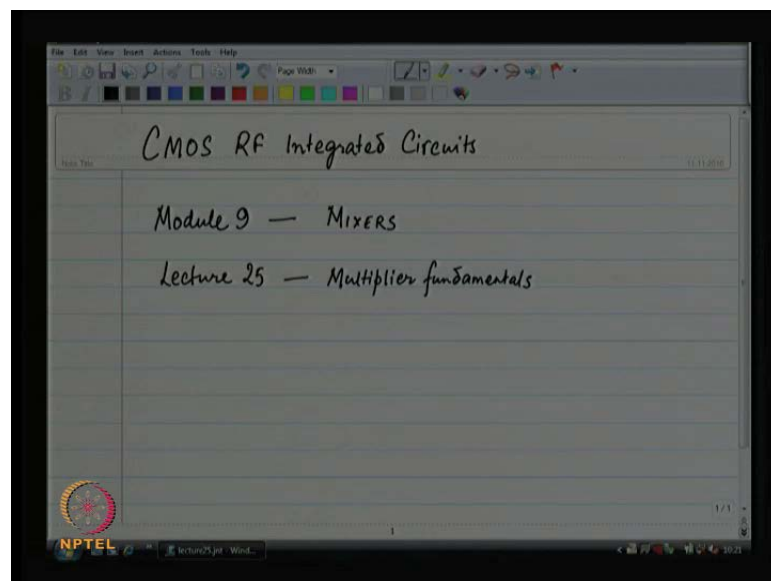


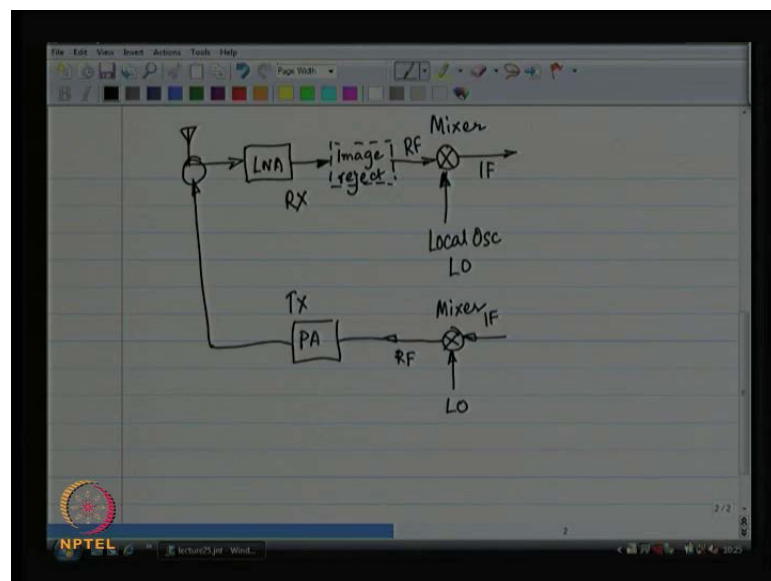
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**Module - 09**  
**Mixers**  
**Lecture - 25**  
**Multiplier Fundamentals**

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Welcome to CMOS RF integrated circuits today is a lecture 25 we are starting a new module today and this is module about mixers or multipliers. So, typically in a receiver the mixer is the second block or sometimes the third block. So, a typical receiver looks like this.

You have the antenna and from the antenna the first block is the low noise amplifier why the first block is the low noise amplifier is something very fundamental because you want to have low noise at the same time gain. So, that the noise figure of the remaining of the following blocks is not important to you anymore that is why the first block is typically a low noise amplifier.

The second block in a super heterodyne receiver or in a heterodyne receiver is going to be something called an image reject filter why you need the image reject filter that we are going to understand in the mixture chapter. In this module we are going to understand the necessity of this image reject filter as well a lot of time the image reject filter is part of LNA itself alright.

