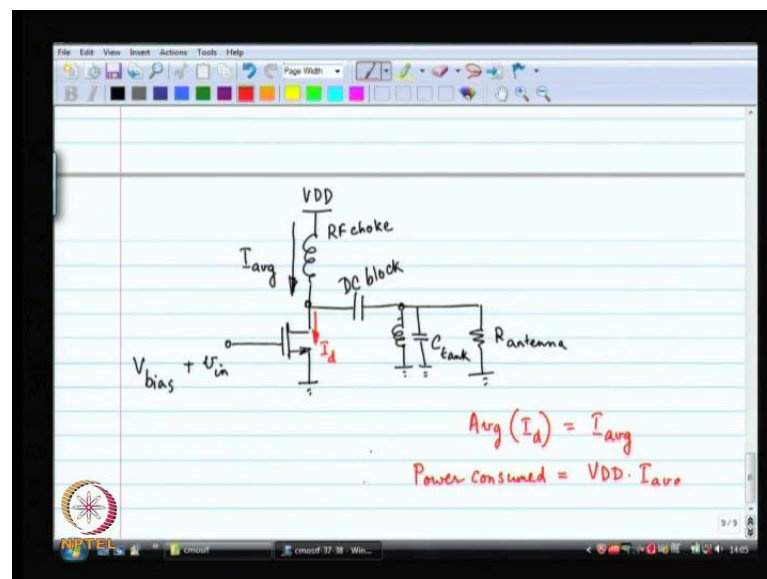


CMOS RF Integrated Circuits
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Module No - 12
RF Power Amplifiers
Lecture No - 38
Class B, C, D Power Amplifiers

Welcome back to CMOS radio frequency integrated circuits we have been discussing power amplifiers in the last class we discussed class A amplifiers and we had started with class B amplifiers although I do not really think we got a full understanding of what is going on. So, we are going to continue with class B class C class D amplifiers and we are going to see we are going to try to get an understanding of how to build our amplifier. So, first of all as we did in the previous class previous lecture.

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The general picture is something that looks like this we have a MOSFET that is amplifying a voltage. So, I am going to call this voltage I think in the previous class I called it V_{RF} , but let us call it V_{in} now of course, there is also a bias voltage associated with the input of the MOSFET now the MOSFET is powered through an RF choke what is an RF choke an RF choke is an inductor that is so, big that it is a short as far as DC is concerned and it is an open circuit as far as RF is concerned.

So, it chokes out the RF. So, it is an RF choke and we have a DC blocking capacitor. So, I am going to call this RF choke and I am going to call this DC block. So, this capacitor is so big that it does not allow DC to go through, but it allows everything else and then we have the load now unfortunately the circuit is going to be non-linear we saw that we are not really planning to have a linear amplifier over here we are just trying to blast out as much as power as possible. So, therefore, I am going to put some sort of tank circuit that is going to filter out all the other components and allow voltage at the output to be only at the desired frequency. So, this is the general block this is the general circuit diagram and the same circuit diagram applies to class A B C amplifiers A B.

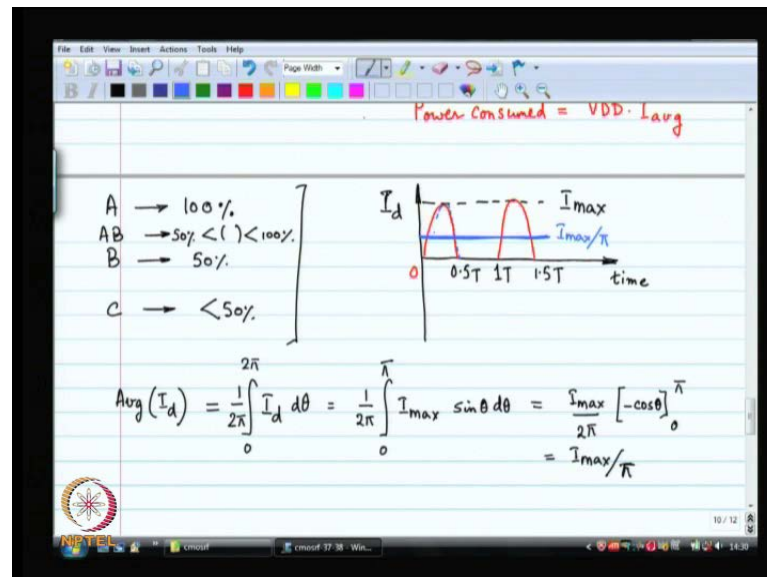
So, I forgot to mention we are also going to discuss A B over here. So, it really is irrelevant what class we are in this is the general picture that I have got now what you have to understand is that this RF choke allows a current at DC it does not allow current at any other frequency it is a short circuit for DC open circuit for AC. So, the current going from coming from the RF choke it is like a constant current no matter what mode of a operation you are looking at A B C whatever you are looking at it does not matter the current through that RF choke is a constant current let us call that current I_{average} why I am calling it I_{average} we will very soon find out the next thing to understand is that the average current through a capacitor through the DC block capacitor has got to be equal to 0.

