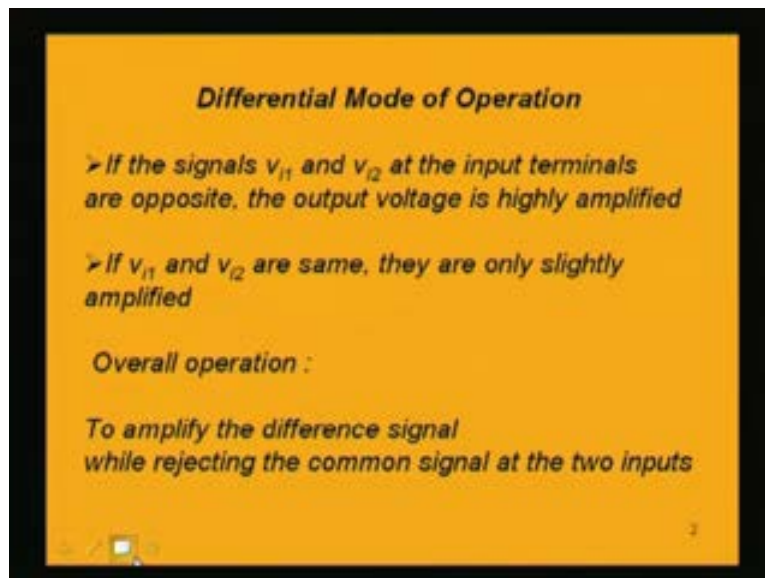


Basic Electronics
Dr. Chitrlekha Mahanta
Department of Electrical and Communication Engineering
Indian Institute of Technology, Guwahati

Module - 4
Operational Amplifier
Lecture - 2
Ideal op-amp

In the last class we have seen that the op-amp has in its circuit basically a differential amplifier and we have actually cascaded stages of differential amplifiers in the op-amp and what we get at the output of an op-amp is a very high gain amplified signal but the magnitude of this output signal is basically dependent on the two input signals that are being applied at the two terminals of the op-amp that is the inverting and non-inverting terminal. If the difference between the two signals applied at the two inputs of the op-amp is high then we get a high output voltage but if the difference between the two input signals is less then we get an attenuated output and if the two signals are equal then we are supposed to get a zero output from the op-amp. So the output voltage of an op-amp will be dependent basically upon the difference between the two input signals.

(Refer Slide Time: 2:47)



Differential Mode of Operation

- If the signals v_{i1} and v_{i2} at the input terminals are opposite, the output voltage is highly amplified*
- If v_{i1} and v_{i2} are same, they are only slightly amplified*

Overall operation :

*To amplify the difference signal
while rejecting the common signal at the two inputs*

For example if we have V_{i1} and V_{i2} at the non-inverting and inverting terminals of an op-amp then the output voltage from the op-amp will be highly amplified if this V_{i1} and V_{i2} are opposite and if they are same then it will be only slightly amplified and ideally we are suppose to get zero when they are equal or same voltages at the two terminals. So the overall amplification or overall operation of an op-amp is to reject the common signal and amplify the difference signal. If we have the difference between the two signals higher then we have an amplified output and the common signal which is present at the two input terminals will be reduced to zero. That is why if we consider the noise which is common to both the input signals being applied at the two input terminals of an op-amp, naturally the noise will be attenuated. That is one big advantage which we have in op-amp that the noise part in the two input terminals for the signals being applied will be attenuated or rejected and the difference between the signals at the input terminals is amplified.

(Refer Slide Time: 4:18)

Output Voltage

The two signals v_{i1} and v_{i2} in general have both in-phase and out-of-phase components

So the resulting output $v_o = A_d v_d + A_c v_c$

where A_d = Differential gain of the Amplifier
 A_c = Common mode gain of the Amplifier
 v_d = Difference voltage = $v_{i1} - v_{i2}$
 v_c = Common voltage = $\frac{v_{i1} + v_{i2}}{2}$

Handwritten: $v_d = v_{i1} - v_{i2}$

Now let us find out the output voltage when you have two input signals V_{i1} and V_{i2} at the two input terminals of the op-amp. These two signals in general if we consider they may have both in-phase and out-of-phase components. Some portion of the two input signals may be common to both of them and some will be out-of-phase. The resulting output of

the op-amp if we consider as V_0 , which is the output voltage from the op-amp, it is given by $A_d V_d$ plus $A_c V_c$ where the A_d term is denoting the differential gain of the amplifier and A_c term is denoting the common mode gain of the amplifier. These two terms are very important when we consider the output voltage or amplified output when we find out, these two play the role for finding out the magnitude. As this A_d is the differential gain and A_c is the common mode gain, these two will determine the output component, which is arising because of the difference between the two signals and the common part of the two signals.

The V_d voltage is denoting the difference voltage between the two signals V_{i1} and V_{i2} that we are applying. So V_{i1} minus V_{i2} is the difference between the two voltages and a common voltage is denoted by V_c . So V_c is the average between the two voltages; half of V_{i1} and V_{i2} . Half of V_{i1} plus V_{i2} is the common signal or the common voltage between the two signals. Then we have the output voltage given by this expression of V_0 equal $A_d V_d$ plus $A_c V_c$.

(Refer Slide Time: 6:45)

Common Mode Rejection Ratio (CMRR)

$$CMRR = \frac{|A_d|}{|A_c|}$$

or, $CMRR(\log) = 20 \log_{10} \frac{|A_d|}{|A_c|}$

Now, $v_o = A_d v_d + A_c v_c$

$$= A_d v_d \left(1 + \frac{A_c v_c}{A_d v_d} \right) = A_d v_d \left(1 + \frac{A_c v_c}{A_d v_d} \right)$$

$$V_0 = A_d v_d \left(1 + \frac{1}{CMRR} \frac{v_c}{v_d} \right) \rightarrow 0$$

Handwritten notes on the right side of the slide:

$$CMRR = \frac{|A_d|}{|A_c|}$$

$$\frac{1}{CMRR} = \frac{|A_c|}{|A_d|}$$

The term which is very important in an op-amp is common mode rejection ratio or CMRR. Basically CMRR is the ratio between the magnitude of the two terms that we

have now described, A_d and A_c differential gain and common mode gain of the op-amp. The ratio between the magnitude of the difference gain and the common mode gain of the op-amp is known as CMRR or common mode rejection ratio. In absolute quantity it is this expression, which is the ratio between the magnitudes of these two gains A_d and A_c or it is also taken in logarithmic unit that is $\log$ of CMRR is also taken. In that logarithmic unit it will be described as $20 \log$ of magnitude A_d by magnitude A_c to the base 10. In these two ways CMRR is expressed.

Now we look back into that expression of the output voltage again. The output voltage $A_d V_d$ plus $A_c V_c$ we do a little manipulation taking this $A_d V_d$ term common. If we take $A_d V_d$ common, then within bracket we get 1 plus $A_c V_c$ by $A_d V_d$. Then it can be written as $A_d V_d$ into 1 plus A_c by A_d into V_c by V_d . The purpose of manipulating in this way is to introduce the term CMRR. That is further made equal to the expression $A_d V_d$ into 1 plus if we look into this term A_c by A_d it is nothing but 1 by CMRR because CMRR is equal to magnitude A_d by magnitude A_c . From this we get 1 by CMRR equal to magnitude A_c by magnitude A_d . We are replacing this A_c by A_d by 1 by CMRR into V_c by V_d . In this expression for the output voltage if we observe closely we find that if CMRR is very high, the value of this common mode rejection ratio for an op-amp is very high, then the output voltage V_0 will be almost equal to $A_d V_d$ because this term being high means the overall term 1 by CMRR into V_c by V_d will be very small approaching zero because the denominator is very high. This whole term will approach zero that means we are left with V_0 equal to $A_d V_d$.

That is a very significant analysis because it tells that for a high CMRR, op-amp output voltage is basically the difference in gain multiplied by the difference in the input voltages, so the common mode part is almost rejected to zero. That is why we have to have a high CMRR op-amp which will amplify the output signal based on the difference between the two input signals and whatever a common part is almost equal to zero.

(Refer Slide Time: 10:40)

Ex.1 Determine the output voltage of an op-amp for input voltages of $v_{i1} = 150\mu V$, $v_{i2} = 140\mu V$. The amplifier has a differential gain of $A_d = 4000$ and the value of CMRR is 100.

