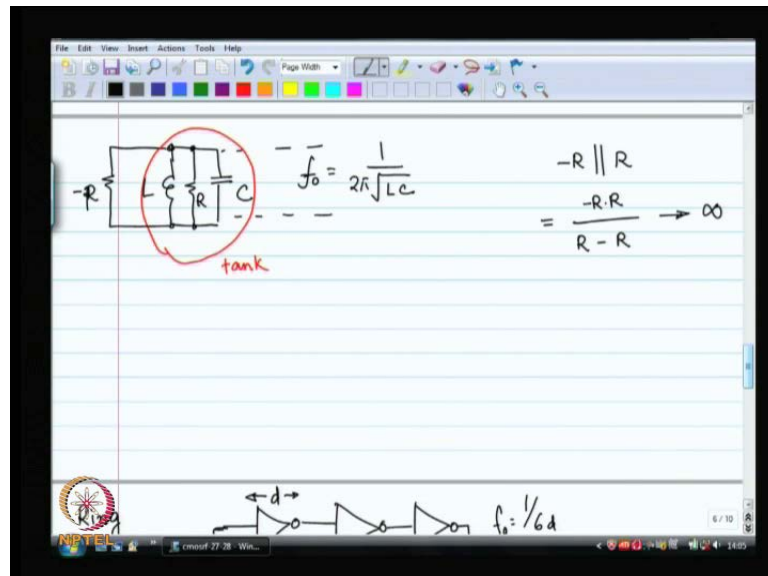


CMOS RF Integrated Circuits
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Module - 10
Oscillators
Lecture - 28
A Tank Based Oscillators

Welcome to cmos RF integrated circuits today we are going to carry on from where we left in the previous lecture and we are going start with the first cut at an oscillators. So, in the previous lecture we saw we are now discussing oscillators just by the way right

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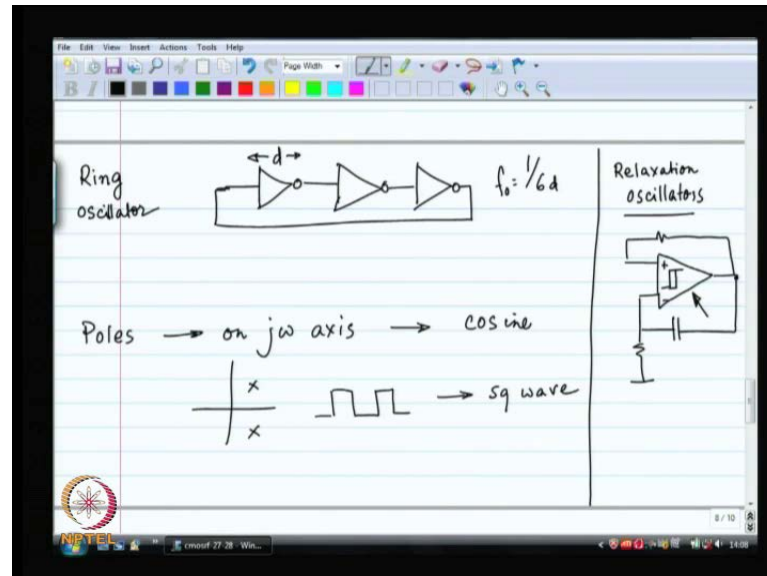


So, the first thing that we saw was most basic oscillators is inductor and sand with the capacitor and never will you get inductor which is a perfect inductor it will always be an inductor in series with some resistance because of the resistance of the wire than there is going to be a d current etcetera etcetera all that stuff you can lump into some form of a resistance right and inductor and series with resistance can be transformed to inductor and parallel with resistance.

So, you I have basically lumping it as an inductor in parallel with some resistance and this is the most basic form of a tank and this is the most basic form of an oscillators unfortunately this will never oscillate. So, to make it oscillate what I am going to do is I

am going to create some sort of a minus R and put it in sand with the entire tank. So, that is my plan right this is the most basic form of an oscillators

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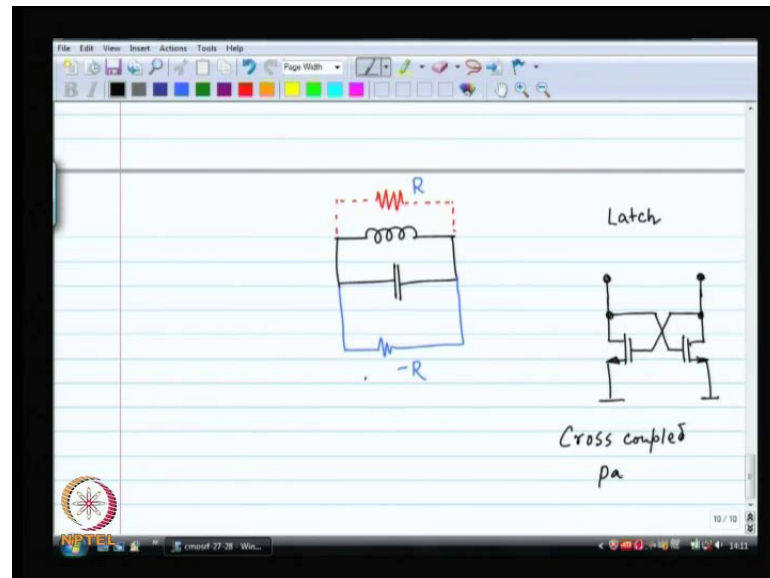


