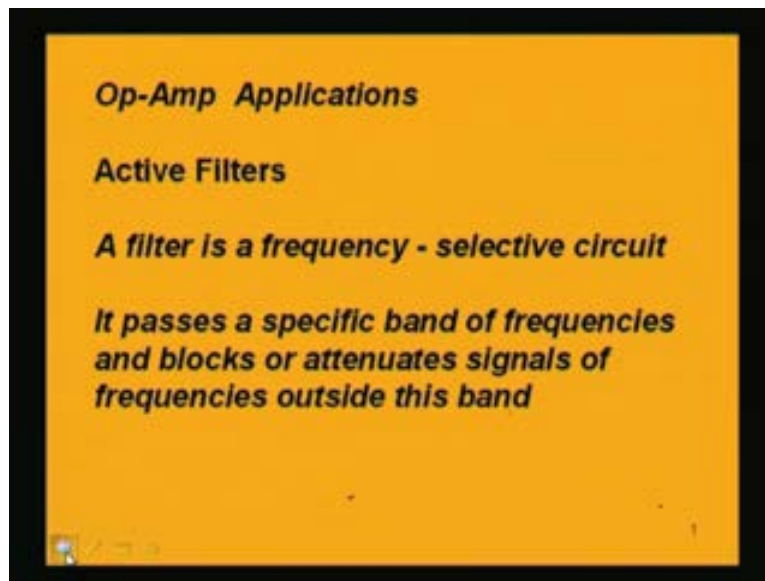


Basic Electronics
Dr. Chitrlekha Mahanta
Department of Electrical Communication Engineering
Indian Institute of Technology, Guwahati

Module - 4
Operation Amplifier (Op-Amp)
Lecture - 5
Op-Amp Applications - Part 3

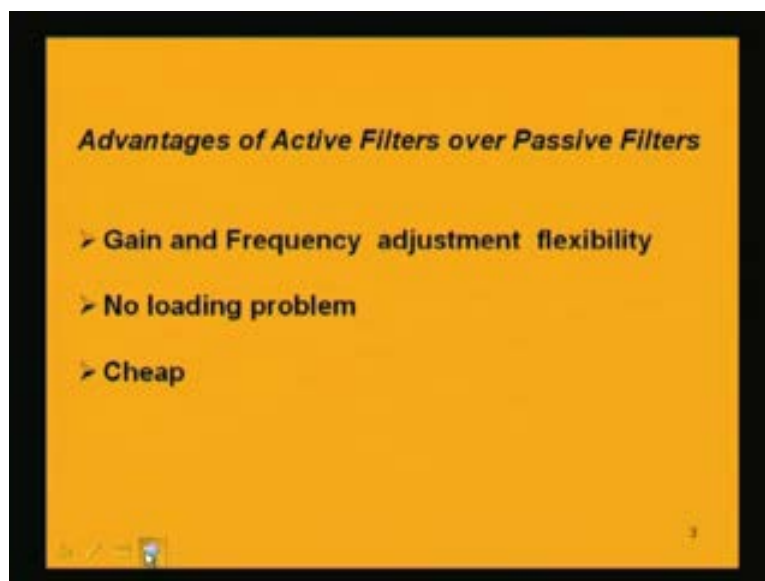
In the last classes we have discussed about the application of op-amp and the various applications we discussed were those of inverting, non-inverting amplifiers, integrators, differentiators and also control sources, logarithmic amplifiers, exponential amplifiers, etc. Today we shall discuss about another important application of op-amp which is active filters. What is a filter basically is that we have to know first. Filter is a frequency selective device or it is a circuit which is used for selecting a particular band of frequencies and the filter can be realized with passive as well as active components. For example we have filters which are made of passive components like resistors, capacitors, etc and we also have filters which are made from a combination of active components like transistors, op-amps also with passive components like resistors, capacitors, etc. In active filters, apart from the resistors, capacitors, etc, we will have active devices like op-amp and transistors. The active filter is the topic of our discussion today.

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The filter is a circuit which passes a specific band of frequencies and it blocks or attenuates signals of frequencies outside this band. So selectively it passes a particular band of frequencies. Depending upon which band of frequencies it allows and blocks the other band of frequencies we have different types of filters, like we have low-pass filter, we have high-pass filter and also band-pass filters.

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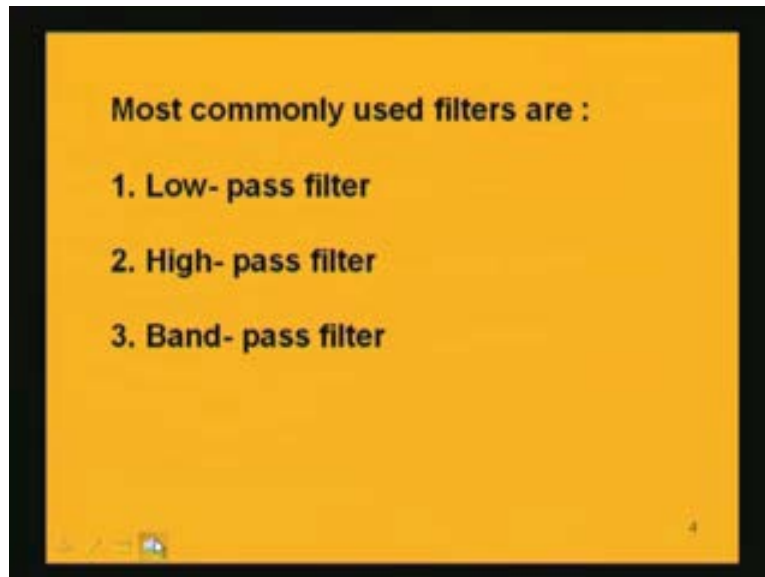


Why we use active filters is because we have certain advantages which we get in using active filters. The advantages which active filters have over passive filters are: first is that, there is gain and frequency adjustment possibility in active filters. In passive filters we have attenuation of the signal which we are giving as input. We selectively allow a specific band but in the process we will have attenuation. But in active filters we can achieve gain. Because we are using an op-amp, it amplifies the signal at the input. So we have gain in op-amp. The overall attenuation of the input signal can be overcome by using active filters. Another advantage is the flexibility in frequency adjustment, we can adjust the frequency easily in op-amp and we will find how over a band of frequencies or over a range of frequencies the response of the op-amp can vary. The gain and frequency adjustment is very flexible which we achieve in active filter and in passive filter this is not so far easily possible as we get in active filter. That is one advantage.

Secondly one great advantage that we achieve using active filter is that the loading problem is overcome. Loading problem comes when you use a device with a source and a load and there is source loading and loading at the output. This source and load offers loading to the device because of which we have reduced input at the source as well as we have reduced output at the source. Voltage drop occurs from the input signal to the actual device at the source because there is a resistance which is in series and at the load also we have another type of loading effect. Because of the presence of the output resistance, there will be another drop at the output. But while we use active filters, because we are using op-amps in the active filters the input resistance offered by the op-amp is very high and the output resistance is very low in op-amp. The output resistance is very small and input resistance is very high. Because of this fact the source loading and the loading at the output is very much reduced. The loading problem is overcome or reduced to a great extent by using op-amp in active filters. That is one advantage we get and another obvious advantage is that the op-amp is cheap and we have a less costly filter possible by using op-amps. These are the major advantages of active filter over passive filters. The filter which is used using op-amp will have apart from op-amp other resistances, capacitances, etc, also.

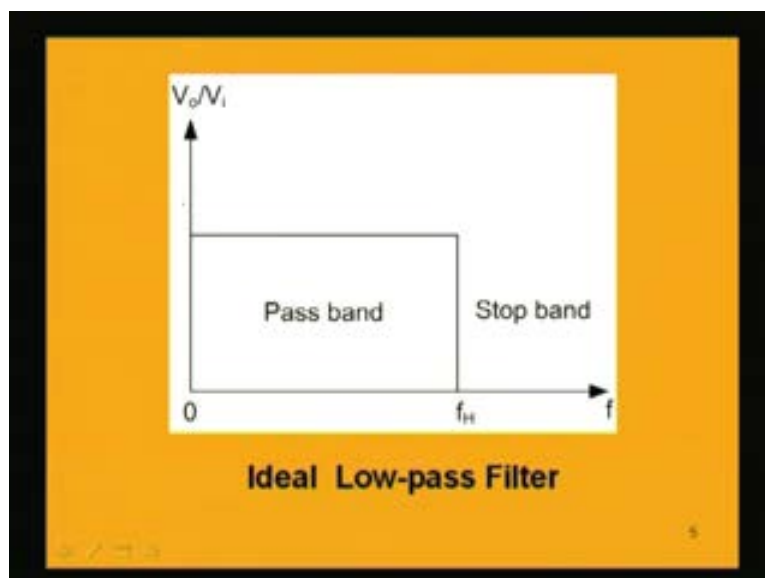
Now we will discuss about the active filter in these three categories: the low-pass, high-pass, as well as the band-pass filters.