Solution :

$$V_o = A_d V_d \left(1 + \frac{1}{CMRR} \frac{V_c}{V_d} \right)$$



$V_d = V_{i1} - V_{i2}$
 $= 150 - 140$
 $= 10\mu V$

For this example you have to determine the output voltage of an op-amp for input voltages V_{i1} is equal to 150 microvolt and V_{i2} is equal to 140 microvolt. In this op-amp, the two voltages which are applied at the non-inverting and inverting terminals are this is V_{i1} and this is V_{i2} . They have the magnitudes of 150 microvolt and 140 microvolt. The amplifier has a differential gain A_d , 4000 and the value of the CMRR is given as 100. We have to find out the output voltage. As per the descriptions just now we have seen V_o , output voltage is given by the expression A_d into V_d within bracket 1 plus 1 by CMRR V_c by V_d . In this expression, we can plug in the given values for the terms. Let us see the values given; V_{i1} is given, V_{i2} is given and the differential gain is given. Common mode gain is not given but we know that CMRR value is 100. If we know the CMRR value and A_d value that is sufficient because the CMRR is nothing but A_d by A_c .

In order to find out the V_o , that is the output voltage let us now plug in the values into these expressions. A_d is 4000, we know it. What is V_d , the difference voltage we can find out. V_{i1} minus V_{i2} that is the difference voltage; 150-140 and that is 10 microvolt. So 10 microvolt is the difference voltage applied between the two inputs of the op-amp and CMRR is 100, which is given and V_d we have found out.

(Refer Slide Time: 13:00)

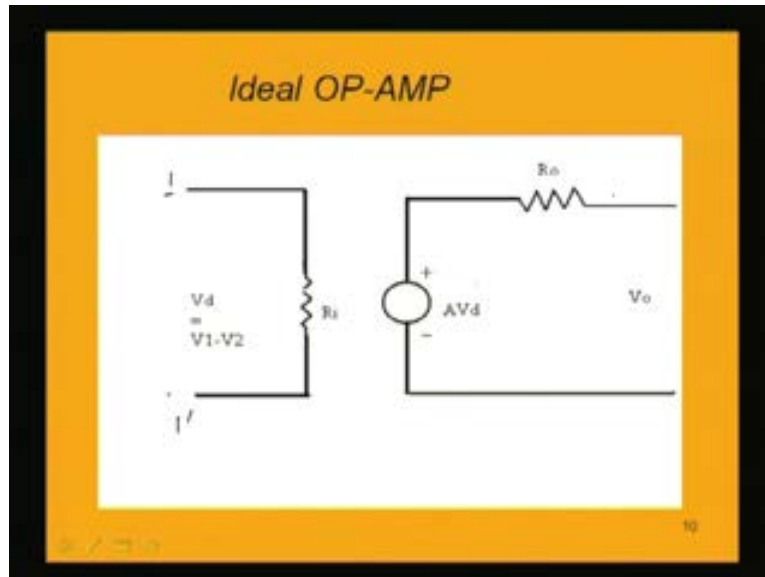
$$\begin{aligned} \text{Sol}^n \quad V_c &= \frac{1}{2}(V_{i1} + V_{i2}) \\ &= \frac{1}{2}(150 + 140) \mu\text{V} \\ &= 145 \mu\text{V} \\ V_o &= A_d V_d \left(1 + \frac{1}{\text{CMRR}} \cdot \frac{V_c}{V_d} \right) \\ &= 4000 \times 10 \left(1 + \frac{1}{100} \cdot \frac{145}{10} \right) \\ &= 4 \times 10^4 (1 + 0.145) \\ &= 45800 \mu\text{V} = \underline{45.8 \text{ mV}} \end{aligned}$$

V_c we can find out because V_c is the common mode part of the two signals and V_c is found out by average between the two signals half of V_{i1} plus V_{i2} . That is equal to half of $150 + 140$; both are in microvolt and that gives the value of 145. 145 microvolt is the common mode part of the two signals and the difference component of the two signals we have found out to be 10 microvolt. We can find out now the output voltage V_o . According to the expression which we know A_d into V_d into 1 plus 1 by CMRR into V_c by V_d , every component of this expression we know. Now we can plug in, so what is A_d ? A_d is equal to 4000; V_d we have found out to be equal to 10 microvolt. The units are written in microvolts. So I need not repeatedly write the units; all are in microvolts. So 1 plus 1 by CMRR is 100. So 100 into V_c is 145, V_d is 10.

Simplifying this expression or finding out the numerical value we now find out what will be this value? 4 into 10 to the power 4 within bracket the terms; denominator is 1,145 by 1000 is 0.145. So 1.145 into 4 into 10 to the power 4 gives the value of 45800 microvolt. To check this value it should come 45,800 microvolt, meaning it is equal to 45.8 millivolt. So 45.8 millivolt output we obtained for this op-amp having the two input signals which are 150 microvolt and 140 microvolt.

Now let us consider an ideal op-amp. You call an op-amp an ideal op-amp because if we consider an op-amp circuit what is inside that circuit is differential amplifier, but they are in stages.

(Refer Slide Time: 15:54)



If we consider the op-amp having input resistance, output resistance and its gain, say A then a simple circuit can be used to explain that op-amp. Here it is a simple circuit having input voltage between these two terminals. This is say 1, 1 dash. These are the input terminals and the input voltage which is applied to the op-amp is a difference input V_1 minus V_2 . V_1 and V_2 are the two inputs at the two terminals of the op-amp, non-inverting and inverting terminal. Then the difference voltage will be V_d , which is equal to V_1 minus V_2 and the op-amp will have an input resistance and this input resistance is very high and let us denote it by R_i . This R_i will represent the input resistance. So the input side is having these two parameters, one is the difference input. This is the input voltage, it has one parameter which is input resistance and what will be the output of the op-amp? Gain multiplied by the difference input into V_d .

So A into V_d is the voltage source at the output meaning that we are getting an output voltage A times V_d and this is the voltage source which is denoting that output voltage

which we obtained at the output. But there is output resistance also. Although this value is very small, we cannot ignore and this is R_0 ; that R_0 is the output resistance of the op-amp. Finally what we get at the output is V_0 which is AV_d minus this drop occurring in the resistance R_0 . Practically this is the circuit which is used to explain the op-amp. We are basically having a simple circuit representing the op-amp in terms of input resistance, output resistance and the gain of the op-amp. These are the parameters which practically we have in an op-amp and which can be known from the data sheet available for an op-amp also.

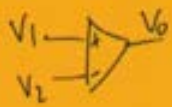
This circuit is a practical circuit of the op-amp but we will now consider an ideal op-amp where we will consider that the input resistance R_i is infinite, output resistance R_0 is zero and the gain A is also infinite. The ideal op-amp consideration is useful for analyzing the circuits having the op-amp and in our analytical expressions which we will be using or in our analysis of the op-amp circuits it is obtained simply or easily by considering the ideal op-amp analysis. That is why we are imagining an ideal op-amp having the requisites of the parameters with certain specification like as we have now told that R_i should be infinite, R_0 should be zero. In this way ideally if these parameters have certain values we consider that op-amp to be an ideal op-amp and in all our analysis from now onwards we will be using ideal op-amp analysis having these properties. So what are the properties of the ideal op-amp or what are the parameter values it should have?

(Refer Slide Time: 19:33)

Ideal OP-AMP

Ideal op-amp has following characteristics -

- Input Resistance $R_i = \infty$
- Output Resistance $R_o = 0$
- Voltage Gain $A = \infty$
- Bandwidth $= \infty$
- Perfect balance i.e $v_o = 0$ when $v_1 = v_2$
- Characteristics do not drift with temperature



11

One is that ideal op-amp has the characteristics of first input resistance, R_i should be infinity, output resistance R_o should be zero, voltage gain should be infinite and bandwidth should be infinity. Bandwidth means the characteristics of the op-amp like voltage gain, should not change with very low or very high frequencies; that means throughout the whole frequency range of application the characteristics of the op-amp should not change. For example if voltage gain is A or infinity, it should be same for all the frequency range, whole frequency range. Because we have seen earlier in the case of transistor, voltage gain if we consider for very low frequencies and very high frequencies the gain drops off or reduces; only in the mid band the gain becomes constant. So, that is not a constant gain. It is varying with frequency and that type of analysis or that type of characteristics should not be there for this ideal op-amp; so ideal op-amp will have an infinite bandwidth. That is within that infinite frequency range the characteristics are same. That means we are not having any changed characteristics like changed voltage gain for frequency range; infinite means, practically for whole frequency range we are getting the same characteristics. That is why it is written that bandwidth is infinite.

