CMOS RF Integrated Circuits
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Module - 01
Introduction
Lecture - 03
Maximum Power Transfer

Hello everyone. And, once again we are at CMOS RF integrated circuits.

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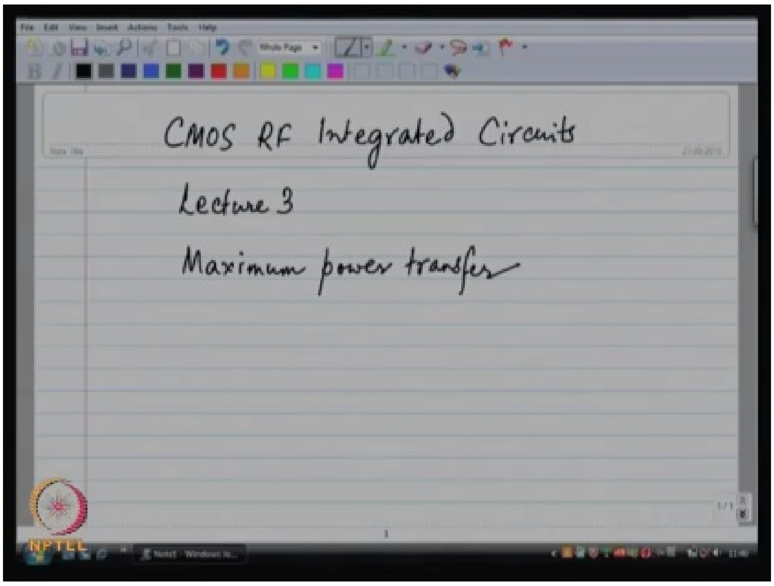

This is the third lecture; and, today's agenda will be to discuss the maximum power transfer theorem. Now, the maximum power transfer theorem – I am sure all of you who are studying this course have already studied it at least once before. And, it is like this.

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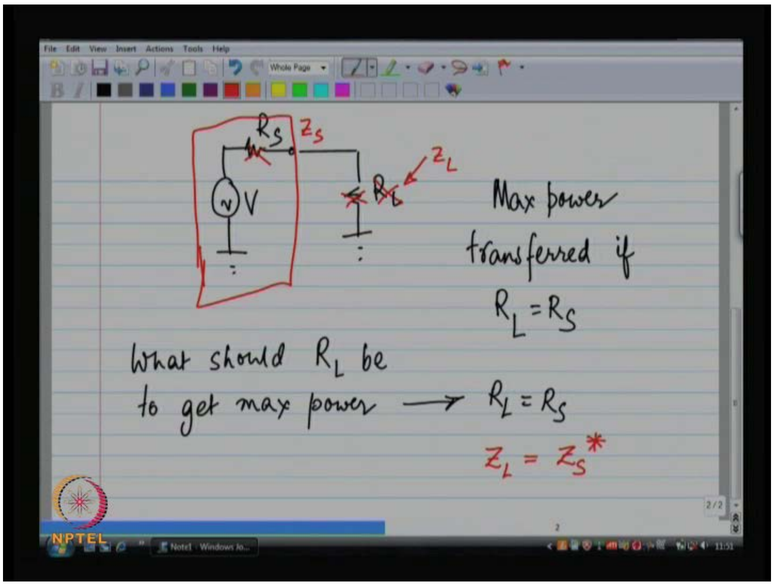

If I have a source; and, there is a source resistance; then, maximum power is transferred if the load resistance is equal to the source resistance. So, it is a very simple theorem. But, it is widely misunderstood. Especially when you study it at your undergraduate level, there are a lot of misconceptions that arise. The first misconception is this. I would not talk about the misconception; let me elucidate this theorem for you. The first most important thing that you have to understand is that, the source resistance is a part of the source.

The source resistance is an integral part of the source; you cannot have the source without the source resistance; it is unfeasible to think of a voltage source without any source resistance. So, this source resistance represents the source resistance that is inside the voltage source. It is unavoidable and you cannot change it. So, this is number 1 that, the source resistance cannot be decoupled from the source.

Now, once you understand that, then the question is what should the load resistance be, what should R L be to get maximum power transfer? And, the answer to that is That Is R L should be equal R S. So, it is important to understand that, the source resistance is part of the source. And, as a result of this, only a limited amount of power can be transferred. If the source resistance was 0, then you can transfer whatever you want.

If this resistance happens to be equal to 0, then you know you choose R L to be 1 ohm; you choose R L to be whatever you want; and, that much amount of power… V squared by that much of power will be transferred to the load. So, there is no limit if the source resistance is 0. But, the source resistance is nonzero, it is finite nonzero number of ohms. So, that is the critical most important point over here. Now, if we talk about impedances and not resistances; so, if this was not R S, if it was Z S; and, if this was not R L; and therefore, I want a Z L; then, the relationship is Z L should be conjugate of Z S for maximum power transfer. So, all this is fine. This is the basic idea. Now, in the last class, we studied the reflections because of transmission media, etcetera.

