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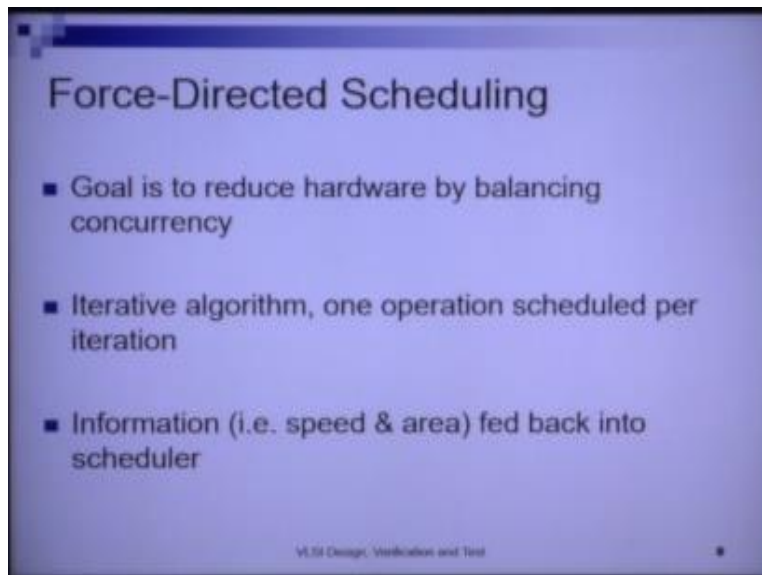
**NPTEL ONLINE CERTIFICATION COURSE
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VLSI Design, Verification & Test

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Hello welcome to module three of lecture five, in this module we will look at the next heuristic time constraint scheduling strategy known as force directed scheduling.

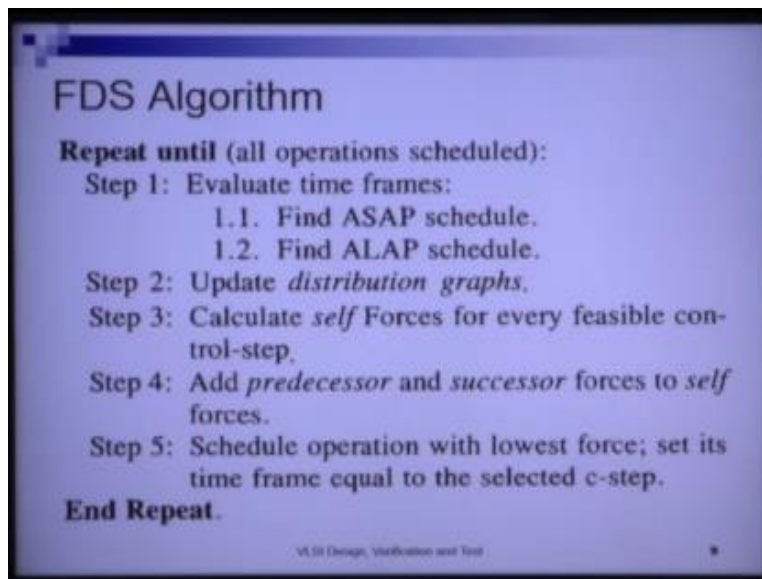
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Now force directed scheduling the goal is to reduce hardware by balancing concurrency so what do we mean by concurrency here, the number of operations that I have to concurrently schedule at a given time step so if we can balance the concurrency distribute the concurrency in two different time steps as evenly as possible we will maximize the usage of resources over these time steps and hence it will lead to a minimum resource scheduling within a given time bound.

So it is an iterative algorithm one operation in schedule per iteration after the iteration the information that is the current on the current speed and area that has been obtained is fed back to the scheduler and the subsequent scheduling decisions are dependent on this information.

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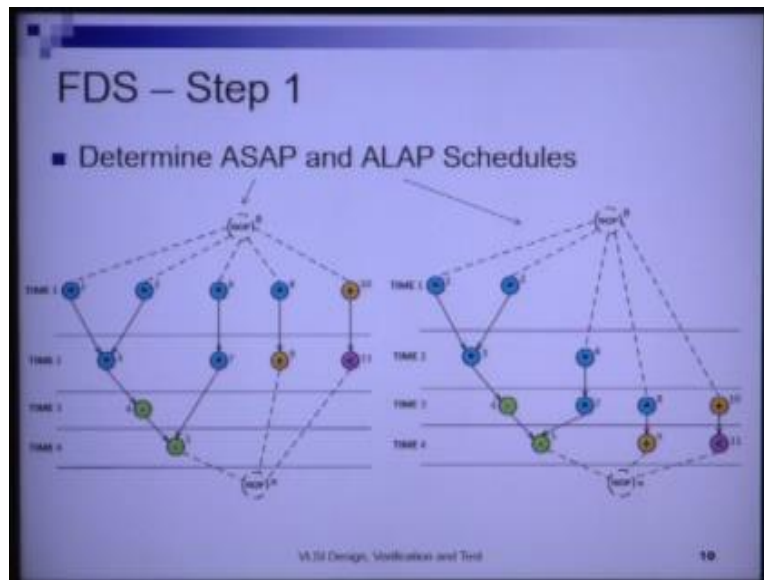


So how does this strategy proceed so we will first look at the scheduling strategy overall scheduling strategy here and then see the different components and explain these different components so repeat until all operations have been scheduled as we said in each step we will have one operation scheduled, so the step operation is to evaluate time frames you will look at what time frames are and to evaluate time frames we will require ASAP and ALAP schedule of the operations at that time step.

