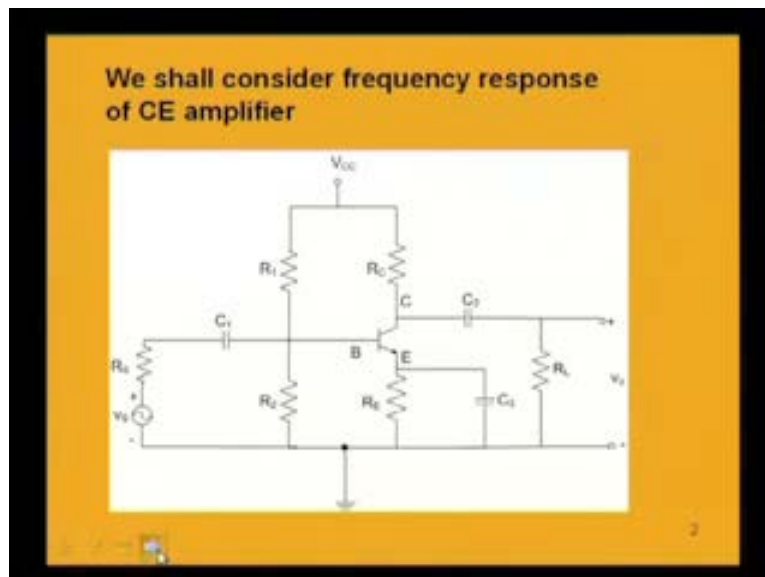


**Basic Electronics**  
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**Module: 2 Bipolar Junction Transistors**  
**Lecture-8**  
**Frequency Response of BJT Analysis - Part-1**

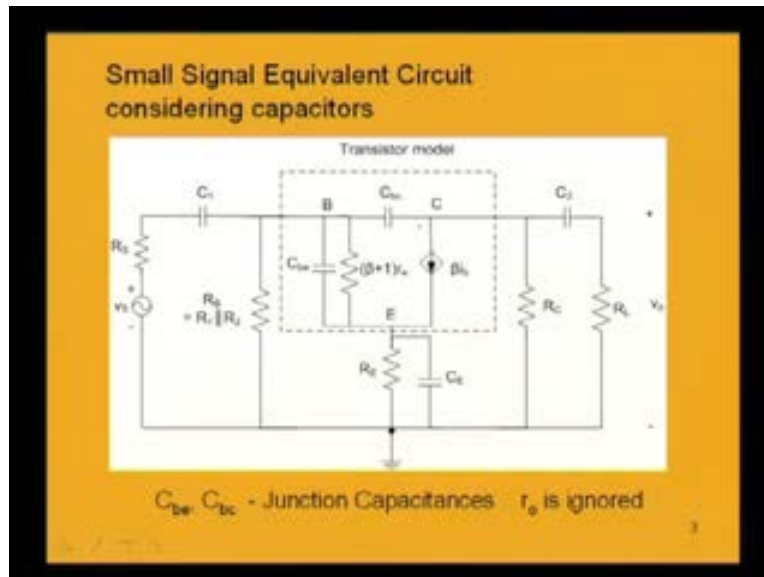
Today we will discuss how the frequency of the applied signal has an effect on the voltage gain of a BJT amplifier. In the last classes we were discussing about the analysis of a BJT amplifier but we were considering that all the capacitances in the amplifier circuit were short circuited. But then this condition is only true for the signal having frequency not very low or not very high; that is in the mid frequency range. But if the frequency of the applied signal is low or high then the voltage gain of the amplifier will not be the same as we discussed earlier because then the analysis has to be done taking into consideration the capacitances which are present in the amplifier circuit. If we recall what we were discussing earlier, the amplifier circuit having capacitances which are coupling capacitors, these capacitors under DC they are open circuit and under signal being applied or under AC condition these were assumed to have a short circuit condition. Because in the frequency at which the signal was applied, at that frequency the capacitive reactance offered by these coupling capacitors were small enough since we have selected high value of the capacitances so that effectively we can very well assume that the capacitive reactance is very, very small and this is like a short circuit part. We revisit the amplifier circuit again and we will consider only the common emitter amplifier circuit.

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Here in the common emitter amplifier circuit, if we look back, the circuit was having the coupling capacitor  $C_1$ ,  $C_2$  and there is an emitter bypass capacitor which is  $C_3$ . When the small signal equivalent circuit is drawn considering these capacitors also then we will get a circuit where  $C_1$ ,  $C_2$  and  $C_E$ ,  $C_E$  being the bypass capacitor, are there. But apart from these three capacitors we also have 2 other capacitances which are  $C_{be}$  between base and emitter and  $C_{bc}$  between base and collector. These two capacitances are coming from the transistor model because when we were discussing the diodes if we remember, the diodes are having a diffusion capacitance in the forward biased condition and a transition capacitance which is prevalent in the reverse biased condition. If we look into the transistor, the transistor is having a forward biased diode and the reverse biased diode, the forward biased diode being between the base and emitter junction and the reverse biased junction is between the collector and base junction.

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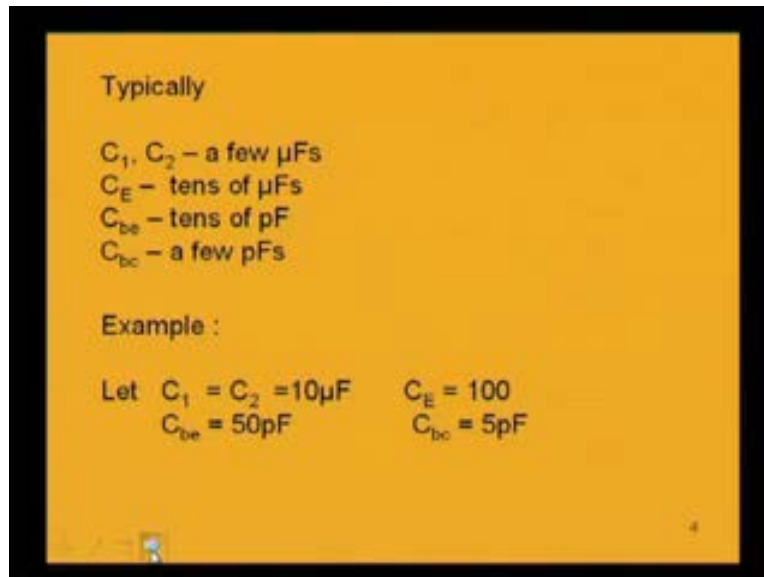


These capacitances will have to be taken into consideration. That is why the transistor model will be having  $C_{be}$  that is the diffusion capacitance in the forward biased condition of the diode, which is the emitter base junction and  $C_{bc}$  is the junction capacitance which is in the reverse bias diode between collector and base. Taking this into account and considering all these capacitances which are present in the amplifier circuit, the coupling capacitor as well as the emitter by pass capacitor, the whole amplifier circuit will be like this. Here all the others are simply what we considered earlier and the extra are only these capacitances. In this circuit we are having the capacitances. We cannot neglect this capacitances or the effect of this capacitances when the frequency of the applied signal that is  $V_s$  is not in the mid frequency range; it is either too small or too high.

What will happen if the signal frequency is in the mid frequency range? What we discussed earlier let us again go into that discussion. We know that typical values of these capacitances are generally  $C_1$  and  $C_2$ , the coupling capacitances are a few microfarads

and the bypass capacitor which bypasses the emitter resistance  $r_e$  is in tens of microfarads, a bit higher than  $C_1$  and  $C_2$  but it is in the microfarad range.