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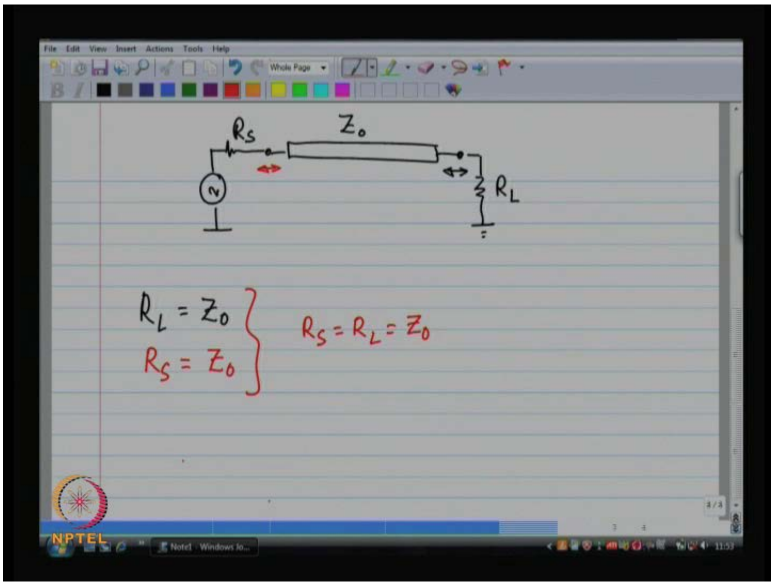

And, we concluded as follows that, if I have a source; then, to avoid reflections, I would like to have R L to be equal to Z naught. To avoid reflections at this interface, I would like R L to be equal to Z naught. And, to avoid reflections at this interface, I would like R S to be equal to Z naught; which eventually means that R S should be equal to R L should be equal to Z naught. Now, this is just a coincidence. It also naturally gives us maximum power transfer.

But, let me remind you that, just a coincidence that it so happens. It so happens that, when R S is equal to R L equal to Z naught, I not only get maximum power transfer; I also do not get reflections echoing back and forth. It is also easy to understand. If I have a black body over there in front; if you have a black body, which observes whatever you send it; it does not reflect back anything. Then, it should be absorbing the maximum power also. So, through network theory, you can show all these that, these two concepts are interrelated. For us, it makes no difference that maximum power transfer leads us to no reflections; no reflections is important for us; maximum power transfer is also important for us.

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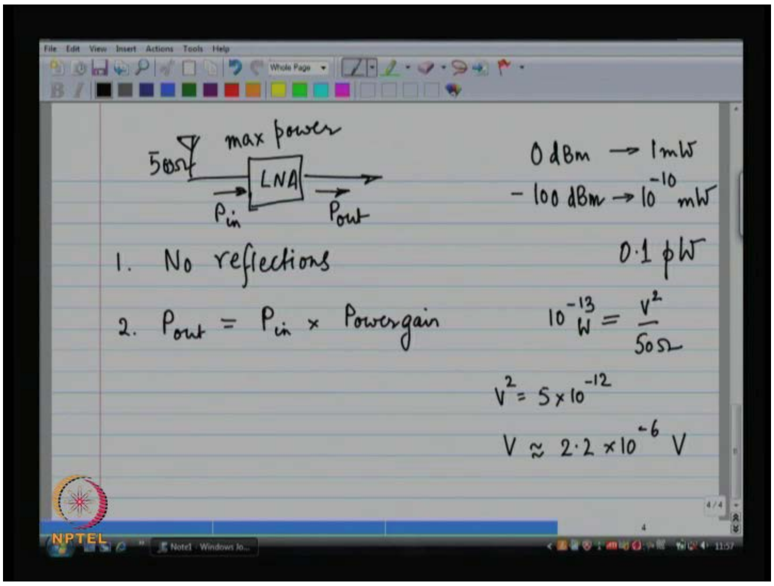

When we are talking about an RF system, we have an antenna and then we have an LNA – a low noise amplifier. We want maximum amount of power possible to be received on the low noise amplifier; why? Why do I want maximum power on the signal to be received on to the LNA. It so happens that, maximum power received on the LNA also means that, there are no reflections; that is a coincidence; that is a coincidence. Next important thing is that, the LNA has a finite amount of power gain.

So, the output power of the LNA is the input power times the power gain of the LNA. Now, if the input power is maximized, then the output power is maximized naturally. So, this is also very important. Why is this important? Because to start with, the input power is very small; the power received on the antenna is very small; we are talking about nano watts of power. So 0 dBm is 1 milliwatts of power.

If I want to make a receiver for a cell phone, when you are farthest from the base station, let us say I am talking about the GSM or something like that; then, typical numbers could be as low as minus 100 dBm. So, a GSM phone has a sensitivity of minus 100 dBm, which is 10 power minus 10 milliwatts. And, how much is that? That is 0.1 picowatts; all right? So, that is a very low amount of power that the LNA has to be sensitive to. The low noise amplifier has to be sensitive to that extremely low amount of power hitting the antenna. Now, what that means, is if I do not do maximum power transfer, then it is even worse for the LNA. So, I want to transfer as much power as possible from the antenna to the LNA and then moving forward to the mixer, etcetera. We are talking about extremely low power levels over here.

So, 0.1 picowatt – if this antenna is 50 ohm antenna, then what this relates to 0.1 picowatt is 10 power minus 13 watts is equal to V squared by 50 ohms. So, it is something like this – 2.2 micro volts. So, that is the amount of voltage that you want to sense for this kind of a power level; that is the amount of voltage that you have. So, that is why maximum power transfer is important for us. Now, you have all studied the circuits – MOSFET. I am going to jump the gun over here and I am going to assume that, you know the MOSFET although in this course, we are talking about MOSFETs later on. But, let me just give you a flavor of what is coming. So, you have all designed amplifiers.