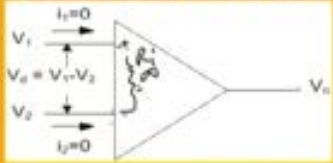
Perfect balance also is one important property which has to be obeyed by this op-amp. That is V_o is equal to zero when V_1 is equal to V_2 . If we consider the op-amp, one is non-

inverting and one is inverting terminal. Let us have these voltages at the inverting and non-inverting terminals V_1 and V_2 . If the output voltage is zero, V_0 is equal to zero, V_1 must be equal to V_2 and that means that the perfect balance is there for the op-amp. This perfect balance will come only because the differential amplifier which is the basic circuit inside the op-amp has the transistors which are perfectly matched. When the two transistors are perfectly matched or balanced we do not have any unbalanced voltage and that means when these two voltages V_1 and V_2 are equal then there should not be any output voltage, output voltage should be zero; that is to be maintained. For op-amp that is one criterion.

Another criterion is that characteristics do not drift with temperature. Temperature should not have any effect on the characteristics change. That is whatever characteristics we are getting for the op-amp, may be the voltage gain or other parameters they will not be dependent on temperature. Even if temperature changes we should not have a varying characteristics. All these properties are to be satisfied for the op-amp when we call that op-amp to be ideal op-amp. Actually this ideal op-amp consideration, make our analysis very easier.

(Refer Slide Time: 23:06)

Ideal Op-Amp analysis



$$\frac{V_0}{V_d} = A$$

$$A = \infty$$

$$V_d = \frac{V_0}{A} = \frac{V_0}{\infty} = 0$$

$$V_1 - V_2 = 0$$

$$V_1 = V_2$$

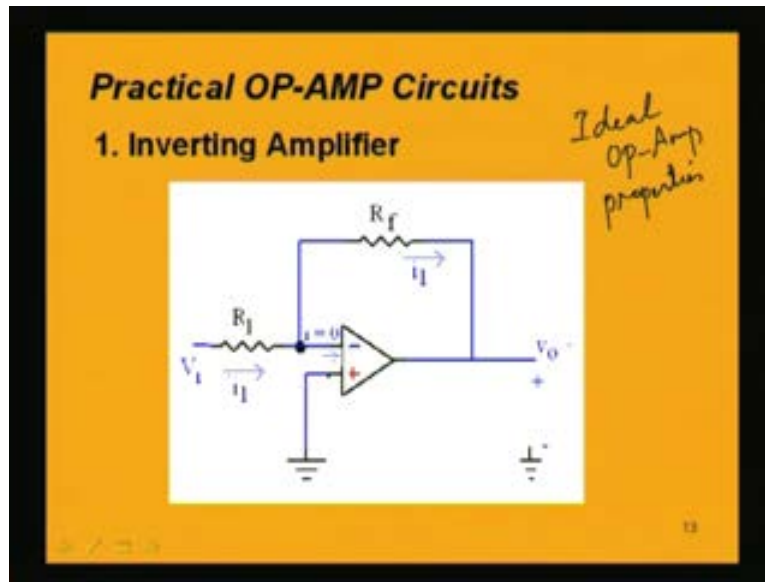
$R_i = \infty$
No current enters into op-amp

Voltage Gain $A = \infty$ or, $V_0/V_d = \infty$
 or, $V_d = V_0/\infty = 0$ [since V_0 is finite]
 $\rightarrow V_1 - V_2 = 0$
 or, $V_1 = V_2$

If we consider ideal op-amp R_i is infinity. So, let us consider this op-amp having the non-inverting voltage V_1 and inverting terminal has a voltage V_2 . Difference voltage is then V_1 minus V_2 . That is the voltage between the two terminals. V_1 minus V_2 is the difference voltage V_d and output voltage is equal to V_0 . This ideal op-amp has the consideration or the parameter R_i infinite, input resistance is infinity. If there is an input resistance here suppose R_i and that is infinity; so if R_i is infinity there cannot be any current entering into the op-amp. The first consequence of this R_i being infinity is that the current entering into an op-amp is zero. This is an important inference which we will use in all our further analysis. This is the first principle which you will have to use when analyzing an op-amp circuit. That is ideal op-amp consideration has the current entering into the op-amp as zero. As there is no current, there is no current entering into the op-amp.

Another characteristic of an ideal op-amp is that gain is infinity. If input voltage is the difference voltage V_d , output voltage is V_0 , this is the gain of the op-amp which is A . As A is infinity for an ideal op-amp and V_0 is finite, we are getting a finite voltage at the output. So, what will happen to the V_d value? V_d is equal to V_0 by A . V_0 by A again becomes V_0 by infinity which is equal to zero. What it means is that for ideal op-amp, the difference voltage is zero means V_1 minus V_2 is equal to zero. That means V_1 is equal to V_2 . For the ideal op-amp, the voltage at these two terminals inverting and non-inverting, this is V_1 , this is V_2 , these two are equal and these two principles of the op-amp that means one is the current entering into the op-amp is zero and V_1 is equal to V_2 will be extensively used in all our further analysis. We will be using these two and find out basically what we will be getting at the output voltage.

(Refer Slide Time: 26:05)



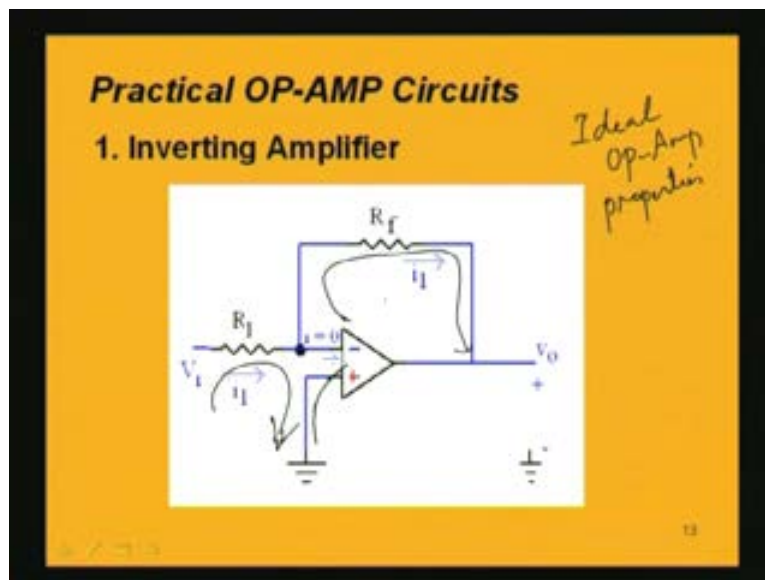
Remembering these two basic principles of ideal op-amp, let us now consider some practical op-amp circuits. The practical op-amp circuits are those circuits which we very often use and one very simplest application of op-amp is inverting amplifier. What is there in an inverting amplifier? This is an op-amp and we are applying a voltage V_1 at the inverting terminal. Inverting terminal is this one which is having a negative symbol. We are having a voltage V_1 and we are having resistance R_1 here in the input side and R_f is a feedback resistance. It is called feedback because it is from the output to the input. So R_f is called the feedback resistance. This is the circuit of an inverting amplifier using the op-amp. What will be the output voltage V_0 ?

In order to find out the output voltage V_0 , we will be using the properties of an ideal op-amp. That means we will be using the ideal op-amp considerations. For all these analysis which we are going to do now will be following ideal op-amp properties. The properties are mainly two; one is that there is no current entering the op-amp and V_1 equal to V_2 . V_1 equal to V_2 means the voltage at the inverting terminal is equal to voltage at this non-inverting terminal. But here we are applying a voltage at V_1 through a resistance R_1 which is connected to the non-inverting terminal. We will be using the first principle that there is no current entering into the op-amp because of the infinite input resistance. That

means what? The current which is flowing from V_1 , let us name it by i_1 . This current cannot go and enter into op-amp. So, where it will go? It will be following this path through R_f . That means the current coming from V_1 will be following through the part R_f and it will not enter into op-amp. If this is so, now we can very easily apply the Kirchhoff's voltage law and find out what is V_0 ? That is what we are going to do

Using the Kirchhoff's voltage law in this circuit, what is Kirchhoff's voltage law? This V_1 minus i_1 into R_1 minus i_1 into R_f minus V_0 equal to zero. We can write in another way also. What is this loop? If we consider this loop V_1 minus $i_1 R_1$ equal to zero because this point and this point has same voltage and it is grounded. If I consider this loop, we can apply Kirchhoff's voltage law in this loop. Also it is a closed loop.

(Refer Slide Time: 29:25)



It is coming from plus terminal of V_1 to ground and minus of the V_1 is also grounded. So this is a closed loop. If we apply this Kirchhoff's voltage law to this loop, it will become V_1 minus i_1 into R_1 and there is no other drop that is equal to zero because this voltage, this voltage is grounded; these are the same voltage. Another loop is starting from this ground, we can go like this in the ground and we can go like this and complete the loop. That means we can start our travel from the ground. Then what will it be? It will be

minus i_1 into R_f and then minus V_1 equal to zero. So there are two equations which we will be taking help of in order to find out V_0 . First is V_1 minus $i_1 R_1$ zero. That gives us an expression for i_1 which is equal to V_1 by R_1 .