So no average current flows through the DC blocking capacitor therefore, the drain current of the MOSFET if I look at the average of the drain current of the MOSFET it will be the same as I_{average} let us call it I_{D} . So, average of I_{D} is definitely equal to I_{average} why because no average current is going through the DC blocking capacitor anyway. So, therefore, the power consumption of this particular circuit is going to be how much the total power consumed is V_{DD} times I_{average} really that is all there is to it. I_{average} current is coming out of a power supply. So, the power is supplying an amount of power that is equal to V_{DD} times I_{average} does not really matter what class the amplifier.

Now all that I need to understand is how much is the power delivered to the load and depending upon the amount of power delivered to the load I will be able to understand what is the efficiency of this particular amplifier by the way amount of power delivered to the load at the desired frequency do you agree at the desired frequency at other

frequencies it really does not matter what power is delivered to the load because it is we do not want it anyway. So, at the desired frequency what is the power delivered to the load that is going to give me an understanding of what is the efficiency of this particular circuit.

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Now, as we said before we have a few definitions over here class A the transistor is on 100 percent of the time class B the transistor is on 50 percent of the time and class AB as the name is suggesting the transistor on for more than 50 percent of the time, but less than 100 percent of the time between 50 and 100 percent. So, these are the broader definitions of the different classes of the amplifiers now when I say that the transistor is 1 for so much percent of the time what does it mean it means that V bias is such that for a given percentage of the time the transistor conducts current for the remaining amount of the time this we discussed yesterday the transistor is cut off and nothing is really flowing through the device.

So, for example, let us take the example of a class B. So, this is if the current was a perfect sinusoid, but we are saying that we are going to arrange the bias voltage such that we are going to arrange the bias voltage such that the transistor does not conduct any current for half the time. So, let us say this is 0 π 2π 3π and so on or if you want to deal with seconds let us say the signal is that point 5 hertz let us say the signal is at 1 hertz then this is 1 second this is point 5 second this is 1 point 5 second and so on or if

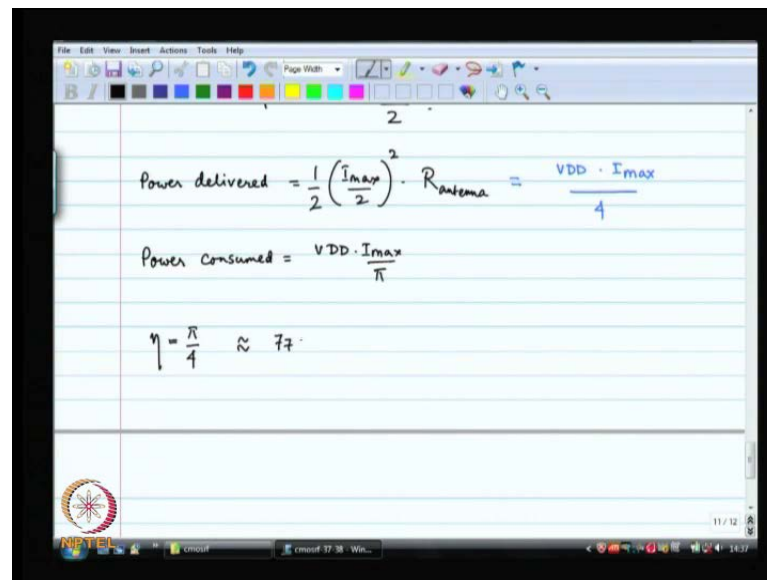
you want more general then 1 point 5 times the period. So, this is the graph of the drain current.