We also picked our brains and we reminded ourselves that there exist other forms of oscillators as well ring oscillators you can even design your own 1 with poles on the j omega axis or outside on the right half length right. So, its possible to do such stuff you can also go for a different class of oscillators. So, there exist second class of oscillators called relaxation oscillators might have studied in the long past you might have studied I C 5555 timer yeah by stable multi vibrators yeah maybe maybe you have studied such stuff right. So, those are relaxation type oscillators the theory for relaxation type oscillators are very very different from the theory we are going to study very different does not work the way we look at it. So, for relaxation type of oscillators you usually use some kind of a. So, I will just draw an example of a relaxation type oscillators. So, when we use a smite trigger .

So, some sort of schematic like this might work as a relaxation based oscillators might have studied sedra and smith or old micro electronics book which talks about bi stable multi vibrators right the theory for this kind oscillators are very different why because of this particular block almost always you are going to use something like a smite trigger and a smite trigger is not really a time invariant circuit smite trigger when the voltage is rising has some characteristics when the voltage is falling has some other characteristics.

So, the system is not time invariant the system is time variant. So, because of this property the theory for this kind of relaxation based oscillators is very different and we are not going to study it at least not here.

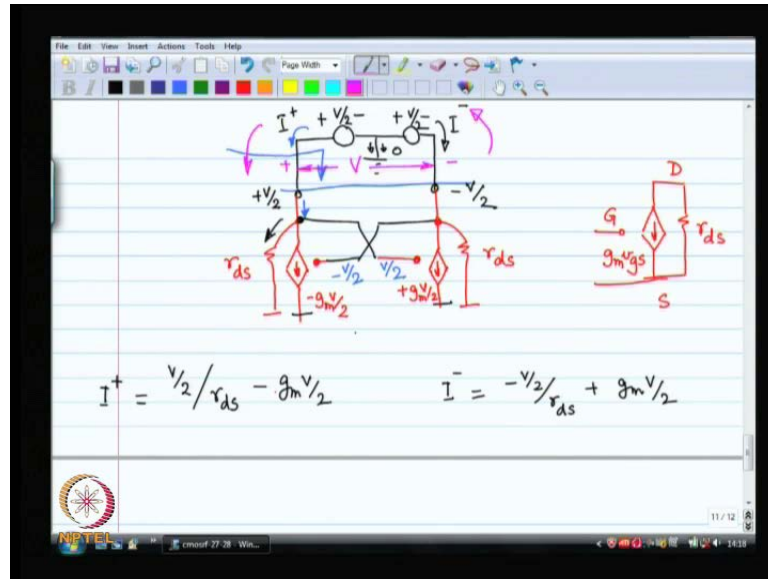
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So, let us first attempt to make a tank based oscillators and as we talked about it if I merely put an inductor in parallel with the capacitor it should work, but I understand that the inductor is never going to be an ideal inductor. So, there is going to be some parasitic resistance. So, this is unintentional this register that I have drawn is unintentional that is why I have put the dashed line over there it just.

So, happens that it is there now that do we do our plan is to put a minus R in sand with the whole dam thing right and hopefully this is going to work now you are going to say that what is this minus R block where do I find such a thing now our savior is the latch circuit I do not know if you recall ever having used such a circuit might have used something like this even if you haven't used let us check it out. So let us analyzed the latch circuit it is called the latch or it is called the cross coupled pair let us just analyzed the latch circuit the cross coupled pair first

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Let's say that differentially I apply a signal of plus V by 2 on this particular knot and minus V by 2 on the other knot alright and the question for you is what is the impedance that is seen looking into the circuit. So, from this from this 2 ports what is the impedance that I see looking in or in other words what is the current that is produced by the plus V by 2 voltage source and what is the current taken by the minus V by 2 voltage source right we have to find out these 2 currents and that divided by the by 2 is going to give me the resistance alright.

