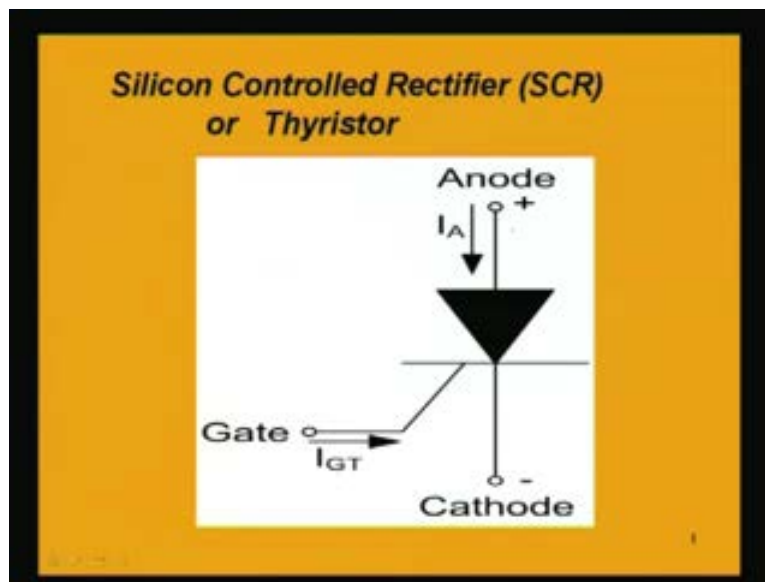


Basic Electronics
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Module - 5
Power Circuits and System
Lecture - 7
Silicon Control Rectifier

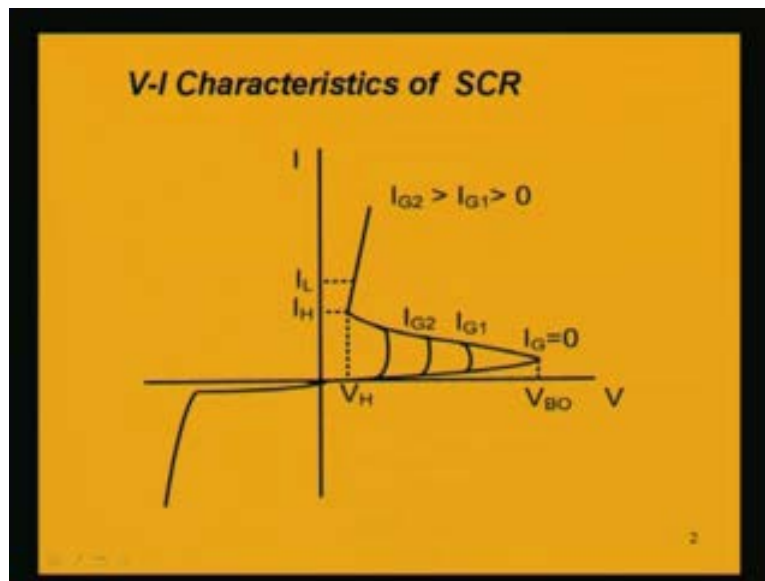
In the last class we discussed about the silicon controlled rectifier which is one of the mostly used p-n-p-n device. It is also known as thyristor. The symbolic representation of the silicon controlled rectifier or thyristor is like as shown in this figure.

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It is having an anode, a cathode and a gate terminal. The anode is kept at a positive potential with respect to the cathode and we trigger the SCR or the thyristor by means of the gate. We give a triggering voltage to the gate and we fire the SCR and due to this gate voltage being applied to the gate terminal, the firing of the SCR takes place at a voltage much lower than the breakover voltage; that we have seen in the last class.

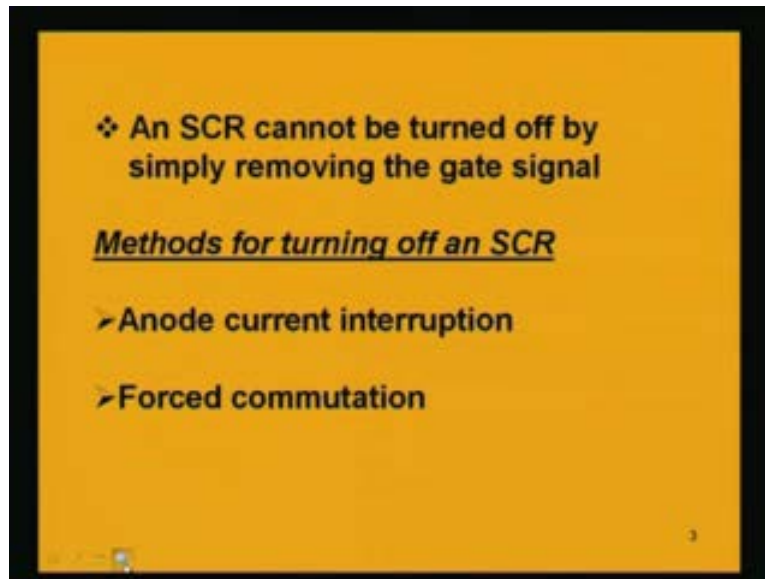
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Again if we recall the V-I characteristic of this SCR, we have this type of characteristic where we have seen that for the gate current I_{G1} greater than zero we see the firing taking place at a value or voltage lesser than the breakover voltage. As you go on increasing the gate current at a smaller and smaller voltage the firing will take place. There are two specific currents which are the holding current and the latching current. These currents are carrying a significant meaning because the holding current is the value of the current below which if the current in the thyristor falls then it will be turned OFF.

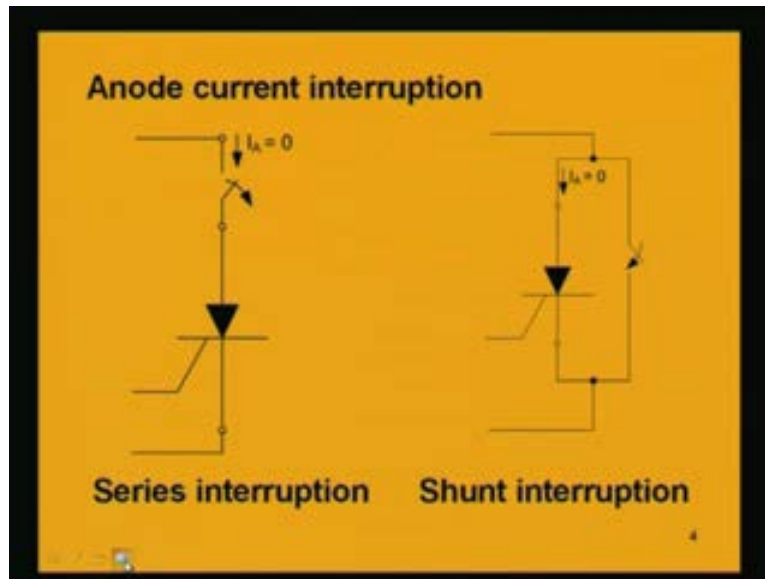
In order to turn OFF the thyristor, the current through the thyristor must be made to fall below the holding current. Similarly there is a latching current. The latching current is important from the point of view that in order to keep the thyristor in the ON condition, the current through it must be above the value of this latching current. At any time when we make the thyristor ON and want to keep it conducting then the current must not fall below the latching current. So, these two currents are important to keep in mind.