So, if this is the graph of the drain current what is the average of the drain current because the average of the drain current is I_{average} . So, the average of the drain current let us first choose a maximum let us say this is called I_{max} or sure I_{max} integral of \cos is \sin or minus \sin integral of \cos is \sin we have a problem here it is not \cos the wave form that I have drawn is not a cosine it is a \sin . So, integral of $\sin \theta$ is $-\cos \theta$ and we go from 0 to π . So, $\cos \pi$ is -1 . So, minus of -1 is $+1$ and $\cos 0$ is 1 . So, $-1 - 1$ so, that is -2 . So, my average current is I_{max} by π very interesting what is interesting is that the average is over here it is not in the middle.

The next thing that I need to understand is what is the power delivered. So, the power consumed is therefore, I_{max} by π times V_{DD} that is the power consumed; obviously, what is the power delivered now to understand what is the power delivered I have to find out what is the fundamental component of this current at the desired frequency I am not interested in the other harmonics I am interested only in the desired frequency.

So, the fundamental component the fundamental component is you can easily distinguish where that is just by I estimation you do not have to work on it very hard because think about it how the Fourier series works the Fourier works you try to place a fundamental and then you try to put a third harmonic you try to put a fifth harmonic second harmonic third harmonic fourth harmonic fifth harmonic and so on. So, just by I estimation you can probably figure out that you will have a fundamental that kind of looks like this just by I estimation I would not go into the mathematics because I better not waste time we will do a generalized version later on now accordingly just by I estimation you can see that the amplitude of the fundamental is something like I_{max} by 2 just by I estimation.

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The image shows a digital notepad with handwritten mathematical derivations. The first equation is $\text{Power delivered} = \frac{1}{2} \left(\frac{I_{\max}}{2} \right)^2 \cdot R_{\text{antenna}} = \frac{V_{DD} \cdot I_{\max}}{4}$. The second equation is $\text{Power consumed} = \frac{V_{DD} \cdot I_{\max}}{\pi}$. The third equation is $\eta = \frac{\pi}{4} \approx 77\%$. The notepad has a toolbar at the top with various drawing tools and a status bar at the bottom showing the date and time.

$$\text{Power delivered} = \frac{1}{2} \left(\frac{I_{\max}}{2} \right)^2 \cdot R_{\text{antenna}} = \frac{V_{DD} \cdot I_{\max}}{4}$$
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$$\eta = \frac{\pi}{4} \approx 77\%$$

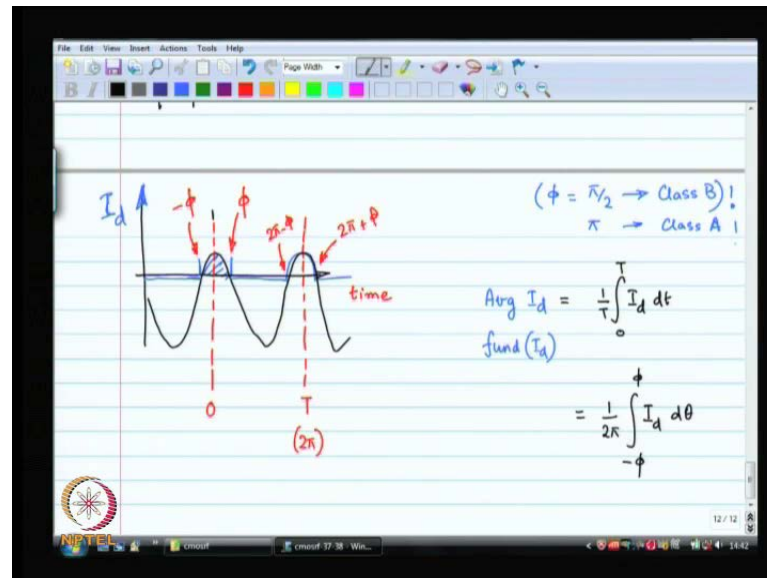
So, this is the current that is coming through the DC block and naturally this is the current that is coming from the load because at all other frequencies the tank is eliminating the current on the load. So, therefore, the power delivered to the load at this frequency is I_{\max} by 2 squared is the amplitude half of that times R I squared R half of I squared R that is the power delivered now what is this R R antenna is there any relationship between R antenna I_{\max} and the power supply remember now that the device cannot possibly go off or the device is going to be off around this from here to here the device is off. So, therefore, I_{\max} by 2 that is how much you have got as far as that is the amount of deviation that you have got in terms of voltage in terms of the average of the voltage.

