Digital Signal Processing

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Lecture - 14

Discrete Time Systems in the Frequency Domain

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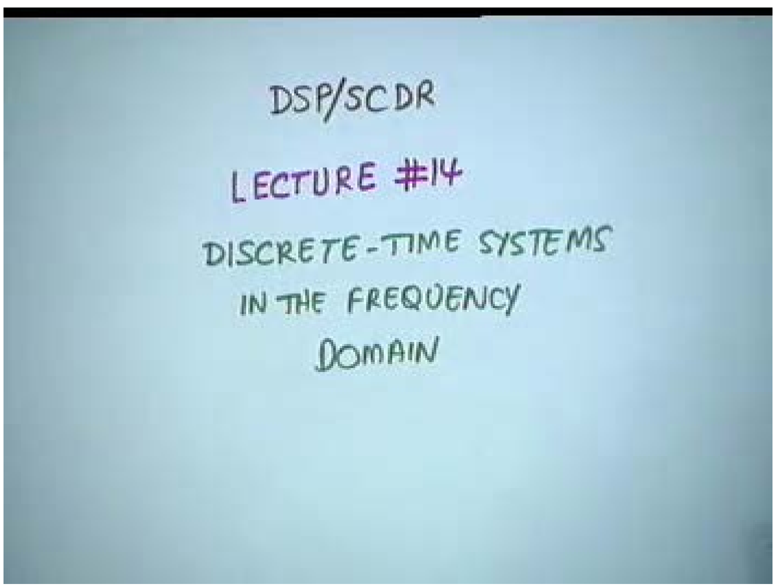

This is the 14th lecture on DSP and today we discuss the topic Discrete Time Systems in the Frequency Domain. We introduced this topic in the 13th lecture and today we shall look at it very carefully. In the previous lecture, we continued our discussion on z transform properties and the property that we particularly discussed in the last lecture was Differentiation, which gives you a clue in finding the inverse transform of $(1-\alpha z^{-1})^r$ where $r \geq 2$.

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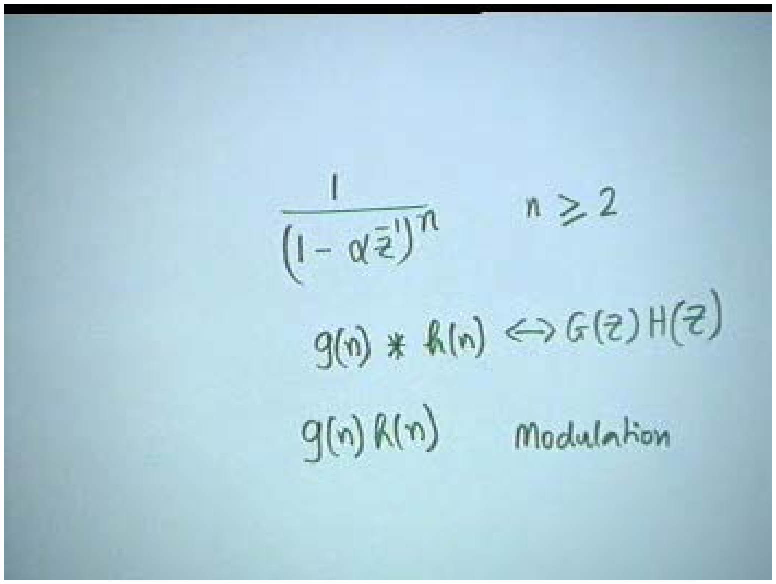

We discussed the convolution property that is, if you convolve g(n) with h(n) (linear convolution) then the z transform of this is simply the product of G(z) and H(z). Then we discussed modulation, that is, x(n) = g(n) × h(n) and what the z transform of this is. That is, in terms of an integral containing a convolution of G and H and a particular case of the modulation theorem in which g and h are identical leads you to the Parseval's relationship for energy.

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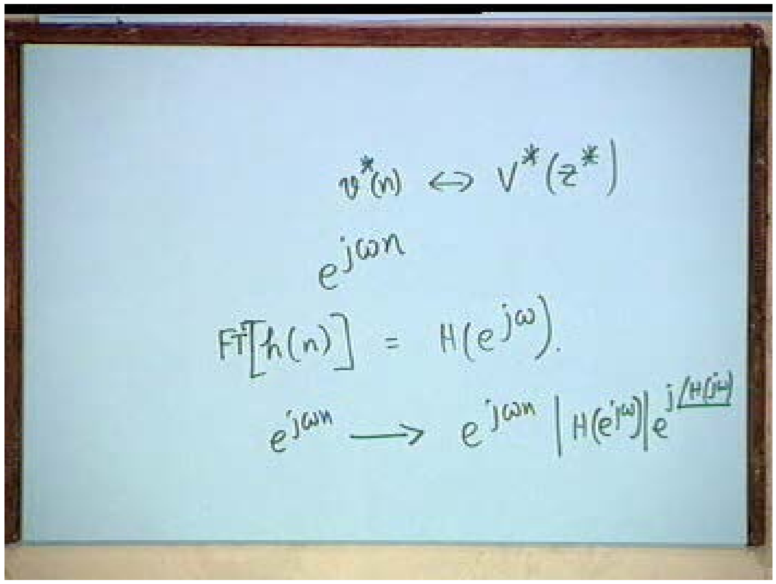

We took several examples; one was to illustrate the fact that v*(n) has the z transform V*(z*). The other one involved multiple poles and we showed how to invert G(z) with multiple poles, without differentiation. That is, you write a set of linear equations in the constants by putting specific values of z and then solve this set, which is much simpler than differentiation, and then putting in the required value of z. We illustrated the importance of the signal $e^{j\omega n}$ and we said that for an LTI system, $e^{j\omega n}$ is an Eigenfunction. An Eigenfunction is any function such that when it is fed as the input to the system, the output is exactly the same function multiplied by a complex constant. In our case this complex constant was the Fourier Transform of h(n) which we denoted by $H(e^{j\omega})$. $H(e^{j\omega})$ is a complex constant in such a manner that $e^{j\omega n}$ fed to an LTI system whose impulse response is h(n) is simply, $e^{j\omega n} \times H(e^{j\omega})$, which I can write in terms of an magnitude and an angle. If I know the response to $e^{j\omega n}$, then I know the response to cosωn and also sinωn, because it is a linear system. If I take the real part of the input, then I must take the real part of the output. Similarly, for the imaginary parts. Therefore this signal is an extremely important one for linear time invariant systems. We illustrated the importance of the signal by considering the example of an M point Moving Average System (M-pt MAS).