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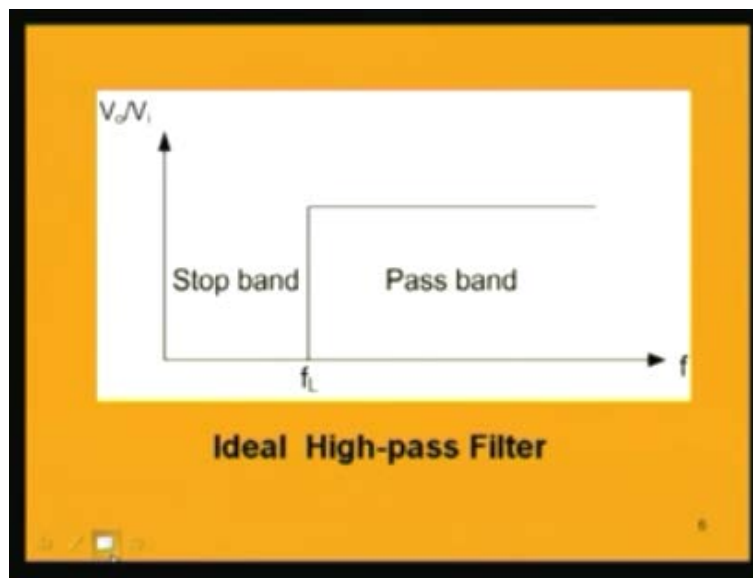
For example, if we take a low-pass filter, in an ideal low-pass filter, the input signal frequencies at the lower range of the band are allowed to pass and it completely stops after a desired cutoff frequency say f_H .

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Ideal low-pass filter should have the characteristic as shown in figure. We are allowing the signal for the lower range of frequencies up to f_H and beyond f_H which is the cutoff frequency, a higher cutoff frequency the signal is totally stopped and this V_O by V_i is the gain basically of this low-pass filter. If we plot the gain versus frequency for an ideal low-pass filter we should get such type of characteristic. The region or the frequency band in which the signal is allowed to pass that is called pass band and beyond that frequency f_H , which is the higher cutoff frequency the signal is totally stopped. During that band of frequencies beyond f_H that is called a stop band frequencies. This is the ideal low-pass filter. If we consider a low-pass filter ideally it should have this type of characteristic.

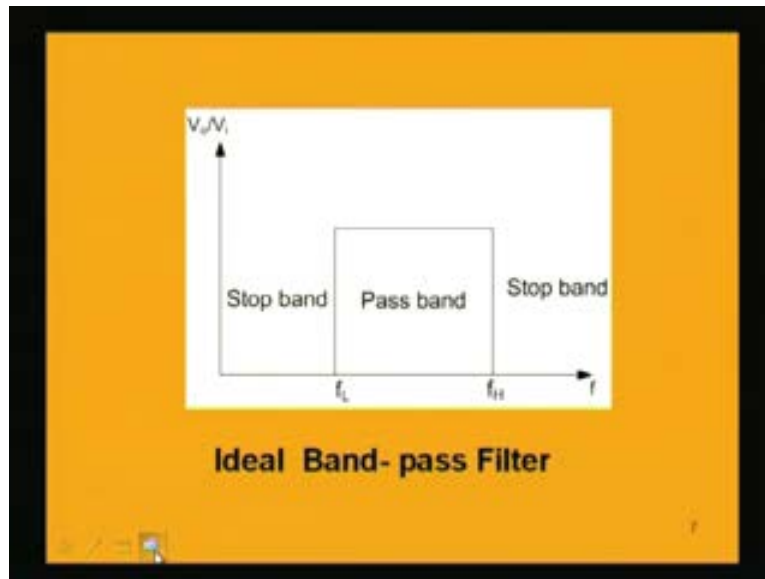
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Next, we consider a high-pass filter. The name suggests that we have the higher band of frequencies which will be allowed and the lower band of frequencies will be not allowed. That is high-pass. The ideal high-pass filter should have the characteristic as shown here. If we plot the gain of the filter V_O by V_i giving an input signal V_i , what will be the output signal V_O ? The gain between the output and the input voltage if we plot versus frequency we should get a characteristic like this and here for the lower range of frequencies up to f_L , the lower cutoff frequency the signal is totally stopped and beyond f_L the signal is

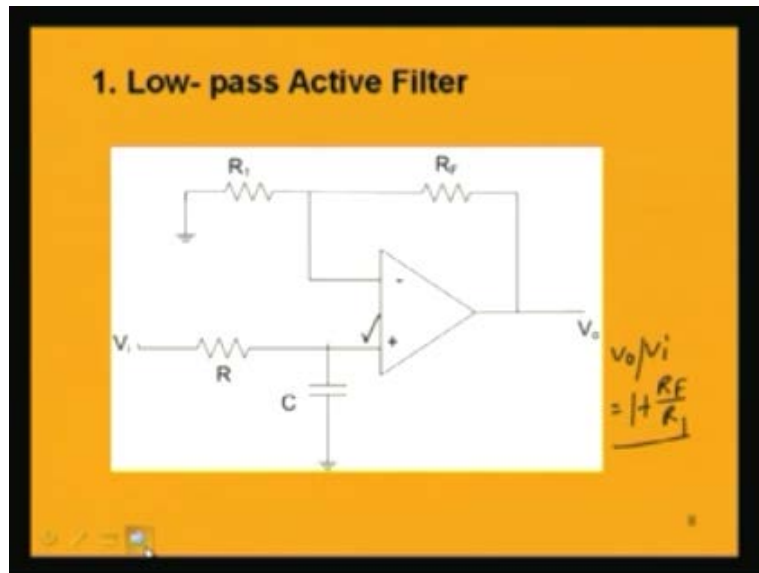
allowed to pass. This band of frequencies from f_L onwards, is called pass band frequencies.

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Similarly if we consider a band-pass filter, ideal band-pass filter will have this characteristic. The plot between the gain, V_o/V_i versus frequency should have such type of behaviour that it allows only a particular band of frequencies to pass. The signal is passed within a particular band which is called pass band. Here between f_L and f_H this band of frequencies is allowed but below f_L , it is totally stopped. The frequencies up to f_L , from zero to f_L that is DC to f_L , these frequencies are called stop band and similarly beyond f_H we have another stop band frequency. Basically this pass band is a combination of the characteristics which we have got in low-pass and high-pass filter. The portion below f_L , this portion is like a high-pass filter and beyond f_H this is like a low-pass filter. They are combined in effect to get a band-pass filter which is ideal band-pass filter.