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The transistor capacitances  $C_{be}$  and  $C_{bc}$  these two are in the order of Pico farads.  $C_{be}$  is typically tens of Pico farads and  $C_{bc}$  is typically a few Pico farads. With this concrete example now let us see the effect of frequency of the applied signal on these values of reactances offered by these capacitances. For example we will take values of  $C_1$  and  $C_2$  to be say 10 microfarad,  $C_E$  is 100 microfarad and  $C_{be}$  is in the Pico farad range, let us take 50 Pico farad and  $C_{bc}$  is 5 Pico farad. With these typical values let us find out what will be the reactances offered by these capacitance when the signal frequency is say 1 kilo hertz that is the mid frequency range; it is neither too small nor too high frequency. 1 kilo hertz is the frequency of the signal which is applied at the input.  $V_S$  is having a frequency of 1 kilo hertz. At 1 kilo hertz value of frequency what will be the capacitances offered by each of this? Let us calculate what will be the capacitive reactances offered by each of this capacitance?

Coupling capacitors are offering a capacitive reactance  $X_{C1}$  and  $X_{C2}$  which are equal because we are taking equal values of  $C_1$  and  $C_2$  which is 10 microfarad. We know that  $1 / (2\pi fC)$  is the value of reactance offered by a capacitance.  $\omega$  is  $2\pi f$ ;  $1 / \omega C$ . We are considering the magnitude. Let us not think about the 'j' part. Actually it is having two components. One is the real and other one is imaginary but let us take the value of  $1 / (2\pi fC)$ . Plugging in the values of this  $C_1$  and  $C_2$  in this expression,  $C_1$  and  $C_2$  value is 10 microfarad.  $1 / (2\pi f)$  is 1 kilo hertz, so  $10$  to the power  $-6$ . We are bringing in hertz order and 10 into 10 to the power  $-6$  because it is microfarad. We calculate this value and it comes to around 15.9 ohm. This value is quite small value of reactance.

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At "mid-frequencies" say,  $f = 1\text{kHz}$

$X_C = \frac{1}{2\pi f C}$

$X_{C1} = X_{C2} = \frac{1}{2\pi \times 10^3 \times 10 \times 10^{-9}} = 15.9\Omega \rightarrow$  Short Circuit

$X_{Ce} = \frac{1}{2\pi \times 10^3 \times 100 \times 10^{-6}} = 1.59\Omega \rightarrow$  Short Circuit

$X_{Cbc} = \frac{1}{2\pi \times 10^3 \times 50 \times 10^{-12}} = 3.18\text{M}\Omega \rightarrow$  Open Circuit

$X_{Cbc} = \frac{1}{2\pi \times 10^3 \times 5 \times 10^{-12}} = 31.8\text{M}\Omega \rightarrow$  Open Circuit

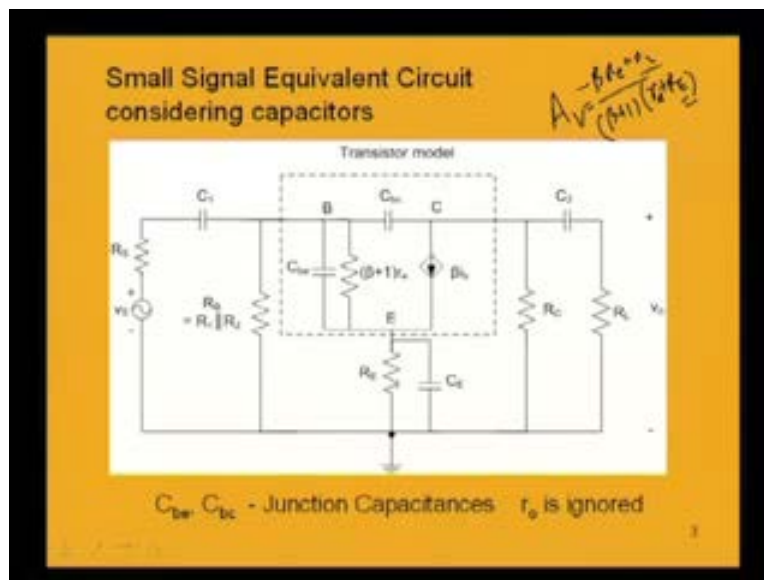
Similarly we calculate this capacitive reactance offered by the emitter bypass capacitor  $X_{CE}$ . Calculating 1 by 2 pi into 10 to the power 3 into, it is 100 microfarad; so it is 1.59 ohm. If we look into these values it is almost like short circuiting the part since these are very small. These values being very small we were very legitimate in assuming that the capacitances were like short circuiting when we were considering the amplifier earlier. We were earlier considering that capacitances  $C_1$ ,  $C_2$  and  $C_E$  are such that they are quite large. In the frequency at which we are considering we can very conveniently assume them to short circuit the part and that was what we were doing earlier and now we are clear that our assumption was not wrong because these values are quite small. We can very conveniently assume them to be short circuiting.

What about the transistor capacitances which are present? Let us calculate the reactances they offer. 1 by 2 pi into f 10 to the power 3 hertz and the value of this diffusion capacitance is 50 Pico farads. Pico farad means 10 to the power -12 farad. It is even smaller value in the denominator, so at this frequency of operation the overall value of this capacitive reactance becomes 3.18 mega ohm. Similarly the capacitance in the collector base junction of the transistor which is  $X_{Cbc}$ , is equal to 1 by 2 pi into 10 to the power 3, 5 Pico farads is the value we are taking which is the typical value. That comes to 31.8 mega ohm. If we look into these two values of  $X_{Cbe}$  and  $X_{Cbc}$  that is the reactances offered by the capacitances present in the transistor model, these are very, very large values and they are in the order of mega ohms. Effectively these are like open circuit. If we look back into the circuit these capacitances are offering such a higher value of reactance that they are almost like open circuit. Since the value of this capacitive reactances are very high, they will practically allow no current to flow through them as if they are just like open circuit. That is why these capacitances were nowhere in picture earlier when we were considering the transistor amplifier in the mid frequency range. We were having a very simplified circuit without these capacitances.

Now we consider the frequency of the applied signal not at mid frequencies or 1 kilo hertz range but at lower frequency. When the frequency is low what will happen to  $X_{C1}$  and  $X_{C2}$ ?  $1 / 2\pi fC$ , this frequency is smaller value. If this frequency is small then the value of  $X_{C1}$  and  $X_{C2}$  will be higher than the earlier mid frequency range. If we consider low frequency signal then the capacitive reactances being offered are significantly high. Similarly the  $X_{C_E}$  also will be significantly higher now because this frequency is small. So these values will be high.  $X_{C1}$ ,  $X_{C2}$  and  $X_{C_E}$  values will be higher now. If these values are higher what will happen to the signal in the circuit? Because we are now having a considerable reactance in this capacitance as well as this capacitance and this capacitance, all three will be offering significant reactance. There will be a higher drop in the capacitances. The signal which is applied here will be having a drop here. Similarly the signal which is available here will be having a drop here. The output signal will be having a lesser value because of these drops occurring at these capacitances.

Similarly the capacitance in the bypassing part of this emitter resistance  $R_E$ , as this capacitance is no longer short circuit, it will not be effectively short circuiting. It will be lesser effective in providing the short circuit part. That is why we will have a current flow in this resistance  $R_E$  also, as a result of which the overall voltage gain of this amplifier will be reduced. The voltage gain of an amplifier is equal to  $\beta R_C$  parallel  $R_L$  divided by  $\beta + 1$   $r_e$  when this capacitance is short circuiting. But when this capacitance is not short circuiting then this resistance and this resistance will be coming together in series. In the denominator of this voltage gain expression we have this voltage gain minus  $\beta R_C$  parallel  $R_L$  divided by  $\beta + 1$ , it will be small  $r_e$  plus capital  $R_E$  because this  $R_E$  is coming in the denominator.