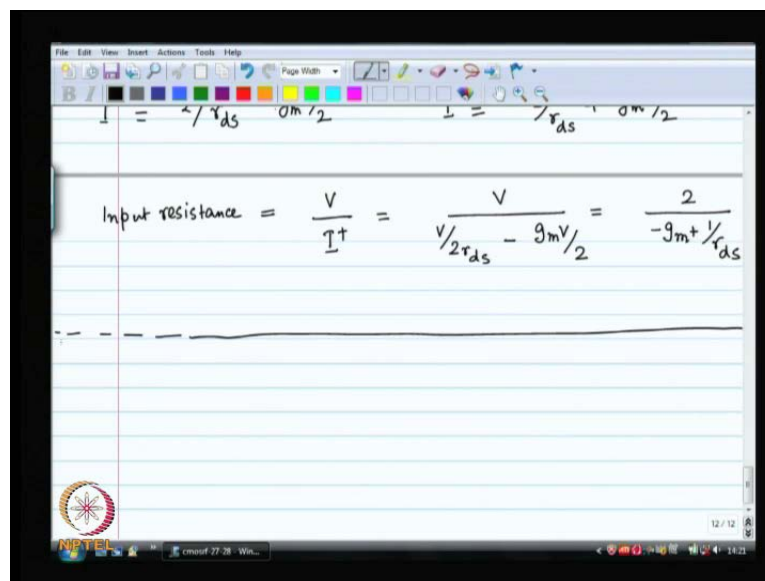
So, let us do this first. So, the voltage here is V by 2 the voltage at the gate over here is V by 2 voltage at the gate over here is minus V by 2 and let's replace the MOSFET with its typical small signal model small signal model of the MOSFET is that I am going to use for now is this this is the typical small signal model of the MOSFET I have thrown out all the capacitors conveniently alright let's throw out all the capacitors it is really not good in the at the radio frequencies you have to consider the capacitors, but I have thrown out just for convenience in our analysis at least as far as d c is concerned we will get the right result in addition to d c we are going to get some parasitic capacitors here and there etcetera etcetera, but our primary concern here is d c right our primary concern here is. In fact, the resistive part of it. So, that is why I have thrown out all the capacitors alright.

So, let us replace the MOSFET with its small signal equivalent model alright. So, this is what I have got as far as the small signal equivalent model is this is R_{ds} R_{ds} now the

values of the current source is this is something that is important alright. So, the 1 on the left hand side is minus g_m times V by 2 the 1 on the right hand side is plus g_m times V by 2 these are the 2 values of the current sources and now let us do KCL to figure out what is the total current over here and it is easy to see that what I have got is V by 2 divided by R_{ds} that is the current going through the resistor minus g_m times V by two.

So, that is your I let's call it I_{in} and let's call this I_{out} I am going to change the direction over here I_{out} interesting to see that I_{out} is going to be the exact opposite of I_{in} which is something that is nice which means if I had put the other direction for I_{out} then no current goes into the ground this current is 0 which is wonderful right. So, then what do I have here I apply a plus V by 2 somewhere and it's taking a current I_{in} . So, what is the resistance looking in. So, the total voltage that I applied across these 2 nodes is v . So, . So, applied V voltage and I_{in} goes in and comes out right. So, what is the resistance.

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The image shows a handwritten derivation of the input resistance of a common-source amplifier. At the top, two current expressions are written: $I_1 = \frac{v}{2} g_{ds}$ and $I_2 = \frac{v}{2} g_{ds}$. Below these, the input resistance is calculated as follows:

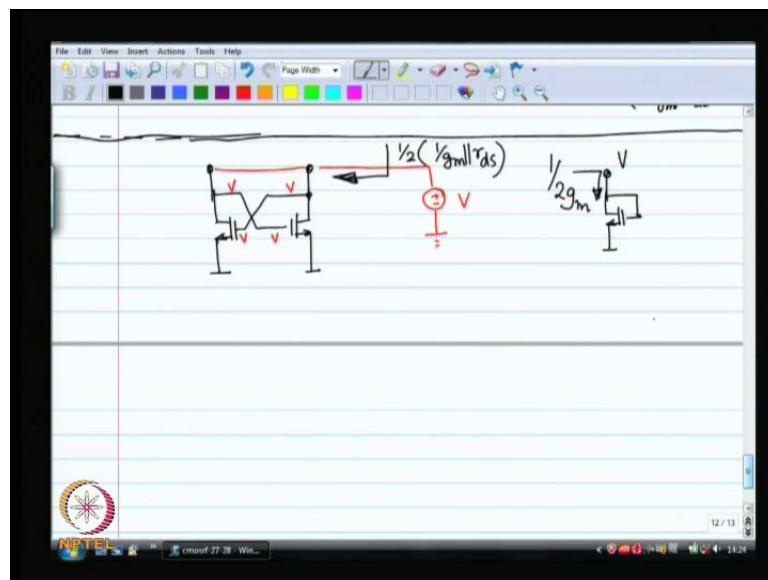
$$\text{Input resistance} = \frac{V}{I_{in}} = \frac{V}{\frac{V}{2} g_{ds} - g_m \frac{V}{2}} = \frac{2}{-g_m + \frac{1}{r_{ds}}}$$