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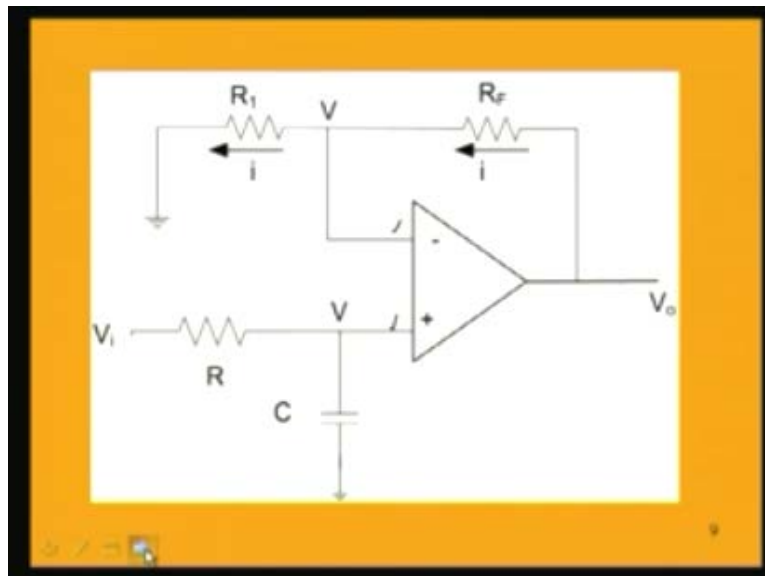


Now let us consider the low-pass active filter. From the characteristics which you have seen for a low-pass filter, we will try to build an active low-pass filter using op-amp and other components. The figure here shows the circuit for a low-pass active filter. This circuit is a familiar circuit which we have earlier seen in the case of the non-inverting amplifier. If we remember non-inverting op-amp then, in that application we were having this type of R_F the feedback resistance, as well as R_I at the inverting terminal of the op-amp and we are applying a signal at the non-inverting end. Here the difference lies in the fact that we are also having a combination of R and C , resistance and capacitance, at the non-inverting terminal. So, the voltage signal which is applied at the non-inverting terminal is V_i . But it is having a circuit which is by R and C . So the signal which is applied at this point is V_i but the signal which will be practically available at the non-inverting terminal is not V_i and the output is V_O .

We analyze this circuit and try to find out what is the output voltage? We will have to find out this output voltage considering it as a non-inverting op-amp, but provided we know what the input voltage at this terminal is? If we can calculate what voltage is available at this non-inverting terminal, then rest of the circuit is similar to the non-inverting op-amp, which has a gain V_O by V_i . We know that it is equal to 1 plus R_F by R_I .

This gain is the gain of a non-inverting amplifier using op-amp. But the V_i , which is here is the V_i at this point. In this circuit which is a low-pass active filter, in order to find out what is the output voltage or what is the gain between output and input let us consider the circuit and apply the ideal op-amp principles.

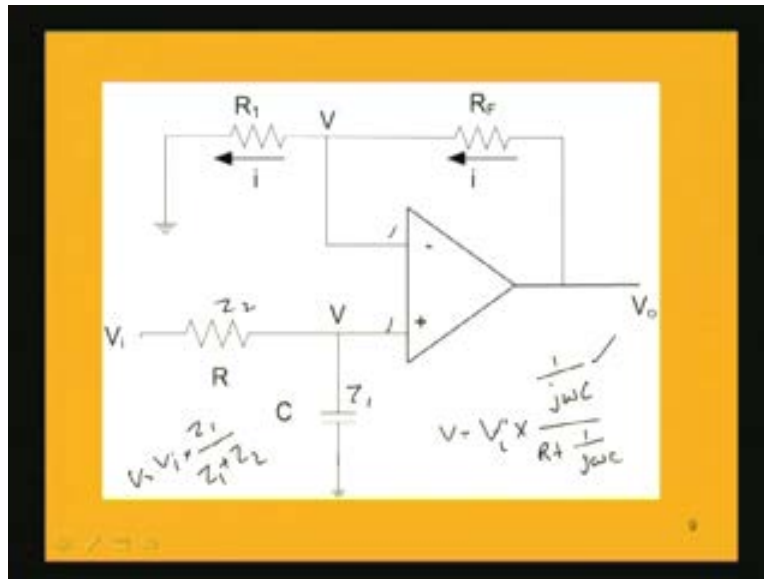
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Let us name the voltage at this non-inverting terminal by V . We are applying the voltage here V_i , but the voltage which is exactly available at the non-inverting terminal let us name it as V and as we know from ideal op-amp characteristics, and we are going to use here also the ideal op-amp property, this voltage here is the same voltage at this point. That means this voltage and this voltage are same and that is why the voltage at the non-inverting terminal V , is equal to the voltage at the inverting terminal V and let the current which flows in R_F be i and this direction is assumed from right to left. The same current will also flow through R_1 because there cannot be any current flow in the op-amp. There cannot be any current entering the op-amp because infinite input resistance in the op-amp. The current has to flow through R_1 only. The same current is flowing so this is the circuit, voltages and currents.

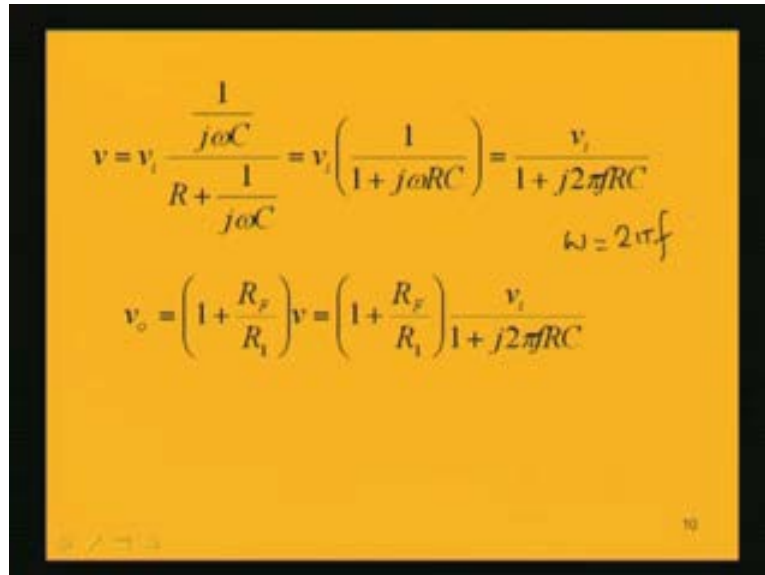
We need to find out what will be the voltage v , this voltage at the non-inverting terminals. This voltage is nothing but which is available across this capacitance. The other point of the capacitance is grounded. The voltage V_i has two divisions of voltage occurring here. One is across this capacitance and the other is across this resistance. This V_i has a voltage division taking place. Now we are interested in finding out the voltage v .

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This voltage is with respect to ground. So this voltage v_i has division taking place in this order that v_i into the impedance across this C divided by this impedance plus this resistance that will give the voltage v . v equal to v_i into and one thing to be noted is that this should be an AC voltage because the capacitance is there. So we are to consider an AC voltage v_i and we are only talking about the filter. It is about the selective passing or allowing of a frequency band. So naturally, we are considering an AC voltage. What is v ? v equal to v_i into the impedance offered by this capacitance and that we know is 1 by J omega C ; 1 by j omega C is this reactance. That will be divided by R plus 1 by J omega C . This is like v_i into Z_1 by Z_1 plus Z_2 . If I consider this is Z_1 , this is Z_2 , this is simple voltage division taking place. v equal to v_i into Z_1 by Z_1 plus Z_2 . That is what is done here. What is Z_1 ? Z_1 is nothing but 1 by J omega C and what is Z_2 ? As it is a resistance Z_2 equal to R .

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$$v = v_i \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = v_i \left(\frac{1}{1 + j\omega RC} \right) = \frac{v_i}{1 + j2\pi f RC}$$

$\omega = 2\pi f$

$$v_o = \left(1 + \frac{R_f}{R_i} \right) v = \left(1 + \frac{R_f}{R_i} \right) \frac{v_i}{1 + j2\pi f RC}$$

Now we know what v is. v , if we know then we can also proceed to find out the gain. This is our v equal to v_i into 1 by j omega C by R plus 1 by j omega C and a little further simplification will lead to this expression as v_i into 1 by 1 plus j omega RC . Here both the numerator and the denominator is multiplied by j omega C . Simplification occurs, so in the numerator we are left with 1 and the denominator will be 1 plus j omega RC . Relating it to frequency, if we write down, that will be equal to v_i divided by 1 plus omega equal to 2π into the applied signal frequency f . Omega is the angular frequency and that is equal to 2π into the frequency which will be in Hertz or kilo Hertz. That is the frequency of the applied signal and so it will be v_i by 1 plus $j2\pi fRC$ and after this point we now know the voltage applied at the non-inverting terminal. That means we now know what is v ? v is the voltage applied at the non-inverting terminals. Now rest of the thing becomes simpler.