So, therefore, let us put it this way if this is my V_{DD} if I_{\max} times R corresponds to V_{DD} then during this time frame it is going to be off you understand what I am trying to say if I_{\max} times R is V_{DD} then at this point of time when the current is the largest the voltage over here will be the smallest you do not want the voltage you cannot allow the voltage over there to go below the saturation voltage of the transistor.

Now, we assumed made an assumption in the last class let us say that the minimum saturation voltage that you possibly can have is 0 volts this was bad assumption, but nevertheless we are using this assumption for our analysis because otherwise it is overly complicated. So, therefore, what I am trying to suggest is that at this point of time at this

point of time the voltage the signal voltage should be V_{DD} , which means I_{max} times R has got to be equal to V_{DD} that is my argument I_{max} by 2 has got to be V_{DD} this deviation has got to be V_{DD} .

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So, this is basically the idea that the power delivered is V_{DD} times I_{max} by 4 though power, but you have consumed is average current times V_{DD} . So, therefore, the efficiency is how much π by 4 π by 4 is something like if say π is 3 then it is 0.75 π is a little more than 3.14. So, it is something like 77 percent.

So, this is basically the idea as far as class B amplifiers are concerned and class C this is where we will work a little harder is a more general version of the class B where we say that our current let us say let us first draw a sinusoid current our current is positive only during this cycle and the remaining part the current is actually equal to 0. So, the idea is that let us approximate the current as the top portion the crest of a sin wave of a sinusoid and let us say that the conduction angle we are going to generalize this let us say this is start of time time T equal to 0 and this is time T equal to capital T and this is 1 period or rather if you want to think in terms of angle then this is 2π and let us say this is ϕ and this is minus ϕ .

So, it conducts when the angle is between minus ϕ and plus ϕ or between 2π minus ϕ and 2π plus ϕ now once again the thought is that the average current I have to find out the average current average I_d it is the same as before so, what is the relationship

between class C and class B amplifiers. So, class C is just a general version when ϕ is $\pi/2$ I get class B when ϕ is π I should get class A. So, we will see if the same results pop out of this or not.

So, here first of all I need to find out the average current this is something that I need to find out and the other thing that I need to find out is what is the fundamental component of I_d because this fundamental component is what is going to get delivered to the load these are the two things that I need to figure out now average I_d is of course, something like this $\frac{1}{T} \int_0^T I_d dt$ why did I put $1/T$ Now, we are going to consider this as our 0. Now this integral is the same as $\frac{1}{2\pi} \int_{-\phi}^{\phi} I_d d\theta$ it is the same pictorially you can see that it is exactly the same.

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The image shows a handwritten derivation on a digital notepad. The first equation defines the drain current I_d as a piecewise function: $I_d = \begin{cases} i_{rf} \cos \theta - i_{rf} \cos \phi & \text{for } -\phi < \theta < \phi \\ 0 & \text{for all other } \theta \end{cases}$. The second equation calculates the average current $Avg(I_d) = \frac{1}{2\pi} \int_{-\phi}^{\phi} (i_{rf} \cos \theta - i_{rf} \cos \phi) d\theta$. This is simplified to $\frac{i_{rf}}{2\pi} \left[\sin \theta \right]_{-\phi}^{\phi} - \cos \phi \cdot 2\phi$, which further simplifies to $\frac{i_{rf}}{\pi} [\sin \phi - \phi \cos \phi]$. The final result is labeled "Power consumed" with a circled 2.