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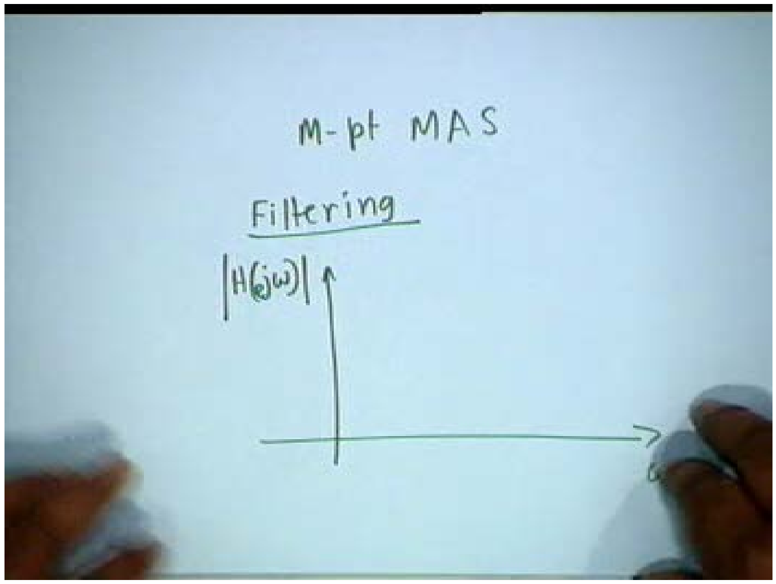

We showed that the magnitude response can be very easily calculated. It is of the form of magnitude sine divided by another sine. But in determining the phase one must be extremely careful to take account of the sharp transitions in phase by an amount exactly equal to 180^0 that occurs whenever the real quantity ratio of sines changes sign. And therefore the phase that you get with an M point Moving Average System is piece-wise linear. That is, it is a straight line up to a certain point then there is a transition and then again linear and so on. This piece-wise linear phase response and the ambiguity in phase response have to be remembered throughout your course on Digital Signal Processing.

We now discuss the concept of Filtering. Once we have come to the definition of frequency response, one can talk of filtering. A filter is an electrical device which selects the appropriate frequencies and rejects others. For example, an ideal Low Pass Filter shall have a frequency response like the one shown in the figure.

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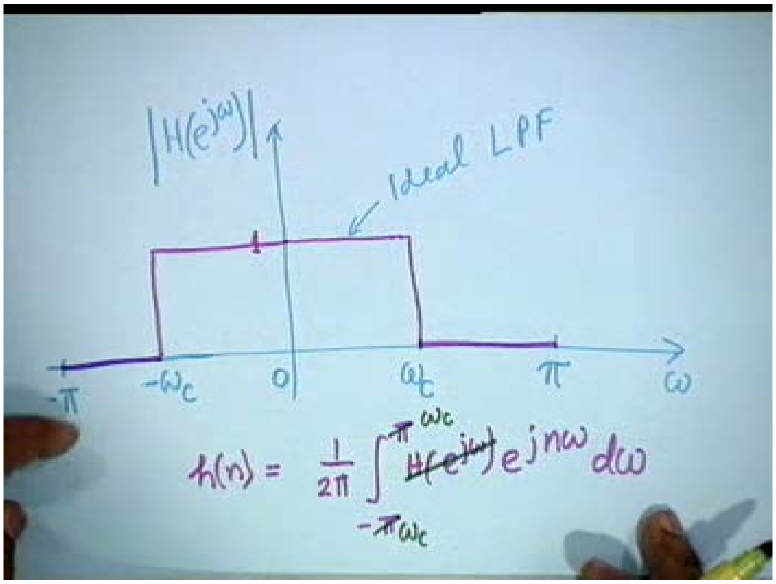

For an ideal Low Pass Filter, between $\omega = 0$ and some frequency ω_c, the magnitude should be ideally equal to 1. The value 1 is a normalized value. Beyond ω_c, the magnitude should be equal to 0 up to the end of our base band, namely π. The plot extends on the other side also in the same manner. We require negative frequencies as well as positive frequencies to make up a real signal. And $j\omega$, $e^{j\omega}$, are all contrived instruments to facilitate our analysis, design and synthesis and making life simpler in general. An ideal Low Pass Filter is not realizable due to a basic philosophical reason, viz. that nature abhors sharp transitions. People who are highly temperamental get angry very quickly but the society does not like them. They moderate them to an extent that they can be tolerated. In the particular case under consideration, there are other more tractable and analytical reasons. One is that if I have a characteristic like this, its impulse response h(n) is non causal. How do you find the impulse response? It is $[1/(2\pi)] \int_{-\pi}^{\pi} H(e^{j\omega})\, e^{jn\omega}\, d\omega$. $H(e^{j\omega}) = 1$ between $-\omega c$ and $+\omega c$ and the rest of it is 0. Hence put $H(e^{j\omega}) = 1$ and replace $-\pi$ by $-\omega_{c;}$ and $+\pi$ by $+\omega_c$.

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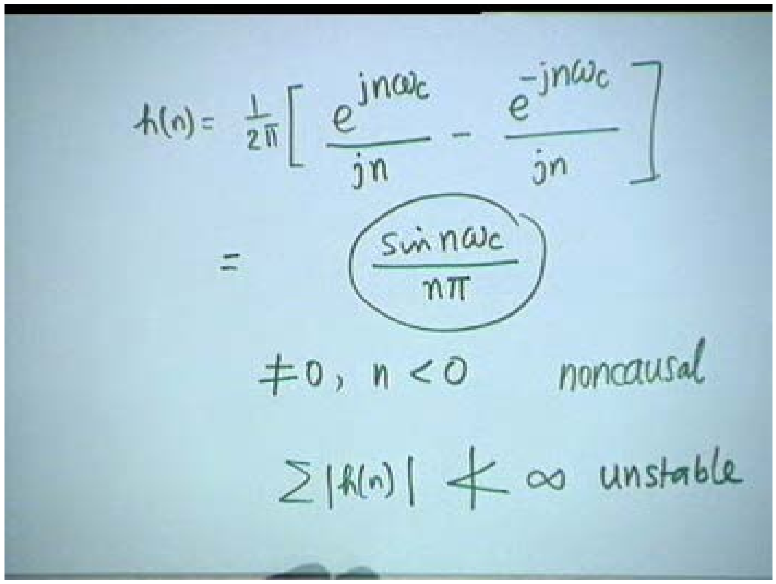

The integration is very easily seen as h(n) $=[1/(2\pi)][e^{jn\omega_c}/(jn) - e^{-jn\omega_c}/(jn)] = \sin \omega_c/(n\pi)$. Obviously at n = 0 it is 0/0 form, but by the usual L' hospital rule, it is found as ω_c/π. But this h(n) obviously is not equal to 0 for $n < 0$. It exists for $n < 0$ and therefore the system is non causal and therefore not realizable. There is another reason why it cannot be realized. This h(n) = $\sin n\omega_c/(n\pi)$ is neither absolutely summable nor require square summable. In other words, the system is unstable. Instability does not require square summability; it only requires absolute summability. This is true for Ideal High Pass, Ideal Band Pass, Ideal Band Stop or any ideal kind of filter. They all are non causal and also unstable, therefore they cannot be realized.