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The image shows a handwritten derivation on a yellow background. At the top, the input voltage v is expressed as a voltage divider: $v = v_i \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = v_i \left(\frac{1}{1 + j\omega RC} \right) = \frac{v_i}{1 + j2\pi fRC}$. A checkmark is next to the final fraction. Below this, the frequency is defined as $\omega = 2\pi f$. The next equation relates the output voltage v_o to v using the non-inverting gain: $v_o = \left(1 + \frac{R_F}{R_1} \right) v = \left(1 + \frac{R_F}{R_1} \right) \frac{v_i}{1 + j2\pi fRC}$. At the bottom is a circuit diagram of a non-inverting op-amp. The input v_i is connected to the non-inverting input (+) through a resistor R_1 . The inverting input (-) is connected to ground through a resistor R_1 and to the output v_o through a feedback resistor R_F . The output is labeled v_o . To the right of the diagram, the gain is summarized as $\frac{v_o}{v} = 1 + \frac{R_F}{R_1}$.

Because now we have simply a non-inverting op-amp, where this voltage we know. This is v and from the negative terminal we have a feedback resistance R_F and the other is R_1 ; this is ground. This v_o we are trying to find out and this circuit is exactly this circuit and that is why we can directly apply the law which we have got earlier in non-inverting op-amp. What is v_o by v ? v_o by v is equal to 1 plus R_F by R_1 . Now if we write down that v_o equal to v into 1 plus R_F by R_1 , this v we can simply substitute from whatever we have obtained. Because this v we have obtained in terms of the input voltage v_i , so doing that, I get now 1 plus R_F by R_1 into this v equal to v_i by 1 plus $j 2 \pi fRC$. This is the final expression for the output voltage in terms of the input voltage and the circuit components.

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$$\frac{v_o}{v_i} = \frac{\left(1 + \frac{R_f}{R_i}\right)}{1 + j2\pi fRC} \quad \text{or,} \quad \frac{v_o}{v_i} = \frac{A_v}{1 + j \frac{f}{f_H}}$$

$A_v = 1 + \frac{R_f}{R_i}$ = passband gain of the filter
 f = frequency of the input signal
 $f_H = \frac{1}{2\pi RC}$ = High cutoff frequency of the filter

That means we know what is the again? v_o by v_i is simply $1 + R_F$ by R_I by $1 + j 2 \pi fRC$. We will use a substitution for the term $1 + R_F$ by R_I and that is denoted by A_v . This A_v actually is the pass band gain of the filter. That means the portion of the frequencies where it is allowing that gain is $1 + R_F$ by R_I and the denominator $1 + j 2 \pi fRC$ is written in a different way. That is $1 + j f$ by f_H where the term $2 \pi RC$ is just replaced by 1 by f_H . f_H is one important frequency that is determined by 1 by $2 \pi RC$ that is known as higher cut off frequency of the filter and that expression is now boiling down to v_o by v_i equal to A_v by $1 + j f$ by f_H .

The denominator term is written as $1 + j f$ by 1 by $2 \pi RC$ and this 1 by $2 \pi RC$, we are writing a different term which is f_H is the frequency. The significance of this f_H is that it is a cutoff frequency for the low-pass filter beyond which it is not allowing the signal and A_v is the gain of the filter when it is allowing the signal.

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$$A = \frac{v_o}{v_i} = \frac{A_v}{1 + j \frac{f}{f_H}} \quad \left| \frac{v_o}{v_i} \right| = \frac{A_v}{\sqrt{1 + \left(\frac{f}{f_H} \right)^2}}$$

1. At very low frequencies, i.e $f < f_H$

$$\left| \frac{v_o}{v_i} \right| \cong A_v$$

2. At $f = f_H$ $\left| \frac{v_o}{v_i} \right| = \frac{A_v}{\sqrt{2}} = 0.707 A_v$ $\frac{a+jb}{\sqrt{a^2+b^2}}$

3. At $f > f_H$ $\left| \frac{v_o}{v_i} \right| < A_v$ Slope -20dB/decade

From this expression we have to further analyze it in terms of the magnitude of the gain. v_o by v_i , which is say, I name it as A of the filter or gain of the filter; if we consider v_o by v_i and focus on the term A_v by $1 + j f$ by f_H , we simply note that this is not simply an expression, which has magnitude. It has both magnitude and phase, because if we look into the denominator term, it is a complex number having magnitude as well as the phase part. That means it has a real as well as an imaginary term. Now we will concentrate on knowing the magnitude. We can find the magnitude of this gain term because what we are interested in knowing is how the attenuation is taking place in the output, how the gain is being modified by the range of frequency. If we want to know the magnitude of this gain, magnitude of the gain A will be denoted by this symbol magnitude of v_o by v_i and this magnitude is nothing but A_v by under root $1 + f$ by f_H square.

The magnitude of the complex term we know. Suppose a plus jb is a complex term, what is the magnitude? Its magnitude is under root a square plus b square and that is what is written here. This term has a magnitude of under root $1 + f$ by f_H square. Let us analyze this magnitude of the gain term because we see in the denominator there is a term f in the expression f by f_H , the frequency of the applied signal. If I vary this frequency over a whole range from zero onwards how the gain of the op-amp will vary is a very

important analysis we have to do because then only we will understand how the filtering action is taking place.

Let us now consider the whole frequency range starting from very low frequencies and go on to high frequencies. First of all at very low frequencies when the frequency is very small as compared to the higher cutoff frequency f_H , then what will happen? Mathematically analyzing this expression then we can correlate it to what actually physically is happening? f by f_H term if we consider, when the numerator f is very small as compared to the denominator term f_H then the whole expression f by f_H will be a fraction only. It will be small as compared to 1. This term is very small compared to 1 means, the whole expression under root we can simply substitute by 1. When f is very small in the low frequency region that is if we start from say f zero, f zero means DC, and if we go on increasing but the portion when this frequency of the applied signal is very small compared to the cutoff frequency and cutoff frequency has a definite value $1 \text{ by } 2 \pi RC$. Depending upon the components which we are connecting in the circuit, R and C we will have a particular cutoff frequency, which we know already. When this input signal frequency is very small as compared to the cutoff frequency f_H , which is given by $1 \text{ by } 2 \pi RC$ then, the whole expression simply boils down to value A_v . That means it is a constant gain and that constant gain is nothing but $1 \text{ plus } R_F \text{ by } R_1$ that we know already from the non-inverting amplifier.

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$$A = \frac{v_o}{v_i} = \frac{A_v}{1 + j \frac{f}{f_H}} \quad \left| \frac{v_o}{v_i} \right| = \frac{A_v}{\sqrt{1 + \left(\frac{f}{f_H} \right)^2}}$$

1. At very low frequencies, i.e $f < f_H$

$$\left| \frac{v_o}{v_i} \right| \approx A_v \quad f = f_H$$

2. At $f = f_H$ $\left| \frac{v_o}{v_i} \right| = \frac{A_v}{\sqrt{2}} = 0.707 A_v$

3. At $f > f_H$ $\left| \frac{v_o}{v_i} \right| < A_v$ Slope -20dB/decade

Handwritten notes: $A \approx A_v / (f/f_H)$

If we have the frequency very small in the low frequency region, the output voltage will be constant and given by the term 1 plus R_F by R_1 . It will act like a simple non-inverting op-amp. What will happen if we now go on increasing the frequency and reach the value f_H ? We are increasing the value of the frequency f and we have seen that in the low frequency region it is giving a constant voltage or a constant gain v_o by v_i which is given by A_v and if we go on increasing and reach the value f_H , what will happen if we replace f equal to f_H in this expression.

