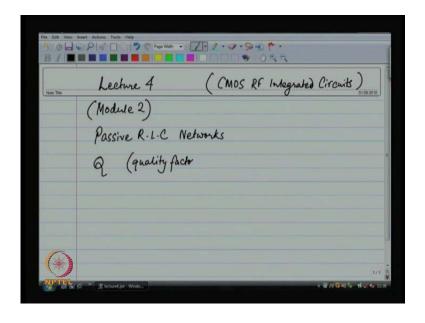
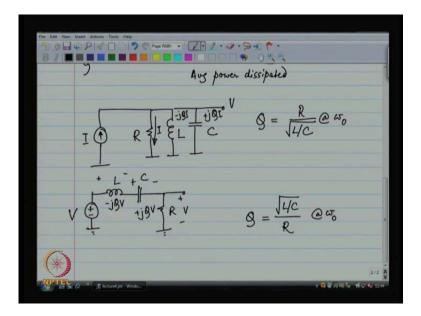
CMOS RF Integrated Circuits
Prof. Dr. S. Chatterjee
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Module - 02
Passive RLC Networks
Lecture - 04
Parallel RLC Tank, Q

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Hello everyone, this is the 4 th lecture in the CMOS RF Integrated Circuits. We are now covering, we are now in the middle of the 2 nd module. This module is about Passive RLC networks and we have been discussing Q or rather quality factor, as it is often called. Now, we go back and the definition of Q was, basically Q is defined at a given frequency.

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So, at given frequency, at the frequency omega, Q is to be defined to be omega times the peak energy that is stored in the circuit divided by the average power dissipated by the circuit. Now, notice the dimensions, omega has the dimension of 1 by seconds, peak energy is Joules. So, the numerator is Joules per second, denominator is average power, power is watts which is also Joules per second. So, over all the quantity is dimensional less, so the quality factor is a factor, dimension less quantity.

So, this is the definition of my quality factor and then, we had briefly analyzed two circuits, the series and the parallel RLC networks. So, we first looked at parallel RLC network, I pushed in a current of I and I absorbed the voltage that was seen across the resistor. And here we found out that, the peak energy that you stored divided by the average power that was dissipated, times the frequency, was given by at the resonant frequency.

Resonance frequency is of course, square root of L by C, so this was Q and then, we also briefly looked out the series RLC network, where I applied a voltage source. And here, we observed that, the Q at the resonate frequency was, I think I am making the mistake, I wrote the exact opposite of what should have been. So, for the parallel RLC network, R is in the numerator, so as the resistance increases, when the resistor is infinitely large, infinitely have quality factor.

And for the series RLC network, the resistance is in the denominator, when the resistance is 0, the design is loss less, in that case the quality factor is infinitely large. So, this is the our result from the previous lecture and this is as far as we did. We also observed that, at resonance the current going through the resistance is I and the current going through the inductors and the capacitors, one was j times Q I, the other was minus j times Q I, etc, I do not remember which is which.

And similarly, as far as the series RLC network goes, the voltage across the resistor at resonance was V. And at resonance, the voltage across the inductor was minus j Q V and the voltage across capacitor plus j Q V, something of this fashion, not exactly sure of which is plus, which is minus, but does not matter, it is irrelevant. What is important to observe over here is that, you can have the voltage source of 1 milli volt and because your Q is 100.

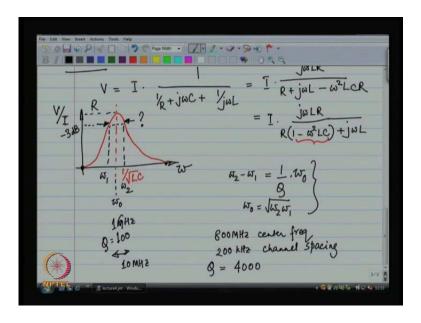
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In that case, the quality factor is infinitely large, this is the our result from the previous lecture and this is as far as we did. We also observed that, at resonance the current going through the reissuance is I and the current going through the inductors and the capacitors, one was j times Q I, the other was minus j times Q I etcetera, I do not remember which is which. And similarly, as far as the series RLC network goes, the voltage across the resistor at resonance was V.

