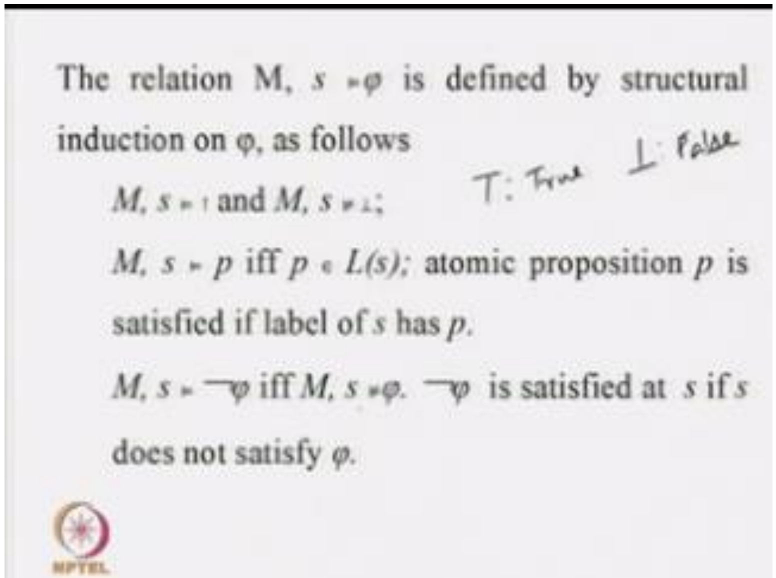

Now that is why I am saying that the relation this m as model φ is define by the structure and indexation of φ s follows so in general what happen we are going to say that in every step it will be mark by this particular prove fellow true and it will not mark by the tool fellow falls because this stop represent that tool fellow tool and button represent the true fellow false so this is basically top is your true and button is you false.

So by dewfall every step will be mark with the tool fellow true and none of them only mark with false that will nature is says that indentify you can sais the true is true in all step and false is false in each and every step, another one we have going to said a check and rule m s model p if p is member of this particular leveling function of this particular step that means we are going to talk about the tool follows of atomic proposition if the step is level with the help of this particular atomic proposition then we said that this particular atomic proposition sis true in this particular step.

Similar if we said that a particular step s of the model m is not a start of φ that means here a negation of this φ is true over here basically it means that it does not this particular step s or model m does not models this particular part so if it does not model φ you can marked it with not of φ also so already I have mention that I basically a negation of atomic proposition we are not going to mark it by defiled it is them.

But in some formula we have to marked it if it is a CTL combination of CTL prompters because this will come as a soft formula of another formula so we need the have the two fellows of this particular negation so if it is mark to it you not of φ it means that φ is not holding in this particular step so these are the three basic components that we have.

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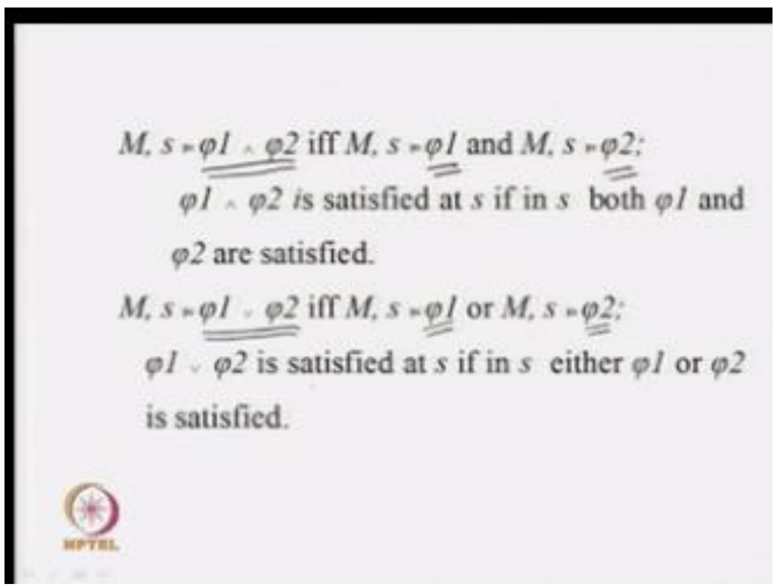

So her you just see that we are having a model so it is a three structure we have unfold a model so s0 s1 s2 s3 now if you look in to this is the labeling function that we are having pq kq so m of so model because it is level it be φ consider model φ, similarly I consider m of s1 models not of p because it is not mark to this particular atomic proportion so in state s1 it models not of p now you are not marking you are consider it models not of p.