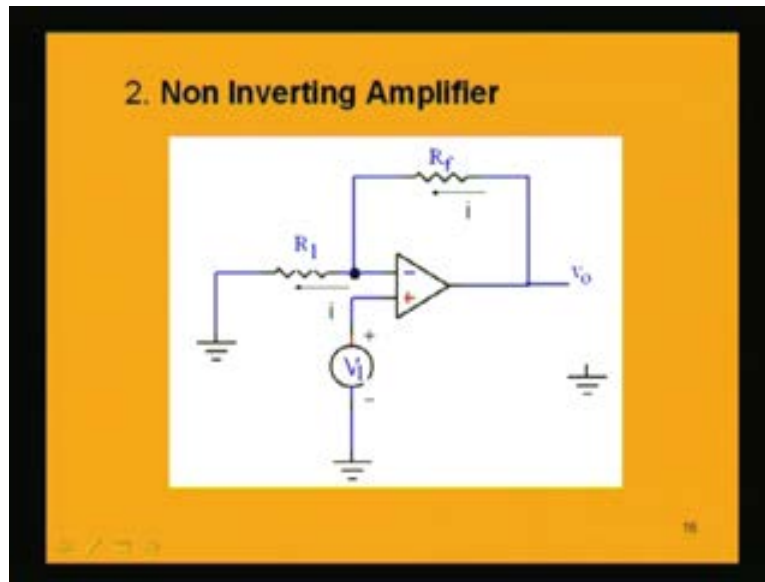
(Refer Slide Time: 30:17)

Using KVL,
 $v_1 - i_1 R_1 = 0$
 $i_1 = v_1 / R_1$
 &
 $0 - i_1 R_f - v_o = 0$
 or, $v_o = -i_1 R_f = -v_1 R_f / R_1$
 $v_o / v_1 = -R_f / R_1$

Diagram illustrating the derivation of the gain of an inverting op-amp using KVL. The equations are written on a yellow background. A curved arrow points from the expression $i_1 = v_1 / R_1$ to the term $-i_1 R_f$ in the second equation, showing the substitution process. The final result is $v_o / v_1 = -R_f / R_1$, which is also written as $-\frac{R_f}{R_1} = \frac{v_o}{v_1}$ on the right side of the slide.

The other equation which we have written from this loop is zero. You can start from ground and write this voltage as zero. This is nothing but the ground voltage is zero minus $i_1 R_f$ minus V_0 equal to zero. So what is V_0 ? Minus i_1 into R_f and minus i_1 , we can replace from here. i_1 is V_1 by R_1 . So it becomes minus V_1 by R_1 into R_f . So V_0 equal to minus V_1 by R_1 into R_f . From that we get the gain V_0 by V_1 . What is that gain? Minus R_f by R_1 . So this is the gain of an inverting amplifier using op-amp. The inverting op-amp circuit gives the gain minus R_f by R_1 . That is equal to V_0 by V_1 ; that is the inverting op-amp.

(Refer Slide Time: 31:33)



If we consider a non-inverting amplifier, as the name non-inverting suggests the voltage will be applied through the non-inverting terminal. This plus terminal is denoting the non-inverting terminal, we are connecting a voltage signal V_1 and there these two resistances R_1 and R_f are connected like this. We are to find out the voltage V_0 . In order to find out the voltage V_0 again we will be proceeding in the same way and let us denote the current i , by this current which is flowing in the R_f that is starting from this point. This direction you can choose in any way. Intuitively, I am choosing the direction from right to left because this is the ground. So it will start and end at ground. It will start from here and end at ground but there is no hard and fast rule because we are choosing the direction of current and we can choose it from left to right also; that will also give you ultimately the same result. But I am just choosing the direction as like this from right to left. Let us denote this current i which is flowing from this right point of R_f to the left, so it will be not entering the op-amp. We know that this is an ideal op-amp analysis. Current entering into op-amp is zero, so it is bound to flow in this direction to the ground. Current direction is this way. Same current will flow through R_1 to ground because there is no current into the op-amp.

(Refer Slide Time: 33:08)

$$v_o - iR_f - iR_1 = 0$$

$$\text{or, } v_o - i(R_1 + R_f) = 0$$

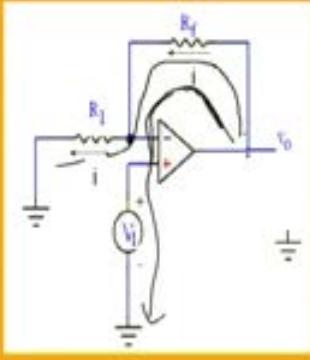
$$\text{or, } i = \frac{v_o}{R_1 + R_f}$$

↓

$$\text{Again, } v_o - iR_f - v_1 = 0$$

$$\text{or, } v_o - \frac{v_o}{R_1 + R_f} R_f - v_1 = 0$$

$$\text{or, } v_o \left(1 - \frac{R_f}{R_1 + R_f} \right) = v_1$$



We can use again Kirchoff's voltage law. What will be Kirchoff's voltage law in this loop? Starting from V_0 , V_0 minus i into R_f minus i into R_1 equal to zero because this current entering into the op-amp is zero and from that, the expression for V_0 is given as V_0 equal to $i R_1$ plus R_f because from this expression I get V_0 minus i into R_1 plus R_f , that i taken common, equal to zero. We can write down the expression for i first. i is equal to V_0 by R_1 plus R_f . The current expression we have found out. Again we can come from V_0 to the ground in this way also. That is another closed loop and that loop if we consider and apply Kirchoff's voltage law what we will get? V_0 minus V_1 equal to zero; it will not lead to a good expression, but we will use V_0 minus i into R_f minus V_1 is equal to zero. That means we will come from V_0 . We will come through R_f , through V_1 to ground.

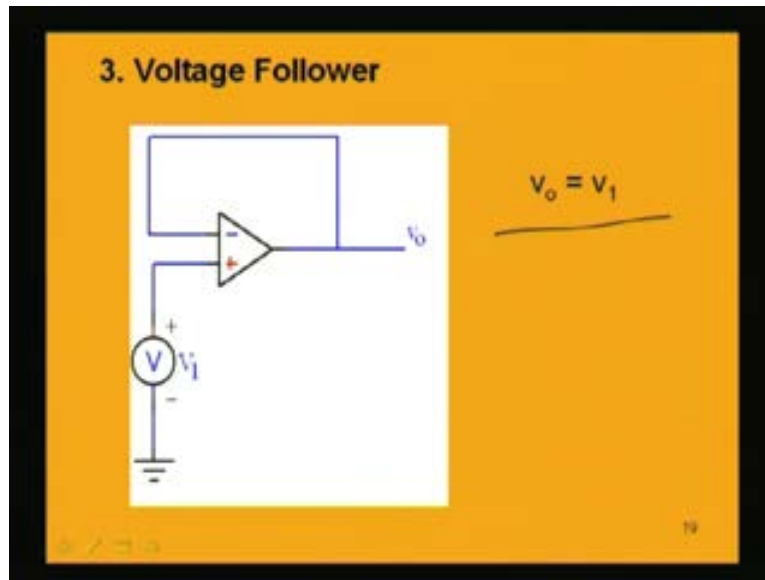
If we start from V_0 , it will be V_0 minus this drop i into R_f and I am coming down. There is no voltage drop in between these portions because these two voltages are same; so minus V_1 equal to zero. That is this expression V_0 minus $i R_f$ minus V_1 equal to zero. We have two expressions; one is for this loop and one for this loop. The second loop we are getting the expression for V_0 . Now replacing i from the earlier expression i is equal to V_0 by R_1 plus R_f into R_f minus V_1 is equal to zero. Taking V_0 common, 1 minus R_f by 1 plus R_1 plus R_f equal to V_1 transferring V_1 to the right side.

(Refer Slide Time: 35:23)

The image shows a handwritten derivation on a yellow background. It starts with the equation
$$\text{or, } V_o \left(\frac{R_1}{R_1 + R_f} \right) = V_i$$
 followed by
$$\text{or, } V_o = V_i \left(\frac{R_1 + R_f}{R_1} \right)$$
 and finally
$$\text{or, } V_o = V_i \left(1 + \frac{R_f}{R_1} \right)$$
 which is underlined.

