Digital Signal Processing

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Lecture - 43

FIR Design by Frequency Sampling (Contd….)

In the last lecture, we discussed the basic philosophy of frequency sampling. We also distinguished between odd length and even length where the samples are different. And we gave the general expression for system function.

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We mentioned that you do not have to compute $h(n)$ from $H(k)$. Once you sample $H_d(e^{j\omega})$ at N number of points, that is, $H(k) = H_d(e^{j2\pi k/N})$, then conceptually you come back to $h(n)$ from $H(k)$. Next you compute the system function and so on. But you can do this without computing $h(n)$ and we showed that the system function $H_1(z)$ can be written into the form $[(1 - z^{-N})/N] \sum_{k=0}^{N-1} H(k)/(1 - e^{j2\pi k/N} z^{-1})$. And we are also tempted to interpret this as a parallel connection of the first order blocks with complex coefficients. But then we desisted from that because poles are on

the unit circle. Therefore the utility of this expression is not in realization but in finding the frequency response, that is $H_1(e^{j\omega})$.

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$$H_1(e^{j\omega}) = \frac{1 - e^{-j\omega N}}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - e^{j2\pi\frac{k}{N}} e^{-j\omega}}$$

$$= \frac{e^{-j\omega N/2}}{N} \sum_{k=0}^{N-1} \frac{H(k)\left[e^{j\omega N/2} - e^{-j\omega N/2}\right]}{e^{-j\left(\frac{\omega}{2} - \frac{\pi k}{N}\right)} 2j \sin\left(\frac{\omega}{2} - \frac{\pi k}{N}\right)}$$

$$= \frac{e^{-j\omega\frac{N-1}{2}}}{N} \sum_{k=0}^{N-1} \frac{H(k) e^{-j\pi k/N} \sin\frac{\omega N}{2}}{\sin\left(\frac{\omega}{2} - \frac{\pi k}{N}\right)}$$

And we can simplify this further by writing $H_1(e^{-j\omega}) = [(1 - e^{j\omega N})/N] \sum_{k=0}^{N-1} H(k)/(1 - e^{j2\pi k/N} e^{-j\omega})$. This can be further simplified to $H_1(e^{j\omega}) = (e^{-j\omega(N-1)/2}/N) \sum_{k=0}^{N-1} H(k) e^{-\pi k/N} \sin(\omega N/2)/\sin(\omega/2 - (\pi k /N))$. Now this expression can be put in a more elegant form (Refer Slide Time: 06.09 min) by recognizing that $\sin(\omega N/2)$ can be written as $(-1)^k \sin[(\omega N/2) - \pi k]$. Therefore I can write the frequency response function in this elegant form $H_1(e^{j\omega}) = [(e^{-j\omega(N-1)/2}/N]$ summation ($k = 0$ to $N - 1$) $(-1)^k H(k) e^{-jk\pi/N} \times \sin[(\omega N/2) - (\pi k)]/\sin[(\omega/2) - (\pi k/N)]$.

The ratio of sines is of the form $\sin(N\theta/\sin\theta)$ which is a familiar function. This is the form that should be utilized for computation of the frequency response. Obviously you shall have to write a program, and compute.

There are some situations in which this sampling procedure cannot be used for $k = 0$ to $N - 1$, because $k = 0$ creates a problem.