The next block is the mixer now this is the symbol for the mixture mixture has 2 inputs 1 of the input is a local oscillator in short call the L o and the other input is coming from the image reject filter or from the LNA or wherever it is the RF signal the RF signal is at a frequency which is act the channel that you are observing may be 1 giga hertz or may be 1 and a half giga hertz I do not where you are what channel you are listening to that is the RF frequency the local oscillator may or may not be equal to the RF frequency if you are designing a direct down conversion reliever then the local oscillator is going to be equal to the RF frequency. So, that after mixing you come down to the basement. So, this 1 possibility.

The other possibility is that the local oscillator is not exactly the RF frequency its somewhat close to the RF frequency and after mixing you come down to something called an intermediate frequency this is what happens in a heterodyne or a super heterodyne receiver you come down to an intermediate frequency which is called IF right.

So, this is the general block diagram for a receiver for a transmitter it is just the opposite you have the L o and you have another input coming in at IF frequency and there you have the mixer and the mixer up converts the signal to the RF frequency and this goes to

a power amplifier which outputs your signals to the antenna at the antenna of course, there should be a block which separates the transmit path from the receive path either some kind of a circulator or some kind of a filter sometimes right. So, this is the general architecture of a transceiver. So, this is the R x path this is the transmit path receive and transmit alright. So, what is this mixer about the mixer is basically a multiplier if you have got 2 signals 1 at a frequency  $F_1$  the other at a frequency  $F_2$  when you multiply these 2 signals

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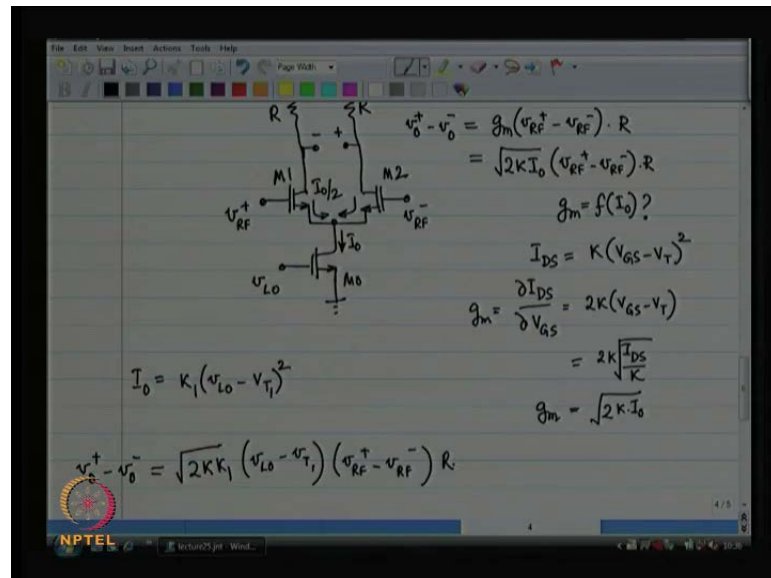
$$\cos(\omega_1 t) \times \cos(\omega_2 t) = \frac{1}{2} (\cos(\omega_1 t + \omega_2 t) + \cos(\omega_1 t - \omega_2 t))$$

Message Carrier Message

$\omega_{\text{carrier}}$

Let's say let us say cosine  $\omega_1 t$  and cosine  $\omega_2 t$  when you multiple these 2 cos times cos is equal to half of right this is your simple trigonometry relationship. So, if your  $\omega_1$  is the carrier and  $\omega_2$  is the message then you get a signal which is a carrier plus the message and carrier minus the message alright. In other words if this is the signal that you want to transmit suppose this is a some audio signal that you are planning to transmit then when you multiply it with the carrier signal it translate all the way suppose the carrier is here then this is what you are going to get right. So, this is the general idea now you can do the same thing in the receive path you receive this and multiply it you receive the high frequency signal suppose you receive the high frequency signal and multiply it with the carrier you go down to the basement. So, this is a general what a general mixture does alright.