And at resonance, the voltage across the inductor was minus j Q V and the voltage across the capacitor plus j Q v, something of this fashion, not exactly sure of which is plus and which is minus, but does not matter, it is irrelevant. What is important to observe over here is that, you can have a voltage source 1 milli volt and because your Q is 100, you can have 100 milli volt across the inductor and across the capacitor or you could have 1 ampere as the current source.

And because your Q is 100, you could have 100 ampere rushing through the inductor and capacitor. So, it is definitely something unexpected, so this was our first observation.

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Let us now look at bandwidth of such a circuit, let us look at the parallel RLC network and there V was equal to the voltage developed. The potential developed across RLC in parallel was I times 1 by 1 by R plus j omega C plus 1 by j omega L, which is also equal to I times j omega L R divided by R plus j omega L minus omega square L C R, so this is what we have in the previous class. Now, if I ask you what is the bandwidth, so I can plot V by I as a function of frequency and at low frequencies at d c, V by I is going to be 0, because omega is 0.

At the extremely high frequencies, I have omega square in the denominator, so therefore, V by I is again going to be asymptotically going to be 0. And at omega square equals to 1 by L C, this factor is going to cancel out, this factor is going to cancel out at omega square is equal to 1 by L C which means that, V by I is just going to be equal to R. So, my curve looks something like this, where this point is 1 by square root of L C, let us not plot in terms of f, let us plot in terms of omega and this value is equal to R, which is 5.

Now, my question to you is, what is the bandwidth, so I understand that, I have the band pass characteristic centered around the frequency 1 by square root of L c, but what exactly is the bandwidth of this band pass characteristic. So, normally when we talk about bandwidth we say that, at what frequency is the response 3 dB below the

maximum or the factor of 1 by square root 2 3 dB is 1 by square root of 2 times the maximum.

So, this is the question and what one can derive is as follows, you can show, you probably have already done this in your control theory or in your earlier circuit classes. So, we would not do it, we would not spend too much time, so one can show that, this bandwidth, let us say this is omega 1 and this is omega 2 you can show that, omega 2 minus omega 1 is going to be equal to 1 by Q times omega naught. So, omega naught is 1 by square root of L C, Q is whatever Q you have, for that particular parallel network.

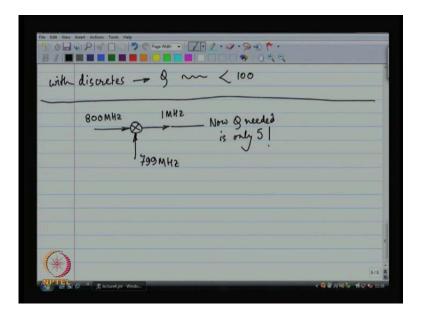
If it is a series network, the same relationship holds without any further modifications then, you just replace the value, put the appropriate value of the quality factor over there. You can also show that, omega naught is the geometric mean of omega 2 and omega 1, so all of this comes from your circuit theory. So, you have two equations, two unknowns and you can compute, what are the values of omega 2 and omega 1. So, this is bandwidth, what does it mean for us, this means that, as I increase Q, the response of my circuit becomes sharper and sharper around the resonant frequency.

So, if I have the Q of 100 at the frequency of 1 Giga Hertz then, that means, so if this is 1 Giga Hertz, if my Q is 100 then, the bandwidth is going to be 10 mega Hertz. If I say that, my channel spacing between two subsequent channels is 100 kilo Hertz, GSM has channel spacing of 100 kilo Hertz or 200 kilo Hertz. And you want to listen to, what is going on in your channel, not in someone else channel, the centre frequency is 800 mega Hertz.

So, if you want to design, if you are asked to design a band pass filters or if you asked to design circuit that can zoom into 200, that can resolve 200 kilo Hertz channel spacing at a centre frequency of 800 mega Hertz then, what is the quality factor that you need. You need the quality factor of 4000, now such kind of the quality factors are very very hard, if is not impossible to achieve. Even with discrete components, the quality factor of 4000 is most probably not going to be possible to achieve or in an IC Integrated Circuit, you can achieve quality factors probably upto 10. So, any one talks to you about designing a circuit, which I has Q of more than 10, you stay away or you have accepted it as big challenge. I mean, first of all Q is proportional to frequency, so may be at higher

frequency, you can achieve better Q's, better quality factor. But, even then, Q of 10, Q of 15 is probably the most you can achieve on an integrated circuit.