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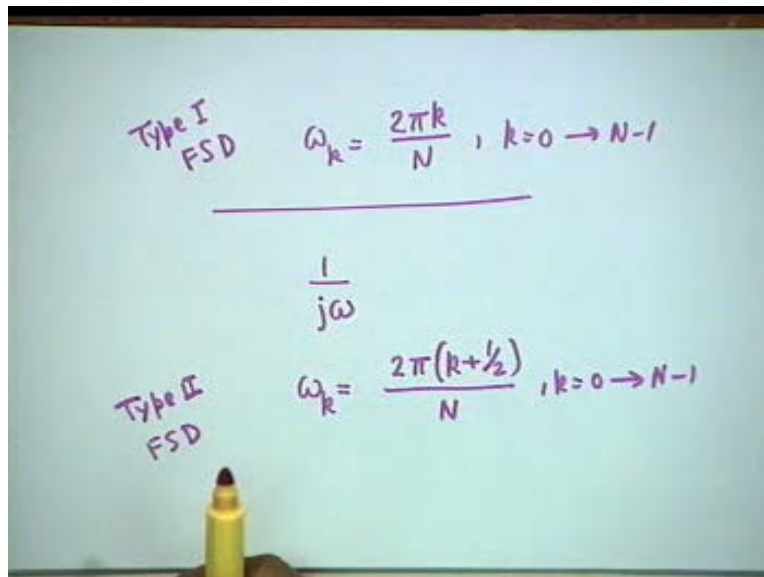
The problem arises when we want to realize an approximation of functions like $1/(j\omega)$ which is an integrator. Suppose we have to design a digital integrator, then obviously we cannot use $\omega = 0$ as one of the sampling points because at this point, the magnitude goes to infinity. There are other situations where we cannot take the sample at $\omega = 0$. In other words, we have to take a sample slightly shifted from 0. The scheme we can use is $\omega_k = 2\pi\,[k + (1/2)]/N$ where $k = 0$ to $N - 1$. This sampling scheme is called type 2 Frequency Sampling Design, the previous one being called type 1 Frequency Sampling Design. If you use type 2 design, then obviously the first sample would be at $\omega = \pi/N$, instead of 0.
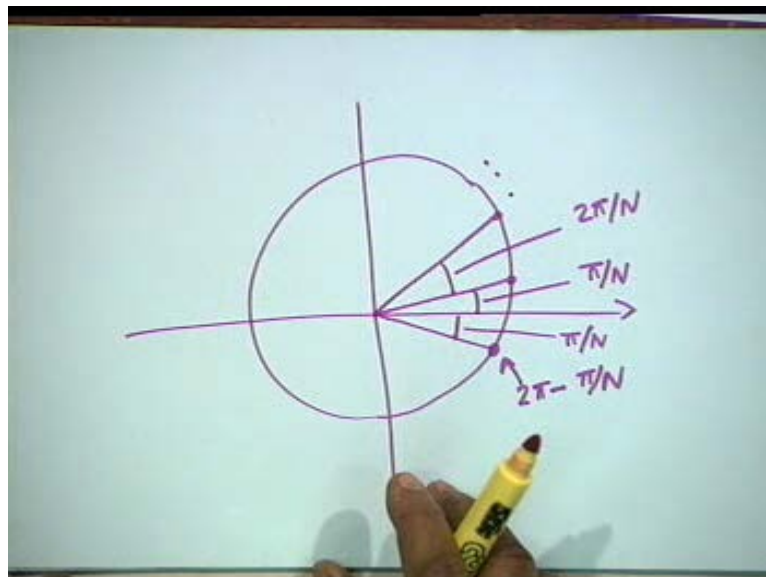
The next one would be at the angle $2\pi/N$ and so on. The last sample would be at $\omega = 2\pi - (\pi/N)$. Now because of this sampling scheme there is no H(0) so we need not have a real term; in the previous scheme, H(0) was required to be real because there was no matching term. But here we can match every term, except when N is odd. In that case, because of the linear phase constraint, the lone term must be zero. Recall that in the previous case, when N was even, the middle sample H(N/2) was required to be 0.

Thus, for h(n) to be real in type 2 design for N odd the conditions are H $(N - k - 1)$ = H*(k). (In the previous case $H(N - k)$ was required to be $H*(k)$). And, in addition, the middle term $H[(N - 1)/2] = 0$. Whereas for N even for type 2, the condition simply is complex conjugation and there is no middle term. For N even you have even number of matches. Every term has a match so there is no middle sample. And when these conditions are satisfied, you can write h(n) for N even and N odd. In the latter case, k = 0 to $[(N - 1)/2] - 1 = [(N - 3)/2]$; for N even, k goes from 0 to $(N/2) - 1 = (N - 2)/2$. We require these upper limits in the summation.