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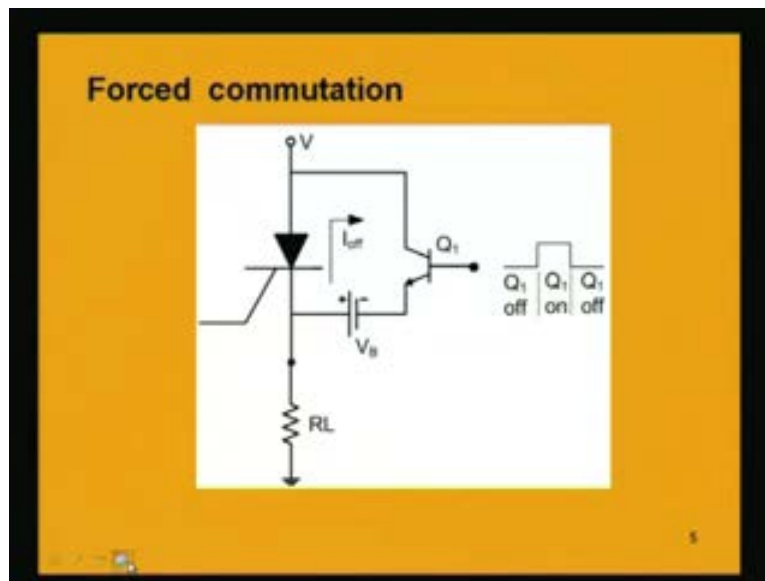
The turning OFF of a thyristor is one important point to be noted because we cannot turn OFF the thyristor or the SCR simply by removing the gate signal. Even if we remove the gate signal, the thyristor will continue to conduct unless and until we specifically apply a method of turning the SCR OFF. These methods are either by anode current interruption or by forced commutation. That is even if we remove the gate triggering signal the thyristor will not turn OFF. We have to apply the method of making the anode current zero or we will have to force the thyristor to turn OFF by making a current flow through it in the reverse direction, by reverse biasing the anode cathode terminals.

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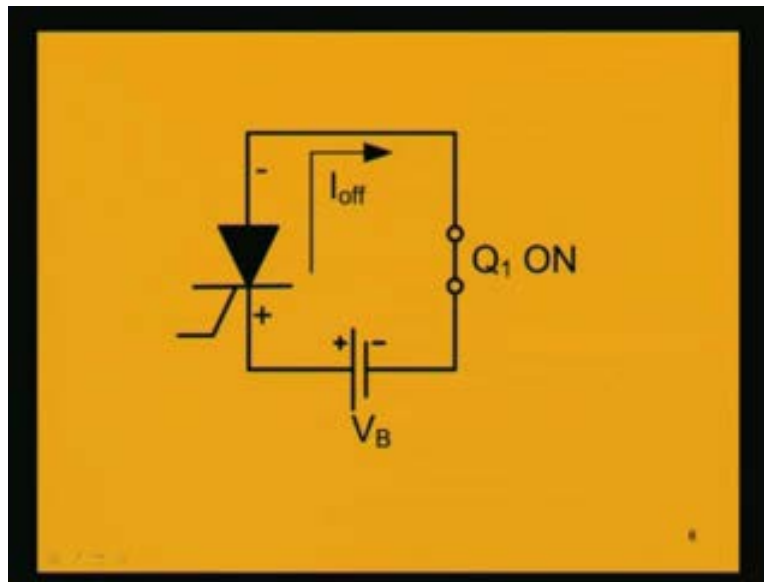
The anode current interruption is simply by making the anode current zero. It can be done by two ways. One is by series interruption or the other is by shunt interruption. In series interruption what is done is that we make the circuit of this thyristor open and then anode current will be zero; thyristor will be OFF or we can connect shunt path through a switch, make that switch close then the current will take the path through the switch which is closed and then the current through the anode of the thyristor will be zero and the thyristor will be turned OFF. These two are simple methods by which the anode current is made zero. It is interrupted either by series interruption or by shunt interruption.

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Another way of making the thyristor turn OFF is by force commutation. What is done here is that we have to make the current through the thyristor in the opposite direction. For example in the circuit we have a transistor Q_1 and there is a DC source V_B which is also connected in series with the thyristor. When we apply a pulse as shown here to the transistor Q_1 when the pulse is high that is the application of a pulse to the base of the transistor will make the transistor turn ON, there will be current flow in the collector. What will happen is that now the current will flow through the circuit in the direction shown by this arrow. What will happen is that this thyristor will now have positive terminal of the battery connected to the cathode and the negative will be connected to anode. We will now have a current flowing through this circuit in such a way that the thyristor will be biased in the reversed way. Anode cathode terminals will not be positive to negative.

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Due to this current flow when the transistor is ON, the polarities of the thyristor are becoming positive to cathode and negative to anode. As we see here the transistor is ON. It is like a very short circuited part. So, conduction is taking place in the transistor. That is why current is flowing and making this thyristor OFF by making the polarities of the anode and cathode like this; just opposite or reverse biased. This is the forced commutation which also makes the thyristor turn OFF.

Now we are going to discuss some important applications of the thyristors. Mostly we find application of thyristor in power control.

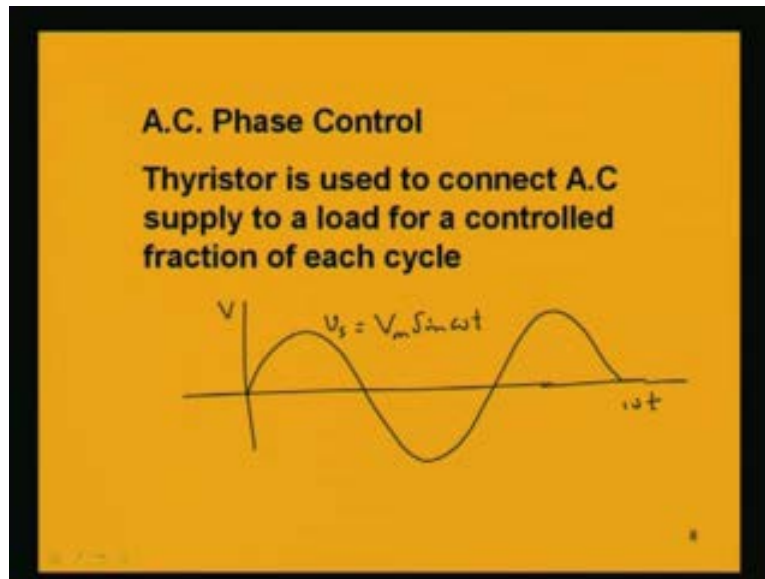
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When we are dealing with power, thyristors are mostly used for applications like rectification, phase control of AC, etc. Here one point to be noted is that we are having a high power. We are dealing with circuits having connected to high power source. So what benefit we are getting by using thyristor? Mostly, the benefit of using thyristor in this circuits which handle high power is that the thyristors can handle high power and even it is more beneficial to use thyristors for power control compared to even controlling by means of power transistors.

It will be even cheaper to use thyristors in those cases of power control where we can use power transistors but that will be mostly from the point of view of economics. We will have cheaper implementation using thyristors in the power control. Let us see how thyristors can be applied for AC power control.

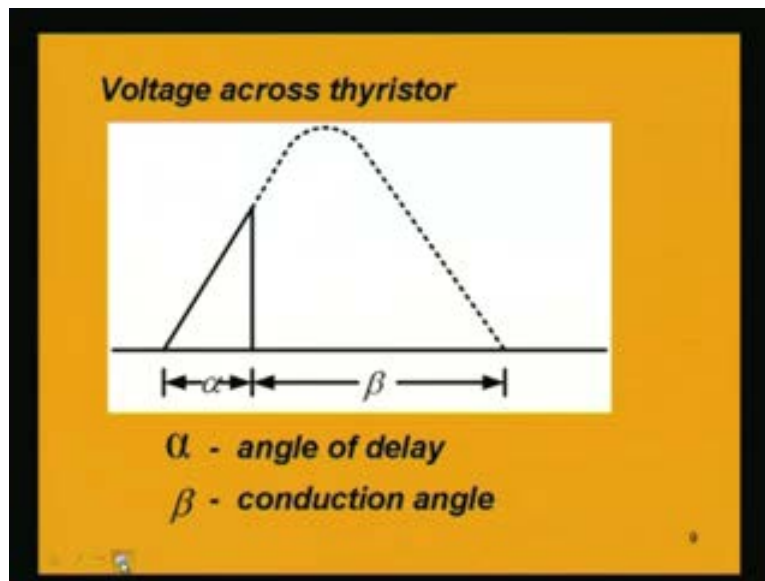
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For example one typical application of thyristor is in AC phase control. The thyristor is used to connect AC supply to a load for a controlled fraction of each cycle. Suppose we are having a load where we are giving an AC supply. But we do not intend to supply the power to the load for the whole input cycle. Only for a fraction of the input cycle suppose we want to feed the power to the load; in those cases we will apply thyristor for this phase control of the AC. For example we are having an AC voltage. Suppose this is a sinusoidal AC voltage we are giving to a load, but we do not want that the power supply to the load to be continuous for the whole input cycle.