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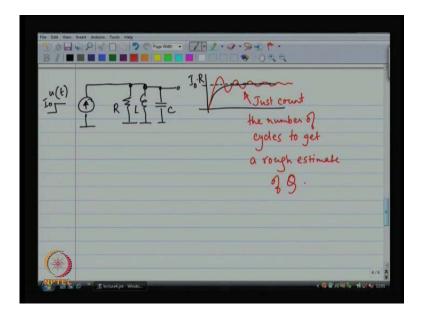
If you use discrete components besides the integrated circuit, if you also allowed to use discrete components, your inductor and capacitor, let us say these are discrete components, not an IC then, ,may be you can achieve Q of 200. These are reasonable numbers, more than this becomes unreasonable, so that kind of tells you that, if someone ask you to design a band pass filter for GSM, which has the 200 kilo Hertz channel spacing.

And you have to resolve this channel spacing in the band filter then, she should be told that, it is unreasonable for him to expect such a filter out of you. Because, the Q of such a filter would be 4000, it is not possible to do this, obviously people do it even then I mean, you do resolve 200 kilo channels spacing. So, there are other techniques to do it, you translate it to base band, where the centre frequency is much lower. So, you use the mixer this is what we do, so instead of trying to resolve 200 kilo Hertz channel spacing at 800 mega Hertz.

We mix the frequency down, let us say I will bring the frequency down, I will mix it with 799 mega Hertz then, the new centre frequency become that 1 mega Hertz. And that 1 mega Hertz, it is very easy to resolve a channel spacing of 200 kilo Hertz, because all I

need is Q of 5. So, this is typically what we do, we do not try to built this super filter at RF frequencies, the next observation that is important is the transient response.

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So, this is my network and instead of continuous current, suppose I apply a pulse or the square wave or a step function then, one can show it is not very difficult. If it was only R and C and you apply a step function then, what is the voltage is going to look like, the voltage is going to look like a, it is going to have the time constant, R C time constant. So, this is what going to happen if it was just an R C and now, we have, what is this value going to be, now on top of it we also have an inductor, so what do you expect, you expect ringing.

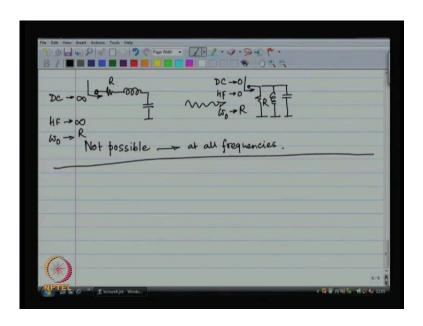
And if the resistance is more then, it shoots to a larger value and the ringing is also going to continue for a longer duration. Also, if the resistance is more then, the Q of the net will becomes more, if the resistance had not been there then, you would basically see a perpetual sinusoidal oscillation as a response. Now, there is resistance, so as a result, it is going to be a damped oscillation, the less this resistance is, the more the damping is going to be.

Typically, if you observe this wave form on an oscilloscope, you can just look at it and estimate the Q of your circuit. So, all you have to do is, count the number of cycles you can see, so over here how I have drawn 1 2 3 4 5 6 crests and troughs. So, I can estimate and of the back my head I can say that, most probably this network has the quality factor

of 6. So, this is observation number 2, this also can be shown, this also can be proved without great difficulty, I am not going to prove it, you can take a look at the books and it is quite a easy derivation.

So, you can look at the frequency response and estimate Q, you can look at the time domain response and estimate Q of the network without any great difficulty. This holds both for series RLC networks, parallel RLC networks, anything and you have shown it in only the case of parallel RLC, in fact I have not worked any proof, what so ever, but you can do it for any configuration.

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Now, the next important thing is a series to parallel transformation, now the question is, that suppose I have RLC network, is it possible to transform this into an equivalent parallel RLC network, is it possible to do this. So, the fine blank the answer is no, why not, why this is not possible I mean, I said no, why not, let us just look at 0 frequency. At 0 frequency, the series network looks like an infinite input impendence, let us look at the input impedance.