When these are satisfied, then for N odd, Nh(n) would be twice summation $k = 0$ to $(N - 3)/2$ real part of H(k) $e^{j\pi(2k+n)/N}$. For even N, all that changes is that the summation upper limit becomes $(N - 2)/2$.

If you want to calculate h(n) (you may not be required to), then is it convenient to leave the real part inside the summation? Real $(a + b)$ is the same as (real of a + real of b) so this is purely a matter of computational convenience. But it is easier to sum up exponentials rather than trigonometric functions. So the real part and summation can be interchanged according to convenience.

(Refer Slide Time: 18.46 - 21.30 min)

$$H_1(z) = \frac{1 + z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - e^{j(2k+1)\pi/N} z^{-1}}$$

$$H_1(e^{j\omega}) = \frac{e^{-j\omega \frac{N-1}{2}}}{N} \sum_{k=0}^{N-1} \frac{H(k) e^{-j\pi(k+\frac{1}{2})/N} \cos \frac{\omega N}{2}}{j \sin\left[\frac{\omega}{2} - \frac{\pi}{N}\left(k+\frac{1}{2}\right)\right]}$$

In this case also, the transfer function $H_1(z)$ can be obtained directly from H(k). You do not have to go via h(n). In this case, instead of $1 - z^{-N}$, as in type 1, you get $1 + z^{-N}$. $H_1(z)$ would be [(1 + z^{-N})/N] summation (k = 0 to N − 1) H(k)/[1 − $e^{j(2k+1)\pi/N}$ z^{-1}]. And if you put z = $e^{j\omega}$, then you shall get a cosine in the numerator instead of sine as in type 1. $H_1(e^{j\omega})$ will be $e^{-j\omega(N-1)/2}$/N $\sum_{k=0}^{N-1}$ H(k) $e^{-j\pi(k+1/2)/N}$ cos ((ωN/2)/(j sine[(ω/2) − π (k + (1/2))]). This expression or previous expression may have to be programmed for calculation.