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So, the idea that I need to be able to multiply and I need a circuit that does this beautiful multiplication job alright let us start with the following let us say that I have got a differential amplifier as I have got a current source a tail current source through the differential amplifier right. Let's say this is our situation to start with let us say the  $R$  of each of these mosfets each infinitely large just for ease of computation may not be infinitely large, but for ease of computation let us say that they are infinitely large this is not really an approximation this is without any loss of any generality we are doing this alright.

So, if this current is  $I_{D0}$  and let us say the input here I am going to call let  $V_{LO}$  plus the input here I am going to call it  $V_{LO}$  minus forget it we are going to call the input here  $V_{RF}$  plus and the input on the other hand  $V_{RF}$  minus ok. Suppose this is the situation both of these are equal registers then the voltage that I am going to observe between these 2 terminals is given by  $g_m$  times  $V_{RF}$  plus minus  $V_{RF}$  minus times  $R$  right.

Now, let us try to understand what those  $g_m$  is what is  $g_m$  does  $g_m$  depend on  $I_{D0}$  may be it does right  $g_m$  does depend on  $I_{D0}$  lets pause a bit over there this function  $g_m$  lets say that we have got a square law kind of mosfet where  $I_{DS}$  is  $K$  times  $V_{GS}$  minus  $V_T$  the whole squared if  $I_{DS}$  follows this relationship then dou  $I_{DS}$

by  $I_{ds}$  that is your  $I_{ds}$  that should be equal to  $2k(V_{gs} - V_t)$ , but  $V_{gs} - V_t$  is really  $I_{ds} k$  square root of that alright.

And now  $I_{ds}$  in our case is really  $I_{naught}$  by 2 fine. So, this is my  $I_{ds}$  for each of these individual mosfets which means that I am going to substitute  $I_{ds}$  over here and what I am going to get is something like this. Now, there is no such things as a current source the current source is really a mosfet and let us say I connect  $V_{LO}$  over here to the gate of this mosfet I connect  $V_{LO}$  now once again let us assume that it is a square law mosfet.

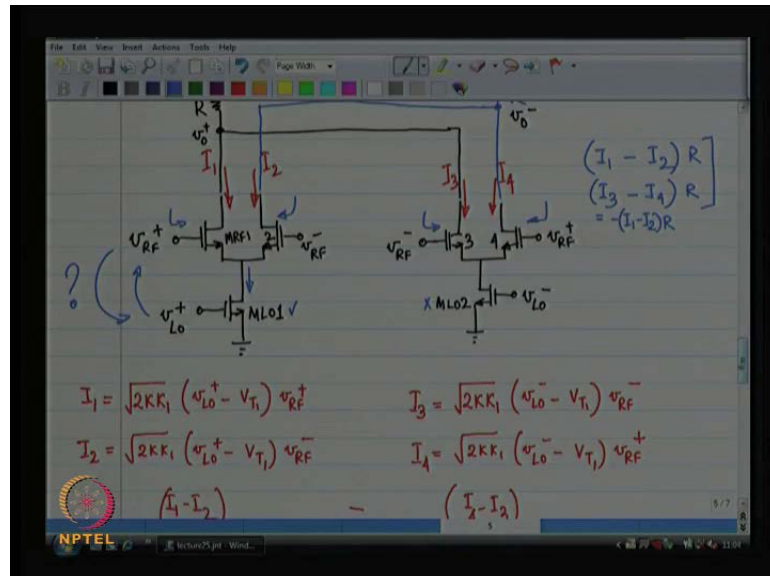
So,  $I_{naught}$  is really equal to some  $k(V_{LO} - V_t)$  the whole square right and this is this could be some other  $k$  I am just writing  $k_1$  and this could be some other  $V_t$  does not really have to track the other  $V_t$  because remember the transistors above the once that are connected to RF those transistor will have higher threshold voltage because the body is not at the source voltage the body is at ground right. So, that the threshold voltages of these 2 sets of mosfets let us call it  $m_0$   $m_1$  and  $m_2$ . So,  $m_1$   $m_2$  will have the same threshold voltage which is higher than the threshold voltage of  $m_0$  right.