So, I apply V voltage I_{in} goes in and comes out means that the resistance is V divided by I_{in} and all the V is cancel out and what I have over here something like this now the good news the good news is that if you have got any kind of a good transistor then typical values of g_m are going to be much larger than $1/R_{ds}$ that is how you get gain from a transistor right g_m times R_{ds} has to be somewhat large something like twenty

years old. So, g_m is going to be larger than $1/R_D$ so now that automatically means that this quantity in the denominator 2 divided by a minus g_m plus $1/R_D$ this quantity in the denominator is negative if the quantity in the denominator is negative I have created something which has a negative input resistance wonderful isn't it this is exactly what you are looking for right.

Now, when does this worth though just 1 catch over here the catch is that you have applied a plus V on 1 side and an sorry a plus V by 2 on 1 side and minus V by 2 on other side only than its working like a negative resistor. So, differentially speaking it looks like a negative resistor if you apply a differential signal on both sides than it is looking like a negative register what happens if you apply the same signal on both sides

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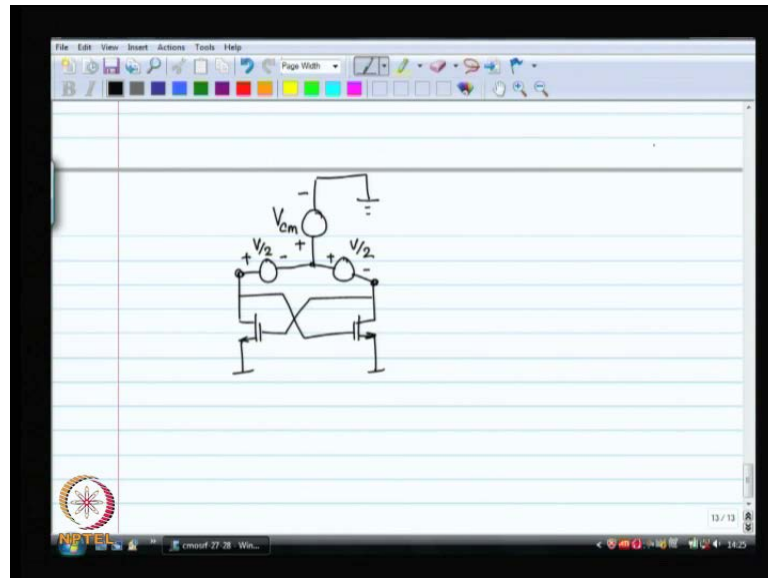


So, let us say I do this is my circuit these are the 2 ports and the point was that if I apply a voltage across these 2 ports then I see a negative resistance if I apply a voltage from these 2 ports with respect to ground what do I see that is the next question its very interesting. So, this voltage is V over here this is V over here this is also V and this is also V and when you replace the transistor with the small signal equivalent model what you get I mean it is easy to reduce this whole circuit into just this 2 devices.

Now, 2 devices means looking in over here the resistance that you see is going to be $1/2g_m$ approximately you can compare with your R_D s. So, that is the impedance that I see. So, now, if I look in I am going to see half of $1/g_m$ in parallel with R_D s earlier

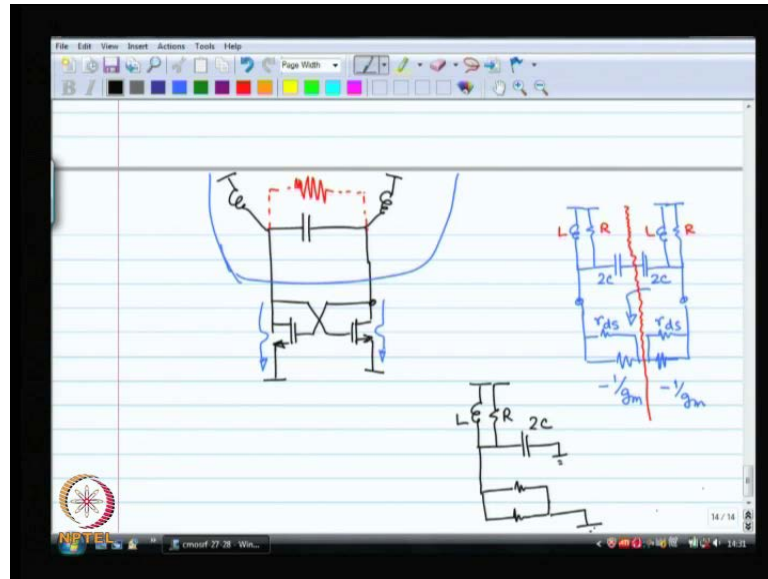
what I was seeing was basically twice of minus 1 by g_m in chant with R_d s that is what I was see earlier now I am seeing something else. So, the resistance the input resistance of this particular circuit depends on how you apply the signal suppose you apply your signal like this.