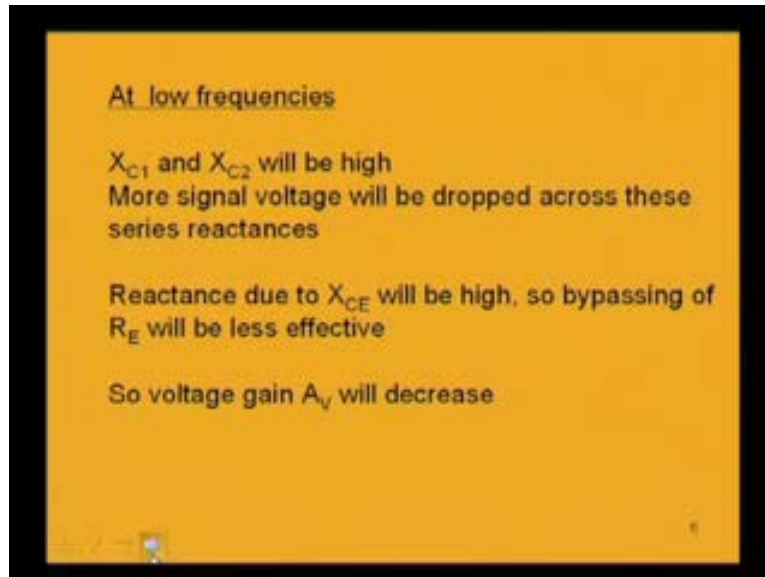
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If denominator is higher overall voltage gain will be lesser. Due to these two factors one being that there will be voltage drop here in these two capacitances, your overall voltage gain which is the ratio between output voltage and input voltage or if we consider the

overall voltage gain with respect to the signal  $AV_s$  it is  $V_O$  by  $V_s$ . This  $V_O$  without this capacitance will be higher than  $V_O$  with capacitance. If there is a capacitance there will be voltage drop here. So the output voltage that we will get is less. We will have a lesser voltage gain. So voltage gain reduces when the signal frequency is low.

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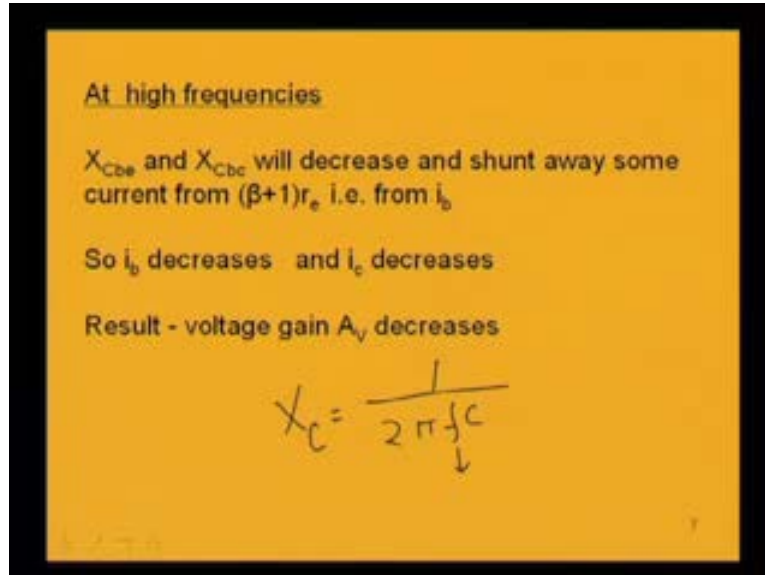


The other capacitances which are present in the transistor, because of this diffusion and transition capacitances this cannot affect because the overall capacitance or the capacitive reactance offered by these capacitances is  $1/2\pi fC$ . Frequency is smaller, so the reactance will be even higher. It will be even higher than the mid frequency range. If this capacitive reactance is even higher, they are more and more like open circuit and they will not come into the picture because the current cannot flow through these two capacitances. In the low frequencies the parameters which are going to affect the voltage gain are the capacitances  $C_1$ ,  $C_2$  and  $C_E$  that is the coupling capacitors and the bypass capacitor.

Now let us consider a higher frequency region. At high frequencies what will happen is that the transistor capacitances  $C_{be}$  and  $C_{bc}$  will offer a lesser reactance. This frequency is higher means the capacitive reactance will now decrease.



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Earlier it was like open circuit, very high. Now it will decrease. When the capacitive reactance is decreasing it will allow more and more current to flow. A part of the current will now flow through the diffusion as well as the transition capacitances and the effect is that they will shunt away a part of the current  $i_b$ . If we look into the circuit again they are these two capacitances. The current coming into this point, the base point without these two capacitances, this current would have flown through these resistances beta plus  $1r_e$ . But now a part of the current is being shunted away by this  $C_{bc}$  and  $C_{be}$  because their capacitive reactances are now low. Earlier it was almost like open circuit, very high. The reactance was in the order of mega ohm. It was not possible for current to flow through them. But now a part of the base current will be shunted away by these two capacitances because they are offering lesser and lesser reactance as a result of which the base current what would have been there, had there not been these capacitances present is now less. Base current is less means collector current is also less, beta times  $i_b$  will be less.

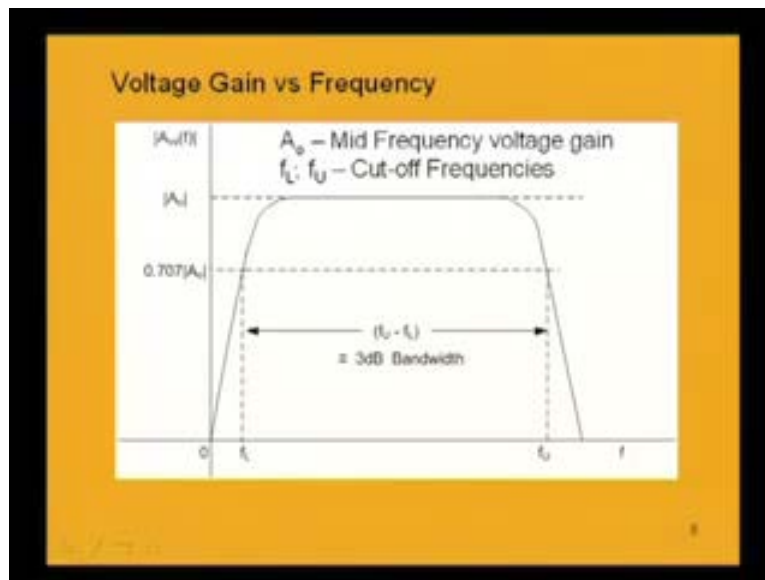
If the collector current is less the voltage gain will be now lesser because the voltage gain will depend upon the output voltage by input voltage. Output voltage is the current through this resistance  $R_L$  multiplied by this resistance  $R_L$  and this current is coming from this current source which is the dependent current source beta times  $i_b$ . If  $i_b$  is less then this dependent current source is providing less current; this current will also be less, so overall voltage gain will be less. From this analysis we have seen that in both the cases when the frequency is low as well as high, the voltage gain of the amplifier is lowered. We cannot ignore the effect of frequency of the signal being applied at the input on the overall voltage gain of the amplifier.

Today we will discuss the frequency response of an amplifier. We will analyze how the gain is reduced with frequency being low or frequency being high and once again at high frequencies the coupling and bypass capacitors will not affect because if the frequency is higher then their values will be smaller and smaller. It will be more and more short

circuiting type because of this fact that it is  $\frac{1}{2\pi fC}$ . If  $f$  is high, the overall capacitance offered by this coupling as well as bypass capacitor will be lesser and lesser and that does not harm our voltage gain. If we consider low frequency the capacitances which will be responsible for the lowering of the voltage gain are  $C_1$ ,  $C_2$  and  $C_E$  and when we consider high frequency of the signal the capacitances which will be responsible for lowering of the overall voltage gain of the amplifier are the junction capacitances of the transistor  $C_{bc}$  and  $C_{be}$ . We can very conveniently divide our analysis into two parts one is when the frequency is low that is at low frequency and then at high frequency, we can consider the response of the amplifier separately. That's what we are going to do now.