Suppose we are applying a v_s supply voltage which is AC which is given by $V_m \sin \omega t$. We are taking typically an example of a sinusoidal source. If we do not want for the whole input cycle, the load should get the AC power then we can control this supply of AC power by means of a thyristor.

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That is the application of thyristor for power control. Here suppose we want to apply the power to the load for only one portion of the whole cycle. Then what is done? By principle how it can be achieved? We will use a thyristor because we can precisely fire the thyristor at a particular angle of the whole input cycle by properly choosing the triggering voltage. The thyristor can be fired precisely at one point. Suppose for example we want the thyristor to fire at an angle of alpha. This angle of alpha is the angle at which we will turn the thyristor ON by using the triggering signal.

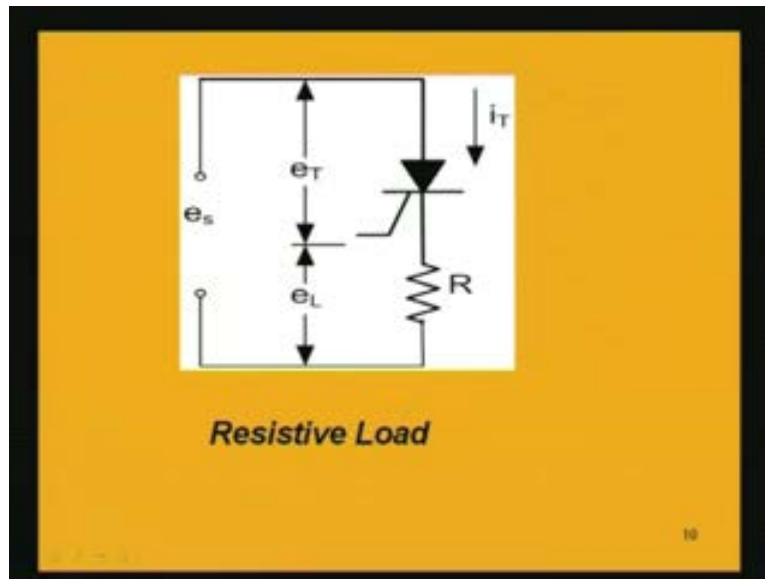
What will happen is that at this angle alpha, the thyristor will conduct; start conducting from this angle alpha which is known as angle of delay and it will be conducting after this alpha angle. Let us take this conduction period or conduction angle as beta. We have the thyristor conduction taking place from this point to this point that is for the conduction angle beta. At the point of angle alpha that means at this angle it is say zero. Here at this point alpha means after angle of alpha that is the phase angle we are firing the thyristor. The thyristor is turned ON. Then after this point it will conduct till the value of or the angle of beta.

What will be the voltage of the thyristor? It is zero although the voltage after turning ON is a very small voltage, not zero to be precise and normally it is greater than the diode ON voltage. Normal diode has the voltage after conducting around 0.7 volt for silicon. But for a thyristor that voltage after conducting or after turning ON, the conduction voltage is greater than normal ordinary diode. But for all practical purposes we will consider this voltage to be zero. Because we are dealing with power circuits, power control, the voltages which we will handle is quite large. In comparison with those voltages this small voltage of the thyristor after conduction can be very well assumed to be zero.

As soon as the conduction takes place, the firing occurs in the thyristor, the voltage becomes zero. Before the thyristor getting fired, the voltage will be equal to the input voltage to the thyristor. Because there is no conduction, no current is flowing in the thyristor. So, the non conducting thyristor will have a voltage equal to the input voltage. So, in this portion you see the voltage is equal to the input voltage. After that point of firing, the thyristor voltage becomes zero. This is the principle which will be applied for power control.

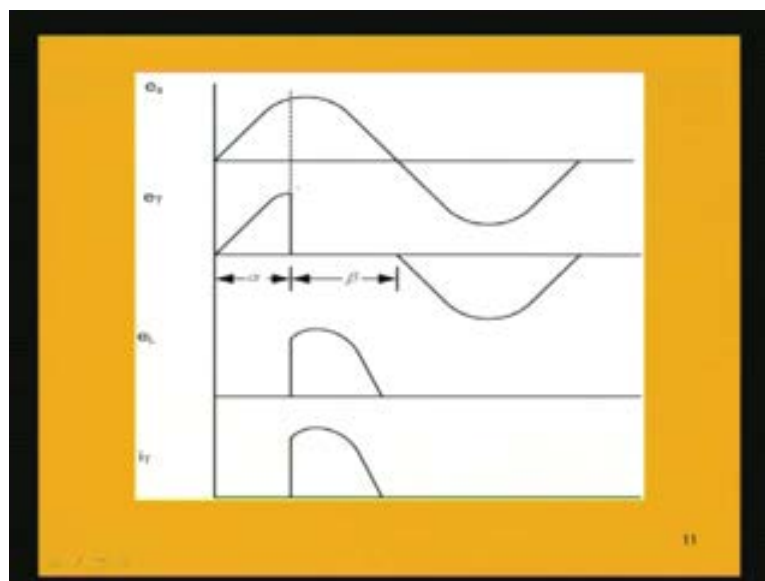
What we are discussing is the AC phase control. We want to supply an AC power to a load; not for the whole input cycle of the input voltage but for a limited portion. Typically one circuit demonstrating the phase control using thyristor is shown here and we are taking a resistive load in the beginning for simplicity.

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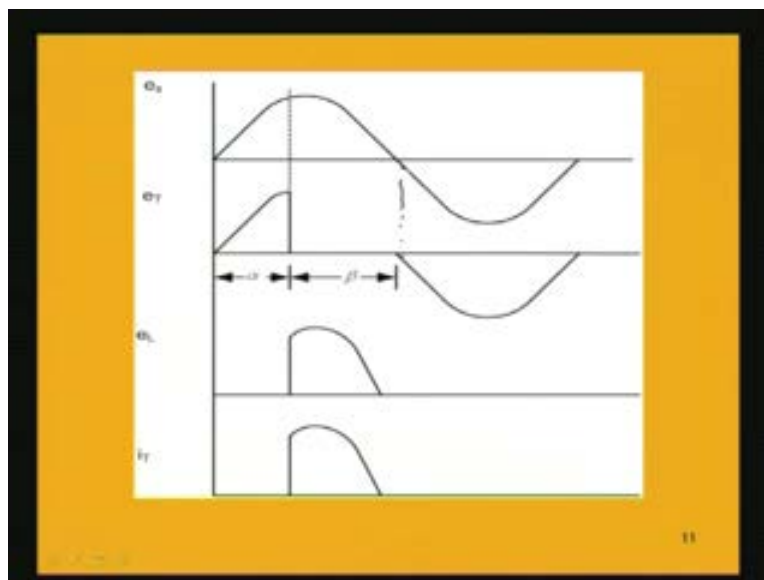
We will understand how the phase control is being achieved using resistive load and then we will also discuss about an inductive load. Now in this circuit we are applying the AC sinusoidal voltage which is e_s and this thyristor is connected to the resistance R and let us take the voltage across this thyristor as e_T and R is the load and the voltage across this load R is say e_L .