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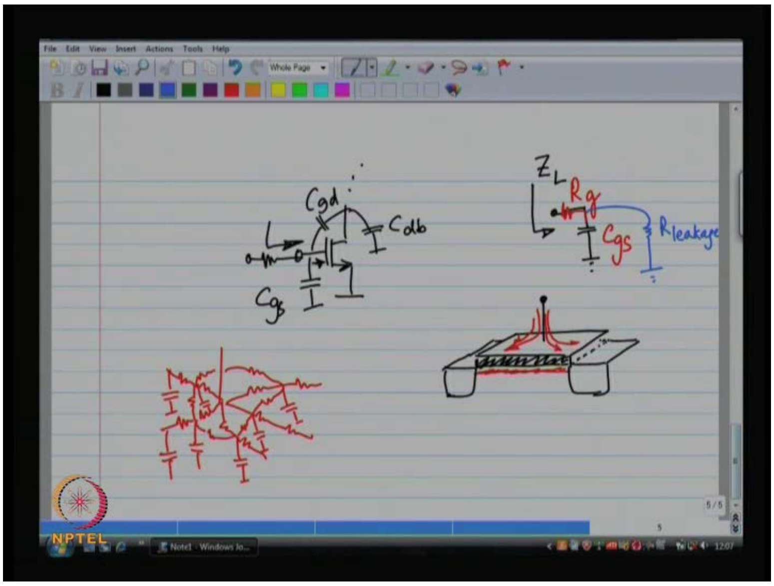

And, amplifiers – low noise amplifier is some kind of an amplifier presumably. So, input of an amplifier what you have designed before typically would like this. Or, if you have designed to differential amplifier, it would look like this, etcetera. So, there is a load to this and so on and so forth. So, this probably how the input of the amplifier looks like. Now, as far as this kind of a circuit is concerned, if you look at the input impudence, the input impedance is infinitely large or is the input resistance is infinitely large. What is the input impedance of this? Let us say differential mode half circuit. So, we forget about all these. So, this is the input – small signal model of the differential mode half circuit of the amplifier.

This is how it looks. Question is what is the input impedance? What does the input impedance look like? The input impedance is primarily capacitive; looks primarily like a capacitor. Over here we have got C g s and there is some C g d, which is through Miller effect is going to be multiplied by the gain of the MOSFET plus 1; and, it is going to come in shunt with C g s, etcetera. So, that is how the input is going to look like. As far as the gate itself is concerned, the gate itself is a perfect insulator. So, the resistance is infinitely large looking inwards further. So, this way, the resistance is infinitely large; you have got some gate to channel capacitance, gate to drain, gate to source capacitance.

Now, if this is the situation, then where is the power going to be burnt? So, the load looks capacitive. If the load looks capacitive, it is never going to burn any power. So, what maximum power transfer are we talking about? If this is the load, which looks like a capacitor; we are always talking about maximum power transfer, this and that; and, how we need to transfer 10 power minus 13 watts on to the MOSFET the input of the LNA. If this is the situation, then where is maximum power transfer going to happen, the input is looking capacitive? The answer to that… I am sure this question is occurring in your minds; it should occur in your minds if it has not already.

The answer to that is that, the MOSFET is not completely capacitive. What I have drawn here is not correct. This is not the end of the story; there are a few more things. So, first most important thing that we are forgetting over here is the gate resistance. There is some series resistance associated with the gate; note: series resistance. So, if I want to spread so much charge on to the gate, the gate looks like a capacitor. This is the channel; we need the gate. So, the gate looks like two parallel plates; gate is 1 – the top plate; the channel is the bottom plate. This is the channel, which is the bottom plate.

Now, if you want to charge the gate to a certain amount of charge; this charge has to travel through this wire and it has to spread all over the gate. And, remember that, although it is a MOSFET, it is no longer metal oxide semiconductor. This gate is made up of material like polysilicon. So, as a result, polysilicon is also used to make a resistor.

So, you can view the gate as if it is a distributed mesh of resistors. Some extremely distributed mesh of resistors and each of these points has capacitance to the channel. So, it is a complicated model. And, probably, this distributed mesh of resistors and capacitors can be substituted by a model that looks like this by a simple RC; where, C is the capacitance gate to channel C ox or whatever C g s or whatever you want over there; and, R g is the gate resistance. This is the gate resistance that I am talking about. It is a distributed mesh of resistors all over the gate. In extremely modern technologies let us say we are talking about 90 nanometers, 45 nanometers, 65 nanometers, 32 nanometers. The gate oxide is so thin that, charges actually tunnel through the gate oxide; and, that causes gate leakage.

Now, in that case, you will additionally get gate leakage. Now, this is not part of our discussion unfortunately. So, it is your homework to figure out what is going to happen when we are talking about super modern technologies. For older technologies, our situation is that, we have a series gate resistance with the gate to source capacitance or what so ever.

So, this R g is what is burning the power. So, this is where you are delivering the power. Now, next question is we talked about something called power gain. What on earth is this? I am just giving out terms and you should not blindly accept these terms. What is power gain? What on earth could power gain be? If you think about law of conservation of energy, you are never going to gain in terms of power.