From this expression basically we get V_o . If this expression is simplified it will be R_1 by R_1 plus R_f equal to V_i . V_o equal to, cross multiplying we get, V_i into R_1 plus R_f by R_1 . Finally we get this expression of V_o equal to V_i into 1 plus R_f by R_1 . So this is also a very important example of op-amp application which is non-inverting amplifier. Its output is given by the input voltage into 1 plus R_f by R_1 . This is for non-inverting; earlier minus R_f by R_1 was the gain for inverting op-amp and here the gain is 1 plus R_f by R_1 .

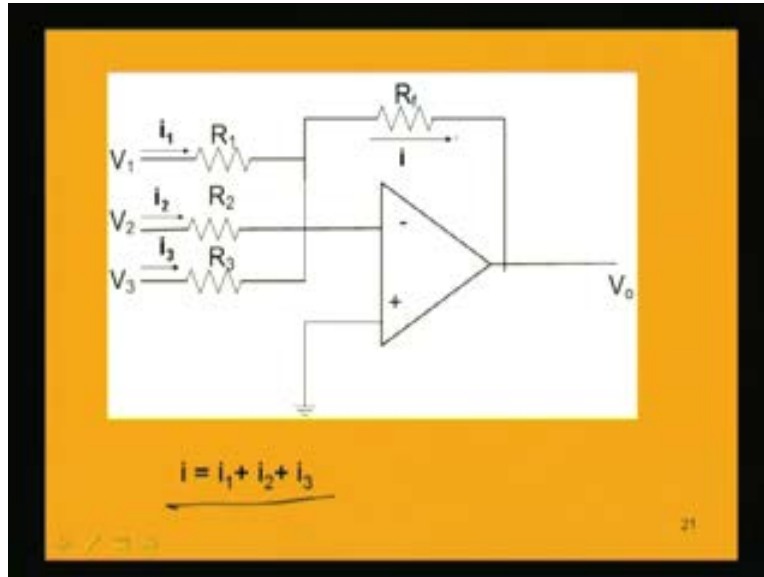
(Refer Slide Time: 36:11)



Now let us consider another application. What is this circuit? If you observe closely, output voltage we will have to find out. What is this voltage? This voltage is nothing but the voltage here. Again this voltage is equal to the voltage here and this voltage is nothing but V_1 , which is the applied voltage. So V_o is equal to nothing but V_1 and that is generally known as a voltage follower. That means whatever input voltage you are applying, output voltage is the same. Only it is used as a buffer; that means whatever input voltage we are giving, we are getting the same at the output. That is why it is called voltage follower where output and input voltages are same.

Now let us consider another application where you can sum up the input voltages. Suppose we are having a number of input voltages given to the op-amp; V_1 , V_2 , V_3 , etc. The output voltage now can sum up the input voltages V_1 , V_2 and V_3 in a scaled manner. That means a scaling will be there in terms of the resistances; not that V_o equal to simply V_1 plus V_2 plus V_3 but it will be scaled with the resistances being applied. By properly choosing the resistances we can make it as an exact summer also.

(Refer Slide Time: 37:43)



In this application, we are applying three voltages. For example I am taking only three voltages. We can extend it to a number of voltages in the same way; that is the same principle is followed. Here V_1 , V_2 and V_3 are the voltages being applied at the input having resistances R_1 , R_2 and R_3 and the feedback resistance is R_f . The plus or non-inverting terminal is grounded. We have to find out what will be the output voltage? In these types of circuits it is always better to follow from the first principle and name the currents flowing. If we name the currents flowing through the three resistances as i_1 , i_2 and i_3 and the current in this R_f is say i , then what is i ? If the current is here say i , the currents flowing in the three resistances we are naming as, say i_1 , i_2 , i_3 . As no current enters into the op-amp, i must be equal to i_1 plus i_2 plus i_3 , the sum of all the currents that must flow through this R_f only because they cannot go and enter into the op-amp. That principle we will be applying. Let us name the currents i_1 , i_2 , i_3 which flows in the three resistances R_1 , R_2 and R_3 and the current which is flowing in R_f , let us name it as i . The principle is that there is no current into the op-amp. So, i_1 plus i_2 plus i_3 is equal to i .

(Refer Slide Time: 39:23)

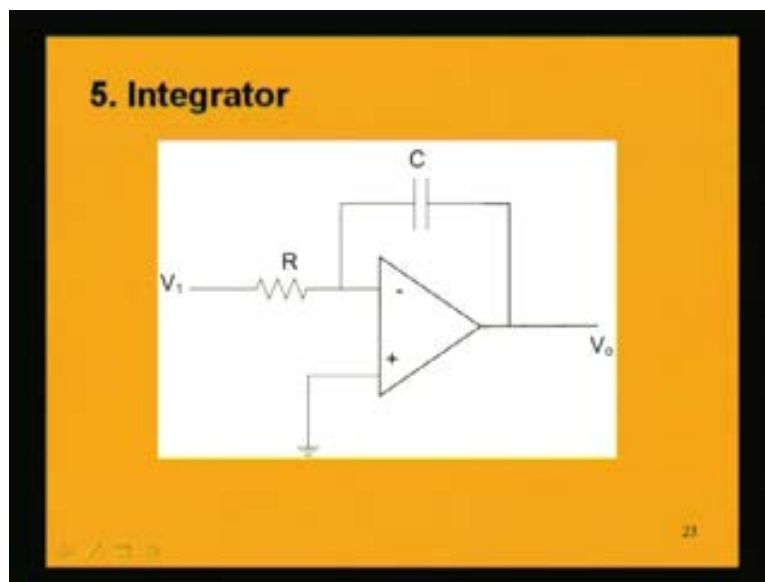
$$\begin{aligned}
 i &= i_1 + i_2 + i_3 \\
 \text{or, } \frac{0 - V_0}{R_f} &= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \\
 \text{or, } \frac{-V_0}{R_f} &= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \\
 \text{or, } V_0 &= -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right) \\
 \text{or, } V_0 &= - \left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 \right)
 \end{aligned}$$

We will find out what is i_1 , i_2 and i_3 , individual currents. What is i ? If we look into the current i , applying Kirchoff's voltage law start from ground; zero minus what is this i ? The drop between these two is zero minus V_0 by R_f is equal to i . The potential drop between these two points that is across the resistance is zero minus V_0 because this point is nothing but this point and this point is nothing but this point which is grounded. So this voltage is zero. So zero minus V_0 minus R_f that is i and what is i_1 ? V_1 minus zero by R_1 because this point is zero; so V_1 minus zero or V_1 by R_1 . What is i_2 ? Similarly V_2 by R_2 . What is i_3 ? V_3 by R_3 . So we get to the right side V_1 by R_1 plus V_2 by R_2 plus V_3 by R_3 . Simple manipulations we will do, so minus V_0 by R_f equal to V_1 by R_1 plus V_2 by R_2 plus V_3 by R_3 .

What is V_0 ? Cross multiplying with minus R_f into this whole term, so finally we get V_0 equal to minus R_f by R_1 into V_1 plus R_f by R_2 into V_2 plus R_f by R_3 into V_3 ; that is the expression for output voltage. It is like a summer. It is like summing of all the voltages V_1 , V_2 , V_3 . There is a minus sign in front because we are connecting at the inverting terminals, all these voltages. If we look here all the voltages are connected into the inverting terminal. So the minus sign will come as this will be 180 degree out of phase. But if we look into the bracketed expression it is like a summer V_1 , V_2 and V_3 are summed up with a scaled down version, by scaling by the resistances; means V_1 will be

multiplied by this ratio of R_f by R_1 . But properly choosing R_f , R_1 , R_2 and R_3 , now we can get whatever sum we desire. For example I want to get V_0 is equal to say minus $3V_1$ plus $5V_2$ plus $6V_3$, etc; some example if we take then we can determine what should be the ratio between R_f and R_1 , R_f and R_2 and R_f and R_3 and properly choosing these resistances, I can get the required sum. That is why it is called the summer. So in analog computation, when summing up is required, MS, in an analog computer where you are summing up the analog voltages, that is done by this type of circuits.

(Refer Slide Time: 42:25)



Another application is an integrated circuit. We want to integrate a voltage. Integration, differentiation, these are the mathematical operations we want to perform by using analog computation. Here it is analog amplifier. We are not considering digital, we are considering an op-amp where the whole operation is analog. So in analog computation, the summer, subtractor, integrator, differentiator, etc can be performed using op-amp and IC 741 is an IC integrated circuit where you get this op-amp. If we want to perform an integration operation on a signal; suppose I want to integrate V_1 with respect to time that I can do with a circuit like this where there will be a resistance and capacitance and they are connected to the op-amp. If we want to find out V_0 given that the voltage V_1 is

applied at the inverting terminal and a resistance is there, which is R and this capacitance is there across this op-amp which is C. What will be V_0 ?