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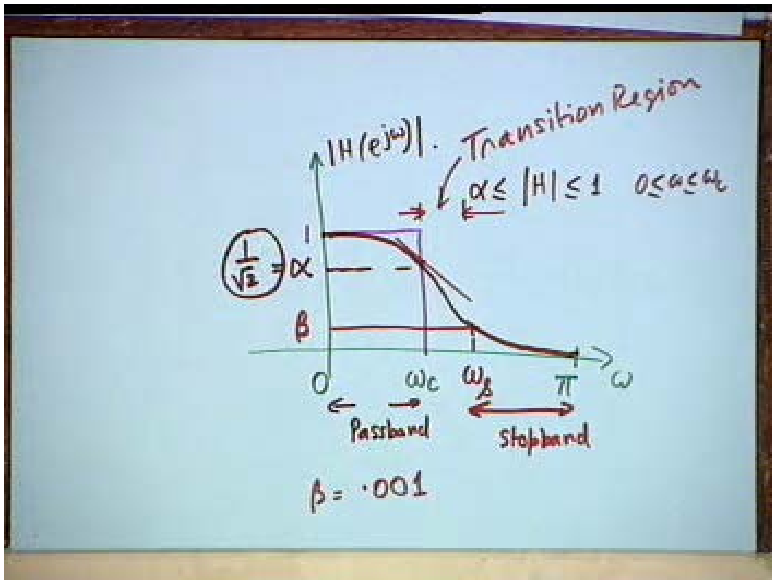

So what we can realize for a Low Pass Filter?

Fortunately, in Digital Signal Processing, we have a finite range of vision of frequency, viz. 0 to π. And when we are considering magnitude or phase it suffices to consider only the positive frequencies because we know for negative frequencies the magnitude is even and the angle is odd. Therefore if we know the positive part we know the other part also. The ideal LPF response has to be rounded off; so what we can have in practice is a smooth curve which is an approximation to the ideal low pass filter characteristic. A typical practical response is shown in the figure; we can have other kinds of characteristics also.

For example, the characteristic within the rectangle (0, 1, 1, ω_c, 0) does not have to be monotonic; it can have ripples, it can go up and down. Similarly in the band in which you do not require any frequencies, it does not have to be monotonic it can also go up and down. But the simplest, of course, is the characteristic shown. We are not considering phase. The magnitude characteristic is monotonic throughout. If it is an ideal characteristic, then ω_c is the cutoff frequency.

We now have to define the cutoff frequency. It is defined by the band of frequencies within which the magnitude does not fall below some quantity α. The cutoff frequency ω_c is defined by $0 \leq |H| \leq \alpha$, $0 \leq \omega_{c.}$ α traditionally is 0.707 or $1/\sqrt{2}$ because $20\log_{10} 1/\sqrt{2}$ is – 3dB. The passband is the band of frequencies within which the magnitude response does not fall by more than 3 decibels. This is the 3dB bandwidth of the Low Pass Filter.

Now we have to define a Stop Band. Ideally, Stop Band is ω_c to π having zero response; however, zero response for this extended range is not achievable. What we could have is zero response at some isolated points if we allowed ripples. But in a monotonic response, we have to define another constant β and the edge of a Stop Band ω_s such that between ω_s and π the magnitude response does not go beyond β. For example, if β is $.001 = 10^{-3}$ then in decibels it corresponds to 60 decibels down $|20 \log_{10}10^{-3} = -60|$. So typically, what you shall specify is that in the Stop Band the attenuation must be at least 60dB. If it is at least 60dB then your β is .001; ω_s shall also be specified.

For example, if it is a Speech Filter for digital telephony, you might specify ω_c (ω_c is a normalized digital frequency) to correspond to 3.3 KiloHertz, and ω_s may correspond to 4 KiloHertz; 04 to 20 KiloHertz (corresponding to the end of the audio band) is called the Stop Band. That is, whatever may happen elsewhere, within the stop band the signal must be attenuated by at least the amount specified which could be 60dB or it could be 80dB. The in between region ω_c to ω_s which, unfortunately, is a fact of life, has to be permitted because from ω_c, the magnitude cannot come down to β immediately. It cannot make an abrupt jump and if it does, then again non-causality shall step in! So you must allow a gap between ω_c and ω_s and this region is called the transition region, transition occuring between Pass Band and Stop Band. The narrower the transition region, the better is the filtering. Transition region sometimes is also specified by the cutoff slope. That is, if you find the slope at the cutoff point ω_c, then this is also an indication of how sharp the filtering is. The more the cutoff slope, the better is the rejection of unwanted frequencies. So a digital or analog filter in practice shall have a Pass Band, a Transition Band and a Stop Band. What typically you shall be specified is the bandwidth of the Pass Band and the Pass Band tolerance. There is nothing sacred about 3dB; it could be 1 db. In a more sophisticated situation, it could be .1dB also. Similarly you shall be specified the Stop

Band and the minimum attenuation in the Stop Band. If you over satisfy the Stop Band you are perfectly in order. But the transition region is a fact of life; it has to be tolerated. The narrower the transition region, the better is the filter. But then you also have to pay for any improvement that you desire. That is, sharper cutoff requires higher order of filtering; higher order means higher cost because of more hardware/software. This is the concept of filtering.

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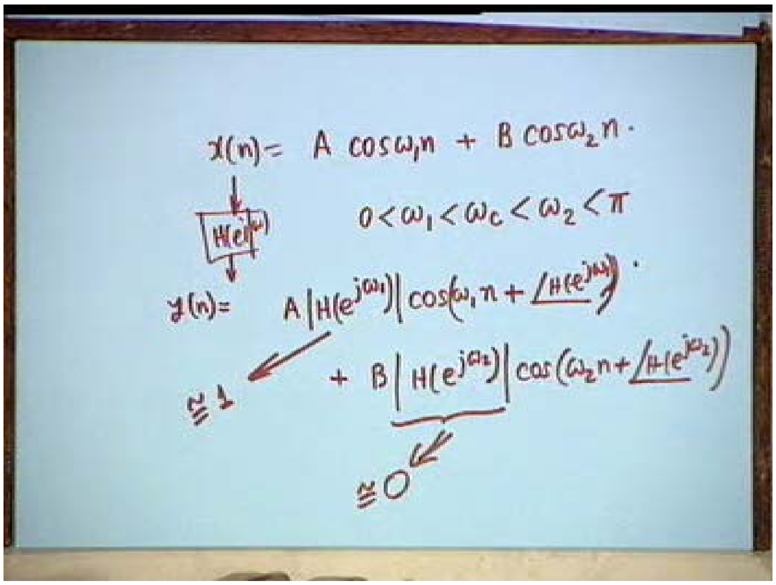