Now let us look at an example. Type 1 and type 2 designs of frequency sampling guarantee linear phase to start with. If you do not want linear phase then you do not have to bother about all these. In linear phase you have to compute half the number of terms. If it is nonlinear phase, then you have to compute all the terms but you may be able to do with less number of terms. But one hardly uses FIR if linear phase is not a constraint.
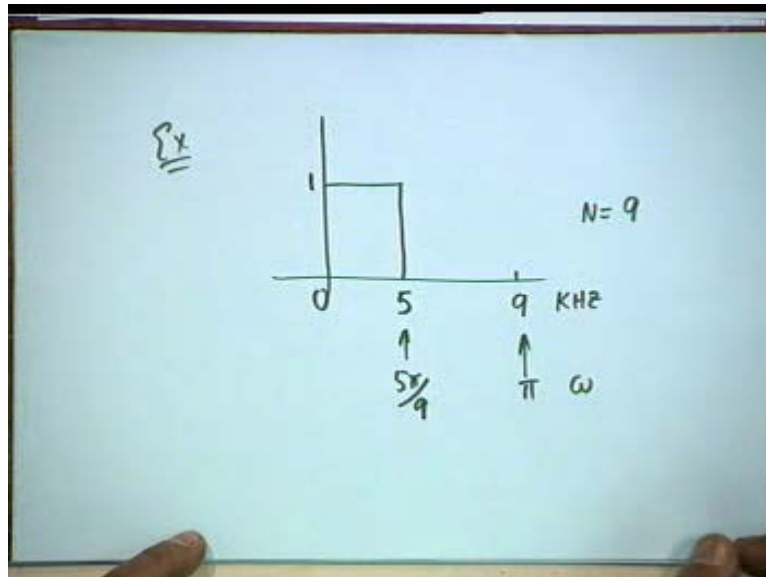
For example, suppose we have to design a low pass filter with the characteristic shown in the above slide: end of the base band is 9 kHz which corresponds to $\omega = \pi$ and the passband is from 0 to 5kHz which corresponds to $\omega = 5\pi/9$. And it is stipulated that you can use only a length $N = 9$. So k should range from 0 to 8.
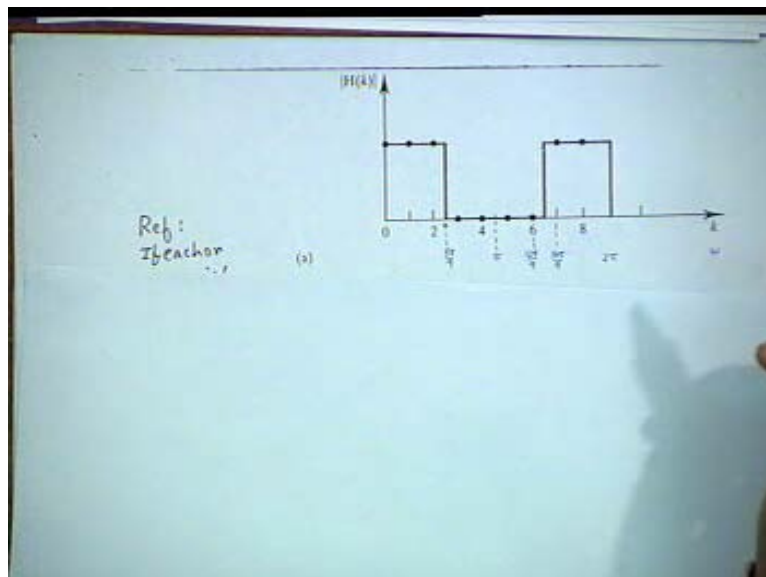
In the above slide, we have shown the characteristic up to $\omega = 2\pi$, and we have shown the sample points. Here I require summation only from $k = 0$ to 2. $H(0)$ has to be real and there are only two additional terms. You must be careful about this. You have to construct the corresponding phase. It is $e^{-j\omega(N-1)/2}\,e^{-j4\omega}$ up to π, and beyond π, it has to be the other way round. But you do not require that. You require only the samples $k = 1$ and 2, so find out the corresponding phases. You do not have to draw the complete diagram, because ultimately it is only these three samples which are required.

(Refer Slide Time: 26.03. – 26.50 min)

Ref: Ifeachor & Jervis / DSP / A-W
p.321

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h[0] =	7.2522627e−02	= h[8]
h[1] =	−1.1111111e−01	= h[7]
h[2] =	−5.9120987e−02	= h[6]
h[3] =	3.1993169e−01	= h[5]
h[4] =	5.5555556e−01	= h[4]

Now, if you do this then the corresponding h's can be calculated based on one real sample (that is 1) and two complex conjugate samples which will give rise to a cosine. The impulse response samples are shown in the above slide. You have to calculate half of them because $h(0) = h(8)$, $h(1) = h(7)$ and so on. There are only four terms in the frequency response and they are complex conjugates of each other. As shown in the slides, this example is taken from Ifeachor and Jervis, Digital Signal Processing, Addison Wesley, starting page 320.