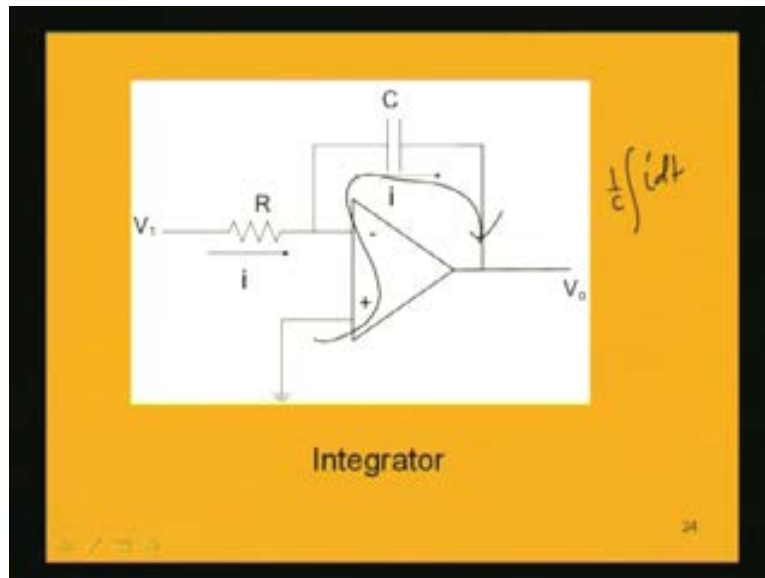
In order to know V_0 , now we will use similar analysis, the way which we followed earlier. We will name the current. Suppose input current i , flowing from V_1 cannot enter into op-amp; so the same i will flow out through C.

(Refer Slide Time: 44:07)

$$\begin{aligned}v_1 - iR &= 0 \\ \text{or, } i &= \frac{v_1}{R} \\ \text{Again, } 0 - \frac{1}{C} \int i dt - v_0 &= 0 \\ \text{or, } v_0 &= -\frac{1}{C} \int i dt = -\frac{1}{C} \int \frac{v_1}{R} dt \\ \text{or, } v_0 &= -\frac{1}{RC} \int v_1 dt\end{aligned}$$

Now if we apply Kirchoff's voltage law, there will be two loops which we can consider. First loop if we consider, this V_1 minus i into R is equal to zero; that is one equation we get and from that we get the value of i and that is equal to V_1 by R . Again if we consider the other loop which is flowing from this ground that will be zero minus i into R minus V_0 ; V_1 minus iR is equal to zero and the other is from here if we consider zero minus the voltage across this capacitance 1 by C integration idt zero minus 1 by C integration idt is the voltage across this capacitor.

(Refer Slide Time: 45:06)

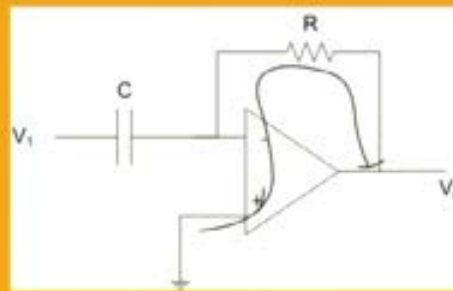


So the voltage drop across this capacitor is $\frac{1}{C} \int i dt$. If I proceed from ground and apply Kirchhoff's voltage law, zero minus $\frac{1}{C} \int i dt$ minus V_0 equal to zero. This is the equation. From here we get V_0 which is equal to minus $\frac{1}{C} \int i dt$. Now I can replace i which is equal to $\frac{V_1}{R}$. Doing that we get minus $\frac{1}{C} \int \frac{V_1}{R} dt$. Finally what is V_0 ?

V_0 equal to what we get? R can be taken out, $\frac{1}{R}$ can be taken out, it is outside the integral. We can take out, it is the constant. So minus $\frac{1}{RC} \int V_1 dt$. That is the value of V_0 . It is out of phase. Minus sign is there that means it will be out of phase with V_1 signal. But if we consider this expression of V_0 , it is nothing but integration of $V_1 dt$ is there so we are integrating V_1 . Basically we are getting an output which is integrating V_1 . This $\frac{1}{RC}$ term is there, it will be scaled down by $\frac{1}{RC}$. It is integrating the input voltage so that is the operation being performed by the connection of this op-amp in such a way with R and C connected in this fashion that we finally obtain the integration operation.

(Refer Slide Time: 46:38)

6. Differentiator



Similarly we can get a differentiation operation. If we want to differentiate a voltage V_1 , then the positions of R and C will be just interchanged. Here it was C here and R here. We will interchange C and R ; C will come to the input and R will be across this op-amp.

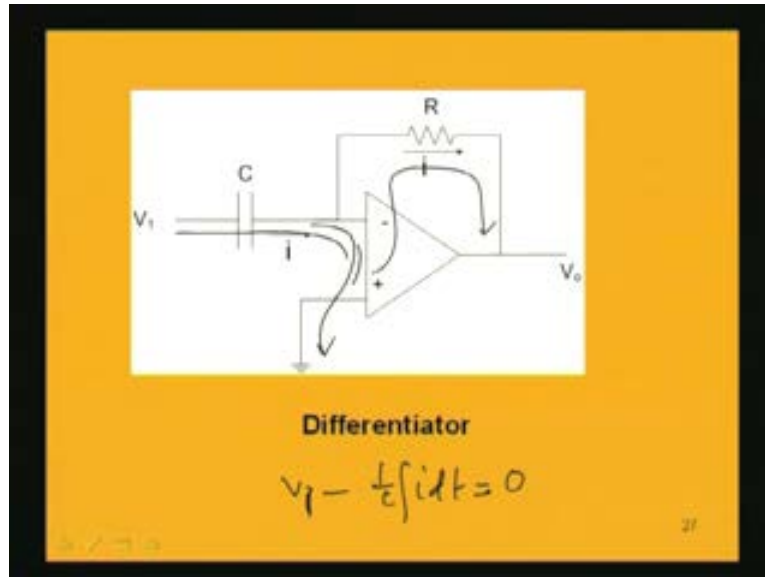
(Refer Slide Time: 47:04)

$$\begin{aligned}
 v_1 - \frac{1}{C} \int i dt &= 0 \\
 \text{or, } v_1 &= \frac{1}{C} \int i dt \\
 \text{or, } \frac{dv_1}{dt} &= \frac{1}{C} i \\
 \text{or, } i &= C \frac{dv_1}{dt} \\
 \text{Again, } 0 - iR &= v_o \\
 \text{or, } v_o &= -C \frac{dv_1}{dt} R \\
 \text{or, } v_o &= -RC \frac{dv_1}{dt}
 \end{aligned}$$

What will be the output voltage? Proceeding similarly, we will be get by applying the Kirchoff's voltage law again. What is this Kirchoff's voltage law being applied? That is V_1 minus the drop across this capacitance. That is this voltage drop across this

capacitance is V_1 by C integration idt. So V_1 minus 1 by C integration idt, you come down to ground. That is V_1 minus 1 by C integration idt equal to zero.

(Refer Slide Time: 47:35)

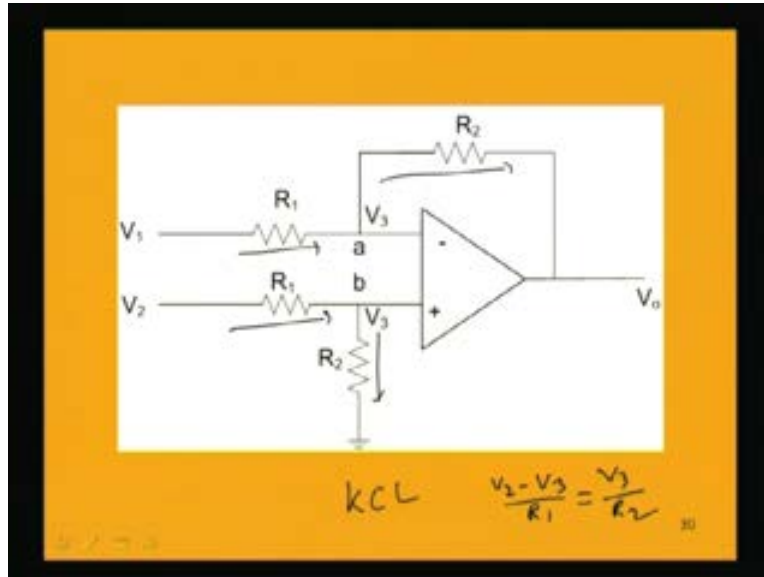


This is one equation when you come to ground starting from here. Another is, you go in this direction. That will give you zero minus i into R minus V_0 equal to zero. So this is the other equation. Manipulating this equation a little, take the differentiation on both the sides; dV with respect to time, dV_1 by dt equal to 1 by C . dt of integration idt means it will be simply free in the term i . So it will be 1 by C into i . What is i ? C into dV_1 by dt .