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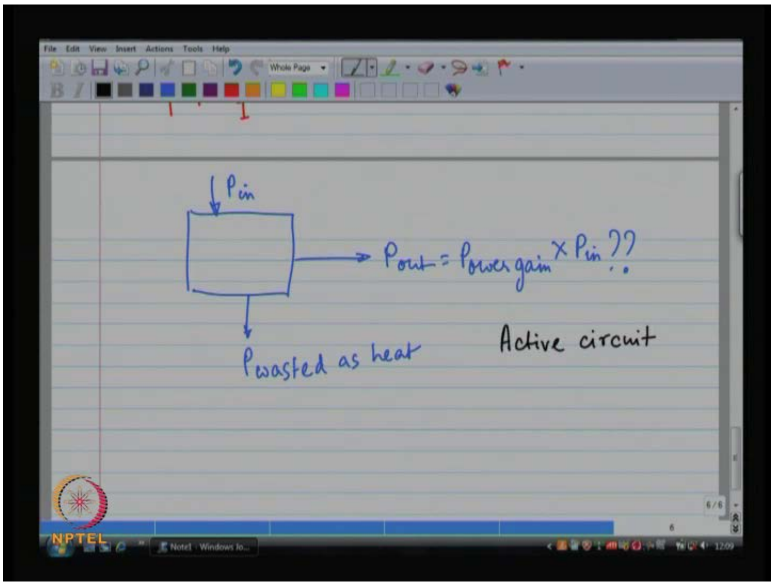

If you put in; you have a system, you put in so much power; either the power is burnt as heat or the power comes out. There is nothing else in between going on; law of conservation of energy holds everywhere. So, the waste power is heat plus the output power should be equal to the input power. Traditional network theory – law of conservation of energy tells us all these. So, what on earth is power gain? So, power gain I have defined as p out by p in. Looking at this picture, power gain has to be always less than 1; which means it is not a again; it is a loss. So, what are we talking about here? Anybody? Any ideas what we are talking about here? This is where you have to put things in a little bit of perfective. We are talking about an active circuit. And, active circuit is something that has an additional power supply.

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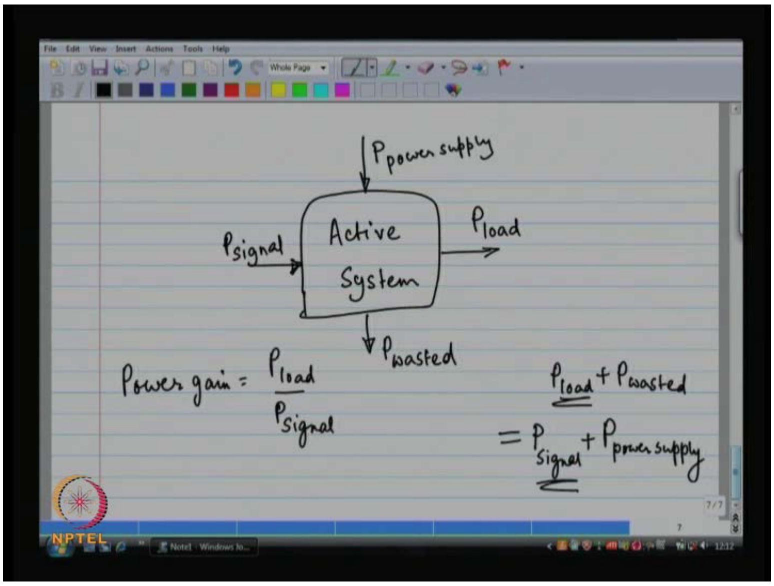

So, we have a MOSFET; probably a MOSFET; probably bipolar junction device, whatever you want it to be; anything that is an active component. And, power into the system; let us say this is the system. Power into the system comes from its power supply; it has a power supply – bias voltages, bias currents. So, these bias voltages and bias currents deliver power to the circuit. It also comes from the signal or the input. And, power is delivered to the load. So, looking at this kind of a system, we can say that, power gain is p load by p signal; and, p load plus p wasted; of course, you want to waste as little as possible; probably you want to waste a 0. So, now that you can see this as your equation, you can conceive of power gain that are more than 1.

Now, I have ticked this. And, this out of this equation; and, I am saying that, p load has to be more than p signal for a power gain to be more than 1; power gain is basically p load by p signal. So, I am pumping in power from the power supply. And, that is how the power delivered to the load is going to be more than the power coming in from the signal. So, this is the basic idea. This is why; this is ok. So far so good; all right. So, with these basic concepts, we are going to close the first module – the introduction module.

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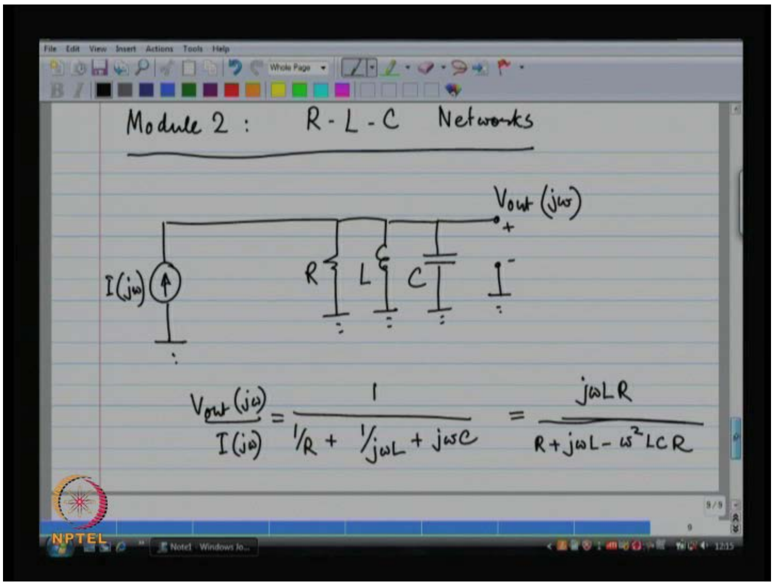

And, we are now going to start the next module of this course, which is RLC networks. So, any questions so far? All right. So, RLC networks – these are also things that you have studied many times at many different places undergraduate, postgraduate. Even before undergraduate, you have probably studied RLC networks. So, the typical RLC network that we are going to study is something that looks like this. And, how does this respond? So, if I is a function of omega, what is the relationship between the output voltage and the current – input current.