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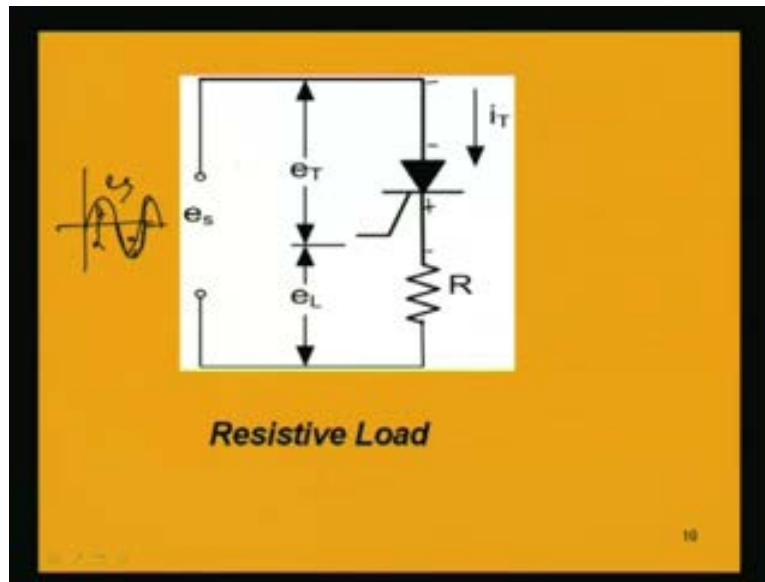
Now let us apply sinusoidal voltage which is e_s . It is a voltage which is say $V \sin \omega t$ type of voltage which is sinusoidal. Now the thyristor is fired at an angle of α . Initially when the thyristor is not fired it is not conducting. The voltage across the thyristor which is e_T will be equal to the voltage e_s . That is why this voltage from this point to this point is exactly same as this voltage. At the angle of α , which is the delay angle the thyristor is fired. When the thyristor is fired then current flows through the thyristor. The thyristor is ON and the voltage across the thyristor is zero. We assume that it is very small voltage and we can very well assume it to be zero. The conducting voltage or the voltage of the turning the thyristor ON is zero. That is why it will drop down to zero.

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It will drop down to zero and be zero till the point when the input voltage becomes negative, because we know that the thyristor is turned OFF if the anode is made negative with respect to the cathode.

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In this circuit we see the voltage e_s is the sinusoidal voltage like this; this is positive half cycle, this is negative half cycle. In the positive half cycle, by means of this triggering voltage at the gate, we are not now discussing the methods of triggering; we are assuming that the triggering voltage is applied to the gate to fire the thyristor at an angle of α ; the delay angle of α is maintained. What method of gate triggering is used is another topic of broad discussion. We are not going to discuss those methods of triggering but simply assuming that is being triggered by suitable method for getting a delay angle of α .

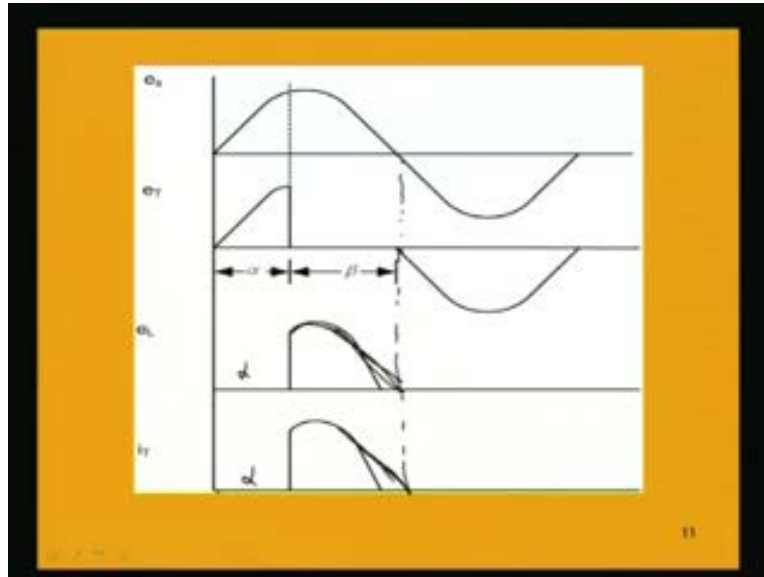
In the positive half cycle, it is triggered at an angle of α . This is angle of α when it is triggered and the moment it is fired or turned ON, the voltage across this thyristor that is e_T , it will become zero. The thyristor will be turned ON and it will continue to be ON, till the point when the input voltage becomes negative. In this negative half cycle, when it is crossing the zero and becoming negative then the anode becomes negative and the cathode becomes positive. There cannot be conduction or the firing of thyristor. It will stop conducting when anode becomes negative with respect to cathode. So, it will stop conducting, current becomes zero and that is why the voltage across this thyristor which is e_T is nothing but this supply voltage when it is not conducting. If we consider

the voltage across these two points, it is nothing but this supply voltage and that is what is shown here.

From this point onwards (Refer Slide Time: 23:15), the voltage across this transistor is equal to the supply voltage e_s and the conduction angle is β . We have already assumed α and β to be the delay angle and the conduction angle. If we look into the voltage across the load resistance R , the voltage across the load resistance, as you can see what will be this voltage? That voltage will be equal to the current into the resistance. The current which is flowing in this resistance will be following the input voltage shape because it is a resistive load.

If we find out this e_L , the voltage across this load we can see here that till firing occurs, till the thyristor is turned ON, the voltage across the load is zero because there is no current flow in the resistor. So, the voltage will be zero from this point till the point of firing and as soon as the thyristor starts conduction, current flows in the resistor. So, the voltage across the resistor is nothing but the supply voltage because the voltage across this thyristor is zero. When it starts conducting the voltage becomes zero. The voltage across the resistance and the supply voltage are same. That is why the shape is exactly same and magnitude is same. That means it is following the supply voltage only. Till when? This will be zero only after this point.

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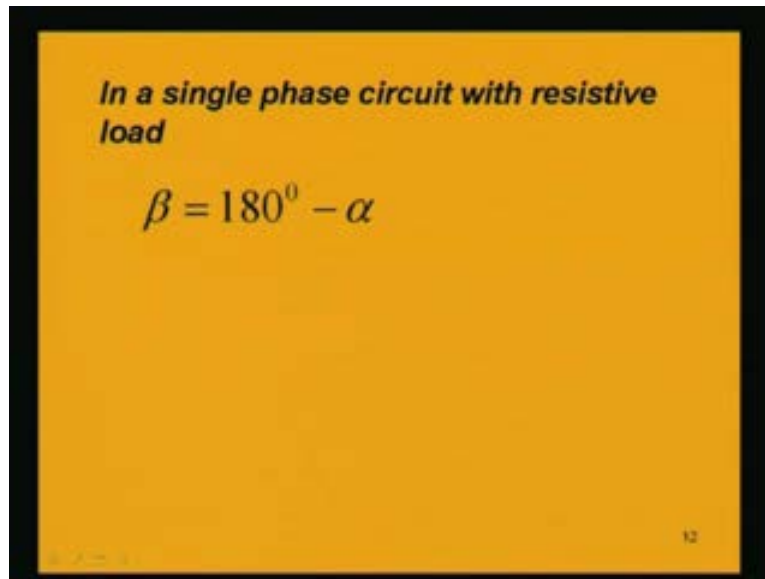


From this point again the thyristor stops conducting. So, the voltage across the load will be zero. Similarly if we consider the current in the thyristor this current i_T (Refer Slide Time: 25:50), which is flowing through the thyristor as well as the load, the current will follow the voltage signal, because it is a resistive load. So, only the magnitude will be less but the shape will be exactly same as the voltage across the resistance. e_L by R will be the current. That current is flowing through the circuit as well as the thyristor. This is the voltage e_L and this will be the current. Till the firing of the thyristor takes place, current becomes zero; current is zero. No conduction is taking place till alpha and after alpha the conduction starts; current starts flowing. The voltage across the resistance is of this shape, so the current will also be this shape only because this is a purely resistive load.