Then we update distribution graphs so we will understand what distribution graphs are then we will calculate self forces for every feasible control step then we will calculate predecessor and successor forces after calculating the cell forces predecessor forces and successor forces we will find out the total force and we will schedule the operation with the least force that means this corresponds to higher stability that we will see.

So the operation which whichever operation is now selected but after step 5 is the time step at which the operation should be scheduled.

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So as we said we have to first determine time frames and in that time frame the first work for us was to determine the ASAP and ALAP schedule the initial ASAP and ALAP schedule is obviously the ASAP and ALAP schedule that we have been we have been considering in all the classes till now the left one produces the ASAP schedule as soon as possible schedule and the right one gives me the ALAP schedule as late as possible schedule.

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FDS – Step 1

- Determine *Time Frame* of each operator ($= \text{mobility} + 1$)
 - Box-Length – Possible execution cycles
 - Box-Width – Probability of assignment $= 1/\text{Box-Length}$
 - Probability of assignment $= 0$ outside the box.
 - Uniform probability distribution inside box; Area assigned $= 1$
 - Operations with probability 1 are bound to start at one specific time-step.

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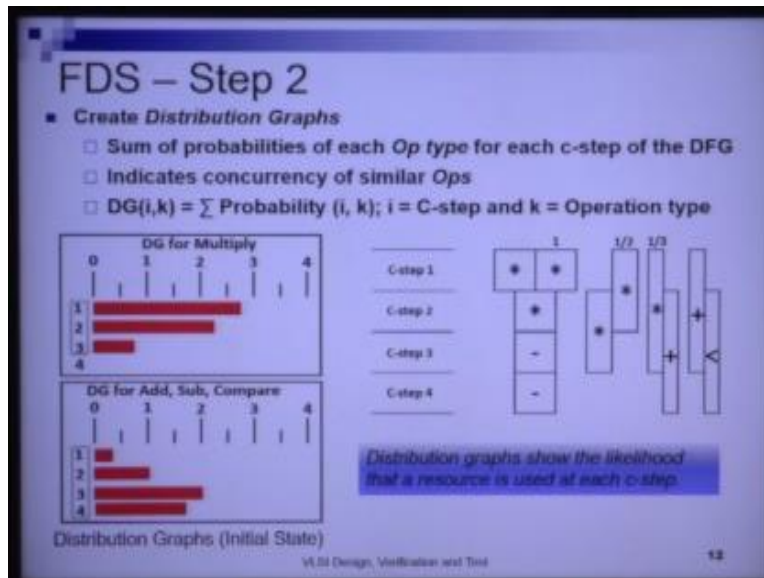
So based on that we obtain the time frame for each operation we will determine the timeframe for each operation and what is the time frame for an operation the timeframe for an operation is equal to its mobility plus one how is the mobility often mobility is given by the ALAP time minus the ASAP time and that is why we found the ASAP and ALAP schedules, now given the time frames we have these boxes where the length of a box that means this is the length of a box.

The length of a box is equal to its possible execution cycles for example the length of this is operation 1 V1, V1 has a length of one because it has to be schedule if I have a latency bound of four if I have a latency boulder for the operation one must be schedule only it must start at time step one only similarly operation two must start in operation time step one operation three must start in time step two operation four must start in time step 3 and operation 5 must start 4, why?

Because these are all the operations in the critical path now let us take operation 6, operation 6 has a length of two time steps why because it is possible to schedule it in either time step one or time step two so here we see that the time step is to because the mobility of this operation is one the all for all the operations in the critical path the mobility of these operations are 0 and hence the timeframe is one here the length is 2 because the mobility is one length means the time frame

the time frame is two because the mobility is one. That means operation 6 can either be scheduled in time step 1 or time step 2 similarly operation 7.

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Similarly operation 7 can either be scheduled in time step 2 or time step 3 and rest for the other similarly operation say operation 8 can be schedule in time step 1 or time step 2 or time step 3 is mobility is too and it is timeframe is there for 3 and these are the possible time steps 1, 2, n 3 are the different possible time steps in which it can be shadowed what is the width of a box, the width of a box is its probability of assignment okay.

And is given by one by the box length so if the box length is one that means the probability of assignment is one that means that operation must be scheduled in that time step okay so that is why the width of these operations are one the width of the multiplication operation v6 operation v6 is half because its box length is 2, so 1 by time frame so 1 by time frame is the probability of the operation.