So similarly now we can look for dot true fellows are beyond this conjunction φ1 and φ2 whether in a model n in a particular steps as where I models φ 1 and V 2 and φ1 and φ2 hold over the end. It holds provided φ 1 and φ2 independently holds ion that particular step okay so we say that φ1 φ 2 satisfies in this particular step is provided this step models φ1 and model φ 2, similarly we can look for the designations φ 1 of φ2 so this things this is this junction so what will happen is say that in particular step as φ 1 of φ 2 holds provided either φ 1 holds in s of φ2 holds in s okay. So these are the basic properties that we are having so this is your φ 1 of φ 2.

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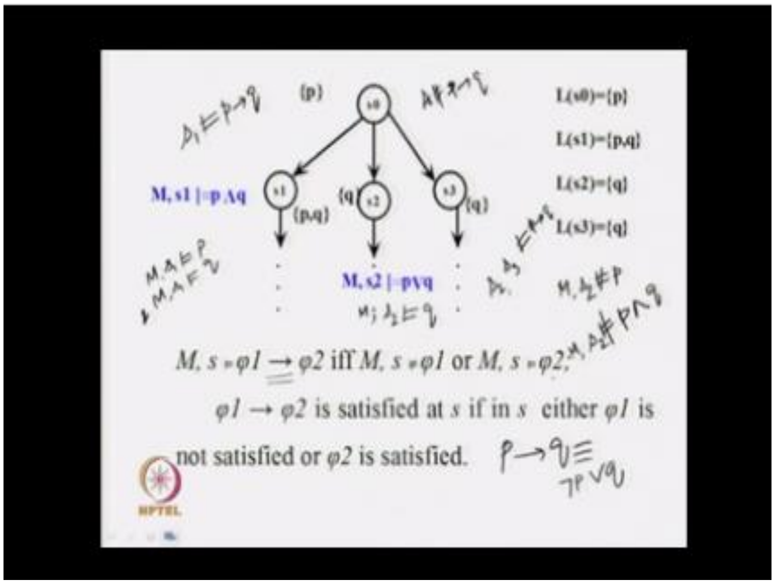

So now you looking to this particular again that model that we are having s0s1s2s3 and we are having the z of tresses structure and these are the leveling function that we have, now if we concerned about this particular state s1 will find that ms1 models p and q because this s1 is leveled a p and q.

So we can say that M S1 models P and M S1 models Q see both P and Q models in S1 we can say that S1 P and Q models okay so that P and Q holds as particular step in S1 if you come to this particular step S2 you will find that it is leveled with Q and it is autonomic power is Q over here so you can say that the M as 2 models P or Q because we use a set up here either P or Q should hold in this particular cell.

So here you will find that M S2 models Q since M is models Q we are going to set up M as to models for P or Q either P or Q but now if you look at the status you will find that M S2 does not model P because it is not leveled to your P since Q is not leveled with P it is not in this particular leveling function that means P is false the atomic position P is false this particular step S2 since P is false so it does not model P so now if you look into this combinations P and Q in this particular step S2 of model m then what we will find that M S2 does not model P and Q because it models Q but that does not model P.

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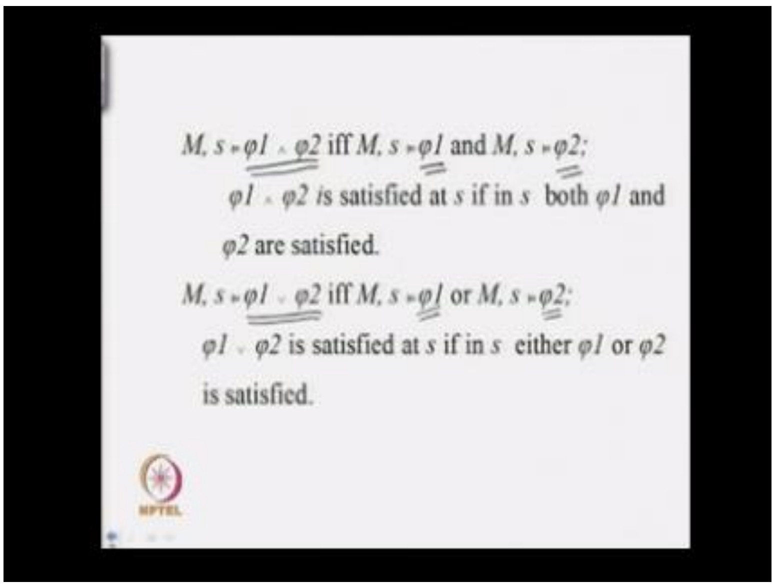

So that how your are saying that M S2 does not model Q just you see the meaning over it instead S models Q1 and Q2 provided that M S model 5 1 model 5 cube so both must be true however so since in this particular step it does not model P so we can say that M S2 does not with PM so similarly we come to this particular connecting implementation 5 1 + 5 2 in a particular step M in model M in status whether it models 5 1 + 5 2 or not so in this particular case what we have to is to say that this set models this 5 1 simplification 5 2 if M as instead it does not models 5 1 or M as models 5 2 okay.