If we now plot the voltage gain of an amplifier versus frequency we get a curve like this. We focus on this curve in the mid frequency range that is the flat portion of the curve. We are plotting the gain versus frequency. Frequency is varied from zero onwards and in the y-axis we are plotting the voltage gain  $A_{VS}$ .  $A_{VS}$  customarily is that voltage gain with respect to the source.  $A_{VS}$  is  $V_O$  by  $V_S$  overall gain of the amplifier with respect to the signal being applied and within bracket 'f' is written because we are now considering the gain as a function of frequency. That is why the parameter or the argument inside the bracket will be f. If gain is plotted versus frequency then during this portion, mid frequency the voltage gain is constant which is denoted by  $A_O$ .  $A_O$  is the mid frequency voltage gain that is the same voltage gain which we were discussing taking into account the capacitors  $C_1$ ,  $C_2$  and  $C_{be}$  short.

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If the frequency is lowered we will get the voltage gain being drooping off. That it is going to be lesser and lesser as well as when frequency is higher then also the gain will be lowering from the value of  $A_O$  as seen here. The lowering of the gain occurs because of the reasons just now explained. One point to be noted in this graph is that we are marking two frequencies  $f_L$  and  $f_U$ ;  $f_L$  and  $f_U$  are called cut off frequencies where the gain of the amplifier drops off to  $\frac{1}{\sqrt{2}}$  times the gain of the mid frequency region.  $\frac{1}{\sqrt{2}}$  by



root 2 means 0.707. The values of the frequencies at which the gain drops off to 0.707 times the constant gain  $A_O$ , which is obtained at mid frequency range, are called cut off frequencies. Basically one is lower cut off frequency one is upper cut off frequency. Lower cut off frequency is the cut off frequency in the low frequency region and upper cut off frequency is the cut off frequency in the higher frequency region. What is the significance of this 0.707 or 1 by root 2 times? It comes actually from practical considerations because generally we apply an amplifier for audio purposes.

If it is an audio amplifier practically what happen is that the gain of the audio amplifier even if it drops off up to 1 by root 2 times the gain then also our ears are not able to percept that change in the gain and we hear almost same gain or we hear the audio signal as having the same amplification. That is there is no distortion perceived by the ears. All ears are naturally immune to the audio signal gain mean dropped off to maximum of 0.707 times the peak value and from that considerations actually this 1 by root 2 times or 0.707 times that value has a particular significance. Even if that voltage gain drops up to that level still it is tolerable and that is why this distance between the upper cut off frequency and lower cut off frequency is called the bandwidth, 3 dB bandwidth. It has to be seen where the 3 dB bandwidth comes from? We are writing in decibel and this is the particular unit which is obtained from a ratio and taking its logarithm to the base 10 multiplied by 20. That is  $20 \log$  to the base 10 of the ratio between the gain and the gain at mid frequency. As shown here this  $20 \log$  10 magnitude of  $A_{VS}(f)$  to  $A_O$  value is in dB unit and this is customarily done to normalize this  $A_{VS}(f)$  gain with respect to the value of gain  $A_O$  and then express this magnitude of frequency response in dB and that is done practically. In the books also we see that the plot is in dB that is in decibel. So that decibel unit we have to take in this way. The voltage gain in the mid frequency range  $A_{VS}$  is conventionally  $V_O$  by  $V_S$ ; in the mid frequency means where there is no effect of frequency of the signal.

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**Voltage Gain at Mid – Frequency  $A_{VS}$**

$$A_{VS} = \left( \frac{R_o}{R_{in} + R_s} \right) \frac{\beta R_C \| R_L}{(\beta + 1) r_e} \quad A_{VS} = \frac{V_o}{V_s}$$

$f_L$  – low -frequency cut-off  
 $f_U$  – high -frequency cut-off

These are the frequencies at which the magnitude of the voltage gain becomes  $1/\sqrt{2}$  of that at mid-frequency

This is a constant and that value, if we recall our earlier analysis, is minus  $R_{in}$  by  $R_{in}$  plus  $R_s$  into beta times of  $R_C$  parallel  $R_L$  divided by beta plus 1 small  $r_e$ . These we are familiar with from our earlier analysis. This is the overall gain of the amplifier in the mid frequency range. Because it is not having any frequency component it is constant. As we have shown in the plot the lower frequency cut off and the upper frequency cut off or higher frequency cut off are two frequencies and we can find out basically from these two frequencies what will be the value of lower cut off frequency and what will be the value of upper cut off frequency for a particular amplifier given the parameters values, because we know that at those frequencies the value of the voltage gain will be 1 by root 2 times or 0.707 times the value in the mid frequency range; that is this value. As it is customary to write it in decibel, the decibel scale if you write then at  $f_L$  and  $f_U$  that is at the lower cut off and upper cut off frequencies the value of the voltage gain  $A_{VS}$ , magnitude of this value will be 1 by root 2 times of  $A_O$ .

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It is customary to normalize  $|A_{VS}(f)|$  with respect to  $|A_O|$  and express the magnitude of frequency response in dB

$$|A_{VS}(f)|_{dB} = 20 \log_{10} \frac{|A_{VS}(f)|}{|A_O|}$$

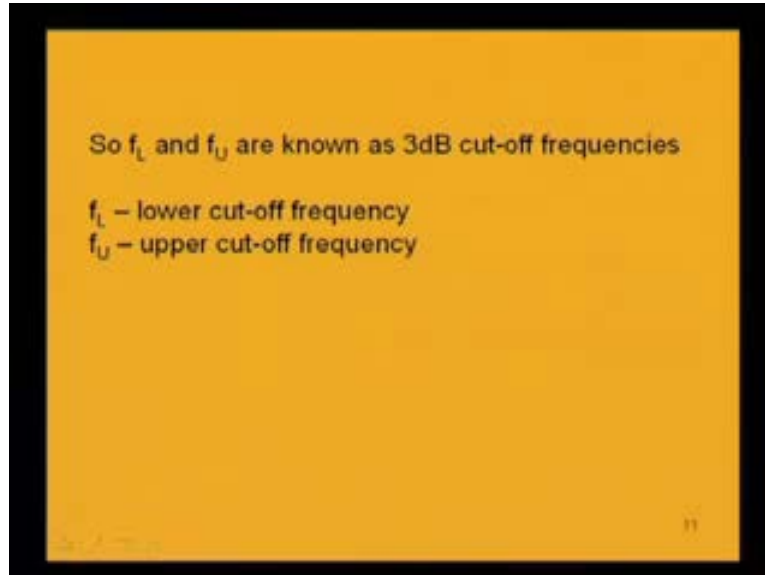
At dB scale, at  $f_L$  and  $f_U$ , we shall have :