To illustrate the concept of filtering, let us take an example. Suppose you have an x(n) which contains two sinusoids A $\cos\omega_1 n$ + B $\cos\omega_2 n$; the frequencies are ω_1 and ω_2. The signal x(n) is not necessarily periodic. Digital signal, even if it is a sine or cosine representation, is not necessarily periodic. For periodicity, $\omega_1/(2\pi)$ must be a rational number; similarly $\omega_2/(2\pi)$ must be a rational number; then the period of the sum is the LCM of the denominator of the two rational numbers. Suppose we have two signals and we want to pass ω_1 and reject ω_2; that is the condition should be that $0 < \omega_1 < \omega_c < \omega_2 < \pi$. This specifies that we require a Low Pass Filter, and 0 to ω_c is our Pass Band, so that ω_1 lies there. ω_2 is greater than ω_c means that it is in the Stop Band.

Now, what would be y(n), if I feed this signal to a linear time invariant system? The magnitude A would be modified by $|H(e^{j\omega 1})|$ and the phase would be modified to $\cos(\omega_1 n$ + angle H $(e^{j\omega 1}))$. All we have to do is to find the frequency response of the system $H(e^{j\omega})$; then we can write directly the output. So $\cos\omega_1 n$ shall be affected in magnitude by magnitude of H at $\omega = \omega_1$ and its angle shall be shifted by angle of $H(e^{j\omega 1})$. Similarly, the second signal shall give the output B $|H(e^{j\omega 2})| \cos(\omega_2 n$ + angle of $H(e^{j\omega 2}))$. All that I have to do now is to choose the system in such a manner that $H(e^{j\omega 1})$ should be approximately equal to 1. Ideally, I can also make it 1. And magnitude $H(e^{j\omega 2})$ should be approximately equal to 0. Let us take a specific example and to bring variety into experience, we consider a High Pass Filter.

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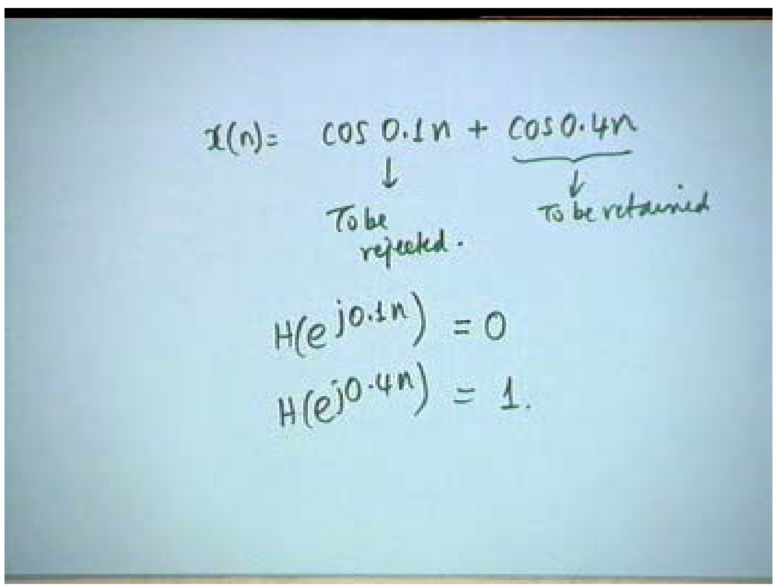

A and B are constants; we need not bother about them. So let x(n) = cos 0.1n + cos 0.4n. Is this signal periodic? No, it is not. .1/(2π) is not a rational number. We want to retain (cos 0.4n) and reject (cos 0.1n). So what we want is that $H(e^{j.1})$ should be equal to 0. Why not make an effort to make it equal to 0 and $H(e^{j.4}) = 1$. Let us see if we can satisfy these. Let us design the simplest possible kind of filter. Let us aim an FIR filler of length 3, arbitrarily; I can also choose length 2.

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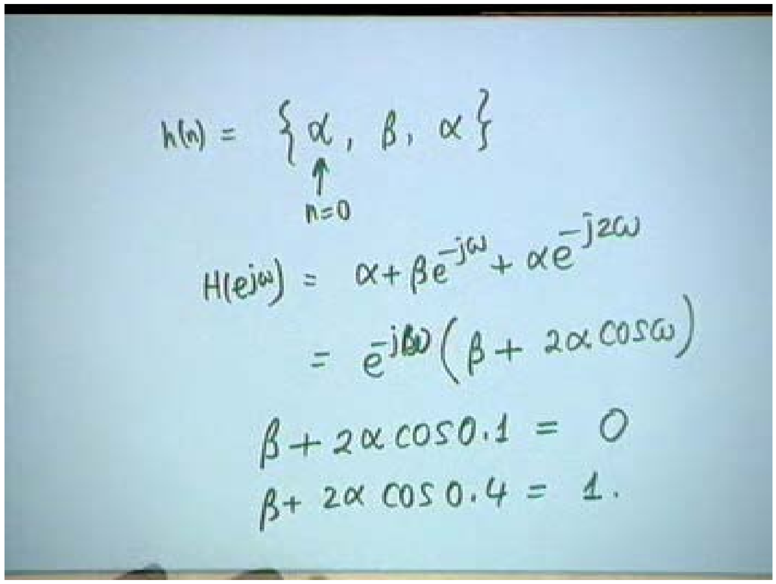

We have length 3 and suppose h(n) = {α, β, α} corresponding to n = 0, 1, 2. I require two degrees of freedom. I require two constants which are to be adjusted to satisfy the two conditions, that is, at .1 the magnitude = 0 and at .4 the magnitude = 1. That is why I chose two constants here. And therefore $H(e^{j\omega})$ which is the Fourier Transform of h(n) can be written as $\alpha + \beta e^{-j\omega} + \alpha e^{-j2\omega}$ which, if I take $e^{-j\omega}$ out, becomes β + 2 α cosω The magnitude of H need not necessarily be this quantity; it is real but it does not mean that it cannot go negative. It can be positive as well as negative. This is a step where you can falter throughout your journey of Digital Signal Processing. Whenever a real quality is given, the fact that it can be positive as well as negative, has to be respected. The negative sign has to be respected by including a π in the angle. So the angle of H is – ω as long as (β + 2 α cosω) is positive and it is – ω + π whenever the quantity changes from positive to negative. That is, whenever it becomes 0 there is an increase of phase by an amount π. Now let us aim at β and α in such a manner that (β + 2 α cosω) quantity shall remain positive. So what I want is β + 2 α cos .1 = 0 and β + 2 α cos .4 = 1. You can subtract one from the other and find α very easily. (cos .1 and cos .4 can be found out from the tables: remember that the angles are in radian, not degree).