So, to understand this, you use the Laplace transforms; you use the traditional technique, phasors, whatever you want to use. And, R, L, and C form three parallel impedances. So, the combined parallel impedance is 1 by 1 by R plus 1 by j omega L plus j omega C. This is the combined parallel impedance. So, V out of j omega by I of j omega is so much; which can be simplified further; and, boils down to something that looks like this. Is this correct? It seems like it is correct. So, how do you know if it is correct or not?

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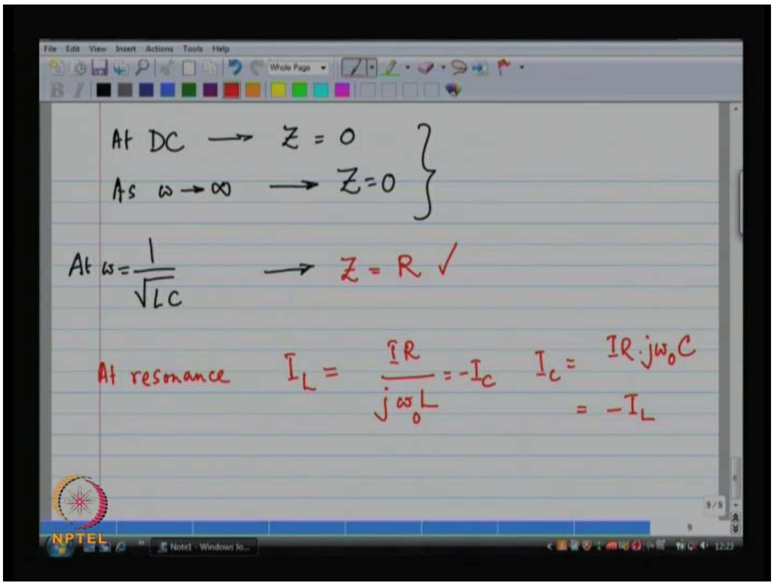

At DC; so, omega equal to 0. What does the V out by I look like; what does the impedance look like? At omega equal to 0, the inductor is a short circuit. So, if the inductor is a short circuit, then the output voltage should be equal to 0; which means that the voltage output divided by the current is also going to be equal to 0. So, I plug in omega equal to 0; numerator becomes is equal to 0; denominator is R. So, naturally, the result is equal to 0. Then, the next step is you check at infinitely large frequencies.

As omega tends to infinity, if you look at the network, the capacitor now looks like a short circuit. And, once again, the response should be 0 volts. So, I plug omega tends to infinity into this expression; and, the numerator is infinity; the denominator is R plus infinity plus minus infinity squared. So, in the numerator, I have omega; in the denominator, I have omega squared. So, net is 1 by omega. So, as omega tends to infinity, 1 by omega tends to 0; which means that, Z is equal to 0. So, this is the sanity check – two sanity checks.

Now, the third important frequency – whenever we talk about L and C in parallel, an important frequency is the resonant frequency. So, we have an inductor and a capacitor in parallel. The inductor stores a current, stores energy in the form of a current; the capacitor stores energy in the form of a charge or a voltage. And, these two keep… The energy keeps cycling back and forth between the inductor and the capacitor. And, this happens most optimally at the resonant frequency. And, at the resonant frequency, which happens to be 1 by square root of LC. What happens at the resonant frequency? So, the first most important thing that you see is that, omega squared LCR; when omega is 1 by square root of LC, omega squared LCR becomes equal to R. So, as a result, R and minus R cancel with each other. So, you are left with j omega L times R divided by j omega L. So, you are left with R. So, it is a very simple relationship.

What happens over here? At the resonant frequency, the inductor and the capacitor in shunt form something of an impedance that looks infinitely large. The shunt impedance of an inductor and a capacitor at the resonant frequency is infinitely large. So, you have R in parallel with something that is infinity large. And, as a result, the net is R. So, this concludes our checking. So, this is my expression. Now, ordinarily, this should be the end of the story. And, this is where I should be concluding this module and going to the next chapter. But, no; there is a lot to study in there. What we need to look at are the currents going through the resistor, through the inductor, and through the capacitor at resonance.

So, suppose I apply a current I at the resonant frequency. So, this is I as I have said. Then, presumably, at resonance, the inductor and the capacitor look like together in shunt look like an impedance, which is infinitely large. So, there will be no current going through this section. Together they look infinitely large. So, no current goes to the inductor and the capacitor together. So, all of the current I is going to go through the resistor. So, the voltage I have is I times R. And therefore, the current through the inductor and the current through the capacitor at resonance is the voltage by the impedance. And presumably, these two cancel each other. I L plus I C should be equal to 0; otherwise, does not work; that is Kirchhoff’s law.