$$|A_{VS}(f)|_{dB} = 20 \log_{10} \frac{0.707|A_O|}{|A_O|} = -3dB$$

Handwritten note:  $|A_{VS}(f)| = \frac{1}{\sqrt{2}} A_O$

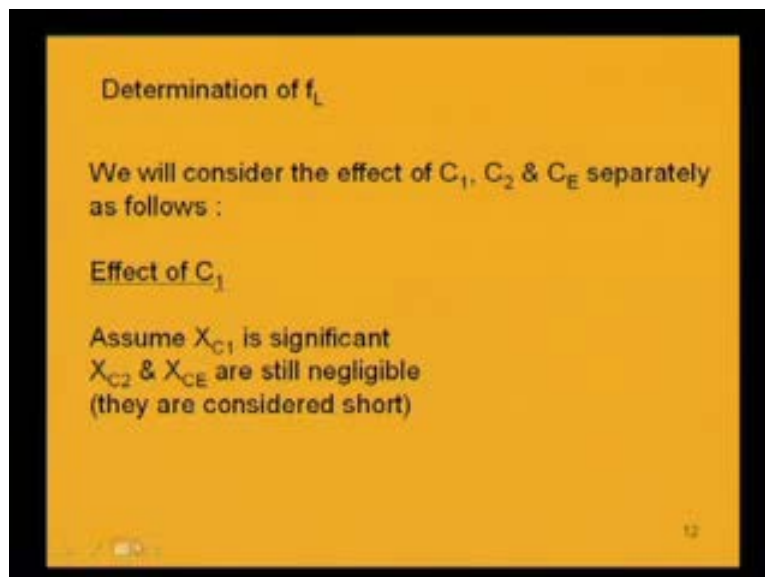
If we put that then we will get  $20 \log_{10}$  of  $A_{VS}(f)$  is 0.707 times  $A_O$  by  $A_O$  and this value is the dB value. If we calculate, this  $A_O$ ,  $A_O$  cancel;  $20 \log_{10}$  of 0.707 becomes -3. The overall quantity becomes -3 dB. That is why this -3 dB is very important as far as amplifiers are concerned. This is the bandwidth and that signifies the distance between the lower cut off and upper cut off frequencies because at those frequencies the gain becomes 1 by root 2 times the maximum gain which is  $A_O$ . This is the dB scale, -3 dB bandwidth we are going to find out. We will find out the lower cut off frequency and upper cut off frequency which are known as 3 dB cut off frequencies. 3 dB cut off frequencies means immediately we know that the gain has reduced to 1 by root 2 times of  $A_O$ .

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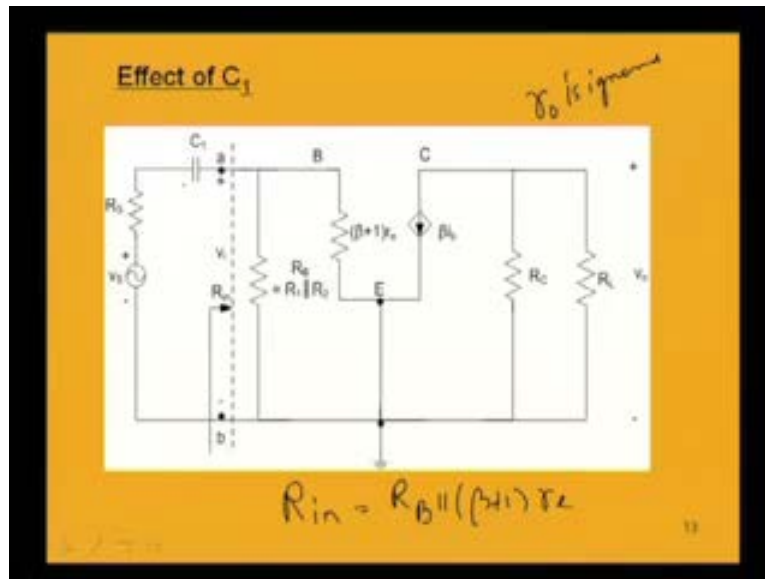
Using this analysis we can now proceed to find out what is the lower cut off frequency? When considering the low frequency region the capacitances which will be there will be  $C_1$ ,  $C_2$  and  $C_E$ . In order to find out the lower cut off frequency we will consider each capacitance separately; we will consider  $C_1$  and neglect the other two and similarly for the others also we will do like that and find out the lower cut off frequency due to each of the capacitances and finally we will find out the overall lower cut off frequency considering all the three. Let us first find out the lower cut off frequency due to  $C_1$ . We are assuming that the other two are short circuited;  $C_2$  and  $C_E$  will be short circuited and  $C_1$  has a significant capacitance.

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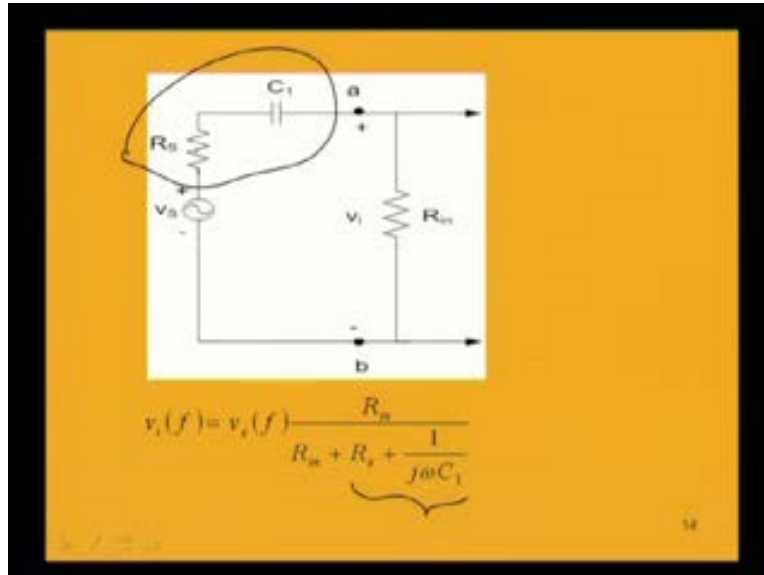
The circuit of the amplifier will have  $C_1$  and there will be no other capacitances.

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If we look into this circuit, it is very familiar to us because earlier also we discussed the same circuit in the mid frequency region. Only in that case this  $C_1$  was not there. We find out what will be the input resistance of this amplifier? Input resistance,  $R_{in}$  can be found out by looking into the input circuit at the terminals AB.  $R_{in}$  is nothing but these 2 resistances in parallel that is  $R_B$  parallel  $\beta + 1 r_e$ . If I consider this  $R_{in}$  and consider the circuit to the left of  $R_{in}$ , we get this circuit where we can see that the input voltage  $V_i$  is across this  $R_{in}$  which is equal to  $V_S$  into  $R_{in}$  by  $R_{in}$  plus this whole series impedance and that is nothing but  $R_s$  plus  $1/j\omega C_1$ .

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Simplifying this expression a little further we get  $V_i$  equal to  $V_s$  into  $R_{in}$  by  $R_{in}$  plus  $R_s$  plus  $1$  by  $j$  omega  $C_1$  can be written as rationalizing by multiplying numerator and denominator both by  $j$ , in the numerator  $j$  will come and in the denominator  $j$  square. We know  $j$  square is equal to  $-1$ . So minus  $j$  by omega  $C_1$  and again omega we know it is  $2\pi f C_1$  for this circuit. Just replacing this I get the expression of this ratio of  $V_i$  by  $V_s$  equal to  $R_{in}$  by  $R_{in}$  plus  $R_s$  minus  $j$  into  $1$  by  $2\pi f C_1$  where  $f$  is the frequency at which we are operating the input signal. That can be further reduced to another expression by writing common  $R_{in}$  plus  $R_s$ ; taking this common in the denominator it will be  $1$  minus  $j$  into  $1$  by  $2\pi f$  into  $R_{in}$  plus  $R_s$  into  $C_1$ .

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$$v_i(f) = v_s(f) \frac{R_{in}}{R_{in} + R_s + \frac{1}{j\omega C_1}}$$

$$\frac{v_i(f)}{v_s(f)} = \frac{R_{in}}{R_{in} + R_s - j \frac{1}{2\pi f C_1}}$$

$$\frac{v_i(f)}{v_s(f)} = \frac{R_{in}}{(R_{in} + R_s) \left( 1 - j \frac{1}{2\pi f (R_{in} + R_s) C_1} \right)}$$

$$\frac{v_i(f)}{v_s(f)} = \frac{R_{in}}{R_{in} + R_s} \frac{1}{1 - j \frac{1}{2\pi f (R_{in} + R_s) C_1}}$$

$j^2 = -1$   
 $\omega = 2\pi f C_1$

That can be further written as  $R_{in}$  by  $R_{in}$  plus  $R_s$  into  $1$  by  $1$  minus  $j$ ; I write  $1$  by  $2\pi f R_{in}$  plus  $R_s$ , whole term is in the numerator and in the denominator keeping this into  $C_1$  in the numerator and keeping only  $f$  in the denominator, this  $f$  is in the denominator only. I can write this one as this one. Look into this term  $1$  by  $2\pi$  into  $R_{in}$  plus  $R_s$  into  $C_1$ . That term let us write it is as  $f_{LC1}$  because this is  $2\pi$  into a resistance into a capacitance.  $R_C$  basically is nothing but a time constant of the circuit and that intention we are keeping in mind. So we are expressing  $f_{LC1}$  as  $1$  by  $2\pi$  into  $R_{in}$  plus  $R_s$  into  $C_1$ . We consider only the magnitude of this ratio because there is a  $j$  term. Both the angle as well as magnitude are associated with this. But we forget about the angle because we are interested in finding out the overall voltage gain which is having the magnitude of the voltage gain.