$$I_d = \begin{cases} i_{rf} \cos \theta - i_{rf} \cos \phi & \text{for } -\phi < \theta < \phi \\ 0 & \text{for all other } \theta \end{cases}$$

$$Avg(I_d) = \frac{1}{2\pi} \int_{-\phi}^{\phi} (i_{rf} \cos \theta - i_{rf} \cos \phi) d\theta = \frac{i_{rf}}{2\pi} \left[\sin \theta \right]_{-\phi}^{\phi} - \cos \phi \cdot 2\phi$$

$$= \frac{i_{rf}}{\pi} [\sin \phi - \phi \cos \phi]$$

Power consumed (2)

So, you now all I need to understand is what is a general expression for I_d . So, I_d is equal to 0 outside of the conduction angle and I_d is equal to sum sinusoid of sum magnitude let us call this magnitude amplitude i_{rf} just calling it i_{rf} plus minus a certain quantity. So, you are downwards a certain quantity and how much are you down by your down by this much or rather this much that is exactly equal to $i_{rf} \cos \phi$.

So, this is my mathematical expression for I_d which basically means that if I want to integrate I_d if I want to find out the average I_d that is my expression and let us work out the integral it is not going to be terribly difficult to work out this integral of $\cos \theta$ is $\sin \theta$ and integral of a constant is just the difference integral of a constant is the

constant times theta. So, that is basically what I have got and I have to apply the limits to this and once I apply the limits to this sin phi minus sin phi and of course, the 2 will all cancel out and this is what you get now what is going to be obvious over here is when you substitute this is your sanity check you said that if we substitute phi equal to pi by 2 we should get back our result for a class B.

So, if I substitute phi equal to pi by 2 sin pi by 2 is equal to 1 phi times cos phi cos of pi by 2 is equal to 0. So, I basically get i r f by pi now under this scenario what is i r f i r f is the amplitude of this sin wave and the amplitude of that sin wave is going to be I max. So, basically I am going to get I max by pi which is the old result. So, I am correct. So, this is my sanity check. So, what I have done so far is I figured out what is the average I d.

The next step is to figure out what is the fundamental component of i d this is my next step average I d will give me an understanding of what is the power consumed the power consumed is the average I d times V D D because the average I d is what the current coming through the through the RF choke now I want to understand what is the fundamental of i d. So, how do you find out the fundamental of i d you do a Fourier series. So, this is the expression for i d fundamental of i d is by doing a Fourier series no it is not i r f it is not i r f do not worry you do a Fourier series expansion and that is what is going to tell you what is the fundamental component of i d.

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The image shows a handwritten derivation on a digital notepad for finding the fundamental component of the drain current i_d using Fourier series expansion. The derivation is as follows:

$$t = \frac{T}{2\pi} \theta$$

$$dt = \frac{T}{2\pi} d\theta$$

$$\omega_s = \frac{2\pi}{T}$$

$$\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$$

$$i_d = \frac{1}{\pi} \int_{-\pi}^{\pi} I_d(t) \cos \theta \, d\theta = \frac{i_{rf}}{\pi} \int_{-\pi}^{\pi} (\cos \theta - \cos \phi) \cos \theta \, d\theta$$

$$= \frac{i_{rf}}{\pi} \int_{-\pi}^{\pi} \left[\frac{1}{2} (1 + \cos 2\theta) - \cos \phi \cos \theta \right] d\theta$$

This is the way to do a Fourier series expansion this is the first component and of course, you replace capital T with something that relates to θ and capital T is the period. So, that should be replaced by 2π . So, I am going to make this substitution this is a substitution I am going to make and as a result 0 will relate to 0 capital T will be relating to 2π or you could shift your integration from minus t by 2 plus t by 2 you could do that also that I think will be helpful in our case $\cos \omega \text{ naught } t \text{ omega naught is } 2\pi \text{ by capital } T$ $1 \text{ by capital } T$ is the frequency 2π times that is ω .