So the probability of the operation is 1 by box length within its time frame and 0 outside it so the probability of assignment is 0 outside the box and is equals to 1 by timeframe within the box,

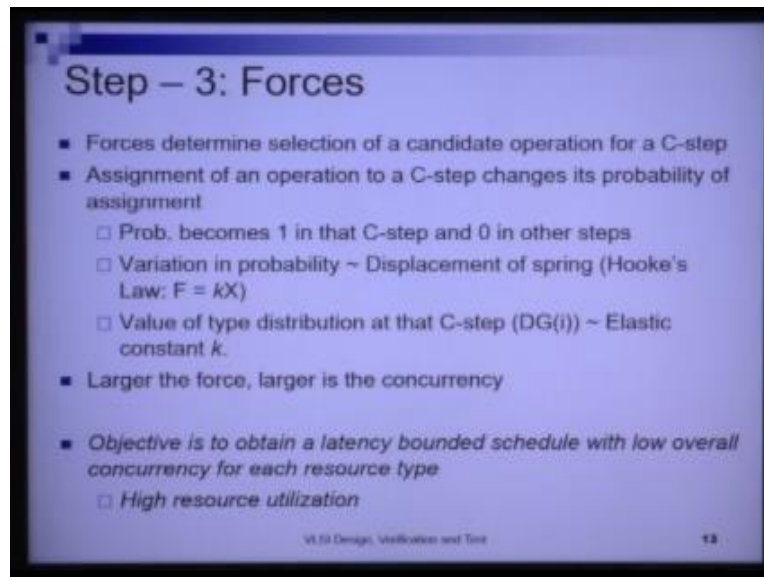
now uniform probability distribution is there within the box which goes to say that the area assignment equals to 1 that means that all operations have this take the same area, so that is why we have we have shown here that the area of each operation is 1 this is and this is a measure of the total area of the box is the measure of the area that this operation will take on the floor of the chip.

So then we said that after we have calculated the time steps of the operations the next work is to create distribution graphs, what is a distribution graph the distribution graph the distribution graph is the sum of the probabilities of each operation type for each control step in the dataflow graph, so it indicates the concurrency of similar operations for example let us see for the multiplication operation the multiplication operation it says that the that at the first time step the value of the distribution type multiple the distribution type multiplication the value is 2.8.

So how does this 2.8 come it is 1 plus 1 plus half plus one-third so the difference so what is was what is it being summed up on it is the sum of the probabilities of all operations that of all operations of a given type that are possible to schedule at a given time step that means at time step one of type multiplication there are four possible operations that can be scheduled in time step one and the sum of their probabilities are $1+1+1/2 + 1/3$ and that is around 2.8, for the second time step what do we have we have $1 + 1/2 + 1/2 + 1/3$ so 2.33 is the DG value is the distribution value of for the multiplication operator at time step two, now what does the sum of these probabilities indicate it indicates the pressure on a resource due to the concurrency of operations.

Hence the pressure of concurrency that means the number of operations that I need that I am that I am I may be forced to scheduled in time step one is high and hence this value of the summation of probability is high so it indicates the concurrency of similar operations at a time step, so similarly we have these values for the addition subtraction and multiplication operations so what does the distribution graphs show the distribution graphs show the likelihood that a resource is used at a given C step.

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Now we will understand the concept of forces so first we understood the concept of time frames which is one plus the mobility and we also understood the concept of the probability of assignment of an operation at a given time step which is one divided by that by the timeframe of that operation and then we understood distribution graphs we said that the distribution value offer distribution graph for a given operator at a given time step is equal to the summation of the probabilities of that operation type of that of operation type at that time step.

So and that indicates the level of concurrency of that operation type at that time step okay, forces determine the selection of a candidate operation for a C step now depending we will find out we will have a concept of forces and that will determine which candidate that will prioritize the operation to a given time step the forces of different operations will attract or repel each other and finally a certain operation will be scheduled at a given time step.

So that the stability of the system is maximized what we mean by that and how we get it I am just coming to, so assignment of an operation to a C step changes the probability of assignment probability becomes one in that C step and 0 in other C steps so when we actually assign an

operation to a C step the probability becomes one in that C step and 0 in other C steps other control steps.

So let us say that I could I could schedule an operation in two time steps so its probability of assignment was half in the first time step and half in the second time step and then we have for that operation to be scheduled in the first time step so the probability now becomes one in the first time step and 0 in the second time step now how does this forces relate to these probabilities now the variation in probability has been equated to the displacement of a spring in Hooke's law.

So Hooke's law tells that the force on a spring is proportional to the displacement X the force F on a spring is proportional to the displacement of the spring and that displacement value is given by X. Here the probability of assignment the variation in the probability of assignment has been equated to the displacement of the spring the value of type distribution at that C step the value of the type distribution at that C step that is the DG value the distribution value has been has been equated to be analogous to the elastic constant k.