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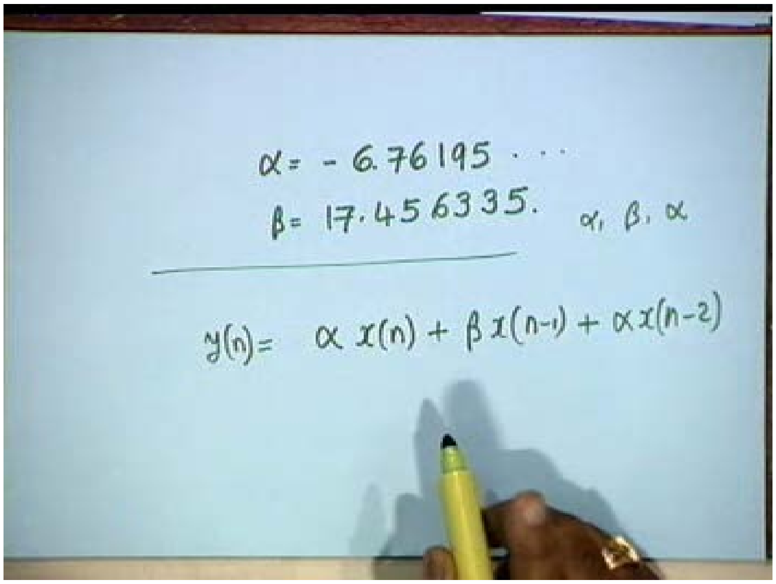

By solving the two equations, we get $\alpha = -6.76195$ and $\beta = 17.456335$. With these values you can verify that $\beta + 2\alpha\cos\omega$ indeed remains positive between .1 and .4. How did we decide about how many digits we retain after the decimal point? It depends on how many bits are available to represent the numbers. If you have a limited amount of cloth, then we have to decide how many pieces of dress can be made out of it. Depending on the number of bits of hardware you have on your computer, you decide the precision on the decimal numbers you can accommodate. For example, if it is an 8 bit number you should not truncate after the first decimal. On the other hand if it is a representation in floating point then you can use more decimals. After we have found out α and β, the system is now known. What would be its difference equation? It is $y(n) = \alpha x(n) + \beta x(n-1) + \alpha x(n-2)$, as simple as that. You can implement this with two multipliers only because $x(n)$ and $x(n-2)$ can be added together and then multiplied by α.

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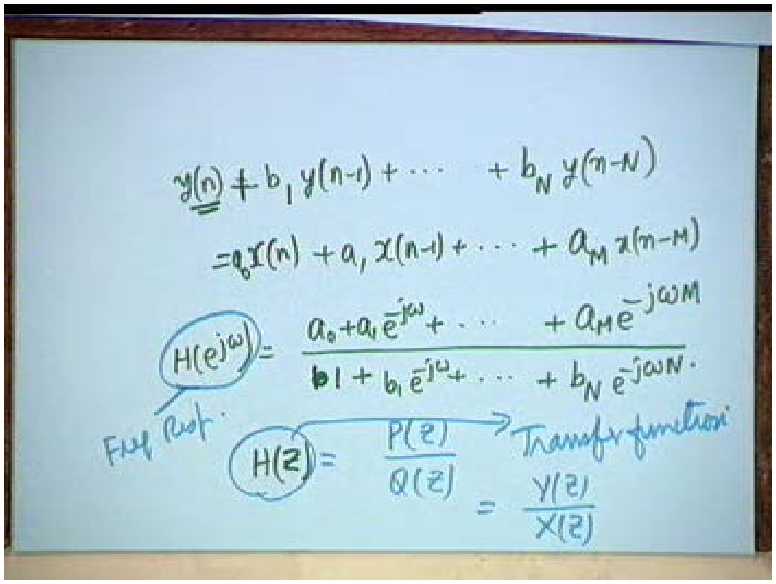

Reducing the number of multipliers is an extremely important criterion in the realization of a DSP. In general, you know that a DSP can be represented by a difference equation like this: $y(n) + b_1y(n-1) + \ldots + b_N y(n-N) = a_0x(n) + a_1 x(n-1) + \;\; + a_Mx(n-M)$. This is the general representation of an LTI Discrete Time System. It can be FIR, or IIR. You can find the frequency response $H(e^{j\omega})$ by taking the Fourier Transform of both sides. The FT of the right hand side is $a_0 + a_1 e^{-j\omega} + \ldots. + a_M e^{-j\omega M}$. Note that we have taken $b_0 = 1$; this is the discipline we have tried to inculcate. $b_0 \neq 1$ can always be handled by dividing both sides so that the coefficient of $y(n) = 1$. The left hand side of the difference equation transforms to $1 + b_1 e^{-j\omega} + \ldots + b_N e^{-j\omega N}$. The frequency response $H(e^{j\omega})$ is the ratio of the two polynomials and is, in general, IIR. There is no reason why it cannot represent an FIR system also by making all bi's equal to zero. This can now be generalized in terms of z transforms.

So far we have considered representation of the Discrete Time System in the ω domain by taking the Fourier transform. There is no reason why you cannot use z transform and generalize H to H(z). That is, instead of taking the Fourier transform, we can also take z transform provided the z transform exists. And H(z) can now be written as $(a_0 + a_1 z^{-1} + \ldots. + a_Mz^{-M})/(1 + b_1z^{-1} + \ldots + b_Nz^{-N})$. It would be, in general, of the form P(z)/Q(z). This is a rational function, being a ratio of polynomials in z inverse (z^{-1}). When we want to find out frequency response, we will simply put z as $e^{j\omega}$, provided the ROC of H(z) includes the unit circle; if this is not the case then we cannot do it. Fortunately, most of the systems that are of our interest can be treated by both z transforms and Fourier transforms. However, there is a change in nomenclature. Why is $H(e^{j\omega})$ called the Frequency response? It is called frequency response because this gives the spectrum in terms of actual ω. To be fair, you must also give a name to H (z). This is simply called the Transfer Function because, as we can see, it is also the ratio of Y(z)/X(z). How X(z) is transferred to the output is characterized by the ratio H(z). One should remember that the z domain Transfer function and the Frequency response are intimately related, provided, of course, both of them exist.