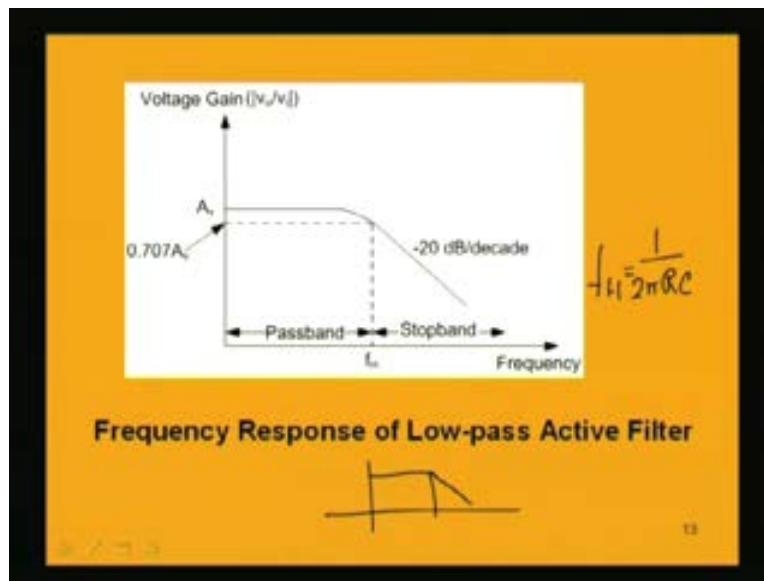
When we give f equal to f_H or when we have the frequency of the applied signal being equal to the cutoff frequency mathematically we see in the denominator term it will be simply root over 1 plus 1. That will be equal to 1; 1 plus 1 means 2. What will be the voltage gain? It will be A_v by root 2 and A_v by root 2 is nothing but 1 by root 2, which is 0.707. It will be 0.707 A_v . When the frequency is equal to the cutoff frequency, the voltage gain drops down to 0.707 times the constant voltage and this is a very important frequency, the cutoff frequency and that is also known as 3 dB frequency because at this frequency the gain drops to the value of the constant voltage by 0.707 times and if you find it in dB, decibel then this will give around 3 dB. 3 dB frequency or cutoff frequency, these names are used for the same frequency, f_H . Sometimes it is known as a 3 dB

frequency and 3 dB frequency name, is coming from the fact that the gain drops down to 3 dB of the voltage gain A_v ; the constant voltage gain is dropping down.

Beyond that what will happen? If you go on further increasing the frequency of the applied signal beyond this cutoff frequency f_H , what will happen is that in the denominator term if we look f is greater than f_H ; very, very greater than f_H . Then the denominator term can be simply written down as A_v by f by f_H . We will get A equal to A_v by f by f ; it is almost equal to because I am just ignoring the term 1 because this f is greater than f_H . Under root square means the term will be released as f by f_H . If we find out the 20 log or the dB decibel of this expression, 20 log of A_v minus, it will be 20 log of f by f_H . If we look into this f by f_H term, then I am increasing the frequency. I go on increasing the frequency and it is in the denominator. So we get 20 log A_v minus 20 log f by f_H . This minus term as it is coming and 20 log A_v is constant dB or decibel value that we are having before the cutoff frequency f_H and the other term minus 20 log f by f_H is giving a slope of minus 20 per decade of change of frequency.

If we go on changing f , suppose we are increasing the value of frequency by 10 times; suppose we are having two frequencies f_1 and f_2 ; f_2 is 10 times of f_1 then, what will happen is that this slope what we will get is minus 20 log of 10 and log of 10 is 1. We get is minus 20 dB per decade. A decade change of frequency means if frequency is increased 10 times from a particular frequency in the high frequency region, the gain term is reducing with a slope of 20 dB per decade. What we observe in the higher frequency region, as we go on increasing the frequency is that the gain of the filter will drop of or reduce with a slope of 20 dB per decade. We have basically three regions that we have seen here in this filter and these three regions are when the frequency is very low and when it is exactly equal to the cutoff frequency which is $1/2\pi RC$ and when the frequency is increasing we are getting a straight line with the slope of minus 20 dB per decade. Combining all these three information, we can now plot the gain of the filter versus frequency and the plot will be like this.

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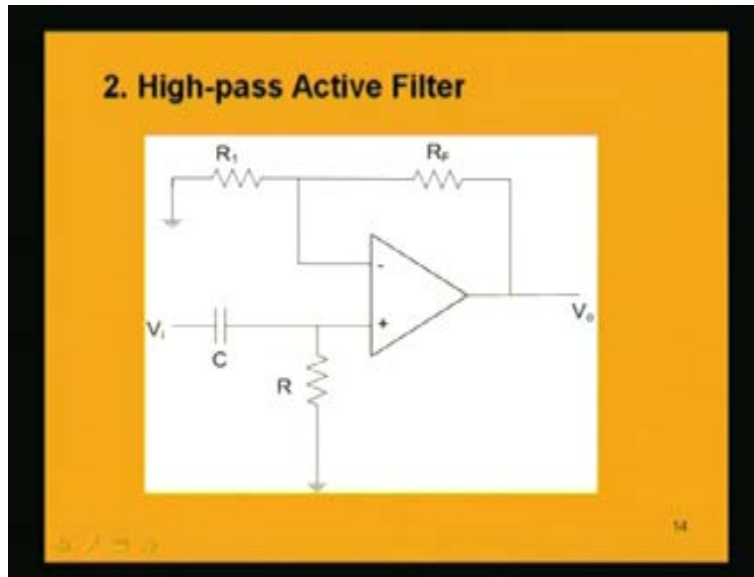


Here we are plotting the voltage gain v_o by v_i versus frequency and we are plotting for the whole frequency region starting from zero. This picture is giving you the frequency response of the low-pass active filter. Here as we have described, initially when the frequency region is very small we are getting a constant voltage A_v that is $1 + R_F$ by R_1 given by the circuit components and then it is dropping off to a value of $0.707 A_v$ at the cutoff frequency f_H and that cutoff frequency f_H is nothing but 1 by $2\pi RC$ and then it is reducing with a slope of minus 20 dB per decade. It is attenuating and this region where this attenuation of the voltage gain is taking place, this is actually not like an ideal filter. Because in the ideal filter, what we have initially discussed, the frequency when it is not allowing it should completely stop; immediately it should become zero, but that is not happening because practically you would not get that ideal characteristic and you always have a response which is approximating the ideal one but it is difficult to get a exact ideal one.

If we look into the whole response, here it is dropping off and when you go on increasing to higher and higher frequencies the lesser and lesser signal will be allowed. This is the stop band frequency and this is the pass band frequency. The frequency response of practical low-pass active filter is like this. Although we are not able to completely stop

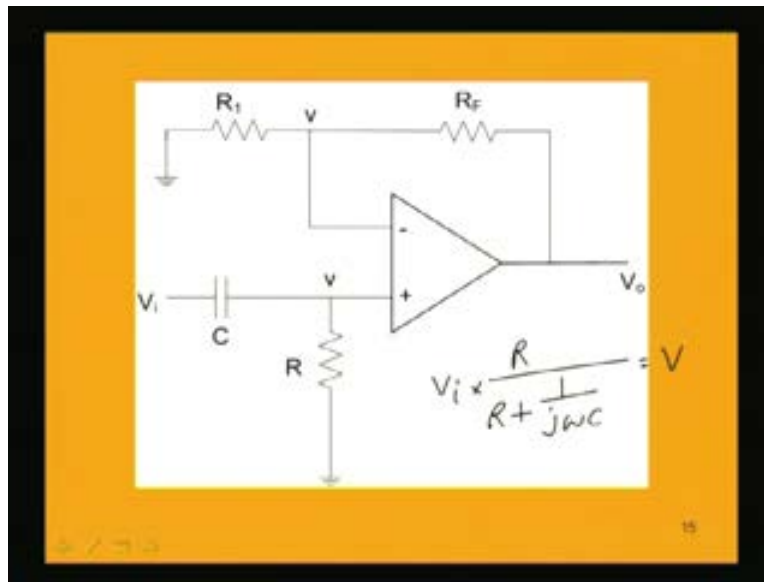
we are not getting characteristics like this, which is ideal. This should be the pass band and this should be the stop band, but it is coming like this. In the low frequency region, we are being able to fully pass the input signal.

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Next let us discuss about high-pass active filter. The low-pass active filter which we studied was allowing the signal in the low frequency region totally and opposite will be the case in the high frequency active filter. It will allow the signal to pass in the high frequency region and it will stop in the low frequency region and one important thing to notice is that simply by interchanging the locations of R and C of the low-pass filter, we can obtain a high-pass filter. The same circuit; only you have to interchange the positions of R and C . Previously where R was, we are placing C now and wherever C was we are replacing R now. Rest of the circuit it is exactly same. Now we find out what is the output voltage? The same principle which we now discussed to find out output voltage will be followed.

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Here we have to find out what is the voltage at this point which is the non-inverting terminal and by the same analysis we will find what is V, voltage division is taking place. Input voltage is v_i ; v_i into R divided by R plus $1/j\omega C$ and that is equal to v. Here this voltage is our interest actually. What is the voltage at this non-inverting terminal?