From the second equation, we get what is V_0 ? Zero minus iR ; that is equal to, i can be replaced from here; C into dV_1 by dt into R . So V is equal to minus RC dV_1 by dt . If we look into this expression, this is nothing but it is differentiating the voltage V_1 . We get a differentiation of the signal V_1 by this connection of this op-amp to the resistance and capacitance in this way and in the earlier case, we were getting a integration operation. These two are examples where the op-amp is being used in a circuit which will give you integration as well as differentiation. These mathematical operations can be performed by the op-amp.

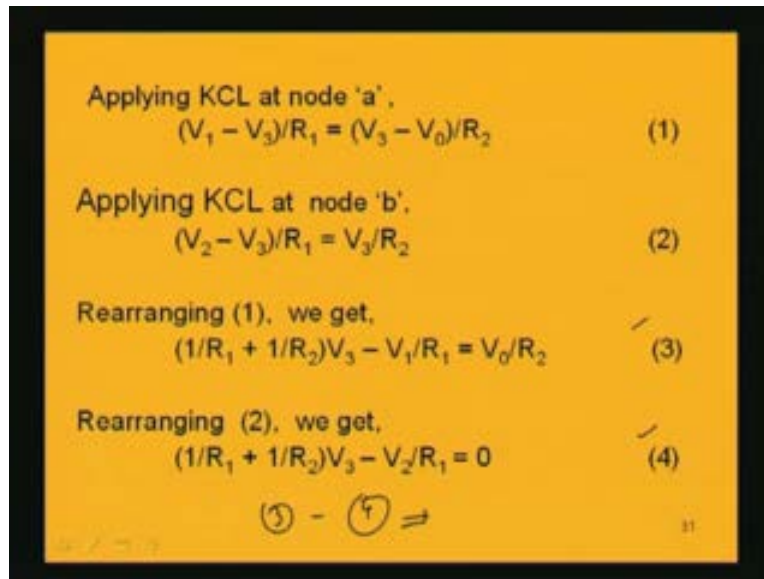
Let us take another example, which will show how the subtraction operation can also be done. We have done addition, we have done integration and we have done differentiation with op-amp. Now subtraction or difference between signals can be found out.

(Refer Slide Time: 49:35)



Here the two signals being applied are V_1 and V_2 , the resistances connected are R_1 and R_2 . First we denote the node voltages which are at the nodes, a and b. For our easy analysis, we will name the nodes as a and b that is the inverting and non-inverting terminal node voltages are denoted by V_3 because we know V_3 the voltage at node a is equal to voltage at node b. Both the voltages are equal. So let us name it as V_3 . This V_3 , we are introducing just to easily analyze the circuit. Because we have the nodes we can apply Kirchoff's current law.

(Refer Slide Time: 50:30 min)



Applying KCL at node 'a',
$$(V_1 - V_3)/R_1 = (V_3 - V_0)/R_2 \quad (1)$$

Applying KCL at node 'b',
$$(V_2 - V_3)/R_1 = V_3/R_2 \quad (2)$$

Rearranging (1), we get,
$$(1/R_1 + 1/R_2)V_3 - V_1/R_1 = V_0/R_2 \quad (3)$$

Rearranging (2), we get,
$$(1/R_1 + 1/R_2)V_3 - V_2/R_1 = 0 \quad (4)$$

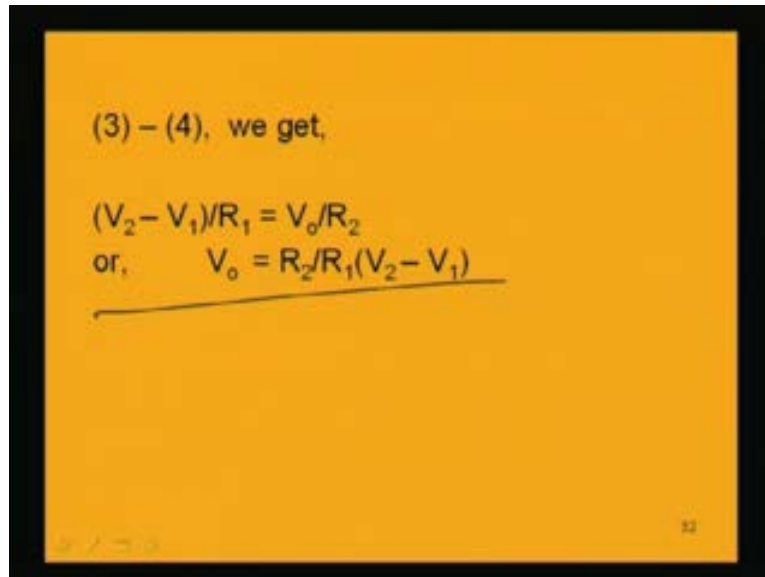
$(3) - (4) \Rightarrow$

What is Kirchoff's current law? It is the algebraic sum of currents at a node equal to zero. Algebraic sum means plus and minus depending upon what direction it will be. We will consider node a and we will apply Kirchoff's current law. The Kirchoff's current law being applied at node a, the sum of incoming current is equal to the sum of outgoing currents. So V_1 minus V_3 by R_1 that is incoming, this direction and that V_1 minus V_3 by R_1 must be equal to the, this is the incoming, the outgoing which is V_3 minus V_0 by R_2 . So that is equal to V_3 minus V_0 by R_2 . This is applying Kirchoff's current law at node a. Applying Kirchoff's current law at node b, V_2 minus V_3 by R_1 is the incoming current. Which one is the outgoing current? There cannot be current going into the op-amp because this is ideal op-amp consideration. The current which will be coming out from terminal b is equal to V_3 minus zero by R_2 because this part is grounded; that is equal to V_3 minus zero by R_2 means V_3 by R_2 . So we get V_2 minus V_3 by R_1 is equal to V_3 by R_2 .

These two are the key equations, which will be rearranged. Rearranging equation 1, transferring this term to the other side, take V_3 common. It will be 1 by R_1 plus 1 by R_2 and then minus V_1 by R_1 that is equal to V_0 by R_2 . I am transferring this to the right side and the term having V_0 by R_2 to the left side. This equation 3 is the rearranged form of equation 1. Similarly rearranging equation 2 I get, taking common V_3 , V_3 into 1 by R_1

plus 1 by R_2 minus V_2 by R_1 equal to zero. Now if we look into the two equations 3 and 4, we can very easily get rid of the term V_3 just by subtracting 4 from 3. If we subtract the equation 4 from equation 3, what will it give us?

(Refer Slide Time: 53:33)



(3) - (4), we get,

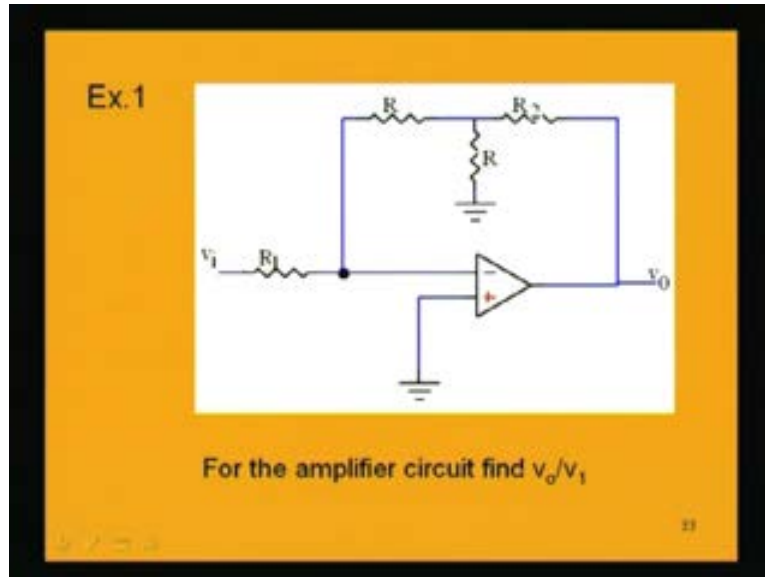
$$(V_2 - V_1)/R_1 = V_0/R_2$$

or, $V_0 = R_2/R_1(V_2 - V_1)$

These two terms will go; it will be positive. So we get V_2 by R_1 positive minus V_1 by R_1 equal to V_0 by R_2 . From this we get what is V_0 ? Cross multiplying by R_2 we get V_0 equal to R_2 by R_1 into V_2 by V_1 . If you now observe this expression what we get is that we are basically having a difference between the two voltages V_1 and V_2 . So V_1 and V_2 were the voltages which were applied. V_2 is the voltage which was applied to the non-inverting terminal, V_1 was the voltage applied to the inverting terminal. V_2 minus V_1 this algebraic difference is obtained at the output. It is multiplied by a factor R_2 by R_1 where R_2 and R_1 are the resistances which were connected in the circuit. One thing that is clear is that we can get this difference operation also from a circuit connecting the op-amp properly with resistances. The factor R_2 by R_1 can be chosen; if we choose the values of R_1 and R_2 appropriately, then we can get the factor you want to multiply that difference or even if you want to make R_2 and R_1 equal that will cancel and we will get only the absolute difference which is V_2 minus V_1 if R_2 equal to R_1 . In this way we have seen that we can get a difference operation between two voltages also done.