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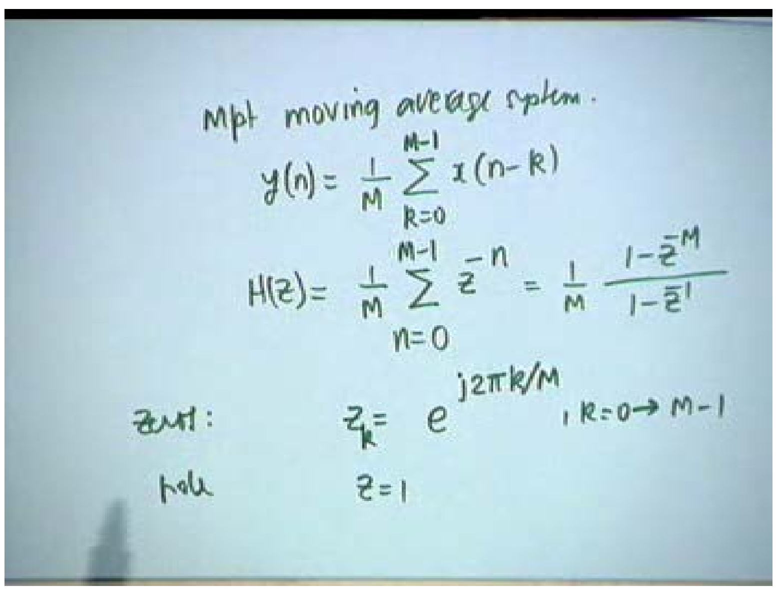

As an example, consider the M point Moving Average System that is y(n) = (1/M) ∑x (n – k) where k goes from 0 to M – 1. What is the impulse response h(n) of the system? It is simply 1/M for n going from 0 to M – 1, otherwise it is 0. You can take the z transform and get H(z) = (1/M) $\sum z^{-n}$,where n goes from 0 to M – 1. This is a geometric series. The z transform exists because the summation is finite. H(z) can also be written as (1/M) $(1 - z^{-M})/(1 - z^{-1})$; it is a rational function but it represents an FIR system because the pole at z = 1 cancels with the zero at z = 1.

The zeros of the system, M in number, are at $z = -e^{j2\pi k/M}$, k = 0 to M – 1. There is just one pole at z = 1 and you notice that in the zeros, k = 0 gives z = 1. So there is a pole zero cancellation and the system is still FIR. To determine the Frequency response, you just put $z = e^{j\omega}$ and then put it in the form of sine/sine and an angle and carry out the rest of it, as we did earlier.

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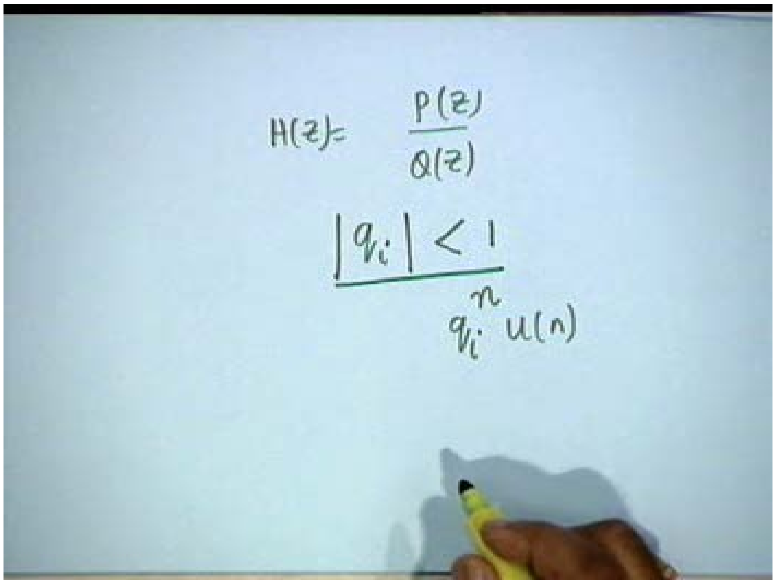

Now, when you express the Transfer function as P(z)/Q(z) the zeros of Q(z) are of course the poles of the system and the zeros of P(z) are the zeros of the system and you know how we represent them in the z plane by means of crosses and small circles. But the important point is that if q_i is one of the poles then the magnitude of q_i must be bounded by unity. What is the logic? It is because the impulse response will have the component q_i^n u(n) and if q_i magnitude is not bounded by unity then it grows without limit, so it becomes an unstable system. It is not that the unstable systems are not useful. If you want to make a digital oscillator, then you do require an unstable system but not for any other purposes. So all q_i's must be inside the unit circle. This is a repetition of what we have stated earlier. We continue our discussion on Filtering and take some simple examples of filters.

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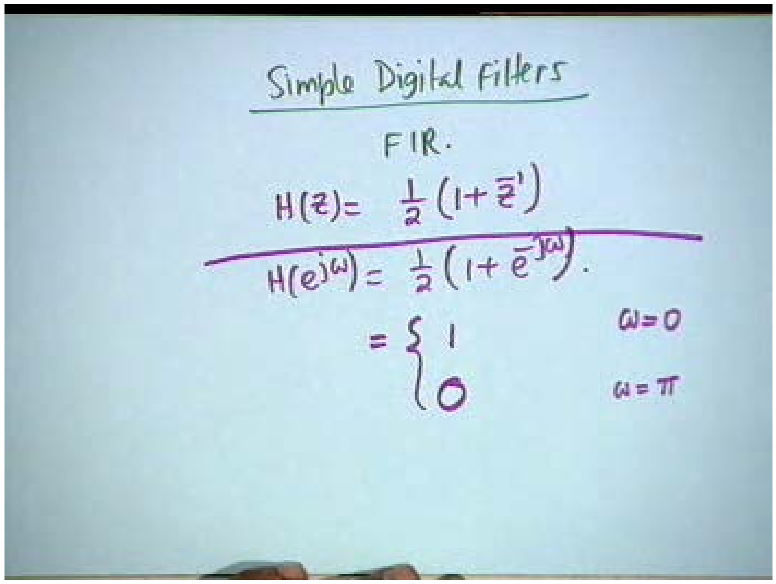

We consider some simple FIR filters to begin with. It is much simpler to consider rather than IIR filters. Let us consider first a function like ½(1 + z^{-1}). Any FIR system will be a filter. In fact, any system which has a frequency response will favor some frequencies and reject others. What kind of filter does this example represent? The best way to do it is to look at the function and put z = $e^{j\omega}$ to consider the Frequency response. Here $H(e^{j\omega}) = (1/2)\ (1 + e^{-j\omega})$ and you see that at ω = 0 it is equal to 1 and at ω = π it is 0; therefore it is a Low Pass Filter. It starts at 1 at ω = 0 and goes to 0 at ω = π. Now, in order to decide whether it is a Low Pass or High Pass or any other kind of filter, later on this step will not be required because ω = 0 corresponds to z = 1 and ω = π corresponds to z = – 1. So you just have to find H(z) at z = + 1 and – 1. Suppose both of them are 0 then we know that in between there must be a maximum; so it must be a Band Pass Filter.