We are writing only the amplitude or magnitude  $v_i$  by  $v_s$  that is equal to  $R_{in}$  by  $R_{in}$  plus  $R_s$  into  $1$  by taking the magnitude of this part. It is a complex quantity of  $1$  minus  $j$  theta. If I want to find out the magnitude of this component, this whole expression will be  $1$  plus theta square under root. That is what is done;  $1$  plus  $f_{LC1}$  by  $f$  whole square under root to the power half. This is the ratio between  $v_i$  and  $v_s$ . When the frequency,  $f$  becomes the frequency  $f_{LC1}$ , the value of a  $v_i(t)$  and  $v_s(t)$ , this magnitude or amplitude of this ratio becomes  $R_{in}$  by  $R_s$  plus  $R_{in}$  into  $1$ ; when these two are equal it will be  $1$ . So  $1$  by  $1$  plus  $1$  means  $2$ ; square into half will go. In the denominator it will be  $1$  by  $1$  plus  $1$ ;  $1$  square means  $1$  to the power half;  $1$  by root  $2$  that means  $0.707$   $R_{in}$  by  $R_s$  plus  $R_{in}$ . That we get at frequency  $f$  is equal to  $f_{LC1}$ .

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The image shows handwritten mathematical derivations on a yellow background. The first equation is:

$$\frac{v_i(f)}{v_s(f)} = \frac{R_{in}}{R_{in} + R_s} \frac{1}{1 - j \left( \frac{f_{LC1}}{f} \right)}$$

Next, it defines  $f_{LC1}$  as:

$$\text{where } f_{LC1} = \frac{1}{2\pi(R_{in} + R_s)C_1}$$

Then, it shows the magnitude of the ratio:

$$\text{Now } \left| \frac{v_i(f)}{v_s(f)} \right| = \frac{R_{in}}{R_{in} + R_s} \frac{1}{\left[ 1 + \left( \frac{f_{LC1}}{f} \right)^2 \right]^{1/2}}$$

At frequency  $f = f_{LC1}$ , we get:

$$\left| \frac{v_i(f)}{v_s(f)} \right| = \frac{R_{in}}{R_{in} + R_s} \frac{1}{\sqrt{2}} = 0.707 \left( \frac{R_{in}}{R_{in} + R_s} \right)$$

Handwritten notes include  $1 - j0$  over  $\sqrt{1+0}$  and  $\frac{1}{(1+1)^{1/2}}$ .

If we consider the overall voltage gain  $A_{VS}$  because this was the ratio between  $v_i$  and  $v_s$ . For the amplifier let us consider the voltage gain  $v_o$  by  $v_i$ . If we look into this amplifier circuit again  $v_o$  is here, this is  $v_i$ . In between  $v_o$  and  $v_i$  there is nothing, no capacitance. The circuit is similar to the mid frequency region circuit.  $v_o$  and  $v_i$  this ratio is same as the voltage gain  $v_o$  by  $v_i$  for the amplifier in the mid frequency region and that is having the relation which is known to us. We know that we can replace that  $v_o$  by  $v_i$  which is  $A_v$  that



means the voltage gain which is nothing but, if we consider the magnitude only minus sign we are not writing, beta into  $R_C$  parallel  $R_L$  by beta plus 1  $r_e$  that is the voltage gain between output terminals and input terminals.

Let us consider the overall voltage gain  $v_o$  by  $v_s$ . By a chain rule I can write it as  $v_o$  by  $v_i$  into  $v_i$  by  $v_s$ .  $v_s$  by  $v_i$  is equal to this expression  $A_v$  into  $R_{in}$  by  $R_s$  plus  $R_{in}$ . This is  $v_o$  by  $v_i$  which is  $A_v$  and this part is equal to  $v_i$  by  $v_s$ . We can also replace  $A_v$ . But keeping it as it is, we are getting  $A_v$  which is the voltage gain  $v_o$  by  $v_i$ . This  $A_v$  is nothing but  $v_o$  by  $v_i$ . Similarly  $v_i$  by  $v_s$  that was just now found out which is from this  $R_{in}$  by  $R_s$  plus  $R_{in}$  into this part. That is what we are expressing and we are considering magnitude that is why I am writing only the magnitude part or I can write this  $v_o$  by  $v_s$  as  $A_{vs}(f)$  considering the frequency. You can write it as  $A_o$  into 1 by 1 plus  $f_{LC1}$  by  $f$  whole square to the power half because  $A_v$  into  $R_{in}$  by  $R_{in}$  plus  $R_s$  is nothing but  $A_{vs}$  that is the mid frequency range overall voltage gain without considering capacitance. That is nothing but again  $A_o$  in the mid frequency region. Mid frequency region voltage gain if it is  $A_o$ , magnitude we are considering; another part is there which is 1 by 1 plus  $f_{LC1}$  by  $f$  whole square to the power half or under root.

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$$\left| \frac{v_o}{v_i} \right| = |A_v| = \frac{\beta R_C || R_L}{\beta + 1} r_e$$

Now  $\left| \frac{v_o}{v_s} \right| = \left| \frac{v_o}{v_i} \right| \left| \frac{v_i}{v_s} \right| = |A_v| \frac{R_{in}}{R_s + R_{in}} \frac{1}{\left[ 1 + \left( \frac{f_{LC1}}{f} \right)^2 \right]^{1/2}}$

or,  $|A_{vs}(f)| = |A_v| \frac{1}{\left[ 1 + \left( \frac{f_{LC1}}{f} \right)^2 \right]^{1/2}}$

At  $f = f_{LC1} = \frac{1}{2\pi(R_s + R_{in})C_1}$

$|A_{vs}(f)| = 0.707 |A_o|$

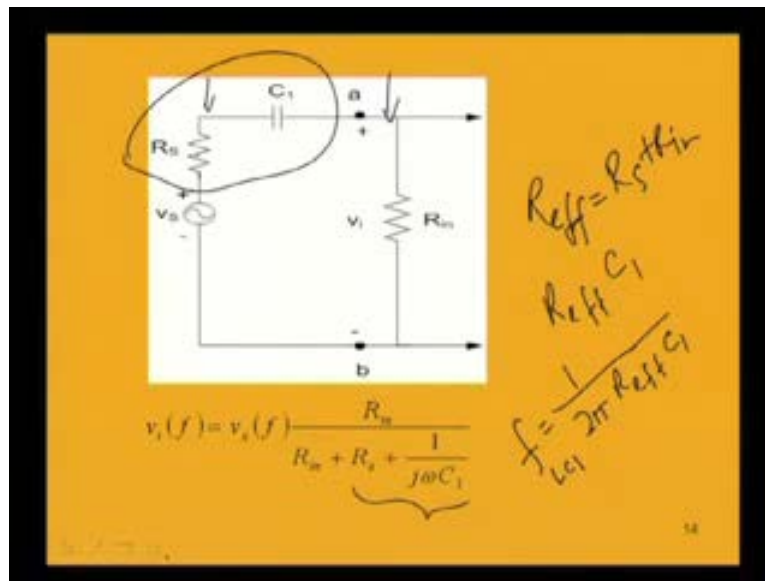
*Handwritten notes:*  
 $A_v \frac{R_{in}}{R_{in} + R_s}$   
 $= A_{vs} A_o$   
 mid freq

At frequency  $f_{LC1}$  when this  $f$  is equal to  $f_{LC1}$ , this part becomes root 2. Overall voltage gain  $A_{vs}(f)$  is equal to 0.707  $A_o$  and that is the lower cut off frequency  $f_{LC1}$  at which this happens. The lower cut off frequency due to  $C_1$  is equal to 1 by 2 pi  $R_s$  plus  $R_{in}$  into  $C_1$  because at that frequency when you substitute  $f$  is  $f_{LC1}$  by this one and make  $f$  is equal to  $f_{LC1}$ , then we get our required criterion that the voltage gain roughs up to 0.707 times the mid frequency gain. This  $f_{LC1}$  or the lower cut off frequency due to  $C_1$  is equal to 1 by 2 pi into  $R_s$  plus  $R_{in}$  into  $C_1$ . This lower cut off frequency can be found out easily.