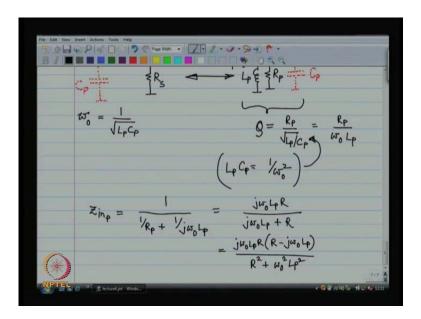
At dc, it is infinity large, because the capacitor is open circuit, at a high frequencies it is infinitely large, because now the inductor is an open circuit. And then, look at the other one, at dc the input impendency is 0, because the inductance is the short circuit. At high frequency, the input in impendence is 0, because the capacitance is short circuit. At the

resonant frequency, the input impedance is R in this case and R in the other case, so the only semblance of similarity is at the resonant frequency.

So, if I want to create a series RLC network, that mimics a parallel RLC network then, the point blank, the answer is, that it is not possible. However, if I want to create series RLC network, that mimics a parallel RLC network at resonance, it is possible to make such a thing. It is definitely possible to make the series RLC network that mimics the parallel RLC network at resonance or at any other given frequency.

So, if you specify one frequency point and say that, I want these two network to be identical at this one frequency point, it is possible to arrange for that to happen, not otherwise, you cannot do over the entire frequency range. Let try to go step by step, so first observation is, if you want to do a series to parallel transformation or a parallel to series transformation then, it is in general not possible to do such a thing. Second observation is that, it is possible to do such a thing at some frequency point or points.

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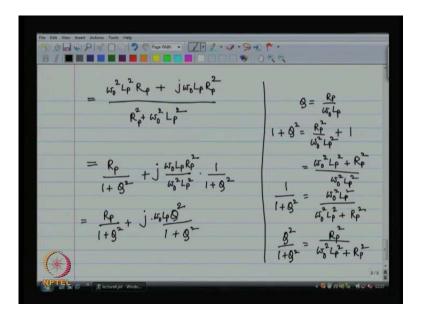
So, given this information, I would like to transform a series L R network, the parallel L R network, this is what I would like to do. I would like to transform back and forth between the series L R network and the parallel L R network. First of all, I want to know if this is possible at some frequency, it should be possible and how are the two networks interrelated. First observation is, it is definitely not possible to transform this over entire

frequency range, a series network can never mimic a parallel network, this is first observation.

So, we have to choose a frequency of interest, let us pick a frequency of interest omega 0, omega 0 so happens to be a resonant frequency of L p with C p, which is not yet there. So, it is not yet there in the circuit, but I am going to choose omega 0 to be the resonant frequency of L p and C p. Now, given that information, what is the Q, the quality factor of my parallel RLC network. Remember, the parallel RLC network, if R increases then, if R tends to infinity then, the resistance is an open circuit then, it should have infinity Q, so if R increases, my Q should increases.

So, that is how I remember what is what, this is the result for my parallel RLC network and I really do not want to involve C p in this. So, what you can recall is that, L p times C p is 1 by omega 0 squared. So, C p is really 1 by L p times omega 0 squared, so this is something that I recall, see you plug in this, so this is the quality factor of my network. Now, with this in mind, what is the input impedance that I see, let us not think about C p, C p is their outside, like I have drawn in the other case. Omega 0 is the frequency of interest, where I want to make sure that, the two networks are identical, so this is the input impedance. Now, if I want to find out the real and imaginary parts of this, I have to multiply the numerator by the complex conjugate of the denominator.

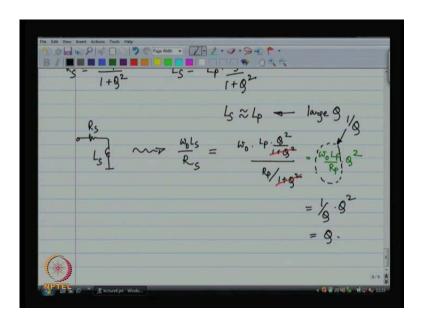
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So, this is what I have got, now notice that Q is equal to R by omega 0 L p from my earlier R p, this is R p. So therefore, Q square is equal to R p squared by omega 0 squared L p squared and 1 plus Q squared is, this plus 1, which happen to be equal to omega 0 squared L p square plus R p square by omega 0 square L p squared. That is wonderful 1 by 1 plus Q squared is exactly what we want, the denominator matches up with what I need.