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Suppose you apply your signal like this than you are going to do some sort of super position trick to figure out what is going to be the current. So, as far as V_{cm} is concerned the common mode voltage V_{cm} is concerned the resistance that you are going to see is 1 by g_m in chant with R_d s divided by 2 because there are 2 such things as far as the differential voltages are concerned you are going to see a different input input resistance right you are going to see a negative input resistance as far as these 2 differential signal are concerned as far as the common the mode signal is concerned you are going to see a positive resistance basically looks just like a diode diode connected musket fine great news because.

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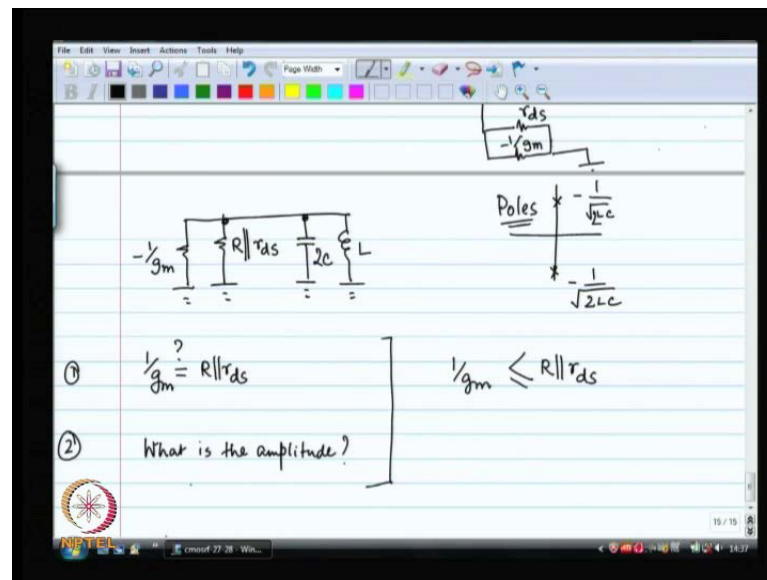
This is something that I like I can put this resistor there in parallel with my tank and I am done looks like I am done just that I am not this works when you think about the small signal what about the biasing of these devices need to be biased right should not forget the biasing of these devices and if they need to have any kind of g_m they need to have some d c current going through them this circuit to be a power supply. So, supposing I need some current like this as far as d c is concerned how am I going to create a current like that.

So, 1 opportunity is that instead of having the inductor over there I do not I delete this inductor from this place and place the inductor to the power supply on both paths. So, this is 1 possibility right this is something that a lot of people would like to do. So, as far as the differential mode picture of this entire circuit goes. So, as far as the differential mode picture of this entire inductor capacitor resistor combination goes this is what you have actually the resistor does not appear there this is what you have right.

I mean this is the circuit you have and looking in from here I see an effective resistance which is really negative and the value of that is something like minus $1/2g_m$. So, now, if you want to break this into a differential half circuit than suppose this is R this is L this is L this is R and this is C how do you break it into a differential mode half circuit first you have break the cross components. So, the cross components can be broken into series components.

So, you place 2 capacitors of size $2c$ each in series in each other and as far as this negative resistor is concerned you treated like 2 resistors 2 negative resistor each of side size minus 1 by g_m is it just minus 1 by g_m no it is not just minus 1 by g_m there is also the R_{ds} portion of it that also can be can easily be broken. So, this is what I have to start with alright and the idea of the differential mode half circuit is that when I apply a circuit on when I apply plus V and minus V on both side it is like a seesaw and the fulcrum point is in between which basically means that if I draw a plane through the axis of cemetery of the circuit than all points lying on that plane will be at 0 potential right that is the way we transform a full circuit into a differential mode half circuit. So, using that concept what I end up with is this

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And if you re draw your thing than this is what you have got alright now it is easy to see that when minus 1 by g_m is exactly equal to R parallel R_{ds} all your left with are the inductor and the capacitor the negative and the positive [re/resistors] when they are in parallel with each other they nullify each other right in this passion this is how they nullified each other it is almost like there is an infinite resistor in chant which is nothing nothing in chant right.

And you are left with a pure inductor in chat with a pure capacitor agreed when you have a pure inductor and pure capacitor in chant with each other the poles of the system lie on the geo mega axis got 2 poles each pole is at what frequency minus 1 by square root of L

$\frac{1}{C} \sqrt{\frac{2}{L}}$ in our case fine and if you have a poles on the geo mega axis than; that means, your system is going to oscillate at that particular frequency. So, all this is wonderful what this is not doing there are 2 problems with this problem 1 is that how do I make $\frac{1}{g m}$ equal to $R \parallel R d s$.