In this figure we have seen that the voltage and current across the load is present only for this portion in the whole half cycle and in the other half cycle it is totally zero. If we consider the input waveform of e_s we are getting the output waveform for only a small fraction or only a portion of the input wave. We are basically giving a controlled power. We are controlling the power input to the resistance because voltage and current these are controlled. We are not allowing the whole input voltage to be supplied to the load. We are only applying a small portion as well as the current is also present for a small portion.

The power which is voltage into current will be also only present for a portion of the whole input cycle.

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In a single phase circuit with resistive load

$$\beta = 180^0 - \alpha$$

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As we are using a resistive load it is very obvious in the figure (Refer Slide Time: 28:14) that beta equal to 180 degree minus alpha. That is the non conducting angle which is alpha is 180 degree minus beta or the conducting angle beta is equal to 180 degree minus alpha. Alpha plus beta is equal to pi. That is because of the fact that we are using a resistive load. But if we do not have a purely resistive load, suppose we are having a load having resistance and inductance in series.

If we are having an inductive load or a complex load then this will not be the case, because in inductive load the current will not immediately stop to become zero as soon as the thyristor stops firing. Because of the inductance the current will still be flowing for some more time and this effect will cause that beta is not exactly 180 degree minus alpha. The non conducting and the conducting angles if we consider as we are getting in this resistive load that beta is equal to 180 degree minus alpha that will be not true for complex loads.

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A.C / A.C Converter

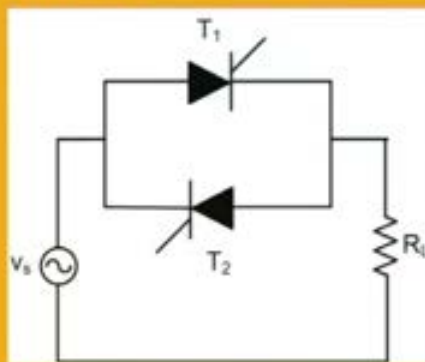
Phase control technique is used to control A.C output power fed to an A.C load

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The AC to AC converter is a device where the phase control technique is used to control the AC output power fed to an AC load. We are considering a load which is getting an AC power. The AC to AC converter using thyristor does the controlling of the AC output power to an AC load. How it does so?

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Single phase A.C / A.C Converter with resistive load



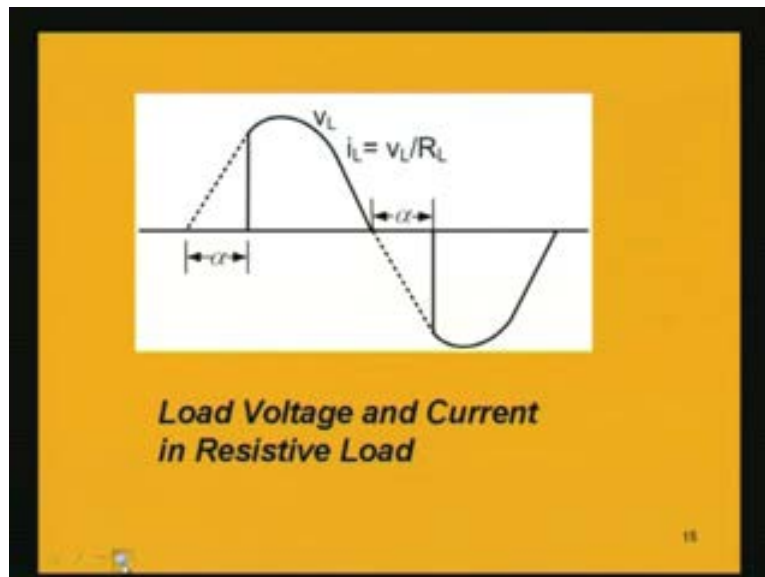
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To understand that let us consider a circuit having two thyristors T_1 and T_2 back to back like this, as is shown in figure. We are considering single phase AC to AC converter. We

are considering a single phase AC to AC converter means we are getting an AC supply. We are supplying AC only to the load. If we recall, a rectifier is AC to DC. We get AC power from a supply, but what we get at the output is DC. But here this is dealing with AC only in the input and output sides both. But it is doing so in a controlled manner. It is an AC to AC converter where we are allowing the AC input to be fed into the load in a controlled manner.

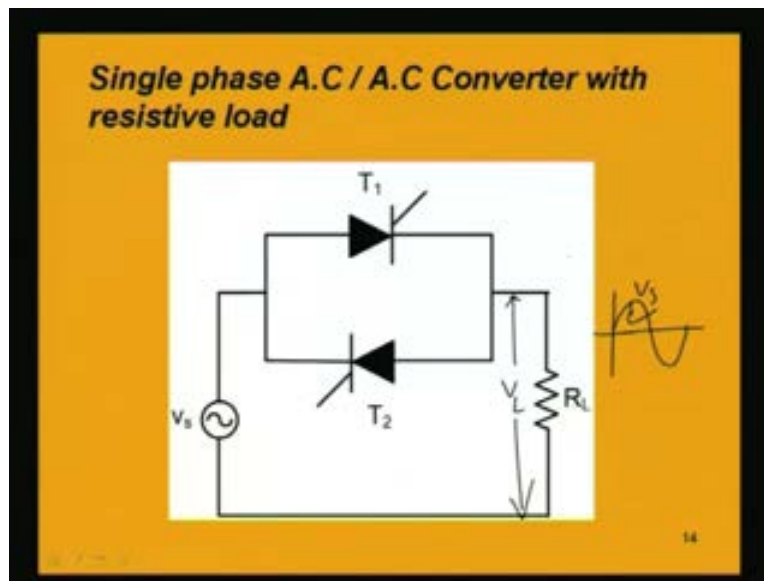
We are not allowing for the whole AC power in the whole input cycle to be fed but we will control it so that the load gets the supply which is an AC supply only for a certain period in the period of the input cycle. This circuit is having a resistive load R_L . We are first of all considering resistive load and we are applying a sinusoidal signal V_s having two thyristors T_1 and T_2 . As shown in figure they are connected.

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This is an AC source, input voltage is sinusoidal. What will happen in the positive half cycle of the signal? In the positive half cycle of the signal this T_1 will have anode positive with respect to the cathode and the lower thyristor T_2 will have the cathode positive with respect to the anode.

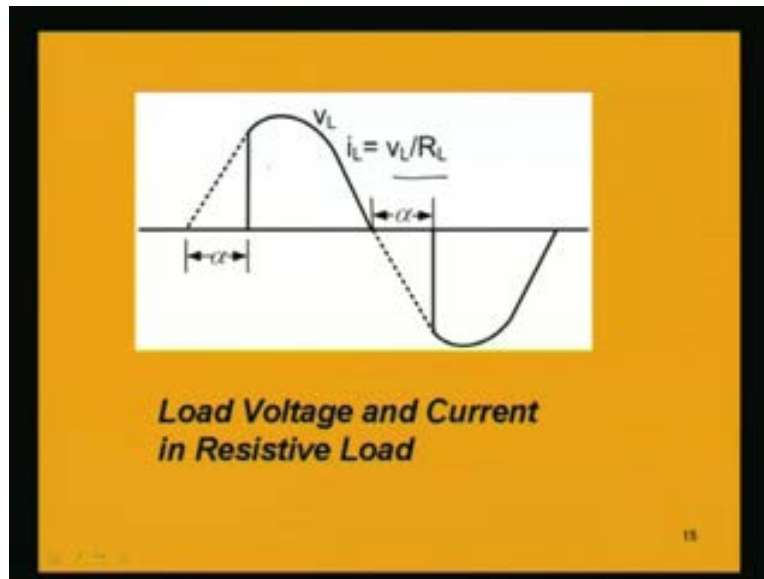
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If we consider this half cycle, positive half cycle, this one, then in this positive half cycle T_1 has anode positive with respect to the cathode. But here the cathode is positive. The thyristor T_2 will not be able to conduct. It will be turned OFF. The conduction can take place only in the thyristor T_1 . But let us assume that a firing angle or the delay angle of alpha is maintained. The thyristor T_1 will fire with delay angle of say alpha and also one point to be noted here which is to be remembered is that the thyristor T_1 and T_2 are identical.