So, this quantity is equal to R p times 1 by 1 plus Q squared plus j times omega 0 L p R p squared divided by omega 0 squared L p squared times 1 by 1 plus Q squared, which is equal to R p by 1 plus Q squared plus j times R p by omega 0 L p whole squared by 1 plus Q squared, which happens to equal to Q squared. So, this is, no this cannot be right, now this is correct, is this correct now, do cross check every result, every step do cross check by yourself to see that, it is dimensionally consistent. Dimensions are very important, especially when you are doing lot of mathematical ((Refer Time: 42:53)) I mean, whenever you are doing manipulations, make sure dimension of each and every term are the same and what you expect them to be. This is usually a very handy sanity check, so I could correct myself just based on dimensions.

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So, this is z in of the parallel network, now z in of the series network is very easy to know. At the frequency omega 0, that is my frequency of interest, this is the z in of the series network and these two inputs impedance have to be equal to each other, that is

when I can transfer the parallel network to series network and vice versa. So therefore, R s has to be equal to R p by 1 plus Q squared, the real part has to be equal to real part and L s has to be equal to L p times Q squared by 1 plus Q squared.

What does this mean for us, this means two things, point number 1, let us look at, let us think about large quantity factors. Let us say my Q is 100 or let even say my Q is 5, if Q is 5 then, the series inductance L s is equal to L p trans 25 by 26, which is almost equal to 1, so L s is almost equal to L p when Q is 5. What happens to the resistor is something very dramatic, the resistance value has to drop dramatically by a factor of Q squared, 1 plus Q square is almost equal to Q squared.

So, let go back ((Refer Time: 45:29)) to what I want to do, I wanted to transform a parallel L R combination to a series L R combination. And what I found was that, the series resistance has to drop by a factor of 1 plus Q squared, the series inductance is more or less almost equal to the parallel inductance. So, there is no great change if the value of the inductance per say, the change is in terms of the value a resistance when you transform it to series, it has to drop dramatically ((Refer Slide Time: 46:15)).

Now, can we do reverse transformation, of course we can do the reverse transformation and just by the way, what is the Q of the series RLC network now. So, if I have R L in series, is also C probably in series with that, I do not care about that, where transforming R and L into parallel contribution. So, when we do this, the Q is inversely proportional to the resistance. So, the Q is inversely proportional to the resistance and what do we have here, so the new Q of the network is also now going to be something else if you are talking about the series RLC network now. The Q has changed, this was my old Q, no you agree with this, so Q of the new series network is going to be equal to the old Q. So, either way it works now that means, that if I am going to transform the series network into the parallel network then, I can just do the same thing.

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$$Z_{in} = \frac{1}{\frac{1}{\sqrt{R_{p}}}} \frac{1}{\sqrt{R_{p}}} \frac{1}{\sqrt{R_$$

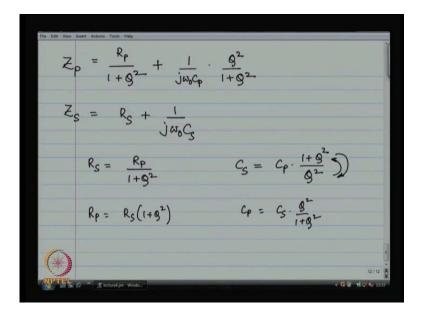
So, suppose, I have R s and L s and I want to transform it to a parallel network with R p and L p then, I just have to do the precise opposite which means, that R p now has to be much larger than R s and L p has to be equal to more or less the same as L s and this Q is the original Q that we started from. So, we can do back and forth between series R L networks and parallel networks at a chosen frequency of interest. And all this is possible by assuming that, you have the required capacitor in parallel with it and then, you do the Q computation in your head.

Presumably, if you do the transformation for series R L and parallel R L networks then, you should also be able to do a transformation between series R C and parallel R C networks and here the inductance is imaginary. The inductance is imaginary, so we will equate the input impedance looking in from these two terminals, real and imaginary parts of these and we will figure it out, how the resistor and the capacitor need to be numerically related to each other.