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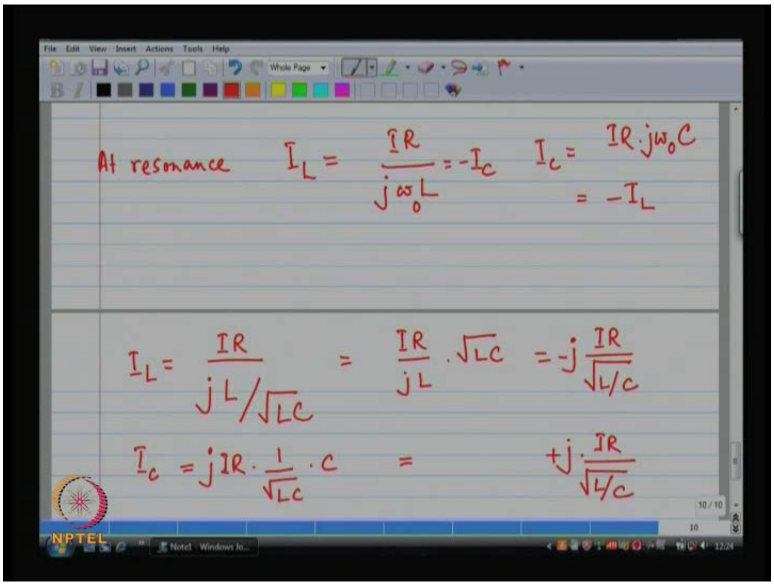

Now, let us see if that works out. Omega 0 is 1 by square root of LC. So, this is the current through the inductor. And, the current through the capacitor is going to be the same with a plus sign hopefully; wonderful. So, they match up; the current through the inductor is the negative of the current through the capacitor. Now, what you observe is a factor. So, the current through the inductor…

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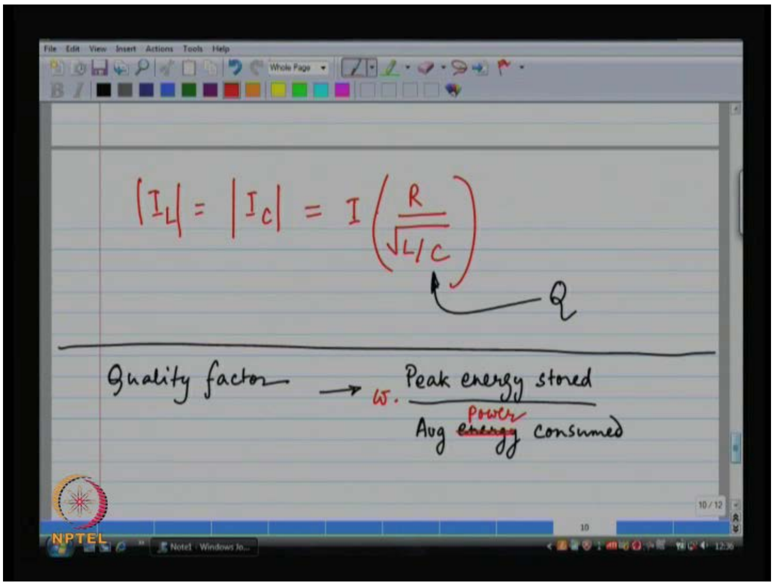

Let us look at the modulus of the current through the inductor, which is equal to the modulus of the current through the capacitor. And, that is going to be I times R by square root of L by C. This factor is defined as the quality factor. Actually that is not the definition of the quality factor; but, it so happens that, we call this… Let us call this Q; let us call this Q; all right? Now, this number can be anything.

You could choose R to be anything such that Q could be 1; Q could be 100; Q could be 0.1; you could choose this quantity to be anything depending on… It basically depends on the values of R, L and C. Now, let the Q is 100; in which case, what do we have here? We have I being pushed into the circuit; I flows through the resistor; but, 100 times I flows through the inductor and flows through the capacitor. Now, that is something amazing. You start with the current I and you get 100 times I through the inductor and through the capacitor.

You can really boost the current to whatever you want depending on the value of Q; agreed? So, this is something that is not normally captured in this expression over here; in this V by I expression, this fact is not being captured; the fact that, the current through the inductor and through the capacitor is some constant times the current that you are pushing in through the resistor. And, this constant is a function of R, L and C; it is basically R by square root of L by C. And, if you make R very large, then you can make this factor arbitrarily large.

This square root of L by C; are you reminded of something; from the previous lecture, from the previous module about characteristic impedance? Yes. So, this square root of L by C; this number is somewhat related to the characteristic impedance. So, it has units of ohms and it is a number; it could be something like 50 ohms; in which case, if I choose R to be 5 kilo ohms, then I have got a Q of 100; all right. If you want the circuit to work well; the parallel RLC circuit to work well, then you want R to be as largest as possible; in which case, the Q is also very large. So, this is something very interesting.

Now, with this, I am going to define Q. What is this Q? I mentioned Q already. But, I have not yet defined it; I just called something – Q. Q or the quality factor is defined as the peak energy stored in the circuit divided by the average energy that is burnt. So, this is the definition of quality factor. It is the peak energy stored; where is the energy stored? Energy is stored in the inductor and in the capacitor.

So, at the peak of the crest… At one point of time, all the energy is stored in the inductor; nothing is stored in the capacitor. At some other point of time, all the energy is stored in the capacitor; nothing is stored in the inductor. And, at some other point of time, energy is shared between the inductor and the capacitor; the amount of energy stored is shared. Where is the energy consumed? The energy is consumed in the resistor. Of all the circuit elements – R, L, C, mutual inductances, etcetera, resistances are the only elements that consume energy. So, now, with this as the background, let us try to find out what is the quality factor of the circuit over here. So, if the current has an amplitude of I amp; let us say I amp is the amplitude of the current; then, the average power that is being burnt in the resistor… What is the average power that is being burnt in the resistor? So, the average current…