So, I have done all these substitutions and these substitutions lead me to believe that this is what I am doing let us remove these brackets now I d of t as before I d is 0 for all other θ an idea is something between minus ϕ and plus ϕ . So, this integral is just basically going to be boil down to an integral between minus ϕ and plus ϕ and between minus ϕ and plus ϕ I d is something it is i r f times $\cos \theta$ minus $\cos \phi$. Now breakup this integral it is a little involved you have got $\cos^2 \theta$ $\cos^2 \theta$ is how do you know this is correct 1 is \cos^2 plus \sin^2 $\cos 2\theta$ is \cos^2 minus \sin^2 .

So, you have got $2 \cos^2 \theta$ half of that. So, do not worry it is not a lot of trigonometry I mean whatever trigonometry there is you can actually work it out on the fly. So, this is the integral that we are working on and now you basically realize that there are three small integrals the first one is a trivial integral it is a integral of half integral of half is half times θ .

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Handwritten mathematical derivation on a digital whiteboard:

$$= \frac{i_{rf}}{\pi} \left[\phi + \frac{1}{2} \sin 2\phi - 2 \sin \phi \cos \phi \right]$$

$$= \frac{i_{rf}}{2\pi} [2\phi - \sin 2\phi]$$

Power delivered = $V_{DD} \frac{i_{rf}}{4\pi} [2\phi - \sin 2\phi]$

$$\eta = \frac{2\phi - \sin 2\phi}{4(\sin \phi - \phi \cos \phi)}$$

Diagram of a sine wave with peak voltage V_{DD} .

Calculated values for ϕ :

- $\phi = \pi/2$ (B) $\Rightarrow \frac{\pi - 0}{4(1 - 0)} = \frac{\pi}{4}$
- $\phi = \pi$ (A) $\Rightarrow \frac{2\pi - 0}{4(0 + \pi)} = \frac{1}{2}$

So, half times 2 theta applying the limits will get 2 phi half times 2 phi which is phi the second integral is half of cos 2 theta integral half of cos 2 theta d theta if you think about this you replace 2 theta with alpha if you replace 2 theta with alpha then twice d theta will be d alpha. So, basically what; that means, is you have got one fourth of integral cos alpha d alpha agreed integral of cos alpha d alpha is sin alpha and what are the limits over here. Limits are minus phi by 2 plus phi by 2.

So, this is the second integral the third integral no I think I am making a mistake over here. So, the limits are going to be from that better 2 phi minus 2 phi to plus 2 phi which is basically going to become equal to sin of 2 phi minus sin of 2 phi. So, that is twice sin of 2 phi. So, this is what I am going to get and the third integral is of cos phi times cos theta d theta there is a negative sign remember that so cos phi times cos theta phi cos phi is a constant integral cos theta d theta is sin theta apply the limits phi 2 minus phi you are going to get minus 2 sin phi cos phi.

So, this is the complete integral and what do you see over here half of sin 2 phi is equal to sin phi times cos phi. So, I have compressed those two terms into one and you can rewrite it in this fashion if you want to. So, this is the fundamental component of the current going into the MOSFET which is the fundamental component of the current going into the load what about the amplitude of the voltage at the load amplitude is V D D same argument as before it is the same argument as before supposing I have some

fundamental of the current going into the load this is the current going into the load I would like it is amplitude to be equal to V_{DD} the amplitude of the voltage to be equal to V_{DD} .

So, that at the output the voltage is swinging all the way from plus $2 V_{DD}$ to 0 over here I am planning to allow the voltage to swing from $2 V_{DD}$ down to 0 this is the idea which means it has an amplitude of V_{DD} and therefore, the power delivered to the load and you compare this expression against this expression which is V_{DD} times that quantity and what you are going to get V_{DD} times i_{rf} by π I think will go away. So, this is the efficiency that I have got and if you look at this expression and compare it plug in first of all plug in ϕ equal to $\pi/2$ that is your class B ϕ equal to π that is your class A you should get back good old results. So, twice $\pi/2$ is ϕ minus \sin of $\pi/2$ \sin of 0 degrees is 0 \sin of π is also equal to 0 \sin of ϕ \sin of $\pi/2$ is 1 minus ϕ times \cos of ϕ $\pi/2$ times \cos of $\pi/2$ that is equal to 0.