Now larger the force larger is the concurrency, so what do we try to do we will we will explain this with an example subsequently. So what do we do the objective is to obtain a latency bounded schedule with low overall concurrency for each resource type why because this ensures high resource utilization.

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FDS – Step 3: Self-Forces

- Two classes of forces:
 - Self Force – Set of forces relating an operation to the different possible C-steps where it can be scheduled
 - Every operation has 'self force' for every C-step of its time frame
 - Predecessor / Successor Forces – Related to operation dependencies
- Force (i, k) = DG(i, k) * X(i)
 - DG(i, k) = Current value of distribution graph
 - X(i) = Change in operation's probability
- Total Self Force associated for assigning an operation at C-Step j:
 - Self force (j) = $\sum_{l=0}^j \text{Force}(i, l)$

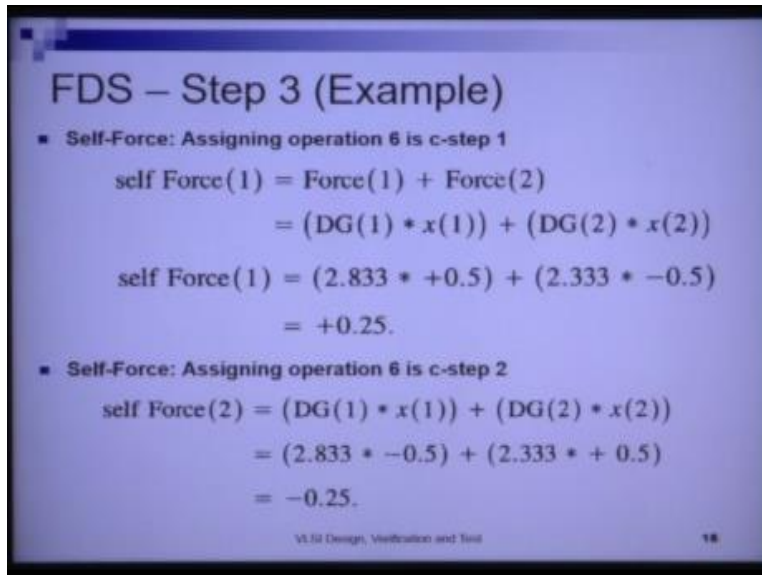
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Now we come to the concept of self forces so basically forces are of two types self force which tells me what it tells me the set of forces relating to an operation to the different possible C steps where it can be scheduled every operation has a self force for every step of its timeframe. There are also along with the self forces there are predecessor successor forces and that is related to operation dependencies.

So how do we compute the force of operation I for a given type of resource it is given by the distribution graph for resource k multiplied by the change in the operations probability. So DG ik what does it tell me it tells me the current value of the distribution graph for resource type k and XI is the change in the operations probability. Now the total self force associated for assigning an operation at C step J is given by summation over all its possible time steps where it can be scheduled and we what do we sum up, we sum up the force.

So self force at time step J is given by the total force for that operation at different time steps where it can be scheduled.

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FDS – Step 3 (Example)

- **Self-Force: Assigning operation 6 is c-step 1**
$$\text{self Force}(1) = \text{Force}(1) + \text{Force}(2)$$
$$= (\text{DG}(1) * x(1)) + (\text{DG}(2) * x(2))$$
$$\text{self Force}(1) = (2.833 * +0.5) + (2.333 * -0.5)$$
$$= +0.25.$$
- **Self-Force: Assigning operation 6 is c-step 2**
$$\text{self Force}(2) = (\text{DG}(1) * x(1)) + (\text{DG}(2) * x(2))$$
$$= (2.833 * -0.5) + (2.333 * +0.5)$$
$$= -0.25.$$

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Now we come to the example, so what is the self force corresponding to operation 6 in C step 1. We said that operation 6 can either be scheduled in time step or C step 1 or in C step 2 in C step 1 it has a probability of 0.5 in C step 2 it has a probability of 0.5. Now how do we calculate self forces we said that the self force is the summation of the forces at time step one and two for operation ϕ_6 .

So what is the force at the first time step the DG value force for operation 6 at the first time step multiplied by its change in probability, what is the change in probability the change in probability we said that previously it was 0.5, and now it has changed to one. So the change in probability is $1 - 0.5$ which is $+0.5$. And what is DG corresponding to that we said that DG corresponding to the first time step was for the multiplication operator was 2.8.

So DG1 was 2.833 multiplied by the change in probability of operation ϕ_6 which is $1 - 0.5$ $0.5 +$ the DG value at operation for the multiplication operation at time step 2.