Likewise I did not have the same width over length issues for  $m_0$  and for  $m_1$  and  $m_2$  which means that the constant  $k$  also might be something else alright. So, does not really matter as far as we are concerned what I need you to observe is that now suddenly I have got  $V_o$  plus minus  $V_o$  minus is equal to square root of  $2k I_{m0} I_0$  is  $k_1$  times  $V_{LO} - V_{t1}$  the whole squared times  $V_{RF}$  plus minus  $V_{RF}$  minus times  $r$ . So, this is great we have got a multiplication between 2 voltages ok. Unfortunately we have a  $V_t$  over here which is a process temperature dependent quantity and we do not like this  $V_t$  coming into the picture, but otherwise we have got a multiplication right.

So, the next step is to say that this is fine and let us get rid of this  $V_t$  business how can I get rid of this  $V_t$  business is by having a differential signal as far as  $V_{LO}$  is concerned. So, I do not want 1 voltage to determine what  $V_{LO}$  what a local oscillator voltage is I want 2 voltages the difference of the 2 should be the voltage of the local oscillator right. So, I want differential signaling and that should be able to. So, thing of it this way I will have 1 term which look like this and other term which looks like this, but the negative

part is as far as  $V_{L0}$  is concerned then I will subtract 1 from the other somehow and I will get rid of the  $V_t$  portion alright.

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So, let us do the following let's connect this to  $V_{L0}$  plus let's connect the gate to  $V_{L0}$  minus and let's connect. So, I am going to name this  $m_{L01}$   $m_{L02}$   $m_{RF1}$   $m_{RF2}$   $m_{RF3}$   $m_{RF4}$  right let's connect the gate of  $m_{RF1}$  to  $V_{RF}$  plus let's connect then  $m_{RF3}$   $m_{RF4}$  to  $V_{RF}$  minus let's connect the gate of  $m_{RF1}$  to  $V_{RF}$  plus let's connect to the minus mirror image and  $m_{RF4}$  let's connect it to  $V_{RF}$  plus right and now before I connect up the registers let us not connect the registers let us just try to evaluate the currents flowing through each of these. So, I am going to call this  $I_1$   $I_2$   $I_3$  and  $I_4$  alright.

What was the current flowing through this and what was the current flowing through this let's call this  $I_1$   $I_2$  the current  $I_1$  was equal to square root of  $2k$  times  $k_1$  times  $V_{L0}$  minus  $V_{t1}$  times  $V_{RF}$  plus right and  $I_2$  was the same thing with  $V_{RF}$  minus ok. So,  $I_1$  over here is equal to this quantity right and  $I_2$  is something very similar, but with  $V_{RF}$  minus and  $I_3$  and  $I_4$  are exactly the same just that instead of  $V_{L0}$  plus now we are dealing with  $V_{L0}$  minus I am sorry  $I_3$  has  $V_{RF}$  minus and  $I_4$  has  $V_{RF}$  plus alright. So, this is my situation.

Now, what is my plan my plan is as follows what I want to do is  $I_1$  minus  $I_2$  gives me  $V_{L0}$  plus minus  $V_{t1}$  times  $V_{RF}$  plus minus  $V_{RF}$  minus fabulous and  $I_3$  minus  $I_4$  gives me  $V_{L0}$  minus minus  $V_{t1}$  times let us do  $I_4$  minus  $I_3$   $I_4$  minus  $I_3$  gives me

the same expression as  $I_1 - I_2$  just that  $V_{L0+}$  is replaced by  $V_{L0-}$  and then what I want to do is  $I_1 - I_2 - I_4 - I_3$