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$$v = v_i \frac{R}{R + \frac{1}{j\omega C}} = v_i \left(\frac{j\omega RC}{1 + j\omega RC} \right)$$

$$v_o = \left(1 + \frac{R_f}{R_i} \right) v = \left(1 + \frac{R_f}{R_i} \right) \left(\frac{j\omega RC}{1 + j\omega RC} \right) v_i$$

$$\frac{v_o}{v_i} = \left(1 + \frac{R_f}{R_i} \right) \left(\frac{j\omega RC}{1 + j\omega RC} \right) \quad \omega = 2\pi f$$

$$\frac{v_o}{v_i} = \left(1 + \frac{R_f}{R_i} \right) \left(\frac{j2\pi f RC}{1 + j2\pi f RC} \right)$$

We are writing it here and now we will do a little bit of simplification and multiplying the numerator and denominator term by $j\omega C$ we get that is equal to v_i into $j\omega RC$ by $1 + j\omega RC$. This is v . This voltage we have found out but now if we want to follow the analysis of a non-inverting amplifier we must express it in terms of the v_o and v_i in order to know the gain and we know $1 + R_f/R_1$ is the gain between v_o and v_i non-inverting op-amp where v_i was at this point, now v is at this point. If we want to find out the gain of this filter, v_o is equal to $1 + R_f/R_1$ into v . But that v can be replaced by this expression which is equal to v_i into $j\omega RC$ by $1 + j\omega RC$. v_o by v_i ratio if we take which is the gain of the filter that is equal to $1 + R_f/R_1$ into $j\omega RC$ by $1 + j\omega RC$ and writing in terms of the frequency in Hertz or kilo Hertz, whatever may be the frequency of the signal at the input, let us rewrite this expression v_o by v_i equal to $1 + R_f/R_1$ into j into $2\pi fRC$ by $1 + j2\pi fRC$. I am replacing ω by $2\pi f$ and now doing a little bit of manipulation in this term, the term 1 by $2\pi RC$ I am replacing here by f_L . We will now rewrite this expression by introducing a term f_L , which is 1 by $2\pi RC$.

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$$\text{or, } \frac{v_o}{v_i} = A_v \left(\frac{j\left(\frac{f}{f_L}\right)}{1 + j\left(\frac{f}{f_L}\right)} \right) \quad \checkmark \quad f_L = \frac{1}{2\pi RC}$$

$$A_v = 1 + \frac{R_f}{R_1} = \text{passband gain of the filter}$$

$$f = \text{frequency of the input signal}$$

$$f_L = \frac{1}{2\pi RC} = \text{Low cutoff frequency of the filter}$$

Like in the earlier case when we substituted 1 by $2\pi RC$ by f_H , here I am writing f_L just to denote that this is the low cutoff frequency. It is a high-pass filter, it is allowing high-

pass or it is allowing the high frequency signal; so, that frequency which is the lowest frequency to cutoff the signal. Beyond f_L , it will allow. Up to f_L , it should not allow. That is why the term is written as f_L ; means it is low cutoff frequency and that is equal to 1 by $2\pi RC$.

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$$v = v_i \frac{R}{R + \frac{1}{j\omega C}} = v_i \left(\frac{j\omega RC}{1 + j\omega RC} \right)$$

$$v_o = \left(1 + \frac{R_F}{R_1} \right) v = \left(1 + \frac{R_F}{R_1} \right) \left(\frac{j\omega RC}{1 + j\omega RC} \right) v_i$$

$$\frac{v_o}{v_i} = \left(1 + \frac{R_F}{R_1} \right) \left(\frac{j\omega RC}{1 + j\omega RC} \right)$$

$$\frac{v_o}{v_i} = \left(1 + \frac{R_F}{R_1} \right) \left(\frac{j2\pi f RC}{1 + j2\pi f RC} \right)$$

$$= \frac{\left(1 + \frac{R_F}{R_1} \right)}{1 + j \frac{1}{2\pi f RC}}$$

$\omega = 2\pi f$

If we want to rewrite this expression I have to divide the numerator and denominator term by $2\pi RC$. I can write it like this; in the numerator term it will be written like this j into f by 1 by $2\pi RC$. Similarly in the denominator I will write 1 plus j into f by 1 by $2\pi RC$ and this 1 by $2\pi RC$ term, we will substitute by f_L . What we get in this expression will be 1 plus R_F by R_1 into $j f$ by f_L by 1 plus $j f$ by f_L and that is what we are getting here (Refer Slide Time: 39:32).

Now if we look into this expression of the gain of the high-pass filter v_o by v_i , I get a term which is complex no doubt because here apart from A_v that is called the pass band gain of the filter which is equal to 1 plus R_F by R_1 that means during that pass band this will be the gain which is constant and the other part of the term is having complex quantities and so we will be now interested in finding of the magnitude of this gain.

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$$\left| \frac{v_o}{v_i} \right| = \frac{A_v \left(\frac{f}{f_L} \right)}{\sqrt{1 + \left(\frac{f}{f_L} \right)^2}}$$

1. At very low frequencies, i.e. $f < f_L$

$$\left| \frac{v_o}{v_i} \right| \approx A_v \frac{f}{f_L}$$

Increases with Slope 20dB/decade

$f_L = \frac{1}{2\pi RC}$

Let us only concentrate on the magnitude. That is what we are interested in. So what is the magnitude of v_o by v_i . Simply finding the magnitude of this term, in the numerator we will get A_v into f by f_L . Because numerator term is having A_v into $j f$ by f_L , the magnitude of this imaginary quantity will be simply f by f_L that is what is written here and the denominator term will be root over 1 plus f by f_L square. Because the denominator term was 1 plus $j f$ by f_L , I am taking only the magnitude that is root over 1 plus f by f_L square.

Similarly analyzing the signal over the whole frequency range, in very low frequencies when I have f is very less as compared to the f_L value because f_L we know it is equal to 1 by $2 \pi RC$. When the applied frequency is very small compared to f_L , in this mathematical expression of magnitude that we are getting for the gain, we can consider the denominator term as simply under root 1 because when f is very less than f_L , when this is satisfied then the denominator term will be almost equal to under root 1 because f by f_L will be very small value. The f value is very small as compared to f_L . It is a fraction. Even if it is squared, it will be a small value and in comparison with 1 , we can ignore or neglect that term. This is an assumption but that assumption will be valid for practical

cases and we will get at very low frequencies the gain is equal to A_v into f by f_L . Simply the numerator term will remain; the denominator term will be almost equal to 1.

What does it signify? From this term we can see that in the low frequency region, when frequency increases the gain will increase with a slope of 20 dB per decade because if we find the 20 logarithmic in dB expression, in decibel expression, it will be simply 20 log A_v plus 20 log f by f_L .

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$$\left| \frac{v_o}{v_i} \right| = \frac{A_v \left(\frac{f}{f_L} \right)}{\sqrt{1 + \left(\frac{f}{f_L} \right)^2}}$$

1. At very low frequencies, i.e. $f < f_L$

$$\left| \frac{v_o}{v_i} \right| \approx A_v \frac{f}{f_L}$$

Increases with Slope 20dB/decade

$$20 \log A_v + 20 \log \frac{f}{f_L} = 20 \log A_v + 20 \log f - 20 \log f_L$$

$f_L = \frac{1}{2\pi RC}$

$\frac{f_2}{f_1} = 10 \Rightarrow 20 \log \frac{f_2}{f_1} = 20$

This is a constant term so the slope can be obtained from this part. If frequency is increased 10 times, it will be 20 log 10 to the base 10; it will be 20 for earlier frequency f_1 and then we are taking say 10 f_1 . In between these two frequencies, if we take this part it will have a 20 dB per decade slope. In the low frequency region when the frequency is increasing we are having an increase of gain and a positive slope is signifying that and that slope is 20 dB per decade.