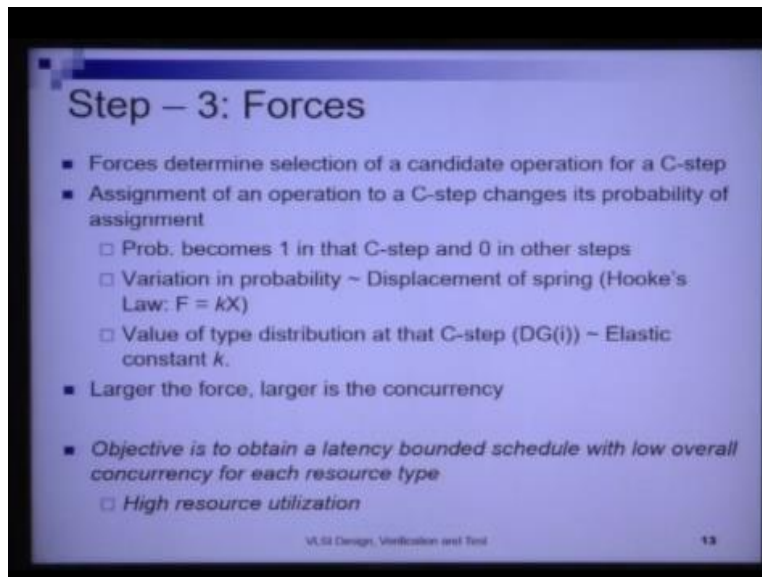
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FDS – Step 3: Self-Forces

- Two classes of forces:
 - Self Force – Set of forces relating an operation to the different possible C-steps where it can be scheduled
 - Every operation has 'self force' for every C-step of its time frame
 - Predecessor / Successor Forces – Related to operation dependencies
- Force (i, k) = DG(i, k) * x(i)
 - DG(i, k) = Current value of distribution graph
 - X(i) = Change in operation's probability
- Total Self Force associated for assigning an operation at C-Step j:
 - Self force (j) = $\sum_{l=c_j}^{c_j} Force(i,l)$

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Step - 3: Forces

- Forces determine selection of a candidate operation for a C-step
- Assignment of an operation to a C-step changes its probability of assignment
 - Prob. becomes 1 in that C-step and 0 in other steps
 - Variation in probability ~ Displacement of spring (Hooke's Law: $F = kX$)
 - Value of type distribution at that C-step ($DG(i)$) ~ Elastic constant k .
- Larger the force, larger is the concurrency
- *Objective is to obtain a latency bounded schedule with low overall concurrency for each resource type*
 - *High resource utilization*

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FDS – Step 2

- Create *Distribution Graphs*
 - Sum of probabilities of each *Op type* for each *c-step* of the DFG
 - Indicates concurrency of similar *Ops*
 - $DG(i,k) = \sum \text{Probability}(i, k)$; $i = \text{C-step}$ and $k = \text{Operation type}$

The slide shows two distribution graphs. The first is titled 'DG for Multiply' and the second is 'DG for Add, Sub, Compare'. Both graphs have a horizontal axis from 0 to 4 and a vertical axis with steps 1, 2, 3, and 4. Red bars indicate the probability of resource usage at each step.

Step	0	1	2	3	4
1	0	1	0	0	0
2	0	0	1	0	0
3	0	0	0	1	0
4	0	0	0	0	1

Step	0	1	2	3	4
1	0	1	0	0	0
2	0	0	1	0	0
3	0	0	0	1	0
4	0	0	0	0	1

The DFG diagram shows four c-steps. Cstep 1 has two multiplication operations (*). Cstep 2 has one multiplication operation (*). Cstep 3 has one subtraction operation (-). Cstep 4 has one subtraction operation (-). Resource usage is indicated by vertical bars above the operations: Cstep 1 uses 1 resource, Cstep 2 uses 1/2 resource, Cstep 3 uses 1/3 resource, and Cstep 4 uses 1 resource.

Distribution graphs show the likelihood that a resource is used at each c-step.

Distribution Graphs (Initial State)

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To the DJ value for the multiplication operation at time step 2 is given by this 1 so it is 2.3.

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FDS – Step 3: Self-Forces

- Two classes of forces:
 - Self Force – Set of forces relating an operation to the different possible C-steps where it can be scheduled
 - Every operation has 'self force' for every C-step of its time frame
 - Predecessor / Successor Forces – Related to operation dependencies
- Force (i, k) = DG(i, k) * x(i)
 - DG(i, k) = Current value of distribution graph
 - X(i) = Change in operation's probability
- Total Self Force associated for assigning an operation at C-Step j:
 - Self force (j) = $\sum_{i \in C_j} Force(i, j)$

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FDS – Step 3 (Example)

- **Self-Force: Assigning operation 6 is c-step 1**
$$\text{self Force}(1) = \text{Force}(1) + \text{Force}(2)$$
$$= (\text{DG}(1) * x(1)) + (\text{DG}(2) * x(2))$$
$$\text{self Force}(1) = (2.833 * +0.5) + (2.333 * -0.5)$$
$$= +0.25.$$
- **Self-Force: Assigning operation 6 is c-step 2**
$$\text{self Force}(2) = (\text{DG}(1) * x(1)) + (\text{DG}(2) * x(2))$$
$$= (2.833 * -0.5) + (2.333 * +0.5)$$
$$= -0.25.$$