We must use identical thyristors so that the output waveform or output voltage which you get across this load resistance should have symmetrical shape. So, T_1 and T_2 must be identical and what will happen in the positive half cycle? At an angle of alpha the thyristor T_1 will be turned ON and when it turns ON, the voltage across it will be zero. What will be the voltage across this R_L till the firing occurs. Till the firing of this T_1 happens the voltage will be zero because there will be no current flow in the resistance R_L . Till the firing takes place the voltage across this resistance is say V_L . This voltage let us name it as V_L , the voltage across the load. So, that load voltage is zero till alpha in the positive half cycle.

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Till this point, the thyristor T_1 is not firing and there is no question of firing in the thyristor T_2 also as this is reverse biased. Cathode is positive where anode should be positive and after this conduction of this thyristor T_1 , the thyristor will conduct. When the thyristor conducts there will be current flow in the resistance R_L . The voltage across this resistance R_L which is V_L will be nothing but the supply voltage, since the conducting voltage or the voltage of the thyristor after conduction or when conduction is going on is zero. That we have been assuming because the ON voltage when the thyristor is ON that voltage across the thyristor is very, very small. It is almost equal to zero. So, the voltage V_L is equal to nothing but V_s as this drop is zero. We will get the output voltage across this load which is V_L to follow the voltage V_s . This voltage V_s whatever is given it will be the same voltage.

This will continue till the input voltage which is a sinusoidal voltage crosses into the negative half cycle. It crosses zero and becomes negative. The lower one thyristor will be properly biased because now in the negative half cycle (Refer Slide Time: 37:11) what will happen is that the cathode of this T_2 will be negative and anode will be positive; so, upper one that is thyristor T_1 will be now OFF. It will be turned OFF because of this biasing being reversed. But the thyristor T_2 will be now properly biased and using a

proper gate pulse and a proper choice of the varying voltage we can now fire the thyristor T_2 identically at an angle of α as in the upper one thyristor.

In the negative half cycle at an angle of α it will start to conduct and then current will flow in the resistance R_L and the voltage across this resistance R_L which is V_L will be same as the supply voltage because this drop in the T_2 is zero. That means in the negative half cycle if we look into, till α that means till firing of the second thyristor, the lower one thyristor, the voltage was zero till this point and then the voltage will follow the input voltage. That means the voltage will be the negative half cycle voltage. So, this will be the load voltage or voltage across the load.

If we consider the current through the load as this is a purely resistive load, the current will be simply V_L by R_L . So, the waveform of the current will be similar to the waveform of the voltage. Only the magnitude will be different. But the shape will be exactly same. We get the current through the load similar to the voltage only that magnitude will be a little less because of the division by R_L . Depending upon R_L , it will have that magnitude. This figure is showing the load voltage and current in the resistive load.

(Refer Slide Time: 39:47)

Sinusoidal A.C. input

$$v_s = V_m \sin \theta$$

$$= \sqrt{2} V_s \sin \theta$$

Thyristor mean current

$$I_T(AV) = \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m}{R_L} \sin \theta \, d\theta = \frac{V_m}{2\pi R_L} [-\cos \theta]_{\alpha}^{\pi}$$

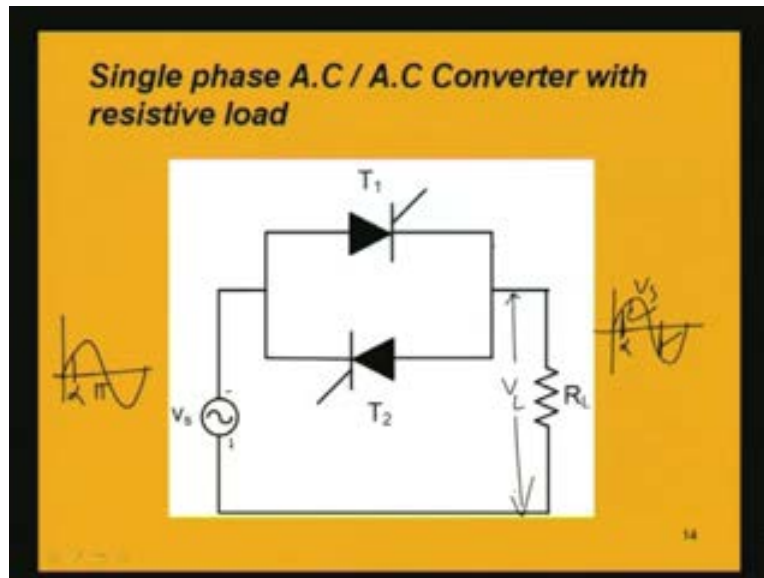
$$= \frac{V_m}{2\pi R_L} (1 + \cos \alpha)$$

Handwritten notes:
 V_m - Peak Value of Input voltage
 $= \sqrt{2} V_{(rms)}$
 $= \sqrt{2} V_s$

If we want to find out what will be the values of the voltage, current, etc., we have to now mathematically proceed and we have to use the expression for the input voltage which is the sinusoidal voltage which is expressed by $V_m \sin \theta$. θ is equal to ωt . So the sinusoidal AC input, which is V_s is equal to $V_m \sin \theta$ where V_m is the peak value or maximum value of the voltage and ωt equal to θ . Again the maximum value or peak value of the voltage is equal to $\sqrt{2}$ into $V_{(rms)}$. The root mean square value if we consider which is say V_s , V_s signifying the root mean square value of the AC voltage which is the average quantity which is used to express an AC quantity and $\sqrt{2}$ times that RMS value will give you the peak value. In another way we can express it as $\sqrt{2}$ into V_s into $\sin \theta$.

If we want to find out the mean current of the thyristor you can see that the thyristor is firing alternately.

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In the whole input waveform cycle, for half of the period thyristor T_1 is conducting and for the other half thyristor T_2 conducts not for the exact half but less than one half. It is firing at an angle of α ; so, basically the thyristor is ON from α to π . The thyristor T_1 if we consider it is firing at an angle of α and it continues to conduct till π .

(Refer Slide Time: 42:30)

Sinusoidal A.C. input

$$V_s = V_m \sin \theta$$

$$= \sqrt{2} V_s \sin \theta$$

*V_m - Peak Value of input voltage
 $= \sqrt{2} V_{s(rms)}$
 $= \sqrt{2} V_s$*

Thyristor mean current

$$I_T (Av) = \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m}{R_L} \sin \theta \, d\theta = \frac{V_m}{2\pi R_L} [-\cos \theta]_{\alpha}^{\pi}$$

$$= \frac{V_m}{2\pi R_L} (1 + \cos \alpha)$$

$-\left[-1 - \cos \alpha\right] = \left[\cos \theta\right]_{\alpha}^{\pi}$

If we want to find out the mean current for each thyristor, the thyristor current if we want to find out, then we have to find out by integrating $V_m \sin \theta \, d\theta$ by R_L because this

is the current expression. $V_m \sin \theta$ is the voltage which is being applied divided by R_L the load resistance; integrating from α to π and dividing by 2π because in the whole period of 2π , 2π is the whole period of the waveform which is given as input, but one thyristor is conducting. The thyristor T_1 if you consider it is conducting from α to π and the other thyristor which is conducting, is from $\pi + \alpha$ to 2π (Refer Slide Time: 43:33).