So, this happens when $\frac{1}{g m}$ is equal to $R \parallel R d s$ right how do I make this equal this is problem 1 problem 2 is when the poles on the geo are on the geo mega axis fine the system oscillates at a frequency of $\frac{1}{\sqrt{2 L C}}$ what is the amplitude of this oscillators is there is anything that is telling me what is the amplitude going to be any anything in my circuit that is deciding what amplitude of my oscillators are going to be we have done some small signal analysis we have figure out small signal input resistance of the cross coupled pair that happens to be negative right is the amplitude also.

Going to be small because if the amplitude is small than its bad news what is this amplitude going to be that is the second question first question is $\frac{1}{g m}$ is equal to $R \parallel R d s$ this is very hard condition first of all R is something which I can only estimate I never really know exactly what R is going to be when I am designing the circuit right I mean I can tell you that if you make an inductor on an integrator circuit the inductor will probably not have a quality factor more than five, but that is not telling you much what exactly the the parallel resistance is going to be equal to right.

So, this is problem 1 problem 2 is $R d s$ $R d s$ is also a big unknown when the process engineers create a semi conductor technology they in general are not that worried about $R d s$ and the tolerances of on $R d s$ the exact value of the $R d s$ could be as largest 2 hundred percent. So, there is no control over here and as a result $\frac{1}{g m}$ equal to $R \parallel R d s$ this is a very very hard condition to satisfy.

So, the next thought is as follows what happens if $\frac{1}{g m}$ is not just equal to $R \parallel R d s$ $\frac{1}{g m}$ is this less than $R \parallel R d s$ I probably cannot guarantee that $\frac{1}{g m}$ is going to be equal to $R \parallel R d s$, but maybe I can make $g m$ large enough maybe I can make $g m$ large enough I can always make $g m$ large I can make $g m$ large enough to drown out $R \parallel R d s$ is it possible to do that yes it is possible to do that what happens in that case what are you going to see. So, you have got a negative resistor in parallel with some positive resistance

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The image shows a handwritten derivation of negative resistance and a corresponding circuit diagram. The derivation is as follows:

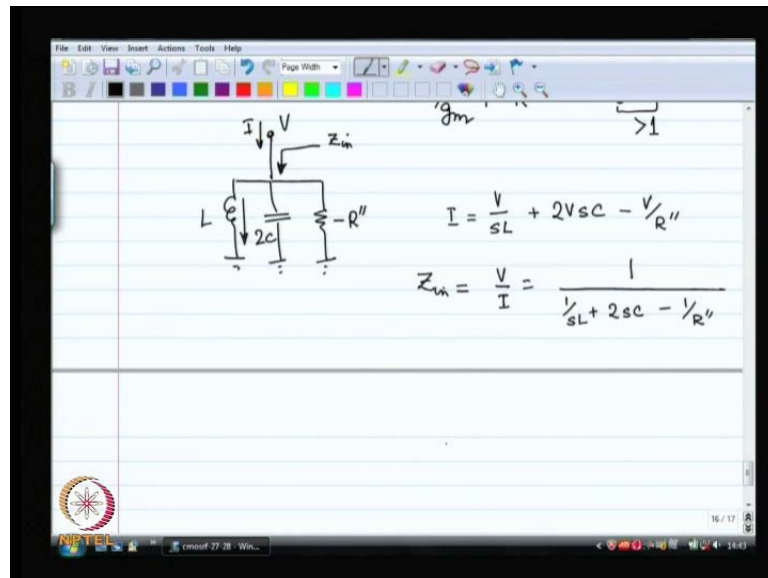
$$-R'' = -\frac{1}{g_m} \parallel R' = \frac{-\frac{1}{g_m} R'}{-\frac{1}{g_m} + R'} = \frac{-R'}{\underbrace{g_m R' - 1}_{>1}}$$

Below the equation is a circuit diagram. It consists of an inductor L in series with a parallel combination of a dependent current source $g_m v$ and a resistor R' . This parallel combination is in series with another resistor $-R''$. The output voltage is labeled $2v$ across the parallel branch.

Let's call it R' and . So, what I am saying is let us make g_m large enough let us make g_m very large if I make g_m very large than g_m times R' is going to be more than one. So, all I am going planning to do is to make g_m times R' more than 1 if g_m times R' is more than 1 then the quantity in the denominator is more than 0 that is its positive if the quantity in the denominator is indeed positive then I have got a net negative resistors some negative resistance is what I have got.