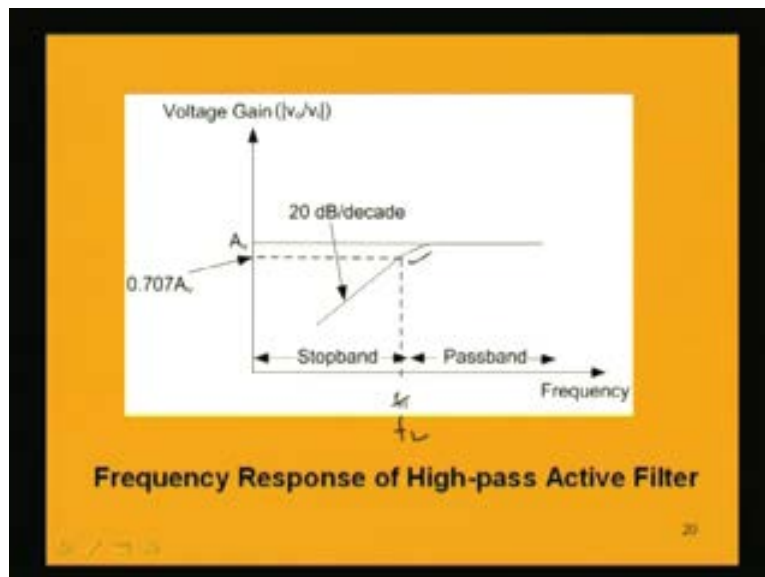
What will happen when f is equal to f_L ? That is when the frequency is equal to the cutoff frequency then in the numerator it will be simply A_v and in the denominator 1 plus 1, it will be simply root 2.

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$$\begin{aligned} 2. \text{ At } f = f_L \quad \left| \frac{v_o}{v_i} \right| &= \frac{A_v}{\sqrt{2}} = 0.707 A_v \\ 3. \text{ At } f > f_L \quad \left| \frac{v_o}{v_i} \right| &\cong A_v \end{aligned}$$

We get at f is equal to f_L , the value of the gain magnitude is A_v by root 2 and that is 0.707 times A_v . This is again known as 3 dB frequency, because $20 \log 0.707$ will give you 3 dB. The gain is increased to 3 dB of the value of A_v .

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If we plot the value, at value of f_L the gain will be 1 by root 2 times of A_v . That is 0.707 A_v we will get here. This is not f_H , this is f_L . We are having a gain of 0.707 times the

constant gain A_v at the cutoff frequency and when f is greater than f_L , in this expression if we see, when f is very high as compared to the f_L in the high frequency region then, this can be written approximately as A_v . If we consider the high frequency region and in the frequency range when the frequency f is greater in comparison with f_L then, this term (Refer Slide Time: 44:30) if we look into again f by f_L whole square plus 1 in the denominator will be simply approximately equal to f by f_L because as f is higher than f_L , so f by f_L whole square plus 1 will be almost equal to f by f_L as 1 is ignored now in comparison with the other term.

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2. At $f = f_L$ $\left| \frac{v_o}{v_i} \right| = \frac{A_v}{\sqrt{2}} = 0.707 A_v$

3. At $f > f_L$ $\left| \frac{v_o}{v_i} \right| \cong A_v$

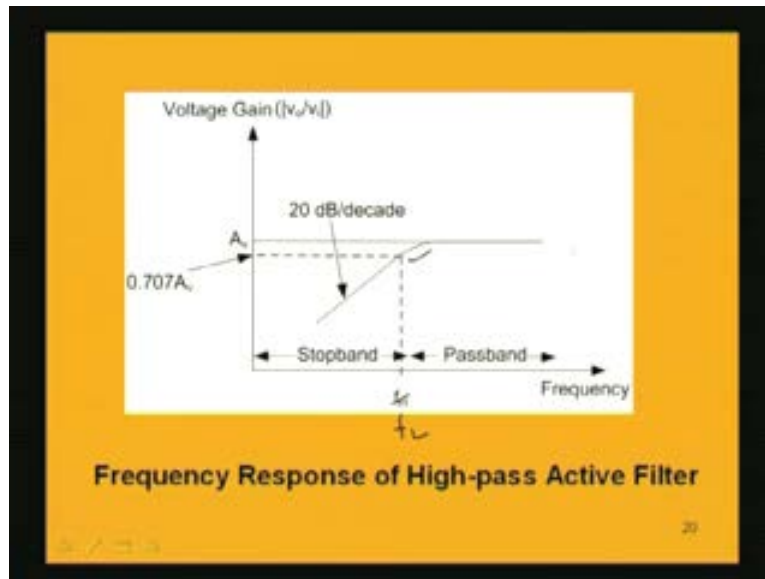
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$\frac{A_v \left(\frac{f}{f_L} \right)}{\left(\frac{1}{f_L} \right)}$

Ultimately we will get A_v into f by f_L in the numerator. This term is there in the denominator also we will get f by f_L and these two cancel. So we get a constant voltage A_v that is $1 + R_f$ by R_1 . In the high frequency region when the frequency is greater than the cutoff frequency we get a constant voltage gain and the signal is totally allowed. It is passed with a gain.

The whole characteristic of this high-pass filter if we plot now we will get a plot like this that is the frequency response of the high-pass filter.

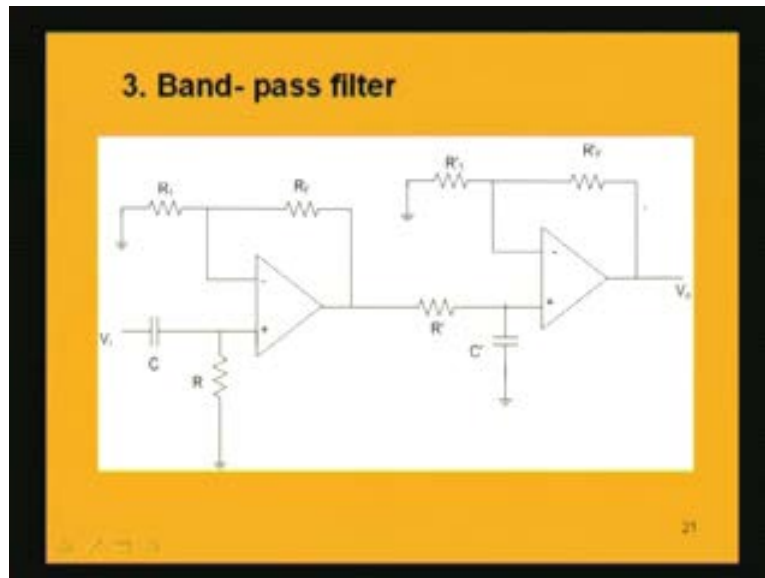
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Here as we have described, in the low frequency region we will get a straight line having a slope of 20 dB per decade that is positive slope; that has to be noted. It is rising means the gain will be rising and it will have a value of 0.707 times the constant gain A_v at the cutoff frequency f_L . Beyond that it will go on increasing and become constant with A_v for the high frequency region. Up to the cutoff frequency a very small portion of the signal is allowed. It is not totally stopping. It is not ideal characteristic that we are getting, but it is stopping and only allowing a small portion of this signal and beyond the cutoff frequency we are having all the signals passed. This is the characteristic of this high-pass filter using op-amp and the cutoff frequency which is obtained is obtained from the circuit parameter because it is $1 / 2\pi RC$. Depending upon what value you are choosing for R and C we are getting the cutoff frequency. While designing a filter you will have to design it according to your requirement and that will depend upon at what frequency you want to cutoff or what frequency region you want to pass. Depending upon that your cutoff frequency will be determined and then the whole response curve you can get like that.

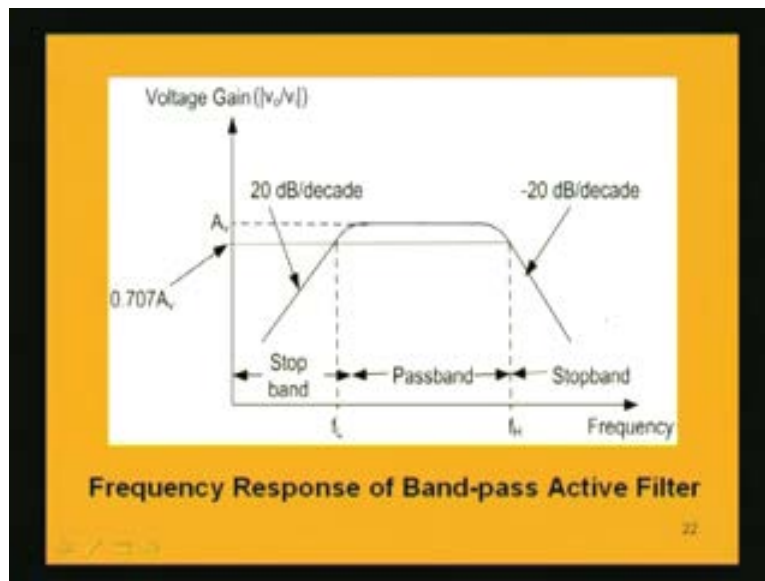
So far we have discussed a low-pass filter and a high-pass filter using op-amp, active low-pass and active high-pass filter and if we now consider the band-pass active filter that will have the two filters that we discussed now; low-pass and high-pass combine. If we combine low-pass filter and high-pass filter circuits and make a complete filter then, that will give the characteristic of a band-pass filter. Let us combine the circuits which we used for low-pass filter and high-pass filter and this will be the band-pass filter.