So what we are going to say if I add models 5 1 does not models 5 1 or modes 5 2 so basically 5 1 + 5 2 satisfy others in either as 5 1 or 5 2 is satisfied either of these conditions so basically we know that if we are going to talk about your some formulas say P implies Q this is equivalent to 0 of P or Q so we know that these particular simplification sign is equivalent to lot of P or Q so in this particular case which are the EP than the Q if P is true then Q must be true P is false Q is silent in this particular case if look that P = Q over here but it is not with Q so0 this particular S0 does not model P implies Q but in this two step S0 and S3.

You will find that it is marked with but it is not marked with P so it is P is false but we said if it then Q if P is false we are is got above it so in this particular S2 and S3 they models you know P implies Q so here you can find that S2 and S3 both models so P Implies Q because Q is parallel P is cordial S does not Q1 or S does models Q2 if you come to this particular one both P and Q that means in this all conditions 5 2 is 2 by so we can say that in S1 also models P implies Q.

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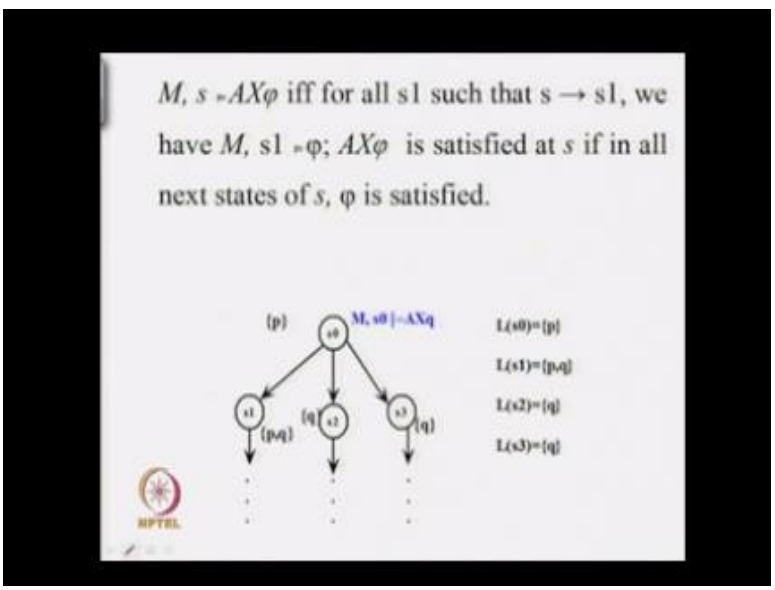

So this is the word we are going to look for the Q of all the above CTL formulas now till now we see that we are discussing about the CTL formulas we have designed a syntax or worked CTL formulas now we are going to define the meaning of our CTL formulas and what we have in our

CTL formula we are having all the proportion connectives and all not exordial simplification area and along with that we are going having the temporal point law and if we are using temporal potter those temporal potter must be preceded by our part quantifier okay.

Then we going to have CTL formulas otherwise we are going to have temporal formulas only and one we are going to talk about CTL formula that prove temporal formula defines what I said that means we have basically step formulas now one we are going to define a symmetric were saying that CTL formulas or the meaning of CTL formulas will be defined over with their structure we have seen what is Kripke structure it is similar to finite set but we are having to additional requirements one is that finishing function must be complete and second one we are having a leveling function it must be leveled by the static atomic proportion later to in this particular steps.