So, what I am going to do is I am going to make this g_m times R' much larger than 1 as a result I am going to have a net negative resistor so; that means, that my small signal picture is going to look like this small signal differential picture. So, let us call this entire thing R'' what happens here what kind of a system is this I have got a negative resistor in parallel with an inductor and capacitor what do you expect to have what what is your expectation from this kind of a system well if you do the analysis lets just do some sort of analysis right

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Let's find out where the poles are or what do you want to do let us say this is V voltage V and I am going to find out if I apply a voltage V what is the current I that goes in let us say that is what we are planning to do. So, this is clearly V by geo mega L let's not use geo mega I am going to use s alright. So, if I planned to look at the input impedance this is what I see which I can simply a little bit you do not need to simply too much just go to multiply the numerator and denominator by s times L

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The equations shown are for finding the poles of the input impedance:

$$= \frac{sL}{1 - sL/R'' + 2s^2LC}$$

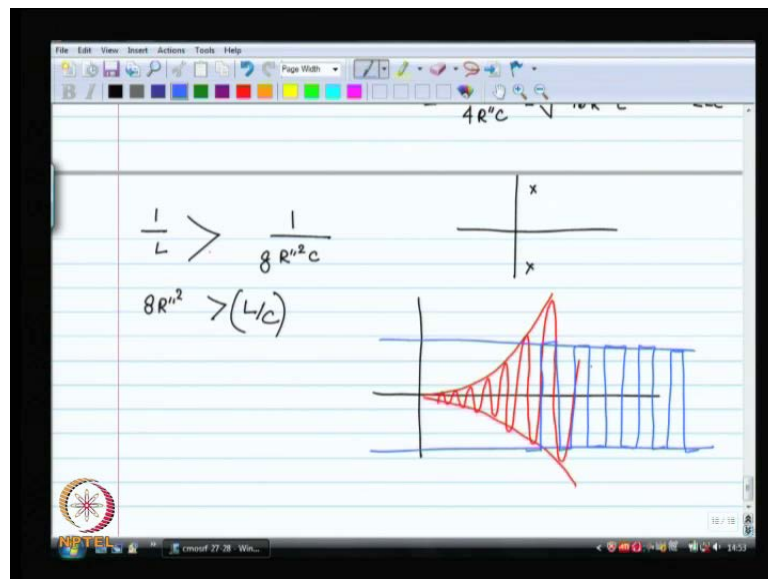
$$1 - sL/R'' + 2s^2LC = 0$$

$$s = \frac{L/R'' \pm \sqrt{(L/R'')^2 - 8LC}}{4LC} = \frac{1}{4R''C} \pm \frac{\sqrt{1/R''^2 - 8C/L}}{4C}$$

$$= \frac{1}{4R''C} \pm j\sqrt{-1/16R''^2C^2 + 1/2LC}$$

And this is what I am going to see. So, you have got a 0 at s equal to 0 which is, but you have poles you have got 2 poles in the system and it is not difficult to see that the 2 poles are going to be on the right half plain right that is where the 2 poles are they are on the right half plain now you want to know exactly where they are to equate $1 - sL/R$ double prime this is not really a meaningful exercise, but just check it out g equal to 0 means s is equal to 0 and just going to further simplify something like this alright. So, if R double prime is equal to infinity than this boils down to 2 poles at on the geo mega axis

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So, as long as you make sure that $1/2 L c$ is greater than $1/16 R$ double prime c squared if you can make sure of this then that will mean that you will have 2 poles which are complex conjugate pair which are in the right half plain, but if this inequality is not maintained that is if $1/2 L c$ is no longer greater than $1/16 R$ prime squared c squared than unfortunately the poles are going to start appearing on the real axis yes you are going to get a pair of poles, but they will be on the real axis which is not particularly exciting why we will just come to it. So, this par[ticular]- this means that this simplify a little bit more .

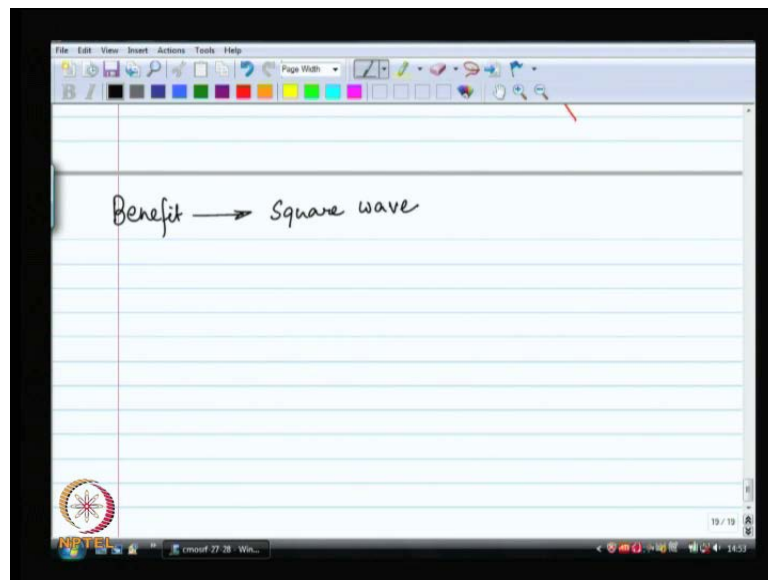
Alright. So, if R double prime is alright. So, it is to have a minus R double prime net negative it is to have such a thing; however, what I am trying to say over is you have to do it with some restriction now what happens when I have 2 poles which are a complex conjugate pair of poles on the right half plain what happens when they are on the right

half plain complex conjugate pair of poles on the right half plain impulse response of such a thing is I do not know if you remember it is a rising exponential modulated by a sign wave. So, this is what happens when the 2 poles are on the right half plain.