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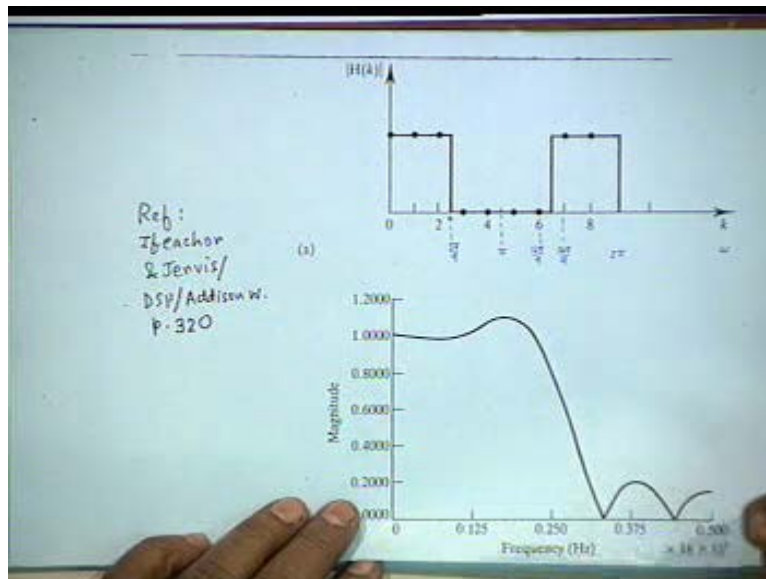
The results are shown in the above slide. You get a wide divergence. Exactly as in windowing technique, where we wanted to avoid abrupt transition, it can be done here also.

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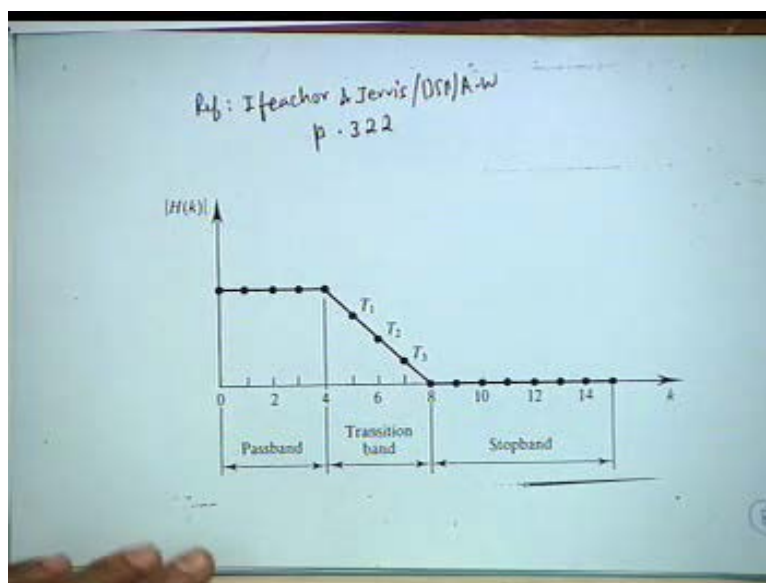
What we do is as follows: Suppose the low pass filter is required to be ideal from 0 to 4 four samples, and then we intentionally taper it between the pass band and the stop band, (allowing a transition band), which for simplicity, has been taken to be linear. Let us see what the effect is.

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We have taken N = 29. We require only the first four samples if you do not allow any transition; or five if you allow one sample in the transition, or six or seven depending on what you want. If you do not allow a transition then H(k) is 1 from k = 0 to 4, H(k) = 0 from k = 5 to 8. If you use one sample in transition band, assumed to be linearly tapered, then obviously the value would be 1 from k = 0 to 3, 0.40406 for k = 4 and 0 for k = 5 onwards. If you use two samples, then in the linear fall region, the first sample would be higher than the next one. The final result has been shown in the next slide; only half of the number has to be calculated for the impulse response samples.

(Refer Slide Time: 33.26 – 33.56 min)

	No transition samples	One transition sample	Two transition samples	Three transition samples
h[0] =	−4.9815884e−02	−1.3766696e−02	−5.7195305e−03	−4.2282741e
h[1] =	4.1202267e−02	−2.3832554e−03	−7.6781827e−03	−7.6031627e
h[2] =	6.6666666e−02	3.9729333e−02	2.3920000e−02	1.8793332e
h[3] =	−3.6487877e−02	1.2729081e−02	2.5763613e−02	2.8145113e
h[4] =	−1.0786893e−01	−9.1220745e−02	−7.3701817e−02	−6.6396840e
h[5] =	3.4076020e−02	−1.8619356e−02	−4.4185450e−02	−5.2511976e
h[6] =	3.1889241e−01	3.1326097e−01	3.0552137e−01	3.0183514e
h[7] =	4.6666667e−01	5.2054133e−01	5.5216000e−01	5.6393334e

Because of symmetry, only the first half of the coefficients are listed here.