We look into the circuit having this capacitance  $C_1$ . Look from this input side across this  $C_1$ . If I find out what is the effective resistance, it is equal to  $R_s$  plus  $R_{in}$ .  $R_s$  plus  $R_{in}$  is the

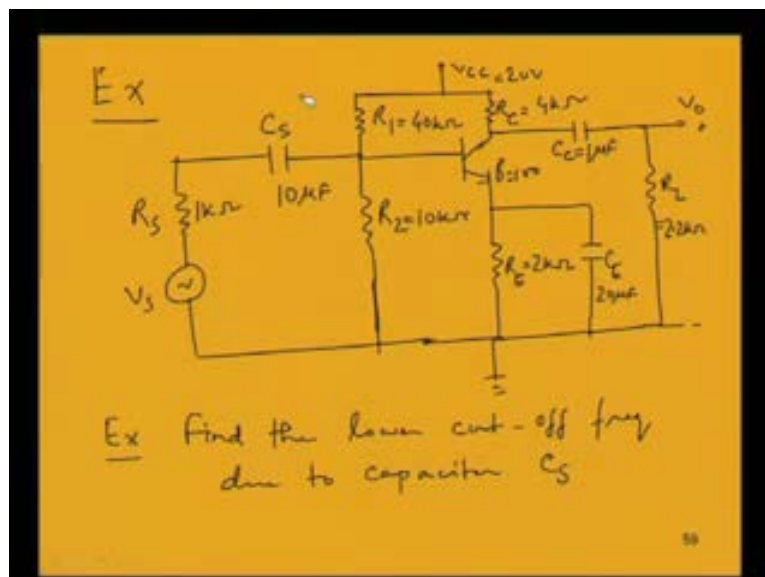
effective resistance looking from the capacitor  $C_1$  by making  $V_S$  zero when you find out effective resistance and  $R$  effective into  $C_1$ , actually this whole term will be determining the time constant of this circuit having capacitance and from that we can find out what is the frequency by  $1$  by  $2\pi$  into  $R$  effective into  $C_1$ . This is actually the lower cut off frequency.

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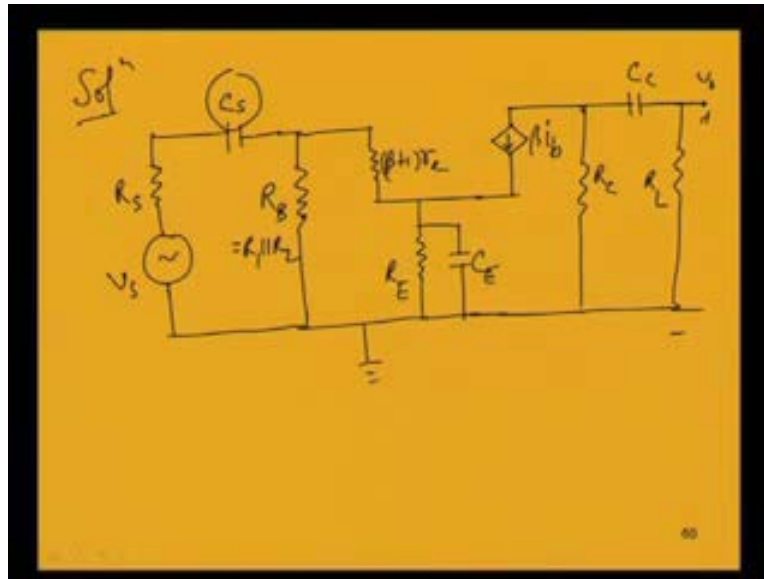
Let us try one problem for finding out the lower cut off frequency due to the capacitance  $C_S$  alone as shown in this figure. In this example we will find out the lower cut off frequency due to the capacitor  $C_S$  alone.

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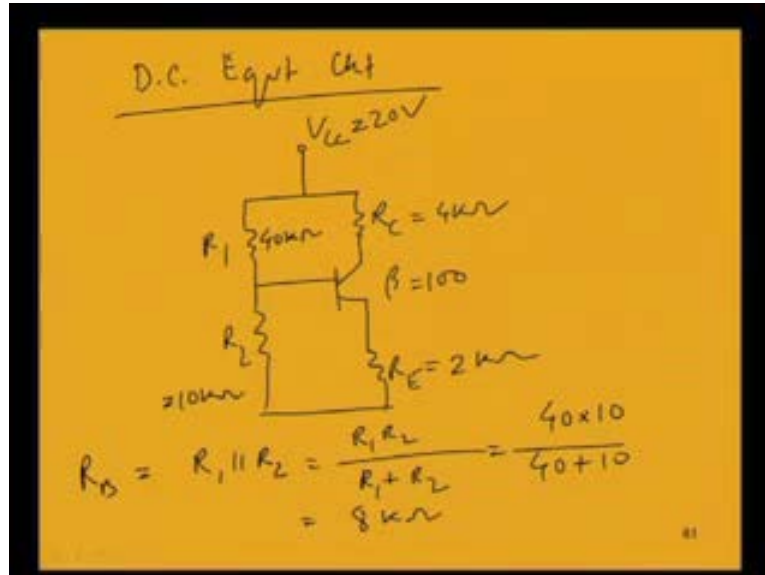
For solving this example we have to first draw the small signal model for this transistor amplifier. The small signal model as we discussed earlier will have the capacitance in the high frequency and the low frequency region also. Let us draw the small signal model including the capacitances because we are concerned with the frequency response and as is the method for drawing the small signal model which we discussed earlier let us proceed. This circuit is having  $R_s$ ,  $V_s$  is the source,  $C_s$  is the series capacitor with the source and in the small signal model we know that the parallel combination of the biasing resistance  $R_1$  and  $R_2$  let us equivalently express it as  $R_B$  and this is beta plus 1  $r_e$  and  $R_E$  the capital R capital E, is the emitter resistance,  $C_E$  is the emitter bypass capacitor and here the current source beta  $i_b$  and the collector resistance  $R_C$  and the collector capacitance, load resistance  $R_L$  will be here. Here in this model due to this  $C_s$  alone we will be finding out the lower cut off frequency.