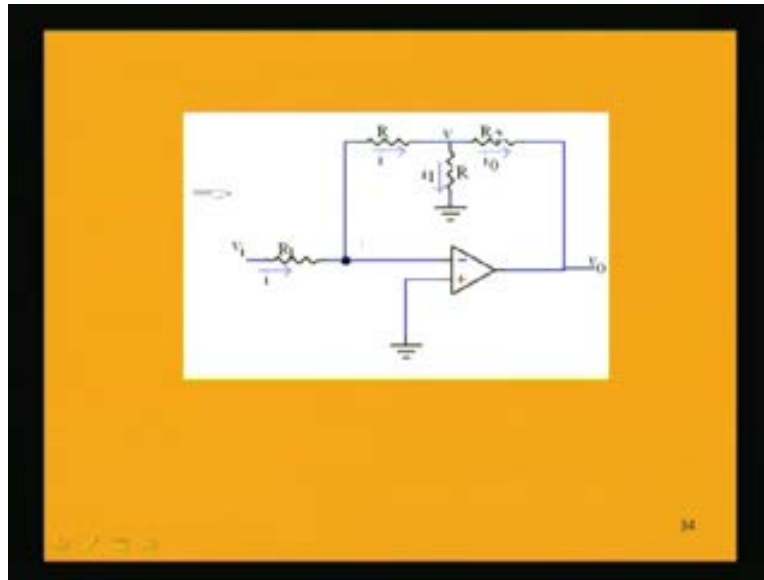
Now let us do one example to sum up all the discussions today.

(Refer Slide Time: 55:18)



In this circuit you have to find the ratio between V_0 and V_i or V_1 that is the input voltage V_1 . What will be the value of V_0 by V_1 ? Given that the resistances are connected in such a fashion, this is R_1 which is connected to V_1 , this is R , R_2 , this is also R . To analyze such a type of circuit the easy way which we have been following we will still adopt, the same way of naming the currents. Input current let us name by i . If this current is I , there cannot be any current into the op-amp following ideal op-amp consideration.

(Refer Slide Time: 56:06)



The current has to go in this direction, this current i has to go in this direction. So i will be following in the direction through R at this point. I am naming this voltage at this point as V . This voltage I am naming for easy analysis as V . So at this point the current will be divided into two parts. One will go to the R to ground downwards which is say i_1 , I am naming and the other is i_0 which is flowing through R_2 . So there are divisions of current at this point.

(Refer Slide Time: 56:43)

Solution :

$$i_0 = i - i_1 = \frac{V}{R_1} - \frac{V}{R} = \frac{V}{R_1} - \left(\frac{-iR}{R} \right)$$

$$= \frac{V}{R_1} + i = \frac{V}{R_1} + \frac{V}{R_1} = \frac{2V}{R_1}$$

$$V_o = V - i_0 R_2$$

$$= -iR - 2V_1 R_2 / R_1$$

$$= -V_1 R / R_1 - 2V_1 R_2 / R_1$$

or, $V_o = -V_1 (R + 2R_2) / R_1$

Hence,

$$V_o / V_1 = -(R + 2R_2) / R_1$$

$R = R_1 = R_2$

$$= - \left(\frac{R + 2R}{R} \right) = - \frac{3R}{R} = -3$$

Applying the Kirchoff's current law or voltage law which ever is suiting, we will now know what is V_0 ? Following in that way now what is the current i_0 ? We can write down that i_0 current is equal to i minus i_1 . That is current division is taking place, so let us find out what is i_0 ? I_0 is equal to i minus i_1 . What is i and what is i_1 . From here I can see what is i ? i is nothing but V_1 by R_1 ; so V_1 by R_1 is i because at this end, voltage is zero. So V_1 minus zero by R_1 is i . What is i_1 ? V minus zero by R or V by R . Replacing this V_i by R_1 , V_i input voltage let us take it as V_i . I am naming it as V_i because this analysis I am doing with V_i . So V_i by R_1 minus V by R that is equal to i_0 and that is equal to V_i by R_1 minus ..

What is V ? If we see what is V , V can be found out if we start from zero. Zero minus i into R is equal to V . Zero minus i into R is equal to V because that is evident from Kirchoff's voltage law being applied like this from this terminal; zero minus i into R is equal to V . So V is nothing but minus iR . Replacing this V by minus iR divided by R is already there what we get is equal to V_1 by R_1 plus, minus, minus plus, this R , this R cancel. So V_1 by V_i by R_1 plus i we get. Again we can write that equal to V_i by R_1 plus, this i is nothing but V_i by R_1 ; we have already seen that. So replacing this i_1 by V_i by R_1 what we get? This is 2 times V_i by R_1 is the expression for i_0 . What is V_0 ? We can write down V_0 as V minus i_0 into R_2 . V voltage here minus this drop equal to V_0 . So V_0 equal to V minus i_0 into R_2 . What is V ? Minus iR we are replacing. What is i_0 ? We have found out 2 times V_i into R_2 by R_1 . This i_0 , I am replacing by 2 times V_i by R_1 into R_2 . Again i is nothing but V_i by R_1 into R minus 2 times V_i R_2 by R_1 . Simplifying this expression taking common minus V_i by R_1 we get R plus 2 times R_2 . If we find the ratio between V_0 and V_i that is required, V_0 by V_i is required we get V_0 by V_i is equal to minus R plus 2 R_2 divided by R_1 . This expression is in terms of the resistances which are connected in the circuit and we have got that the ratio between the output and input voltage equal to minus into the resistances R plus $2R_2$ by R_1 . These resistances which are connected in the circuit we are taking these resistance values as this is R , this is R , this is R_1 and this is R_2 .

Now if we choose particular values of these resistances, actual values which we will be using, it will give say 1k, 2k, etc, whatever values we apply. Then your voltage ratio will be in terms of those values because you can see here if all are equal suppose R is equal to

R_1 equal to R_2 , so what we will get? Minus R plus $2R$ divided by R and that is equal to minus $3R$ by R , so -3 . So V_o by V_i becomes -3 . That means output voltage will be -3 times the input voltage. It will be out of phase with the input voltage and it will be amplified 3 times. If we have a signal which is 1 millivolt, we will get 3 millivolt but minus that is out of phase with the input voltage. So properly choosing these resistance values, whatever, we are connecting in this circuit; this is the typical circuit, where the ratio between these two voltages that is output and input voltage will be determined by the values of the resistances which you are connecting. Properly choosing these resistance values in terms of what you want to get, how much ratio between the two voltages you want to get in that way you will be choosing these resistances.

For example just I have shown that if we want to get output voltage to be 3 times the input voltage with a minus sign because we are connecting the voltage at the inverting terminals. We will have to choose all resistances equal that will give you 3 times. In this way you can choose the resistance values and you can find whatever required output voltage you want in comparison with the input, how many times you wanted to be greater.

In today's discussion we have seen basically the ideal op-amp characteristics or properties of an ideal op-amp that it should have infinite input resistance and voltage gain should be also infinite and we have used these properties extensively in finding out the output voltages for different various types of applications. Practical op-amp applications we have seen starting from inverting, non-inverting op-amp, up to summing or difference amplifiers, also integrator or differentiator etc. In all these practical op-amp applications we have used the ideal op-amp analysis and we have used the principle of an ideal op-amp that input current entering into an op-amp is zero and the gain is infinite that is why the two voltages at the input terminals, that is negative and positive terminals, these two voltages are equal. These analyses we have extensively used. So the analog computation can be performed using an op-amp in a proper combination of resistances as well as capacitors and we can get the different analog computation starting from summing,

difference up to integration, differentiation, etc. This is the versatility of op-amp which gives us analog computation.