In terms of frequency response, we require only up to 7 at the most.

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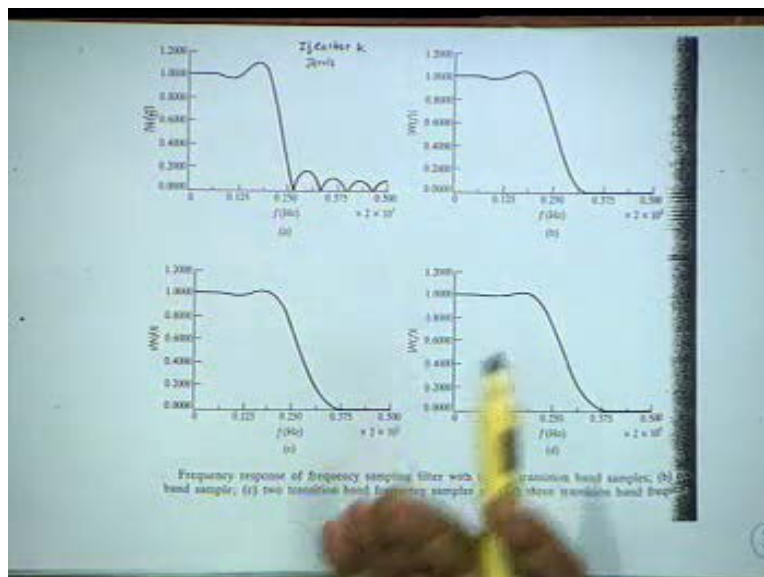
If you do that, the final results are shown in the above slide. The first diagram shows the frequency response without any sample in the transition band. There is Gibbs phenomena in the

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transition region. The next diagram shows what you get with one sample, where Gibbs phenomenon has been considerably reduced. The third diagram is for two samples and the last one applies to three samples in the transition region. If you use more, then you are widening the pass band but you are achieving a better pass band response. The next slide shows some formulas for estimating the required N.

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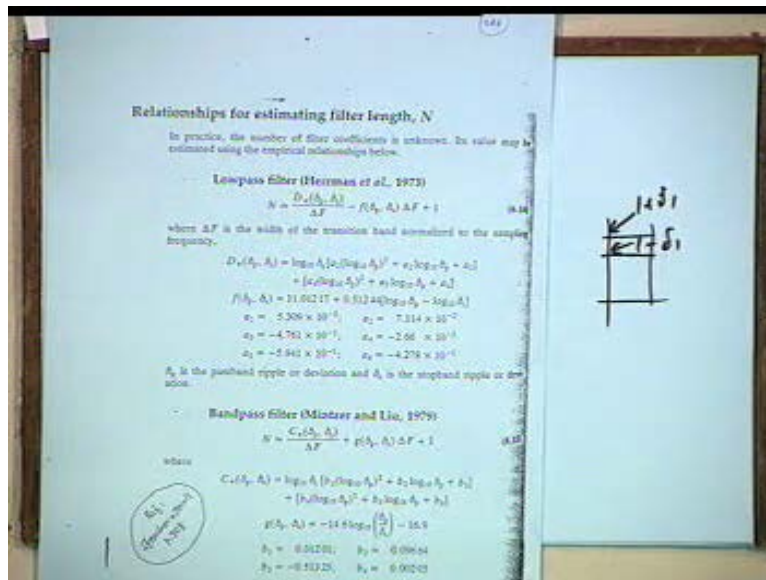
These are the best formulas that are known so far. For a low pass filter, which apparently is also applicable to high pass, the estimation of the order is slightly complicated as it requires a little bit of calculation, but once you program it, then it is not a problem. The only thing that one has to be careful is that the symbols are slightly different. In our case δ_p stands for the smallest transmission in the pass band but here δ_p is the ripple that is the difference between the maximum and the minimum. If the maximum is 1 then $1 - \delta_p$ is the minimum transmission in the pass band. So this δ_p is the pass band ripple. The δ_s remains the same as in our notation. On the other hand in FIR, the usual procedure is to specify $1 + \delta_1$ and $1 - \delta_1$. If that is the specification, then δ_p has to be twice δ_1. So δ_p and δ_s are different; also δf is as defined earlier, i.e. $(\omega_s - \omega_p)/2\pi$. In the case of a high pass filter it is $(\omega_p - \omega_s)/2\pi$.