So, this is the same result as before where was I. So, that is why I skip this step of computing all kinds of things in doing so many integrals in the same class kind of makes you things less. So, we did one and you can back calculate and prove that our assumption was correct or we did an I estimation and said that looks like it is going to be half of the swing what about if ϕ is equal to ϕ if ϕ is equal to π . So, I have got 2π minus \sin of 2π is equal to 0 divided by 4 times \sin of π is equal to 0 minus ϕ times ϕ is π times \cos of π that is equal to minus 1. So, you get 2π by 4 π which is exactly equal to half remember we did this in the previous lecture the efficiency of a class A system is at max 50 percent why is this at max because of this you need a little bit of room to allow for the transistor to remain in saturation.

So, little bit of room is needed over there if you do not allow that room then you are device is going to cut off at the place where you do not want it to cut off when the current is maximum that is where the device is likely to cut off. So, this is the general expression for a class C amplifier efficiency of a class C amplifier. And this kind of tells you that as you decrease the conduction angle. So, for a conduction angle of $\pi/2$ you get an efficiency of $\pi/4$ what if the conduction angle is $\pi/4$

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$$I_{avg} = \frac{I_m}{\pi} (1 - \cos \phi)$$

$$\phi = \frac{\pi}{4} \Rightarrow \frac{1.57}{\pi} (1 - \cos \frac{\pi}{4}) = \frac{1.57}{\pi} (1 - 0.707) = \frac{1.57}{\pi} \times 0.293 = 0.7$$

So, twice pi by 4 will give you pi by 2 minus sin of 2 phi 2 phi is pi by 2. So, sin of 2 phi if 2 phi is 1 divided by 4 times sin of phi sin of pi by 4 is 1 by root 2 minus phi times phi is pi by 2 pi by 4 cos of phi cos of phi is again 1 by root 2. So, if you do quick computation over here pi by 2 is something like 1.57 1.57 minus 1 divided by 4 times 1 by root 2 is 0.7 minus pi by 4 is pi is 3.1. So, 3.1 by 4 is how much 0.77 0.78 into 0.7.

So, let us approximate it as 0.8. So, 0.56. So, it is a little more than 0.14 and 4 times 0.145 is some or I think it is about point it will be 0.15 this is very close to 100 percent. So, what you see over here I just made the conduction angle smaller and I got a better efficiency what is going on over here what is going on what comes to your mind by saying that I am making the conduction angle is smaller and I am getting better and better efficiencies what is going on over here think about it. What is the deal in this wave form I am making this is all that the current is let us choose some other color over here this is the current waveform.

Now most of the time the current is 0 for a little blip of time you have got some amount of current; that means, that the average current drawn from the power supply remember here we got an average current of I max by pi that average current is dropping if you conduct all the time the average current is something it is going to be I max by 2 something like that we are going down as far as the average current is concerned what is average current when the conduction angle is phi by 4 when the conduction angle is pi

by 4 the average current is $i_{rf} \sin \frac{\pi}{4}$ is $\frac{1}{\sqrt{2}}$. So, 0.7 minus phi which is again $\frac{\pi}{4}$ something like 0.8 times 0.7.

So, I have got something of the order of 0.15. So, the average current is going down I am getting lesser and lesser average currents by conducting for lesser durations of time now when you are getting lesser and lesser average currents what is happening to the power delivered to the load the current delivered to the load is also going down what does that mean; that means, that the amount of power that you are delivering to the load is; obviously, also going down.

So, you are going down in terms of the power consumed you are also going down in terms of the power delivered. So, the sad news is that these class C amplifiers although it looks like you can achieve whatever efficiency you want it looks like that by making the conduction angle narrower and narrower the tragedy of the matter is that you end up delivering lesser and lesser power to the load your devices everything is all everything is always off.

So, of course, if everything is off your efficiency is going to be 100 percent no power delivered no power used efficiency is 100 percent or tiny amount of power is used tiny amount of power is delivered. So, this is the tragedy of the situation and we have to deal with this. Now with this I am going to close the lecture and we are going to talk a little bit further about the class C amplifier and then about other amplifiers in the next lecture.

Thank you.