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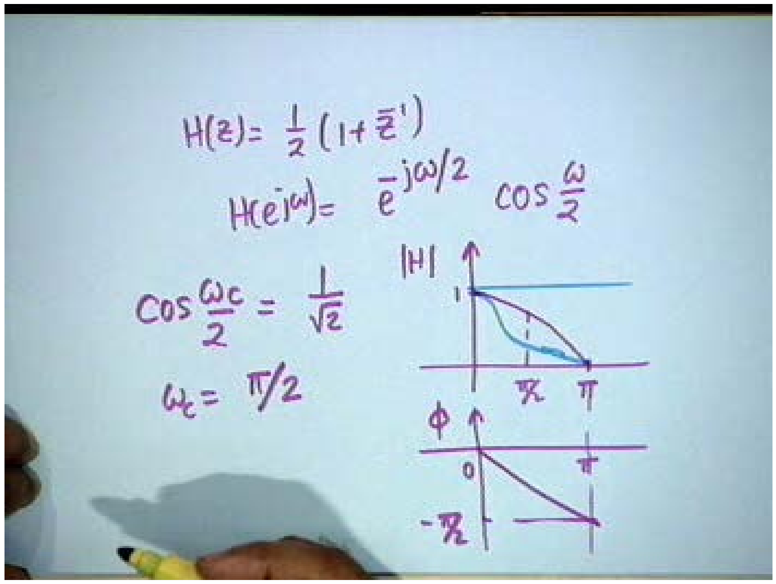

For this Low Pass example $H(z) = ½(1 + z^{-1})$ the frequency response is $H(e^{j\omega}) = e^{-j\omega/2} \cos(\omega/2)$. Now $\cos(\omega/2)$ in the range 0 to π is positive. So if I plot the magnitude it starts from 1 and at π it goes to 0 so the characteristic would be something like what is shown in the figure, which is indeed a Low Pass Filter. And if I plot the angle φ from $\omega = 0$ and π, the angle will be strictly $-\omega/2$. So it starts from 0 and goes to $-\pi/2$ at $\omega = \pi$. In the range of vision the real quantity does not change sign, so we are comfortable.

Now if this is a Low Pass Filter, we want to find out the 3-dB cutoff frequency ω_c which satisfies $\cos\omega_c/2 = 1/\sqrt{2}$; thus $\omega_c = \pi/2$, which is exactly half way between 0 and π. This Low Pass Filter has a large bandwidth and large Stop Band. This is not a very good one but it is the most elementary FIR Low Pass Filter. Is there a way to sharpen the characteristic, or make it better, for example like the second curve in this figure? Yes, the way is to cascade, say two of them. Obviously, since $|H|<|$ for $\omega>0$, if I square $|H|$, the frequency response will come down. If in Stop Band, $|H|$ is 0.1 somewhere, then $0.1^2 = 0.01$; therefore by cascading two such filters, I can get a better characteristic.

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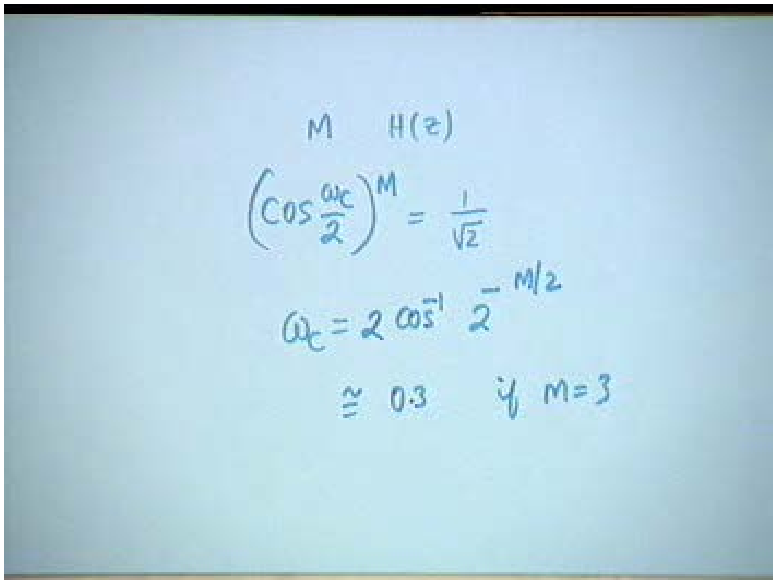

If M number of such H(z) are cascaded, where would be ω_c i.e. what would be the cutoff frequency? This is easily found out from $|H|^M = 1/\sqrt{2}$. So ω_c = 2 cosine inverse $2^{-M/2}$. This is approximately equal to 0.3 if M = 3 (three sections); so the cutoff frequency comes closer to ω = 0. This is the way people still design, people who do not know much of Digital Signal Processing or wish to keep life simple. For example, for medical signal processing, $\frac{1}{2}(1 + z^{-1})$ is still a very important filter.

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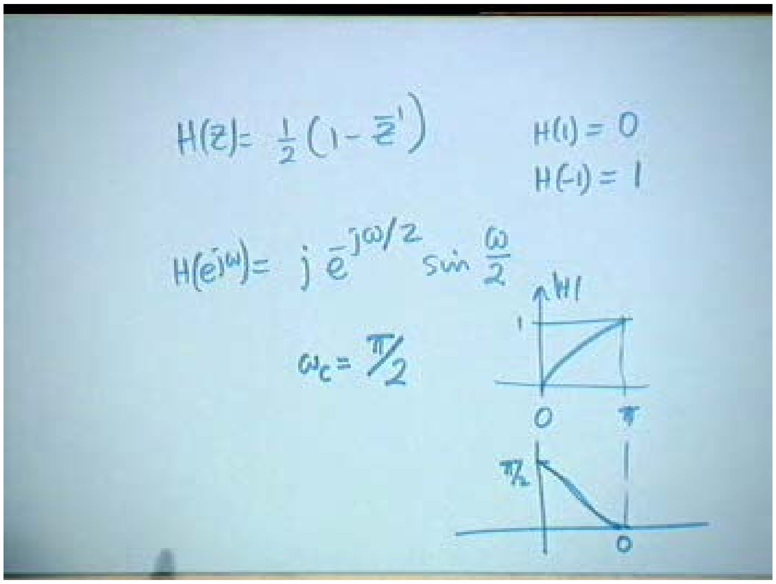

Next consider $H(z) = ½(1 – z^{-1})$; without going through any Fourier Transform, we can conclude that this filter is High Pass, because H(1) = 0 and H(– 1) = 1. You can also show that $H(e^{j\omega}) = j \times e^{-j\ \omega/2}\ \sin\omega/2$ by putting z = jω. Here magnitude is the magnitude of sin ω/2 which does not change sign in the range 0 to π. You can show that ω_c, the 3db cutoff frequency is once again π/2 but what about the phase now? Phase at ω = 0 is π/2 contributed by the term j; beyond ω = 0 the phase will be π/2 – ω/2 so it comes linearly down to 0 at ω = π. One fact of great importance is the following: that $H(z) = ½(1 + z^{-1})$ was a Low Pass Filter but $(1/2)\ (1 – z^{-1})$ is a High Pass Filter. This is in general true, i.e. if in a LPF transfer function, you change the sign of z, you get a HPF.