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If we look into the first part, it is the high-pass filter. If we look into the latter part, it is the low-pass filter. We are combining them. What we will get at the response curve? If we plot the gain versus frequency, we get such type of response because as we are combining the low-pass and high-pass filters together, there will be two frequencies which are the cutoff frequencies.

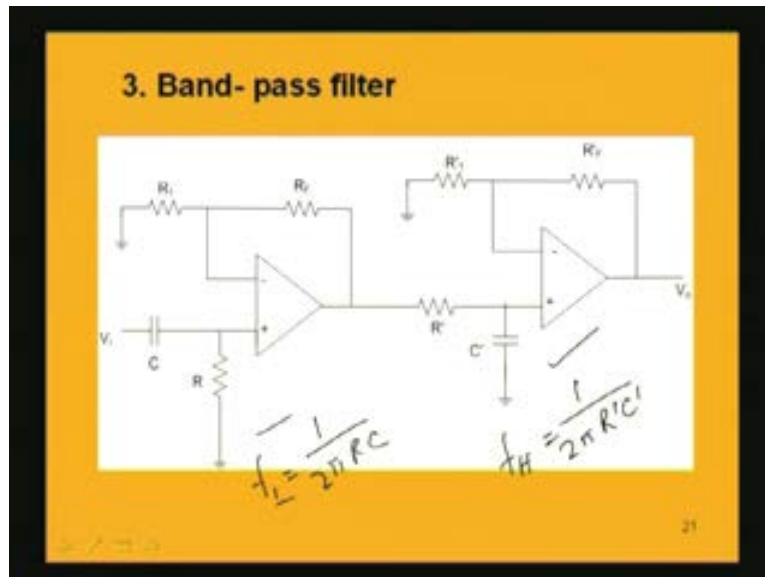
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One is the low cutoff frequency and one is the high cutoff frequency. The high cutoff frequency is f_H , which will come from the low-pass filter circuit and the low cutoff frequency will come from the high-pass filter. We are combining these two characteristics. We will be initially getting the characteristic of a high-pass filter; the signal will be attenuated very much and then after this low cutoff frequency we will get an almost constant voltage A_v and up to the low cutoff frequency the slope is positive with 20 dB per decade slope and then at another frequency which is determined by the low-pass filter circuit, we will have a high cutoff frequency that is f_H . After that the gain will attenuate or reduce. These two regions are stop band regions and the in between region between f_L and f_H is the pass band. This is the characteristic of the band-pass filter. What is f_L and f_H will be determined by the two circuits.

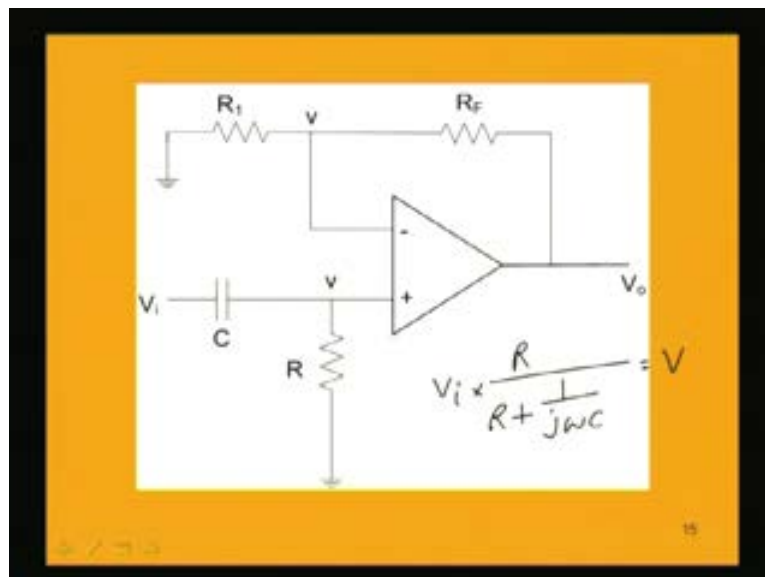
We see here that the low-pass filter circuit is this one.

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This will determine the high cutoff frequency f_H and that will be given by 1 by $2\pi R$ dash C dash and this is giving you the high-pass filter.

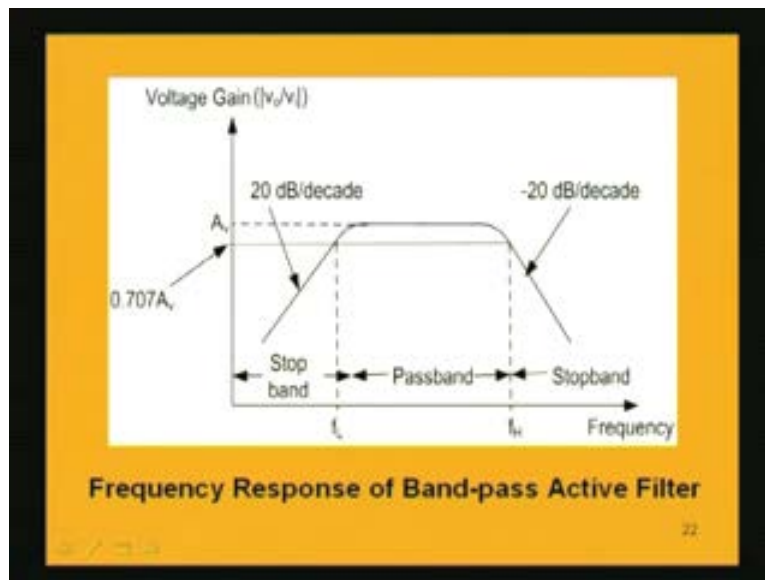
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Because if we look back into the high-pass filter circuit, this is the high-pass filter circuit and this circuit was giving you the low cutoff frequency. The low cutoff frequency is

given by f_L is equal to $1 / 2\pi RC$. Here f_H is obtained as $1 / 2\pi R \text{ dash } C \text{ dash}$. These two are the determining frequencies for this band-pass filter. We have now seen that band-pass filter is nothing but a combination of low-pass and high-pass filter circuits. We can make it from the two circuits by combining them and basically while designing this band-pass filter we have to know what band of frequencies we are going to allow? That will determine the f_L and f_H .

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f_H , here as you can see is higher than f_L . The frequency f_H has to be made higher than f_L , if within this region of f_L to f_H we want to have the pass band and below and above these two frequencies we get the stop band.

The discussion we had today was about the active filter using op-amp and other passive components to get a frequency selection and we have to design a filter and actually the designing of a filter is a very complex issue because you have to select the proper components in order to give us the required frequency band which we want to select and in designing some of the things we have to actually consider and remember that we cannot have the freedom of selecting both R and C . Because you have to see that $1 / 2\pi RC$ if it is giving you the cutoff frequency then if we choose initially C that will give

you R value. What value to choose and what value of C and R will be proper that we have to basically consider and generally the standard values of resistances you have to take while designing what is available, but sometimes we may not get the exact value. Then we will have to go for a variable resistance. Suppose I am having 19.5 kilo ohm per hour and it will not be available as a discrete component 19.5 because we have standard values of resistance. Then we will have to have a variable resistance also and generally the capacitance, which is selected, C, it is less than 1 microfarad. We generally take the values of capacitance selected for the circuit; say the C value and C dash value generally we select less than 1 microfarad and then we go for finding out R and use standard values.

This discussion will help you in designing a filter depending upon your requirement whether you want it to be low-pass filter or high-pass filter or band-pass filter, you can design following this discussion. We will have to also remember certain things while designing because we cannot arbitrarily have any value of R and C and we will have to depend on what standard values are available with us.