You can find out any way; either you can find out the current in thyristor T_1 or you can find out the current in thyristor T_2 , but we will have to integrate properly that is within the proper limits. For the thyristor T_1 will have to take the limits between α to π and for thyristor T_2 we have to take the limit between $\pi + \alpha$ to 2π . Doing this integration what we will get is V_m by $2\pi R_L$. This is the constant part taken outside the integral and integration of $\sin \theta d\theta$ it is equal to $-\cos \theta$ and putting the limits of α to π , we get this expression. Evaluating $\cos \theta$ between α to π what we will get? Minus $\cos \theta$, minus can be taken out, so $\cos \theta$ between α to π ; $\cos \pi$ is minus 1, minus $\cos \alpha$. So, minus can be taken out. What we will get finally is $1 + \cos \alpha$. So, this is the expression for the average current in a thyristor which is V_m by $2\pi R_L$ into $1 + \cos \alpha$.

(Refer Slide Time: 45:16)

$$\begin{aligned}
 I_T^2(RMS) &= \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m^2}{R_L^2} \sin^2 \theta d\theta \\
 &= \frac{V_m^2}{2\pi R_L^2} \int_{\alpha}^{\pi} \left(\frac{1 + \cos 2\theta}{2} \right) d\theta \\
 &= \frac{V_m^2}{2\pi R_L^2} \left(\frac{\pi}{2} - \frac{\alpha}{2} + \frac{\sin 2\alpha}{4} \right) \\
 I_T(RMS) &= \frac{V_m}{2R_L} \left(\frac{\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi} \right)^{\frac{1}{2}}
 \end{aligned}$$

This is the average current and if we want to find out the root mean square current I_T RMS square, let us first find out the square and then you can find out the square root to get $I_T(RMS)$. Our aim is to find out $I_T(RMS)$. You know that the root mean square value can be found out by under root 1 by 2 pi into integration alpha to pi by square of this I, whatever we have found out here. This integration we can find out the RMS. Basically the $I_T(RMS)$ if we find out that will be under root 1 by the whole period. Period is 1 by 2, integration within the same limit alpha to pi. The square is of I_T mean square; which is actually this is we have to find out the square of this term which is this one (Refer Slide Time: 46:46) and integrate between alpha to pi; this is I square d theta.

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The image shows a handwritten derivation of the RMS current formula on a yellow background. The equations are as follows:

$$I^2_{T(RMS)} = \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m^2}{R_L^2} \sin^2 \theta d\theta$$

$$= \frac{V_m^2}{2\pi R_L^2} \int_{\alpha}^{\pi} \left(\frac{1 + \cos 2\theta}{2} \right) d\theta$$

$$= \frac{V_m^2}{2\pi R_L^2} \left(\frac{\pi}{2} - \frac{\alpha}{2} + \frac{\sin 2\alpha}{4} \right)$$

$$I_T(RMS) = \frac{V_m}{2R_L} \left(\frac{\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi} \right)^{\frac{1}{2}}$$

There is a handwritten note on the left side: $\frac{1}{2\pi} \int_{\alpha}^{\pi} \sin^2 \theta d\theta$ with an arrow pointing to the integral in the first equation.

Let us first find out the square of this term. That means free this root and then we will find the root. To proceed in an easy manner, we are doing that; so, basically we are removing the root first. So, that means only the square value we are taking 1 by 2 pi. Integration from alpha to pi I square or the I average square whatever we have got is V_m square by R_L square sin square theta d theta. Doing this integration will lead to, first of all taking the constant parts outside, it will be V_m square by 2 pi R_L square; now integration alpha to pi sin square theta d theta.

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$$\begin{aligned}
 I_T^2(RMS) &= \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m^2}{R_L^2} \sin^2 \theta d\theta \\
 &= \frac{V_m^2}{2\pi R_L^2} \int_{\alpha}^{\pi} \left(\frac{1 - \cos 2\theta}{2} \right) d\theta \\
 &= \frac{V_m^2}{2\pi R_L^2} \left(\frac{\pi}{2} - \frac{\alpha}{2} + \frac{\sin 2\alpha}{4} \right) \\
 I_T(RMS) &= \frac{V_m}{2R_L} \sqrt{\frac{\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi}}
 \end{aligned}$$

Handwritten notes on the left side of the derivation include:

- $\frac{\pi}{2} \text{ (from } \int \sin^2 \theta d\theta)$
- $\int \sin^2 \theta d\theta = \int \frac{1 - \cos 2\theta}{2} d\theta$
- $\int \sin^2 \theta d\theta = \frac{\theta}{2} - \frac{\sin 2\theta}{4}$
- $\frac{d}{d\theta} \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) = \frac{1}{2} - \frac{\cos 2\theta}{2} = \frac{1 - \cos 2\theta}{2} = \sin^2 \theta$

So, integration sin square theta d theta between alpha to pi that we have to do. For that we will use the technique of cos twice theta equal to 1 minus twice sin square theta. From that we can find out what is sin square theta. Sin square theta is equal to 1 plus cos twice theta by 2. If we use this sin square theta and represent this by 1 plus cos twice theta by 2, then we can now easily find out the integration. What will be this value? You take the integration of d theta which will give you theta. So, theta; then half will be there plus integration cos twice theta d theta. So, integration cos twice theta d theta will be sin twice theta by 2 and within the limit alpha to pi. If we now replace the values of the integration limit pi divided by 2 plus sin 2 pi by 2, sin 2 pi is equal to zero; so that we will remove; minus alpha by 2 minus sin of 2 alpha by 2. If we consider this we will get pi plus sin 2 pi by 2 minus alpha minus sin 2 alpha by 2. What we will get? I think there is a mistake in plus and minus sign because we have sin square theta d theta. If we do this sin square theta d theta what we will get? Sin square theta integration and we know that sin twice theta is equal to twice sin theta cos theta and cos twice theta is equal to 1 minus twice sin square theta.

So, twice sin square theta will be 1 minus cos twice theta; it will be minus I think. If it is minus then it will be integration cos twice theta. The differentiation of sin theta is equal to cos theta. So, cos twice theta is equal to d theta of sin twice theta, which is equal

to cos twice theta. Here integration of cos twice theta will be simply sin theta. So, this is minus sign and this will be plus. Because of this integration, sign of minus is there; finally we will get this result. You have to check this result. Just see this plus and minus sign. Finally what we will get? This is correct pi minus alpha plus sign twice alpha by 2 by pi. From limit alpha to pi when I put, I will get this result but this is I square. If I want to find out the root mean square, it will be square rooting of this figure. It will be V_m by 2 R_L . I can be taken out inside and we will take the root of this I as well as root of this whole thing. Actually what is done? This 2 is taken out, it becomes 4; so its root will be 2. So, it will get rid of this 2 and here it will be kept only 2. Finally this is the root mean square.