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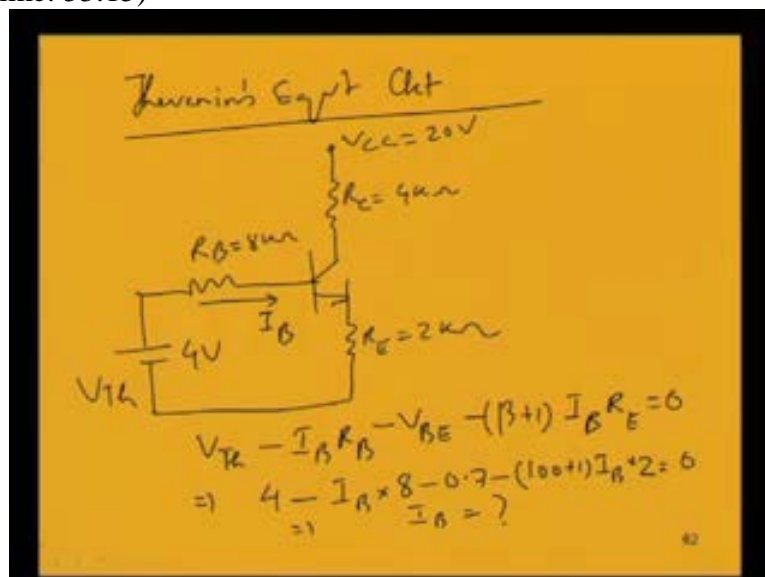
Replacing these values we have to first of all find out what is  $r_e$ ? That we do not know. In order to know  $r_e$  we have to be concerned about the DC equivalent circuit. DC equivalent circuit if we draw then it will be having this  $R_1$  and  $R_2$  parallel. This is  $R_C$  and this is the transistor emitter resistance  $R_1 R_2$ ;  $V_{CC}$ , 20 volt,  $R_C$  is equal to 4 kilo ohm and for this transistor beta value is given as 100. In this example actually beta is given by 100. We have to note beta value in order to find out the parameter  $R_s$ .  $R_E$  value is 2 kilo ohm,  $R_2$  is 10 kilo ohm,  $R_1$  is 40 kilo ohm. What will be this  $R_1$  parallel  $R_2$ ? Let us write that as  $R_B$ . It is  $R_1 R_2$  by  $R_1$  plus  $R_2$  and putting the values 40 into 10 by 40 plus 10, we get 8 kilo ohm.

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This is the value of the parallel equivalent resistance. The Thevenin's equivalent circuit will give us the value of  $R_E$ . We draw the Thevenin's equivalent circuit replacing this  $R_B$  by whatever we have got which is 8 kilo ohm. This is  $V_{CC}$ ,  $R_C$  is 4 kilo ohm;  $V_{CC}$  is equal to 20 volt,  $R_E$  is equal to 2 kilo ohm and we need to know the Thevenin's equivalent voltage. Thevenin's equivalent voltage can be found out from this;  $V_{Thevenin}$  equal to  $V_{CC}$  into  $R_2$  by  $R_1$  plus  $R_2$ ; 20 into 10 by 10 plus 40. This is equal to 4 volt. We can now replace here 4 volt. Now we can find out what is  $R_E$ ? This is  $I_B$  base current. In this circuit  $V_{Thevenin}$  minus  $I_B R_B$  minus  $V_{BE}$  minus  $\beta + 1$   $I_B$  into  $R_E$  that is equal to zero. Replacing the values 4 minus  $I_B$ , we do not know;  $R_B$  is 8 minus 0.7 minus 100 plus 1 into  $I_B$  into  $R_E$  is 2 is equal to zero. This equation will give the value of  $I_B$ .

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If we do this we will get the value of  $I_B$ . You should check and get a value of 0.0157 milli ampere. This is the value of  $I_B$  which we get from this equation. Solving this equation we get a value of  $I_B$  which is equal to this much. If we know  $I_B$  we can also know  $I_E$  which is nothing but beta plus 1 into  $I_B$ . Beta value is 100; 100 plus 1 into  $I_B$  is 0.0157. This is in milli ampere. What we will get is 1.586 milli ampere. Knowing the value of  $I_E$ , we can now find out what is the value of small  $r_e$ ; 26 milli volt by capital I capital E,  $I_E$ . Putting these values we get 26 milli volt by 1.586 milli ampere and that is equal to 16.39 ohm. Because this order is milli volt by milli ampere, it will be in ohm. 16.39 ohm is small  $r_e$ . Beta plus 1  $r_e$  equal to 100 plus 1; that is 101 into 16.39 and that comes to 1.655 kilo ohm.

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$$\begin{aligned}
 I_B &= 0.0157 \text{ mA} \\
 I_E &= (\beta + 1) I_B \\
 &= (100 + 1) \times 0.0157 \text{ mA} \\
 &= 1.586 \text{ mA} \\
 r_e &= \frac{26 \text{ mV}}{I_E} = \frac{26 \text{ mV}}{1.586 \text{ mA}} \\
 &= 16.39 \Omega \\
 (\beta + 1) r_e &= (100 + 1) \times 16.39 = 1.655 \text{ k}\Omega
 \end{aligned}$$

This is ohm. But in kilo ohm we will get 1.655 kilo ohm. We now know this beta plus one  $r_e$ . We look into the small signal model and find out what is the input resistance  $R_{in}$ ? We can find this out and that is equal to  $R_B$  parallel beta plus 1  $r_e$ . In the mid band region, that is when we ignored the effect of this capacitance, that  $R_{in}$  the input resistance we have to find out. In the mid band that  $R_{in}$  is equal to  $R_B$  parallel beta plus 1  $r_e$ . Replacing the values  $R_B$  is equal to 8 kilo ohm and beta plus 1  $r_e$  we have found out to be 1.655; so 8 into 1.655 by 8 plus 1.655 is equal to 1.371 kilo ohm. We know this  $R_{in}$  that is the input resistance for the mid band frequency region. Then in order to find out the lower cut off frequency we know that, we have just now found out the lower cut off frequency due to  $C_S$  alone. Individually we are finding out the lower cut off frequency and that we know.  $f_{LCS}$  if we denote that is 1 by 2 pi into effective resistance as looked by this capacitance  $C_S$  and that is nothing but  $R_S$  plus  $R_{in}$  into  $C_S$ , because the effective resistance is as looked by capacitor  $C_S$ .

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Handwritten calculations on a yellow background:

$$\begin{aligned} R_{in} &= R_B \parallel (\beta + 1) R_E \\ &= 8 \parallel 1.655 \\ &= \frac{8 \times 1.655}{8 + 1.655} \\ &= 1.371 \text{ k}\Omega \end{aligned}$$

Lower Cut-off Frequency due to  $C_S$

$$f_{LCS} = \frac{1}{2\pi(R_S + R_{in}) \times C_S}$$

This is the capacitors  $C_S$ . What will be the effective resistance? That is  $R_{in}$  plus  $R_S$ . From this side we will look into; so it will be  $R_S$  plus this  $R_{in}$ . What will be this value? That value we can find out;  $f_{LCS}$  is equal to  $1$  by  $2\pi$  into  $R_S$ , series resistance here with the source is  $1$  kilo ohm, so  $1$  plus  $1.371$  that we have found out, into the capacitance  $C_S$  which is  $10$  microfarad. But this is in kilo ohm. So into  $10$  to the power  $-6$  into  $10$ ; that is  $10$  microfarad is the capacitance  $C_S$ . If we do this calculation then we will get the value of  $f_{LCS}$  to be  $6.71$  Hertz. This capacitance  $C_S$  is offering a lower cut off frequency of  $6.71$  Hertz. In order to know the lower cut off frequency due to this capacitance we have seen that we have to find out the effective resistance which is looking from the capacitor what effective resistance is there and for finding that out we have seen that we have to find out the input resistance is in the mid band region and that input resistance is found out by considering the small signal model and after finding out that  $R_{in}$  we found out the effective resistance because it is simply the series resistance between  $R_{in}$  and the  $R_S$ . This effective resistance due to the  $C_S$  only that has been obtained which is equal to  $1.371$  kilo ohm and then it will give you the value of the lower cut off frequency.

In this discussion we have today analysed the transistor amplifier in the common emitter configuration. We are considering first the low frequency and then we will be also considering the high frequency analysis. But today we have discussed the lower cut off and upper cut off frequencies and we are trying to find out what is the cut off frequency value for both the lower and upper frequency regions and for the lower frequency regions we have found out the lower cut off frequency considering one coupling capacitor  $C_1$ . Also we will be finding out next the lower cut off frequency due to the other two capacitances and then we will be also finding out the overall cut off frequency in the low frequency region and then we will also find out the higher or upper cut off frequency.