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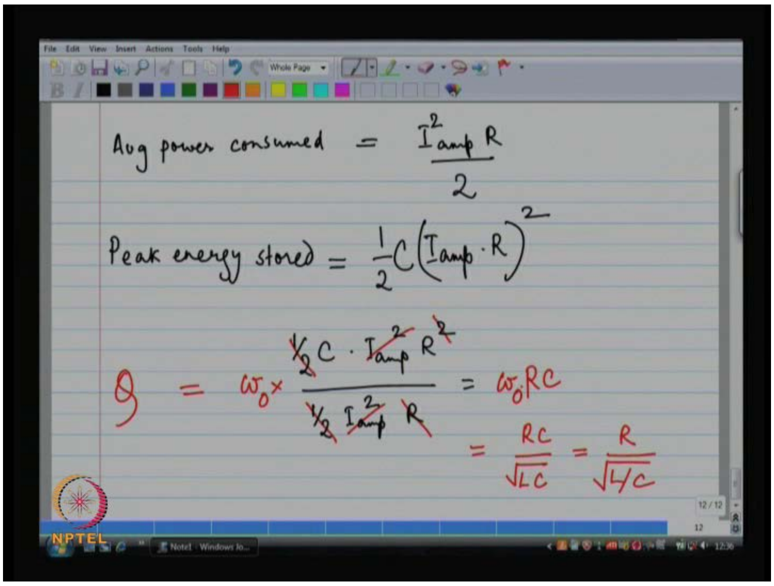

So, the current through the resistor is I, which has an amplitude of I amp. So, the average power burnt is equal to I amplitude square times R divided by 2. The 2 comes because of the RMS; all right? What is the peak power that is stored? At some point of time, the inductor stores all the power in the form of a current. At some point of time, the capacitor stores all the energy in the form of a voltage. Let us look at the capacitor. Both of these energies are equal. When the inductor has all the energy, the capacitor has nothing; when capacitor has all the energy, the inductor has nothing. So, let us look at the capacitor.

The energy stored in a capacitor is half C V squared. The peak amplitude across the capacitor, peak voltage across the capacitor, is I amplitude times R. That is the voltage across the capacitor. So, I amplitude times R – that is the peak voltage; at that point of time, this amount of energy stored in the capacitor is half CV square. So, half C times V square; all right? So, the ratio of the energy stored to the average power being burnt is going to give me the quality factor. So, what it this ratio? This is not right; this has units of time. I am dividing a power; I am dividing an energy by a power. So, it will have dimensions of time. So, this cannot be right. What I wrote was correct; average energy consumed per cycle. This is the average power consumed; all right?

So, what is the average energy consumed? No, this is not right. So, please correct your… So, this is correct now. Please make this correction. If the quality factor is omega times the peak energy stored divided by the average energy consumed – average power consumed. And, we are talking about the resonant frequency. At the… All these computation is valid at the resonant frequency; it is only at the resonant frequency that, the power consumed is so much, etcetera.

So, at resonant frequency, this is omega 0, which makes it RC by square root of LC. And, that is equal to R by square root of L by C; all right? So, this is the quality factor. The quality factor at a given frequency is the omega times the peak energy that is stored divided by the average power that is consumed. Now, with this in the background, can we now look at series RLC circuits.

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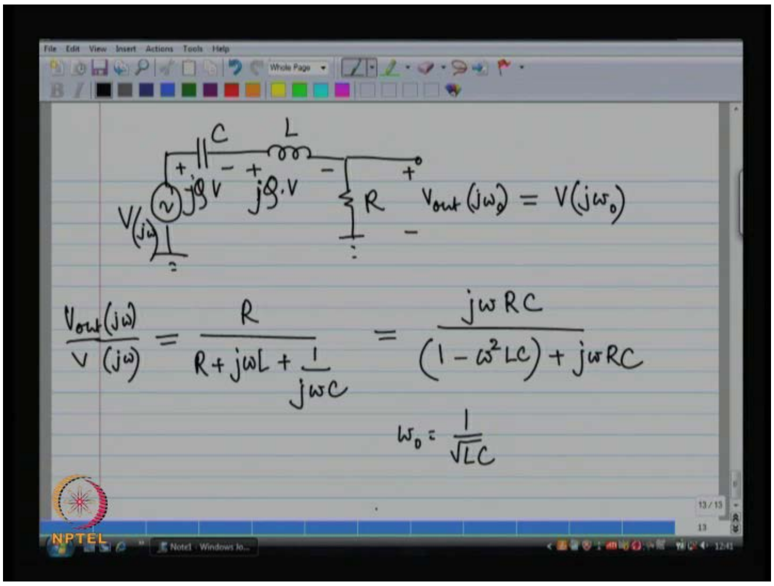

It is easy to now look at series RLC circuits. The same definition of quality factor applies here; same old definitions apply. Let us apply a voltage and exchange the configuration a little bit. Let us apply a voltage. Last time we applied a current; now, let us apply a voltage. And, we are going to measure the voltage that I developed across the resistor. So, this is a resistive divider. And, this is what we have got.

So, that makes it j omega RC by 1 minus omega square LC plus j omega RC. This is the final expression for the output voltage as a function of the input voltage. Now, crosscheck sanity checks. When omega is 0, the inductor is a short circuit; the capacitor is an open circuit. If the capacitor is an open circuit, whatever you apply at input is never going to reach the output. So, the output should be equal to 0 volts at omega equal to 0. The expression also says the same thing.

At infinitely large frequencies, the inductor is an open circuit; which means once again, whatever you apply at the input is never going to reach the outputs. So, the output should be 0 volts. You look at the expression; you have omega in the numerator; omega squared in the denominator; which means 1 by omega overall. So, at infinitely large frequencies, 1 by omega will tend to 0; which means that, you will get 0 at the output. Somewhere in between at the resonant frequency, which is 1 by square root of LC; at this resonant frequency, surprisingly enough, omega square LC becomes equal to 1. So, j omega RC is in the numerator; j omega RC is in the denominator.