Now, unfortunately we cannot have the output of our circuit rising forever because you have got a limited power supply eventually sooner or later non-linearity's of the transistor are going to come in to effect the system is not going to look like 2 poles on the right half plain anymore as a result the voltage wave form coming out of the circuit is going to be clipped at the power supply voltage.

So, it is not going to look like a rising exponential what it is going to look like is a clipped version of that which is a square wave right its wonderful why is this wonderful lot of reasons number 1 solves this problem I do not need 1 by g m to be exactly equally to R parallel R d s number 2 it solves this problem amplitude is the power supply voltage almost I mean basically where the non-linearity is set in that is the amplitude that you are going to get coming out of this circuit.

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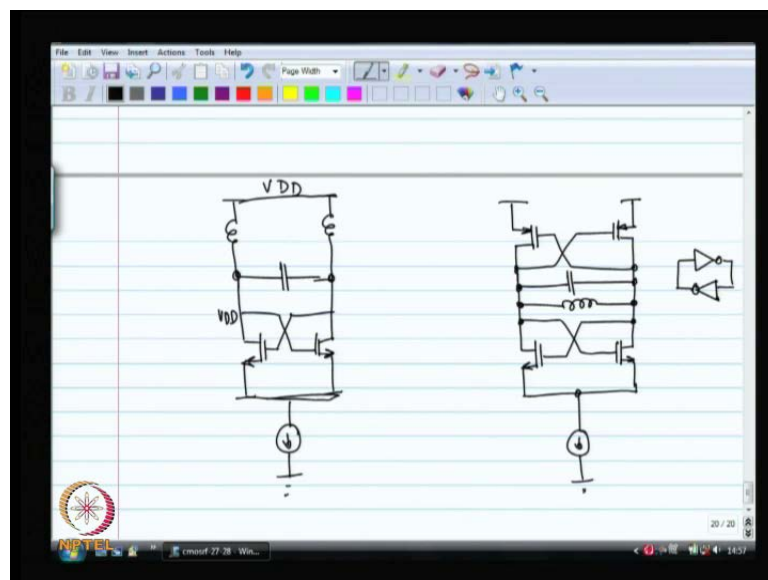


Now, the additional benefit is that you are getting a square wave and not a sign wave alright why do I like square wave I think we talked about this before we like square waves because the noise is only the phase noise the amplitude noise is irrelevant because it is a square wave right whatever wherever there is any noise in the amplitude its

cleaned out because you have got plus and minus VDD or the power and ground as your voltages.

So, there is no noise as far as the amplitude is concerned all the noise is in the phase of the signal the 0 crossing in the signal which basically means coming back from thermodynamics statistical thermodynamics that you get a 3 d b noise improvement over traditional sign wave signals. So, the noise in this noise power in a square is 3 d b less than the noise power in an equivalent sign wave wonderful this is an extra benefit alright. So, this is how we make oscillators this is just 1 example of an oscillators. So, what I did was basically this.

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Now, of course, you do not like this because there is no control over how much power this is consuming if you look at the d c picture the d c picture is voltage is at VDD which means you have got just 1 end moss as a diode between VDD and ground. So, its going to sink a lot of current you do not really like that you would want a current source over here to control how much current is going through the entire thing that also gives you a handle to what is the g m of each of this individuals musket this particular current source right. So, this is 1 example of a of an oscillators tank based oscillators there are variations in the topologies you can use p mosses you can use N mosses and p mosses for example, you could contemplate doing this.

So, instead of using just cross coupled pair of N mos devices I am going to use 2 inverters which are cross coupled. So, effectively this is what this is. So, that is why this is called latch if you redraw the circuit you will find that this is exactly the same as what I have drawn over there anyway. So, you can use this over here and then you can just put the inductor and capacitor right across.

So, this also is quite popular right and in this case the negative resistance that you are getting is even better than before because now you have the g_m of the N mos plus the g_m of the p mos right of course, you also have the R_{ds} of the N mos and the R_{ds} of the p mos, but if that is not of concern compare to the resistance in the inductor than this could be a potentially better topology anyway. So, these kinds of topologies are used routinely these are all tank base oscillators in the next class we are going to start talking about other kinds of oscillators which also use the tank circuit at its heart called pit oscillators pairs oscillators and. So, on.

Thank you for your attention.