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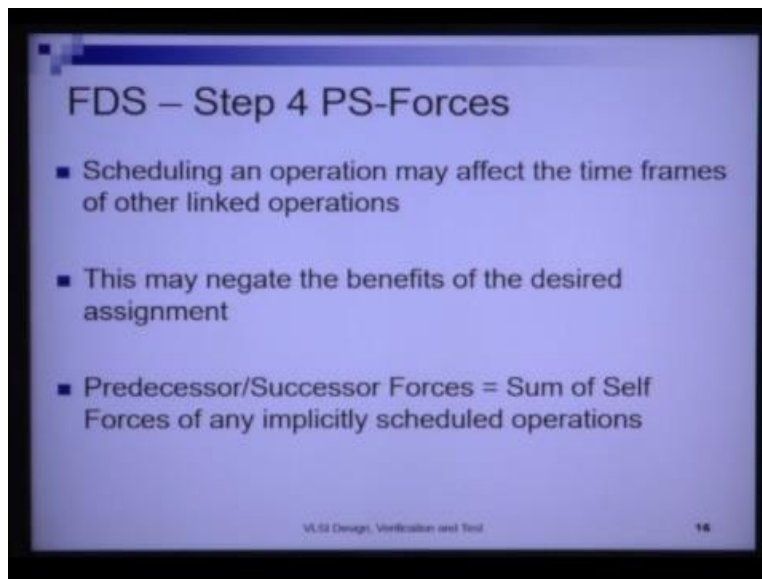
So DG2 is 2.33 and X2 is -0.5 why because I have not scheduled that operation here my probability is zero now, what was my probability previously it was 0.5 so $0 - 0.5$ gives me -0.5 . So the total self force for scheduling operation v6 at time step 1 becomes 2.833 which is the distribution value from the distribution graph of the multiplication operation at time step one in 2 plus x + multiplied by + 0.5 which is the change in probability of operation v6.

If it is scheduled at time step 1 plus the distribution value for the multiplication operation at time step 2 multiplied by the change in probability for not scheduling the multiplication operation at time step 2 which is -0.5 and therefore the total self force at the first time step becomes + 0.25. The self force for assigning operation v6 at C step 2 what is that we found out the self force for assigning the operation in C step 1.

And now what is the self force total self force for assigning operation 6 in C step 2, the total self force is given by again $\text{DG}(1)X1 + \text{DG}(2)X2$. So now what does this give me here the DG remains same 2.833, however now I have not scheduled the operation v6 in time step 1 so previously my operation probability assignment probability was 0.5.

Now it is 0, so $0 - 0.5$ gives you gives me $- 0.5$ + the DG at time step 2 which is 2.333 multiplied by $+0.5$ which is why because now it becomes $1 - 0.05$ the $1 - +0.5$ that is the change in its probability of assignment and this value is $- 0.25$. So we find that the self force corresponding to operation 6 is lowered in time step 2 than in time step one.

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After we have found out the total self force for the operation we have to find out the predecessor successor forces. Now what are these scheduling and operation may affect the time frames of other linked operations right. So when I have fixed let us say operation v6 in time step 2, let us say then I am forced to schedule operation v7 which is the linked operation here at time step 3 right must be scheduled at time step 3.

And hence the linked operation the time frames of the linked operations now change. Now due o this, this may negate the benefits of the desired assignment. So if we consider only self forces we could do the scheduling based only on self forces, so what would that do it would find the self force of each candidate operation each ready to schedule operation at a given time. And then it would take the least self force the one with the least self-worth and schedule it.

However, if we consider only self forces I am not considering the effect of the of the operations that this schedule that this assignment of my current operation has affected on other linked operations, and that may negate the benefit of the assignment of this operation at the current time step.

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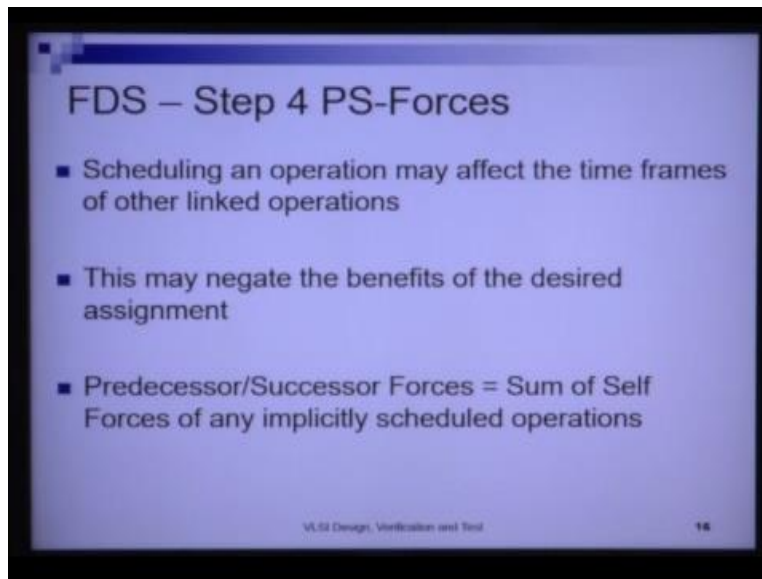
FDS – Step 4 (Example)