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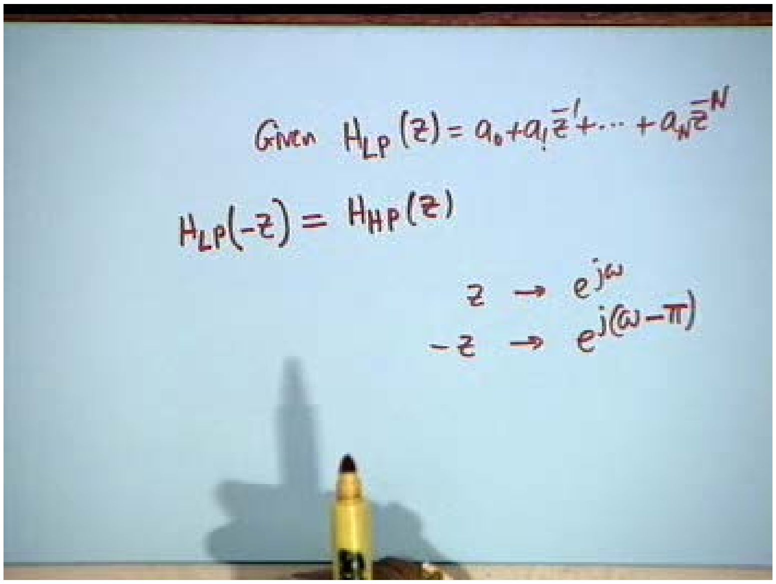

We repeat: Given a Low Pass Filter $H_{LP}(z)$ you can always derive a High Pass Filter $H_{HP}(z)$ if you change z to – z. The logic is not difficult to find. You see z corresponds to $e^{j\omega}$ and –z corresponds to $e^{j(\omega \pm \pi)}$. In $\omega \pm$ pi, whatever occurs at $\omega = 0$ shall now occur at $\omega = \pm \pi$. The magnitude was 1 for the Low Pass Filter at $\omega = 0$ and now the magnitude will be 1 at $\omega = \pm \pi$. So the characteristic is just flipped over. It is in general true that in any Low Pass Filter, if you change z to – z then you shall get a High Pass Filter.

In terms of an FIR filter for example suppose the Low Pass Filter is $a_0 + a_1 z^{-1} + \dots + a_N z^{-N}$; then changing z to – z is also equivalent to changing the sign of alternate coefficients. The coefficient of z^{-2} remains a_2 but the coefficient of z^{-1} becomes $-a_1$. Changing the sign in alternate coefficients means that you modify the impulse response such that the sign of alternate samples are changed. So it is very simple to transform a Low Pass to High Pass filter. The characteristics are complementary as the cut off frequency remains the same. They would be ω_c and $\pi - \omega_c$. Does it remain symmetrical? Not necessarily. For the simple case $1 + z^{-1}$ or $1 - z^{-1}$, they remained identical but it is not necessarily true. The next thing I would discuss is a Band Pass Filter.

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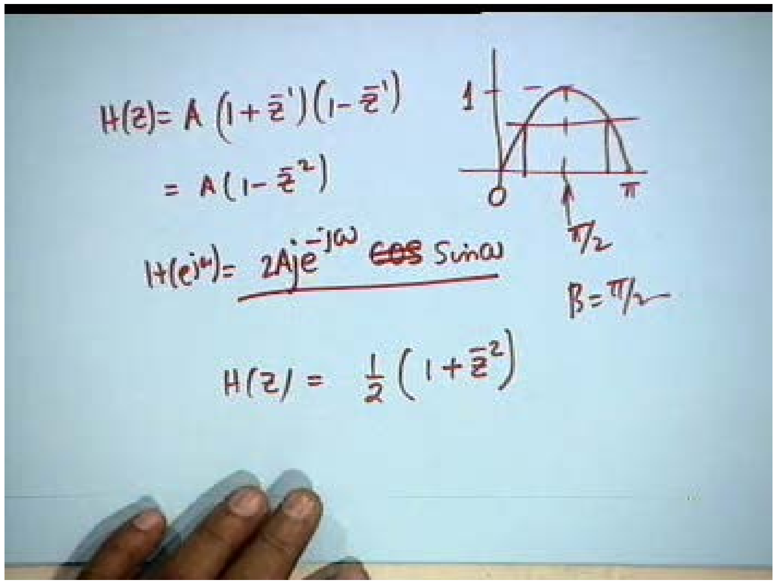

I want a characteristic like this: at π it should be 0, at 0 it should be 0 and somewhere in between it should be 1. One of the very simple things I can do is to cascade a low pass with a high pass. That is, I multiply $(1 + z^{-1})$ by $(1 - z^{-1})$.what would be the constant multiplier: 1/4 or ½? Suppose I take a function like this $H(z) = A (1 + z^{-1}) (1-z^{-1})$, then obviously at $z = 1$ it is 0, at $z = -1$ it is also 0; where does the maximum occur? For that we have to write this as $A(1 - z^{-2})$ and therefore $H(e^{j\omega}) = 2Ae^{-j\omega}$ j sinω. Obviously this frequency response shall have a maximum magnitude 2A; thus 2A = 1 or A = 1/2 at π/2. It is a simple Band Pass Filter.

Now we shall have to be careful with the phase. sinω remains positive between 0 and π. Hence phase = $\frac{\pi}{2} - \omega$. It starts at $\frac{\pi}{2}$ at ω = 0, becomes zero at $\omega = \frac{\pi}{2}$ and $\frac{-\pi}{2}$ at ω = π. What is the bandwidth of this band pass filter? You can easily show, by putting $\sin \omega = \frac{1}{\sqrt{2}}$, that this is satisfied by ω = π/4 as well as 3π/4. Hence the bandwidth is π/2. Bandwidth is the difference between the two frequencies at which the magnitude is $\frac{1}{\sqrt{2}}$. It is a poor Band Pass Filter, nevertheless it is Band Pass. Suppose I wanted a Band Stop then what should I do? One method is; All Pass minus Band Pass. Another method is to use $\cos\omega$ as the frequency response because $\cos 90 = 0$, $\cos 0 = +1$ and $\cos \pi = -1$.