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$$I = \sqrt{2kK_1} (V_{RF}^+ - V_{RF}^-) (V_{L0}^+ - V_{L0}^-)$$

$$V_0^+ - V_0^- = \sqrt{2kK_1} \cdot R \cdot (V_{RF}^+ - V_{RF}^-) \cdot (V_{L0}^+ - V_{L0}^-)$$

$$\sqrt{2 \cdot \frac{\mu C_{ox} \cdot W}{L} \cdot \frac{\mu C_{ox} \cdot W_1}{L_1}}$$

$$\frac{\mu C_{ox}}{\sqrt{2}} \sqrt{\frac{W \cdot W_1}{L \cdot L_1}}$$

Which also happens to be equal to  $I_1 + I_3 - I_2 + I_4$  if I do this job then what I am going to get is something like this fine I have jumped 1 step. So, just check for yourself that this is correct ok. And. So, the understanding is that I have to sum  $I_1$  and  $I_3$  how do you sum 2 currents just join the 2 currents you have got the sum of the 2 current brilliant right keep jobs current law and I will sum  $I_2$  and  $I_4$  and I need to look at the difference of these 2 currents now instead of looking at the difference of these 2 currents I am going to look at the voltage degenerate differentially which is the same thing alright.

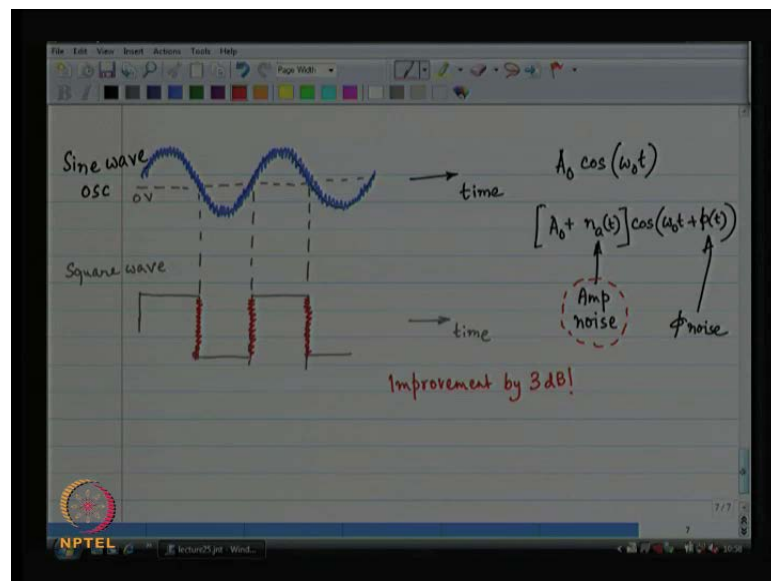
So, net what we are going to get is differentially what we are going to get is the same currents the sum and difference of the currents times  $R$  and we have successfully gotten rid of all the threshold voltages etcetera etcetera all process and all that stuff we have got rid of all of those the only thing that is still process and temperature dependent is the gain the gain contains some process and temperature dependent quantities.

So,  $k$  is really  $\mu c_{ox} w$  by  $L$   $\mu c_{ox}$  by  $2 w$  by  $L$   $k_1$  is  $\mu c_{ox}$  by  $2 w_1$  by  $L_1$  and we have got a  $2$  also. So, the square root of these is really  $\mu c_{ox}$  by square root of  $2$  times  $w$  times  $w_1$  by  $L$  times  $L_1$  minus right. So, this is what we have got alright. So,  $\mu$  and  $c_{ox}$  over here  $c_{ox}$  is not heavily dependent on temperature it dependent on the

process right, but  $\mu$  is dependent both on process and temperature the mobility of the electrons what can be done ah.

Suffice is to say that we have achieved successfully a multiplication operation we have successfully multiplied 2 numbers over here right. Questions there should be a few big questions over here. The first big question is we assumed that RF is a small signal and  $L_o$  is a large signal. So, when I did my analysis I assume that this particular current over here is a large signal current and the differential current  $I_1$  and  $I_2$  are small signals what is this is this justified  $L_o$  is a large signal RF is a small signal its absolutely justified I mean look at the names  $L_o$  and RF  $L_o$  is a local oscillator local oscillator is residing on the chip it is a large signal RF is something which has been picked up by the antenna it is not a large signal it is going to be quite quite small I mean very very rare occasions unless you are sitting next to the antenna you would not really get a larger signal over there ok.