- Assignment of operation v_6 to C-step 2 implies the assignment of operation v_7 to C-step 3

$$\begin{aligned}\text{succ Force}(3) &= (\text{DG}(2) * x(2)) + (\text{DG}(3) * x(3)) \\ &= (2.333 * -0.5) + (0.833 * +0.5) \\ &= -0.75.\end{aligned}$$
$$\begin{aligned}\text{Total Force}(2) &= \text{self Force}(2) + \text{succ Force}(3) \\ &= -1.\end{aligned}$$

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FDS – Step 4 PS-Forces

- Scheduling an operation may affect the time frames of other linked operations
- This may negate the benefits of the desired assignment
- Predecessor/Successor Forces = Sum of Self Forces of any implicitly scheduled operations

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So predecessor force, successor forces is the sum of the self forces of any implicitly scheduled operations.

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FDS – Step 4 (Example)

- Assignment of operation v_6 to C-step 2 implies the assignment of operation v_7 to C-step 3

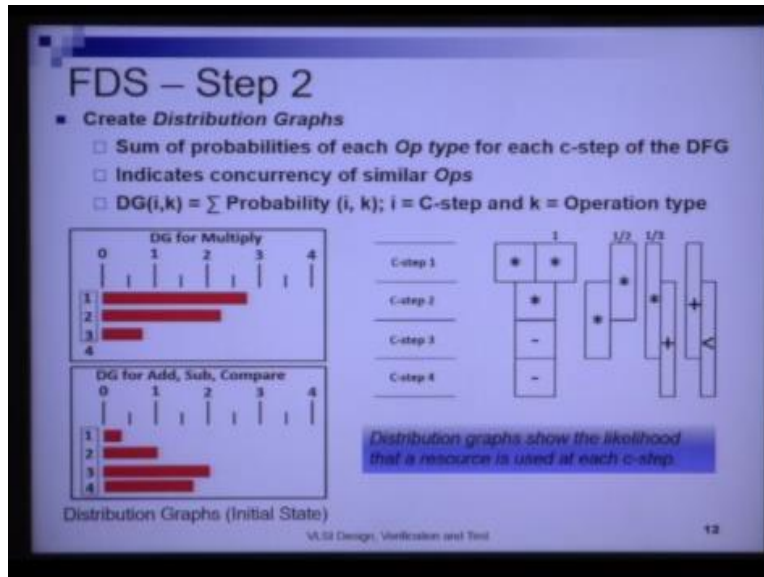
$$\begin{aligned} \text{succ Force}(3) &= (\text{DG}(2) * x(2)) + (\text{DG}(3) * x(3)) \\ &= (2.333 * -0.5) + (0.833 * +0.5) \\ &= -0.75. \end{aligned}$$
$$\begin{aligned} \text{Total Force}(2) &= \text{self Force}(2) + \text{succ Force}(3) \\ &= -1. \end{aligned}$$

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So what does this tell me here now assignment of operation v_6 to C step 2 implies the assignment of operation v_7 to C step 3 as I was saying. Now what is the self force of v_7 when it is scheduled on C step 3 right, so that will be this the $\text{DG}_2 \times X_2$ because I have not scheduled it is possible to schedule operation v_7 at time step 2 and time step 3, but I have not scheduled this operation in time step 2 because my time step, it has been my time steps have reduced.

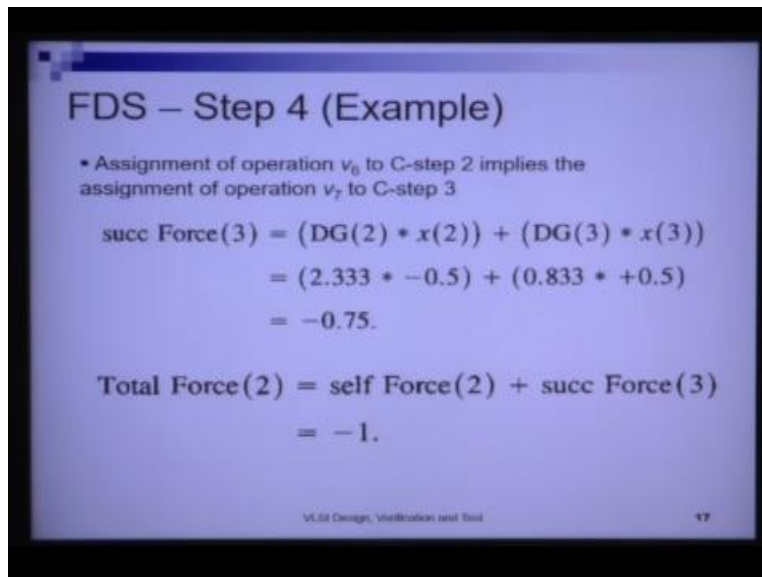
And I have actually scheduled will return time step 3 if I do that then DG_2 is 2.333 multiplied by X_2 which is - 0.5. So $2.333 \times -0.5 + 0.833 \times +0.5$, so -0.5 because the probability of v_7 at time step two was half and I have not scheduled it on time step two so the change in probability becomes 0 - 0.5 so it is - 0.5 the change in probability and for time step three the DG value that means the concurrency value is 0.833, let us just look back and see why is that so.