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R.M.S Output Voltage

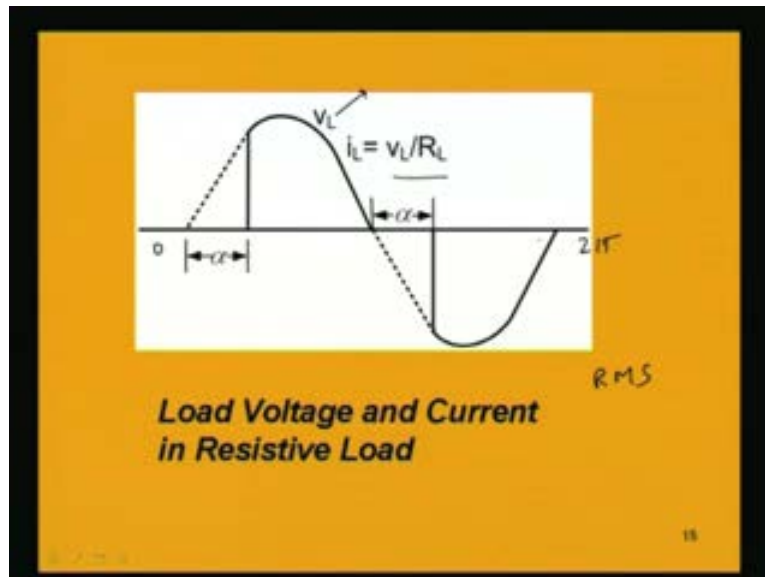
$$V_o^2(RMS) = \frac{1}{\pi} \int_{\alpha}^{\pi} I_m^2 \sin^2 \theta d\theta$$

$$V_o(RMS) = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} I_m^2 \sin^2 \theta d\theta} = \sqrt{\frac{I_m^2}{\pi} \left(\frac{\pi}{2} - \frac{\alpha}{2} + \frac{\sin 2\alpha}{4} \right)}$$

$$= \frac{I_m}{\sqrt{2}} \left(\frac{\pi - \alpha + \frac{1}{2} \sin 2\alpha}{\pi} \right)^{\frac{1}{2}} = I_s \left(\frac{\pi - \alpha + \frac{1}{2} \sin 2\alpha}{\pi} \right)^{\frac{1}{2}}$$

If we want to find out the RMS output voltage similarly we can proceed and we have the voltage $V_m \sin \theta$ is the input voltage and the output voltage which we will get is following the input voltage only. So, that sinusoidal voltage only but we will have to take into note that its value when we take the limit is alpha to pi but we are finding out the RMS value. The integration alpha to pi $V_m^2 \sin^2 \theta d\theta$ but it will be 1 by pi, not 1 by 2 pi because here we can see that we are finding out the RMS value of the output voltage.


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RMS value of the output voltage when I find out output voltage is there or available for whole period from zero to 2π . There are in between non-conducting periods from zero to α but we are finding out the RMS value or root mean square value. For the whole period we will have to divide it by 1 by π here because we are integrating between α to π for one half cycle.

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R.M.S Output Voltage



$$V_o^2(RMS) = \frac{1}{\pi} \int_{\alpha}^{\pi} I_m^2 \sin^2 \theta d\theta$$

$$V_o(RMS) = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} I_m^2 \sin^2 \theta d\theta} = \sqrt{\frac{I_m^2}{\pi} \left(\frac{\pi - \alpha}{2} + \frac{\sin 2\alpha}{4} \right)}$$

$$= \frac{I_m}{\sqrt{2}} \left(\frac{\pi - \alpha + \frac{1}{2} \sin 2\alpha}{\pi} \right)^{\frac{1}{2}} = I_o \left(\frac{\pi - \alpha + \frac{1}{2} \sin 2\alpha}{\pi} \right)^{\frac{1}{2}}$$

But the consideration of the whole period is to be done because the root mean square value when we find out, we are finding out the root mean square value of the output voltage V_L for this whole cycle between zero and 2π and out of this whole cycle from zero to 2π we are finding out from α to π because from zero to α there is no conduction. The voltage will be zero. Only the voltage will be here and again here.

When finding out the root mean square value, we will use the consideration that we are finding out from α to π and the integration of V_m square sin square θ because here it is same shape of $V_m \sin \theta$. This is the voltage. So we will get V_m square sin square $\theta d\theta$ and multiplied by 1 by π . This is the V_o square RMS and in order to find out the root mean square we have to take the square root, so, square root of this whole thing. By replacing the value V_m square by π by 2 minus α by 2 plus $\sin 2\alpha$ by 4 , it will give you V_m by root 2 because 2 can be taken out and inside this root π minus α minus $\sin 2\alpha$ by 2 this π can be taken in. This will be the result of this RMS value of the output voltage.

If we now replace that V_m by root 2 is the root mean square value of the voltage which is nothing but V_s , we can represent V_m by root 2 by V_s and the rest of the quantity is same. This is the representation of the RMS value of the output voltage.

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R.M.S Output Current

$$I_s(RMS) = \frac{V_s(RMS)}{R_L} = \frac{V_s}{R_L} \left(\frac{\pi - \alpha + \frac{1}{2} \sin 2\alpha}{\pi} \right)^{\frac{1}{2}}$$

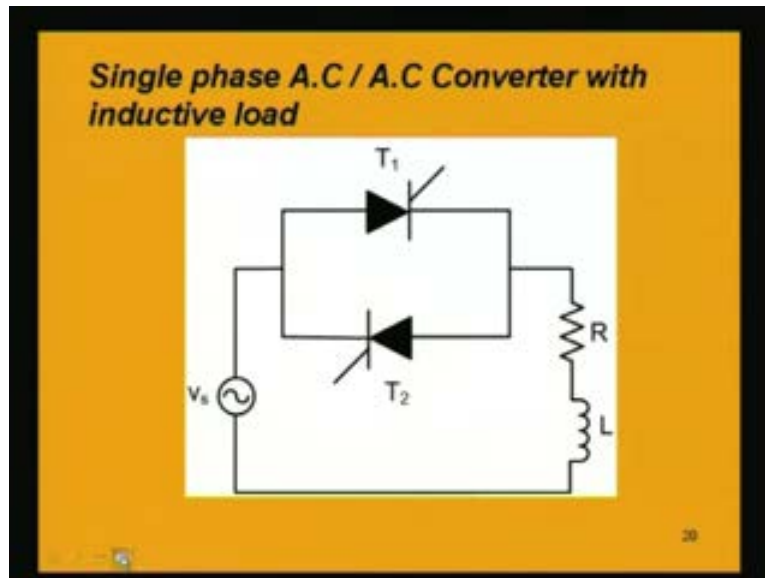
R.M.S Output Power

$$= \frac{V_s^2}{R_L} \left(\frac{\pi - \alpha + \frac{1}{2} \sin 2\alpha}{\pi} \right)$$

$= I_{rms}^2 R_L$

The RMS value of the output current can be found out simply by dividing the RMS value of the output voltage by R_L . That is what is done, V_s by R_L into this other part. If we are interested in finding out the RMS value of the output power, output power if we want to find out $I^2 R$ is the output power. So I_{rms}^2 into R_L this is the output power. Replacing this I_{rms} by this value which we have just now seen multiplied by R_L so if we square this value, this half will go. V_s^2 square will be there R_L^2 square into R_L will be simply R_L in the denominator. This term gives the value of the output power in RMS.

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Here using a resistive load, we have seen that the output across the resistive load is following the input voltage and there is conduction taking place for a controlled period and by that we are basically controlling the power fed to the load. We are not giving the input power to the load for the whole time or the whole period. But we are only supplying the input power to the load for a certain portion of the time and that is basically used for those applications where we are interested only in using the power for a certain period of time, not the whole period. That is going to reduce the power wastage also. Where we do not want that the power should be consumed through out the whole period but we want that it should be only having the power only for a certain period, in that case we will use this converter, AC to AC converter.

One thing is important here. Till now we are only considering resistive or pure resistive load. If we have pure resistive load things are easy or simple because the current which flows through the load immediately stops to zero as soon as the thyristor does not conduct. As soon as the thyristor is turned OFF, the current through the resistive load is zero. But we will find in the next classes that this is not true if we have a load which is not purely resistive, which is an inductive part, suppose. If we are dealing with a complex load having resistance and inductance both then we will find that this condition is not going to happen. That is the current through the load will not immediately become zero

as soon as the thyristor is turned OFF. Because of this inductor being present, inductance is there. We know there will be a flow of current even beyond the non conducting moment. So, even if the thyristor stops to conduct then also there will be current flow for a certain time depending on the time constant of the circuit and because of which we will have certain problems in the output voltage; because output voltage also will vary accordingly. As we know that the output voltage across the load will be the product of the current and the impedance, if current is there the output voltage will also be present even when the thyristor is not conducting and we will try to get rid of these methods by using some techniques; so, that we will discuss in the next class.