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That's sorts out that 1 problem that  $L_o$  why we did large signal analysis for the bottom transistors and we did small signal analysis for the top transistor kind of short set up right  $L_o$  is larger signal RF is a small signal ok. Next big question very big question over here  $L_o$  is most probably a square wave does this multiplier work for square waves as well is  $L_o$  is square wave or sine wave what is the local oscillator output you would think that the oscillator output is sine wave, but no when you design an oscillator



you want it output to be a square wave why why would you want the oscillator output to be a square wave think about this this is my sine wave output of the oscillator x axis is time.

Now, the oscillator output is never going to be perfect there is going to be noise on top of the oscillator output now when you have noise on the top of the oscillator output what it is really going to look like is something like this it is an analog signal of course, these are all random it is a random noise it is not really deterministic. So, what we have got over here you can classify this noise into 2 things 1 is the amplitude noise the other is the phase noise. So, really the oscillator output can be written as ideally it is  $A \cos(\omega_0 t)$ , but this amplitude has some noise and the phase also has some noise. So, really it is  $A + n_{\text{amplitude}}(t) \cos(\omega_0 t + \phi + n_{\text{phase}}(t))$ . So, this is the amplitude noise and this is the phase noise right. So, this is how a sine wave oscillator is going to oscillate I have exaggerated the noise, but that is how it is unless I exaggerate you would not really understand what is going on ok.

Now, let us pass this sine wave through a comparator a comparator that compares the output of the oscillator to whether it is above or below 0 volts right c g in order to build the square wave such a comparator in that case what is going to happen is this is what you are going to get the amplitude noise is going to get cleaned up, but as far as the phase noise goes there is going to be an error where the 0 crossing is there is going to be some little bit of disturbance as to where the 0 crossing is and the comparator will have to figure out where exactly it is that will get affected by the noise at that point of time.

So, a lot of the noise is basically getting thrown out alright. So, this is an important reason why we like square waves as opposed to sine waves it's very interesting. Here you do not have this amplitude noise has all gone and all you are stuck with is the phase noise which basically determines where the 0 crossing exactly was and that is all you are left with. So, you have gotten rid of a huge quantity of noise if you think about statistical thermodynamics etcetera.

If there are 2 degrees of freedom for the noise to happen in or rather if molecules are vibrating in 2 different directions then those 2 directions will carry equal amounts of energy this is your statistical physics this is what it tells you. So, you carry forward several steps from there and you can show that the phase the energy in the phase noise

is going to be equal to energy in the amplitude noise which means that if you cut out the amplitude noise then the amount of noise you're left in the signal goes down by a factor of two. So, it is a several step argument starts from Boltzmann who concluded that if you have several degrees of freedom for molecules to vibrate then each of those degrees of freedom will carry equal amounts of energy as far as the vibration goes gas molecules goes right.

Here we are not talking about gas molecules anymore we are talking about molecules in a wire which are free to move around similar to gas molecules it is an ocean of molecules right. And then there are several steps in between to show that the phase noise and the amplitude noise are the 2 unique degrees of freedom in which the noise can be distributed and from that you can show that the energy contained in the noise in both of these 2 unique degrees of freedom are going to be equal not really the energy the power spectral density right and as a result you can show that if you can get rid of the amplitude then what you are left with as far as the phase noise is concerned is half of the total noise which means that you have improved by 3 dB it is a lot alright that is a lot of dBs 3 dB is a lot of dBs factor of two.

So, it is important that we talk about square waves and not sine waves now unfortunately when you finally, design your oscillator the output comes out it would not really look like a square wave it is very hard to create a square wave, but the wish is to create wave not a sine wave more importantly when you target to make a square wave what actually comes out more looks like a sine wave anyway it really does not matter our objective is to work with square waves if we can work with square waves successfully makes us the happiest if we can generate square waves as far as the local oscillator is concerned we are doing good job good job ok.