So, how are we going to do this, same procedure, first of all we have the observation that, Q of the parallel network is proportional to R p, R p by square root of L by C and I do not want to really use L over here. So, let me try to get rid of L, so Q is R p times omega naught time C p, that is the quality factor of my parallel R and C combination. So, z in is equal to 1 by 1 by R p plus j omega naught C p, this is at a frequency of interest omega naught and this can be furthered simplified to R p by 1 plus j omega naught C p R

p. However, we need to convert this to real and imaginary, so we have to multiply by the complex conjugate of the denominator and final result is, this is my final result. And what is very neat over here is the fact that, Q is equal to omega naught C p R p. So, this is Q squared

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And Q squared by 1 plus Q squared, so something like this, so this is my input impedance of the parallel network, input impedance of series network is easy to figure out and that kind of tells you what is going to be what. So therefore, R s has to be equal to R p by 1 plus Q squared and C s has to be equal to C p into 1 plus Q squared by Q squared, so that is the only difference between the earlier, from the earlier result this is the only difference.

So, earlier with the inductor, Q squared was in the numerator, 1 plus Q squared was in the denominator in this part, so this is the only difference, otherwise it is the same thing. If I have a large resistance in the parallel network then, for the series to have the equal amount of Q, I need 1 plus Q squared times less resistance in series, the capacitance value does not change much when Q is large. So, if I have the Q of 5 let us say then, the capacitance value is going to remain more or less the same, just a little bit more than the original value.

You can also do the parallel the series to the parallel transformation, R p is going to be equal to R s times 1 plus Q squared and C p is going to be equal to C s times Q squared

by 1 plus Q squared, so this is also going to follow. The Q of the network is going to remain unchanged between the series and parallel to transformations. So, just like before, the Q of the series network was, so earlier the Q of the series network was this much, the Q of the parallel network was Q, itself was the Q of the parallel network. And we showed that, the Q of the series network also remains to be equal to Q.

Now, in this case, you will get the same exact result, the Q of the network will remain unchanged over a series to parallel transformation or a parallel to series transformation. What is important over here is that, these transformation can be done at one frequency of interest that is, omega naught. Omega naught can be anything, because the inductor can be chosen to be anything, the inductor is imaginary remember or the capacitor in the earlier case was imaginary.

This capacitor over here is something that I imagined, depending on what omega naught I want to do this transformation at, I choose this capacitor in my head and place it in my head. I imagine the presence of this capacitor and then, I do the transformation and then, everything falls into place. The capacitor is useful only to visualize, what Q is going to be, the capacitor is useful only to do the appropriate, to use the appropriate formulae, otherwise the capacitor is meaningless.

As far as the derivation goes, equating z in parallel to z in series, have not use the capacitor at all. The presence of the capacitor is useful only for the Q, the definition of the Q. So, with this I will summarize what we have done today, so to start with, we talked about ((Refer Time: 61:23)) the bandwidth, what is the relationship between the quality factor and the band width of the circuit. And ((Refer Time: 61:34)) you can go back to your network theory and it can be proved that, the band width is going to be the center frequency divided by quality factor, so that is number 1.

Number 2 is, ((Refer Time: 61:55)) we talked about the transient response, in the transient response, the number of times the circuit rings, gives you a rough estimate of what the quality factor of the circuit is. I have not proved this, you can look at the books, you can look at Thomas book which is very good reference for this course and the derivation is done there. After that, ((Refer Time: 62:31)) we talked about the series to parallel transformations, first of all it is obvious that, the series to the parallel

transformation is meaningless when we want it to work over all frequency range, over the entire frequency.

A series to parallel transformation ((Refer Time: 62:49)) can work at a specific frequency, so given that, we are interested only at a specific frequency can be transform a series L R network to a parallel L R network, we can. And the way it works is that, the parallel L R network should have 1 plus Q squared times the resistance of the series L R network. As far as the inductance itself goes, the inductance itself is unchanged, more or less unchanged.

We can also ((Refer Time: 63:30)) transform a series R C network to parallel R C network and vice versa. Once again, the capacitance value itself remains more or less unchanged, the value of the resistance in the series R C network has to be 1 plus Q squared times lesser than the value of the resistance in the parallel R C network, so this is what we have covered today.

Thank you.