In the case of a band pass filter, which apparently is applicable to band stop also, the calculations are almost of the same complexity. δp and δs have the same interpretation except that in this case, the transition band δf is the transition width normalized to the sampling frequency.

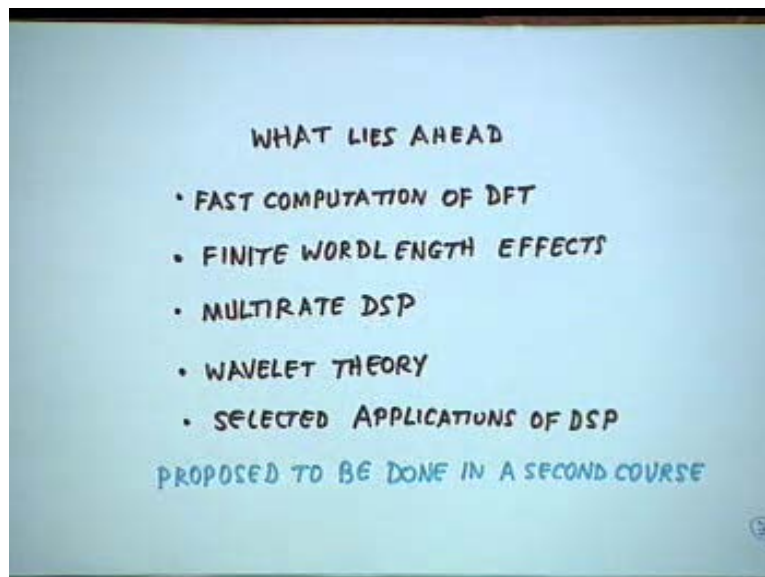(Refer Slide Time: 38.36- 40.20 min)
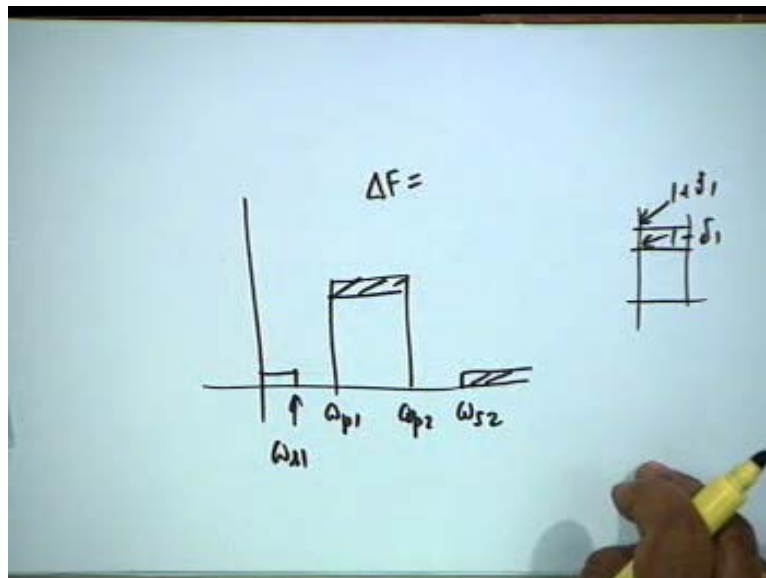
One has to make sure that (refer to the above slide), $(\omega_{s2} - \omega_{p2})/(2\pi)$ has to be equal $(\omega_{p1} - \omega_{s1})/(2\pi)$. If this is not the case, then you have to adjust them. You have to adjust the two transitions to be equal by adjusting either ω_{s1} or ω_{s2}. Do not change ω_{p1} or ω_{p2} and only then you can apply this formula. The formula is only an estimate. Application of the formulas may make it slightly easier to converge to what you ultimately want. In the case of a band stop filter, you shall have to exercise the same caution. In the case of a band stop filter δf would be $(\omega_{s1} - \omega_{p1})/(2\pi)$ which has to be adjusted to be equal to $(\omega_{p2} - \omega_{s2})/(2\pi)$. These are the only formulas. First estimate the order and depending on the required order, you can make your calculations.