So, you happen to get V out is equal to V. So, at the resonant frequency, you except V out to be equal to V. And, what is not being told to you is what are the drops across the capacitor and the inductor at resonant frequency. Now, this you are going to find out; they are going to be equal to Q times V. Actually, there is the j somewhere. But, j is relevant as far as the magnitudes are concerned. So, one of these will be j Q times V; the other will be minus j Q times V; and, they will cancel out with each other. And, you are finally going to see V at the output across the resistor. So, you apply 1 volt at the input. As long as the Q is huge… Let us say the Q is 100; you are actually producing 100 volts across the capacitor; 100 volts across the inductor. So, that is something very interesting that happens at the resonant frequency. Once again, energy is shared between the inductor and the capacitor and the definition of Q is as before.

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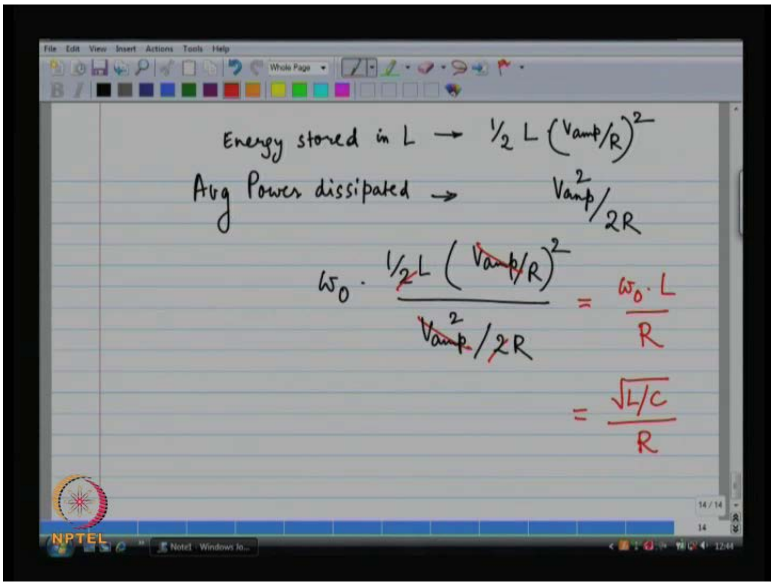

So, if you look at Q as before, energy stored…

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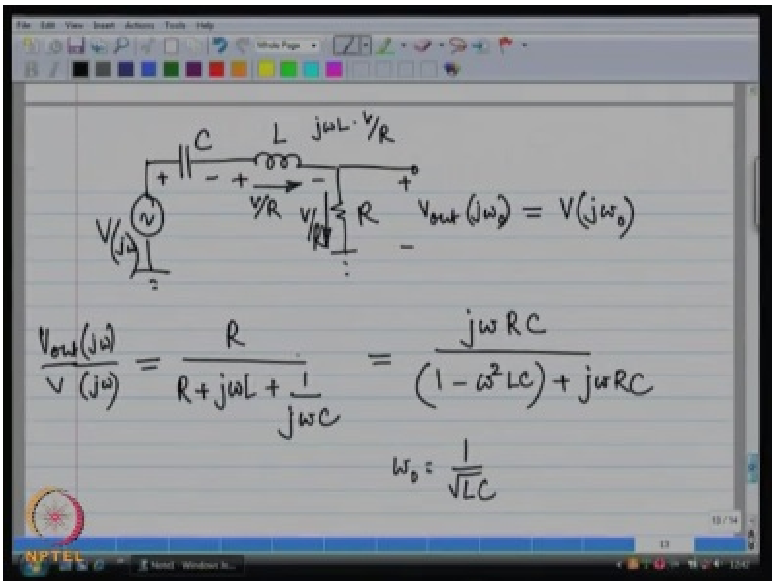

Let us… Instead of writing Q times V, let us do this properly. The current at resonance… The voltage across the resistor is V. So, the current is V by R. So, the drop across L is j omega L times V by R. The drop across the capacitor… Actually, that is enough. And, the current is V by R. So, the energy – the peak energy stored in the inductor is half L I squared right. So, the peak energy stored in the inductor is half L times the amplitude of I squared; the amplitude of I is the amplitude of V divided by R. And, power burnt – that is equal to V squared by 2R. So, Q is omega 0 times half L V amp by R the whole squared divided by V amp squared by 2R. So, 2 and 2 cancel out; V amplitude squared cancels out. And, what you end up with is going to be omega 0 times L by R. And, you substitute omega 0 is 1 by square root of LC. What you will get is square root of L by C divided by R. And, this happens to be the exact inverse of the previous formulation. Previous formulation was R by square root of L by C; this one is square root of L by C by R.

I want you to ponder about these thoughts. So, what we have covered today. First of all, we have finished the earlier module. We talked about maximum power transfer; how the source resistance is an integral part of the source. You cannot tamper with it. And, maximum power is transferred if the load impedance is the conjugate of the source impedance. It so happens that, when maximum power transfer occurs, there is also no reflections; all right.

So, this is the situation that we had for the LNA. If I manage to do maximum power transfer, I do not get any reflections; I also get the maximum output power because the amplifier has a finite power gain. Then, we talked about what is this power gain we are talking about, because the input of the amplifier is capacitive primarily; there is also a small resistive component as far as the gate is concerned. Most of this power is coming from the power supply. So, we will stop here.

Thank you.