So, the question lets come back to the question the question that does this work does this scheme of things work when  $L_o$  is a square wave take a look at it it does work when  $L_o$  is a square wave. So, when  $V_{L_o \text{ plus}}$  is 1  $V_{L_o \text{ minus}}$  is 0 when  $V_{L_o \text{ minus}}$  is 1  $V_{L_o \text{ plus}}$  is 0 this is the idea. So, when  $m_{L_o 1}$  is on  $m_{L_o 2}$  is off. So, when this on this is off which means that when current is flowing through the first pair the second pair there is no current flowing right.

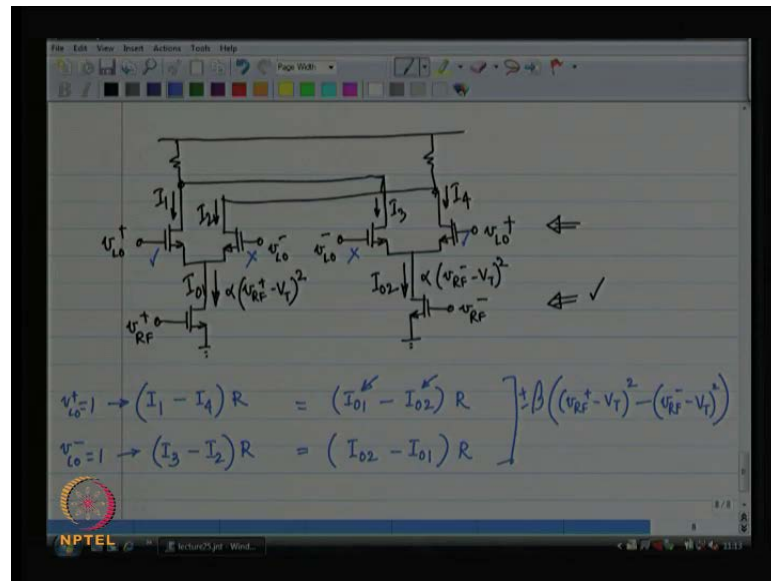
Which means that sometimes the output  $V_o$  plus minus  $V_o$  minus  $V_o$  plus minus  $V_o$  minus is  $I_1$  minus  $I_2$  at sometimes  $I_1$  times  $R$  this is at sometimes when  $V_L$  o plus is 1 and when  $V_L$  o minus is 1 the output is  $I_4$  minus I am sorry  $I_3$  minus  $I_4$ . So, there are 2 possible outputs and that depends on which side of the oscillator I am sorry which side of the mixer is on right.

Now, let us examine does this really work. So,  $I_1$  minus  $I_2$  is  $V_{RF}$  plus minus  $V_{RF}$  minus something that looks like that and  $I_3$  minus  $I_4$  looks like  $V_{RF}$  minus minus  $V_{RF}$  plus it is the opposite of it which means that sometimes I have got some output at other times I have got the negative of that output and that is exactly what you want it to build right that is exactly what you want it to build when you are multiplying by a signal that is either plus 1 or minus 1 you are trying to either keep the signal or you are trying to create the negative of the signals. So, 1 you first multiplied by 1 half of the times other half of the times you are multiplying by minus one. So, this is exactly what you want it to build in the first phase when you multiply it by a square wave. So, it works ok.

So, the next step the next question is are there any other question over here there is there is a very big question matching what are we going to do about matching remember something like the LNA is driving this mixer mixer inputs looks capacitive mixer input looks like a capacitor could be a problem right could be a problem because you cannot do a maximum power transfer any more. So, it is a potential problem you could potentially get rid of this problem by resonating with this capacitor with with another inductor that is 1 way of doing it.

However it might not be the best way you probably want to do maximum power transfer next thing is  $V_{RF}$  this is the signal that is coming from the LNA you are trying to drive 2 sets of the differential amplifiers is it possible to drive 1 set of differential amplifiers instead. For example is it possible that I interchange  $L_o$  and  $R_f$ . So, instead of  $L_o$  being the bottom and  $R_g$  being on the top can I have  $RF$  at the bottom and the  $L_o$  at the top.