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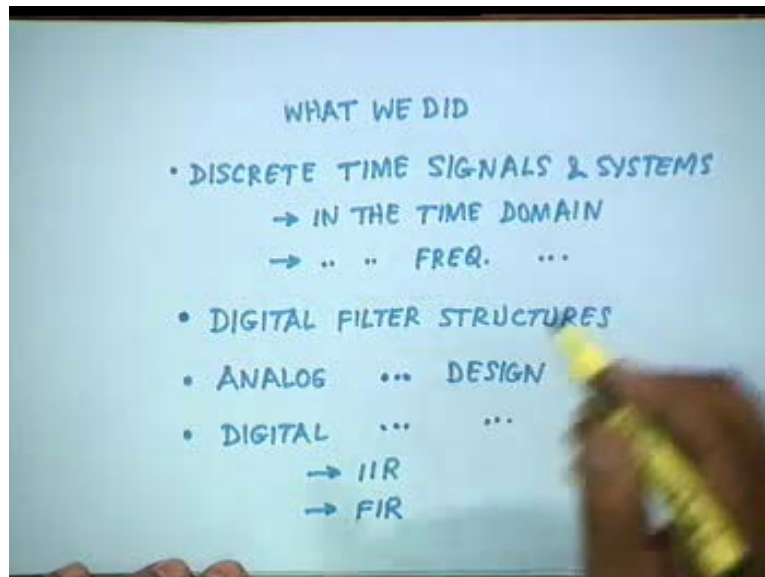
To conclude this course of lectures, we spent time on discrete time signals and systems; we considered the time domain and the frequency domain. In the frequency domain we uncovered Fourier Transform, Discrete Fourier Transform and Z transform. In the time domain, we showed how to calculate the convolution, we gave some tricks and then the rest of the semester we spent on filters. First we discussed digital filter structures and then as a prelude to digital filter design, we discussed Analog filter design thoroughly and then talked about digital filter design, IIR as well as FIR. We only discussed the analytical techniques. The computer aided techniques are modifications of numerical optimization procedures. And in FIR, Remez exchange algorithm is very much used which you can get in any book.

(Refer Slide Time: 42.28 – 43.50 min)

WHAT LIES AHEAD
- FAST COMPUTATION OF DFT
- FINITE WORDLENGTH EFFECTS
- MULTIRATE DSP
- WAVELET THEORY
- SELECTED APPLICATIONS OF DSP
PROPOSED TO BE DONE IN A SECOND COURSE

In the next course, DSP II, we propose to start with fast computation of DFT. We will do FFT and also number theoretic transforms which offer a better means of speeding up the computation of DFT. There is also a technique called WFT, the Winograd Fourier transform, which is still better. We then propose to do finite word length effects and how to minimize them. We will come across different kinds of structures in finite word length.

When we do finite word length effects, we will look into their minimization. And then we talk about multirate DSP where at different locations in a system, there are different sampling frequencies. In the interface between two sampling frequencies, decimation and interpolation are required. They pose a challenging problem in filter design and we shall do it with the help of procedures already provided. Then we shall do wavelet transforms in some details and finally some selected applications of digital signal processing. All these, as I said, are proposed to be done in the second course, DSP II and with that we close this course.