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This is because in time step three I can only schedule either v7 or v8 in time step 3 and the probability of v7 is half and the probability of v8 is one-third. So one-third plus half is the total distribution value at time step three.

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FDS – Step 4 (Example)

- Assignment of operation v_6 to C-step 2 implies the assignment of operation v_7 to C-step 3

$$\begin{aligned}\text{succ Force}(3) &= (\text{DG}(2) * x(2)) + (\text{DG}(3) * x(3)) \\ &= (2.333 * -0.5) + (0.833 * +0.5) \\ &= -0.75.\end{aligned}$$
$$\begin{aligned}\text{Total Force}(2) &= \text{self Force}(2) + \text{succ Force}(3) \\ &= -1.\end{aligned}$$

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And that is equal to 0.833 so the total successor force the total successor force is - 0.75 why because the successor force is now given by the scheduling of operation v_7 at time step 3. So the total force for scheduling the operation v_6 at time step two is self force of v_6 at time step two.

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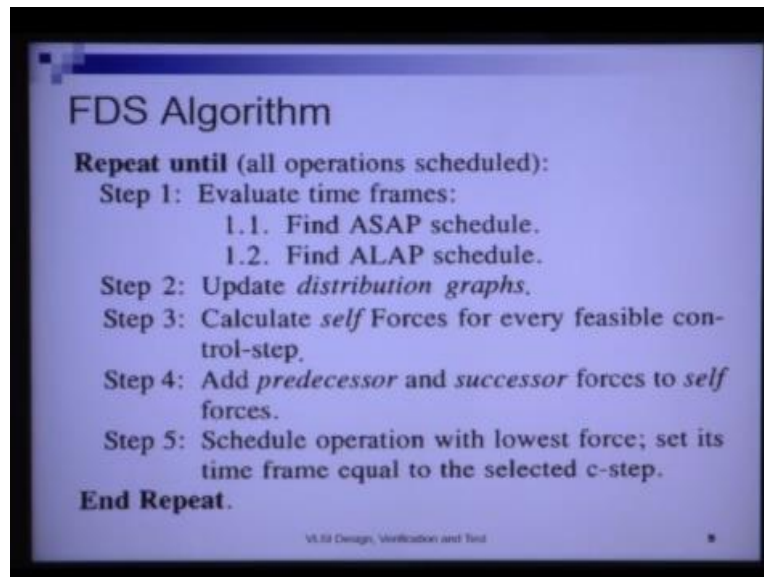
FDS – Step 3 (Example)

- **Self-Force: Assigning operation 6 is c-step 1**
$$\text{self Force}(1) = \text{Force}(1) + \text{Force}(2)$$
$$= (\text{DG}(1) * x(1)) + (\text{DG}(2) * x(2))$$
$$\text{self Force}(1) = (2.833 * +0.5) + (2.333 * -0.5)$$
$$= +0.25.$$
- **Self-Force: Assigning operation 6 is c-step 2**
$$\text{self Force}(2) = (\text{DG}(1) * x(1)) + (\text{DG}(2) * x(2))$$
$$= (2.833 * -0.5) + (2.333 * + 0.5)$$
$$= -0.25.$$

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Which was -0.25 which was - 0.25 here and plus the successor force which induces on time v7 which is - 0.75. So the total force becomes -1 now likewise for all operations I will at each step for all operations I will calculate the total force I will find out the minimum force and I will schedule that operation with the minimum force at each time step. And that until all operations have been scheduled. Now if we go back to the algorithm we should understand it better now.

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So what are we doing so repeat until all operations are scheduled evaluate timeframes find the ASAP schedule find the ELAP schedule so based on the ASAP and ELAP schedule now you find time frames, the timeframes is equal to the mobility plus one find out probability of assignment which is one divided by the time frame value.

Now given the probability of assignments find out the distribution graph for each operation type and that distribution graph for each operation for a given operation type will give the distribution value for that operation type at each time step, then calculate self forces for every feasible control step then add the predecessor and successor forces to the self forces and then find the total force, find the operation with the least total force and schedule that operation with the least force.

Set the time step of that operation to the selected C step. So if I schedule operation v6 in time step 2 because let us say assume that the total force with respect to v6 at time step two is lowest then the start time of v6 will be assigned equal to 2 with this we come to the end of module 3 of lecture five.

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