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So, this is the other big question that we are left with over here well the answer to that is it is possible let us take a look how. So, the proposal is I am not showing all the connections it becomes very complicated alright. So, the proposal is that instead of RF on the top and L o at the bottom why can I have RF here and L o over here is it to do such a thing.

Probably not because we kind of depended on the fact that this current is going to change with the square of L o V L o minus V t the whole squared right that what we wanted we wanted this current to be proportional to V if if if if this does happen then we are in good shape if it is proportional to V RF plus minus V t the whole squared we are in good shape, but V RF plus is a small signal. So, what do we do about that we can add a bias voltage to it it will work if we add a bias voltage will it work it is no reason why it is not going to work if we add a bias voltage to it right ok.

So, if we manage to add a bias voltage to V RF plus and V RF minus this is going to work now the output of the LNA already contains that bias voltage presumably. So, it is going to work right any other problems. Next problem is the V L o and V L o minus they are square waves 1 0 signals. So, is it still going to work we did a small signal analysis for the top transistor remember we did not really do a large signal analysis over there we did a larger signal analysis which is also valid for small signals if you do a

large signal analysis it is also valid for small signals, but if you do a small signal analysis it is not valid for large signals.

So, over here we had done a large signal analysis which is very good, but over here we had done a small signal analysis which is not really going to cut the ice for us right. So, what do we do about it let's check what happens when I do 1 zeros if  $V_{L0}$  plus is 1 let us say this current is proportional to  $V_{RF}$  plus minus  $V_t$  let us say let us say this current is proportional to  $V_{RF}$  minus minus  $V_t$  whole squared. So, this is  $I_0$  over here  $I_0$  I am going to call it this is  $I_0$  alright.

So,  $I_1$   $I_2$   $I_3$  and  $I_4$  when  $V_{L0}$  plus is 1  $V_{L0}$  minus is 0  $V_{L0}$  plus is 1  $V_{L0}$  minus is 0 means this transistor is on this is off off on. So, the output that I get is going to look like  $I_0$ . So, I get  $I_1$  minus  $I_4$  times  $R$  as my output. The other case when  $V_{L0}$  minus is 1 when  $V_{L0}$  minus is 1 I am going to get  $I_3$  minus  $I_2$  times  $R$  now  $I_1$  minus  $I_4$  both of these  $I_1$  is really  $I_0$  and  $I_4$  is really  $I_0$  and  $I_3$  minus  $I_2$  is  $I_0$  and the other 1 is  $I_0$  alright.

So, first of all it's nice to see that you are getting something in when  $V_{L0}$  plus is 1 and then just the opposite of that when  $V_{L0}$  minus is one. So, you are getting that chopping behavior the 1 minus 1 multiplication by plus 1 multiplication by minus 1 that it is nice to see. What is not nice to see is the fact that  $I_0$  and  $I_0$  these are really proportional to the square of  $V_{RF}$  plus minus  $V_t$   $V_{RF}$  could also be a somewhat large signal if you go close to the if you go close to the base station then  $V_{RF}$  plus could be a large signal you do not know may be you are making the transmit path in which case  $V_{RF}$  plus could be a large signal there is no problem for it to be a large signal.

So,  $I_0$  and  $I_0$  these are suddenly proportional to the squares. So, really what I am getting I am getting the plus 1 minus 1 behavior which is all nice what I am not particularly happy about is the fact that I am getting  $V_{RF}$  plus minus  $V_t$  the whole squared minus  $V_{RF}$  minus minus  $V_t$  the whole squared times some gain .

So, this is something which I am not happy about I would like to get rid of these squares. So, this we are going to see in the next class how we can linearized this and also how do we get the matching behavior we are going to see in the next class and ah we are going to stop here for now and let me summarize we started off with a differential amplifier and we made a fully balanced mixer this is called the fully

balanced double balanced mixer also known as the gilbert cell mixer the gentle man who came up with this topology was gillbert it has been around for a long long time this is the gilbert cell mixer is the basic building block for all mixers on I c e s. So, we are going to stop over here.

Thank you.