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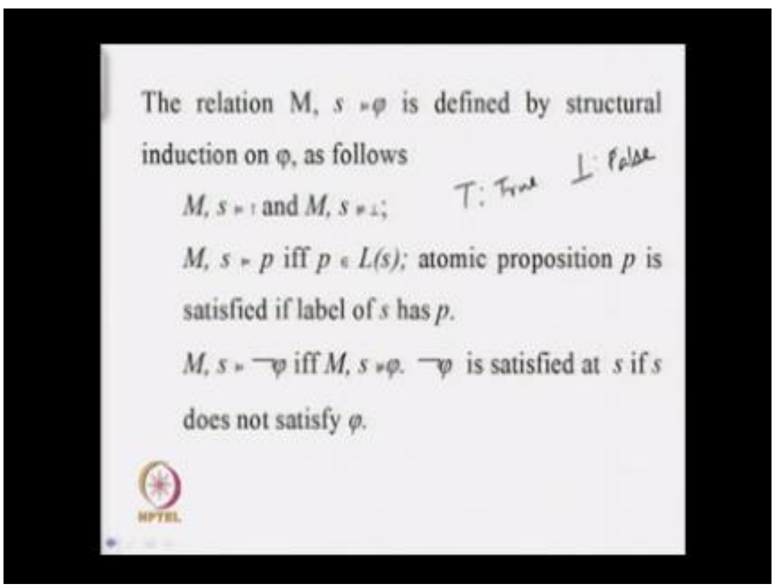

Now we have seen how to define meanings so in this lecture we have talked about the meaning or how to send true fellows of temporal for CTL formulas with respect to our this particular proportional connectives so we have seen an or negation and your distance simplification and your line depth now we have to see the hoe we going to assign the true values of our temporal operations next class we are going to discuss about this things so up till now we have talked about a true fellows of this particular proportional operators only.

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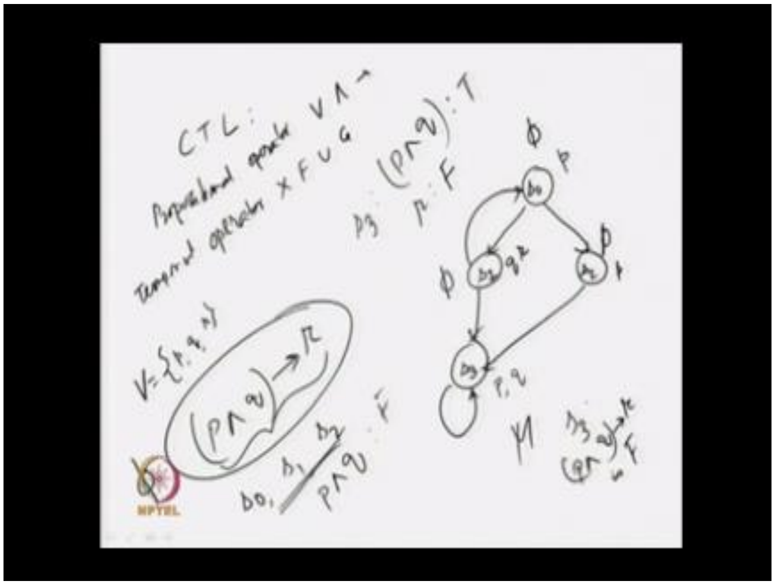

So next class we are going to talk about our something what we call Carlson to look for the entire temporal operator now we see that now what happens we are talking about say CTL formula it is having proportional operator like that or an simplification and temporal operator next step fusion until and your globulin now today you have seen the meaning of those particular proportional operator only so next class we are going to talk about this particular how to define a meaning of this particular temporal operator.

Now you just take a simple model say this is my one so this is a Kripke structure if you look into it you will find that we are having this particular forth step as 0 as 1 as2 as 3 now if you look into this illustration you will that the every sates are having a successor steps so9 that’s why the condition illustration is complex we can set up this model M in a Kripke structure now we said we are talking about atomic proportion V Q R.

The leveling function is r we can say that the VQR P PQ now if I give a temporal formula P and Q implies say R so if you look into this particular temporal formula whether this formula is true in this particular which are the steps this particular formula is true because we will find that everywhere this particular component is instead as 0 as 1 as 2 this component P and Q is your false you said P is true but Q is false Q is true but P is false P is true but Q is false.

So P and Q is false in this particular tree step so since this is false so P and Q implies R will be true in this particular tree step now when you come to this particular tree step + S3 then you will find that P and Q is true over here but what is the all the step of your r you will find that r is false so true implies false so in this particular case you will find that instead as 3 the given formula P and Q until R is false basically because this is the P and Q is true but R is false.

So true implies false which is basically false now what will happen if we are having this particular tree four step they are leveled with this proportion now when we are going to look for this particular temporal formula then we will find that this particular formula say if I say this is you 5 then what we can get say this 5 is true instead as 1 as 2 as 0 as 1 as2 but this is not true over here so like that what we do we can say that we know this particular component is true.

Now if I say it going to set up A F 5 that means we will know the level of this particular 5 it is leveled in this particular now we can look this particular of this formula so prior to that we are going to see how we are going to define the meaning of F so in next class we are going to talk about or we are going to discuss about how to define the meaning of this particular temporal operators along with part quantify in our Kripke structure okay thank you.

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