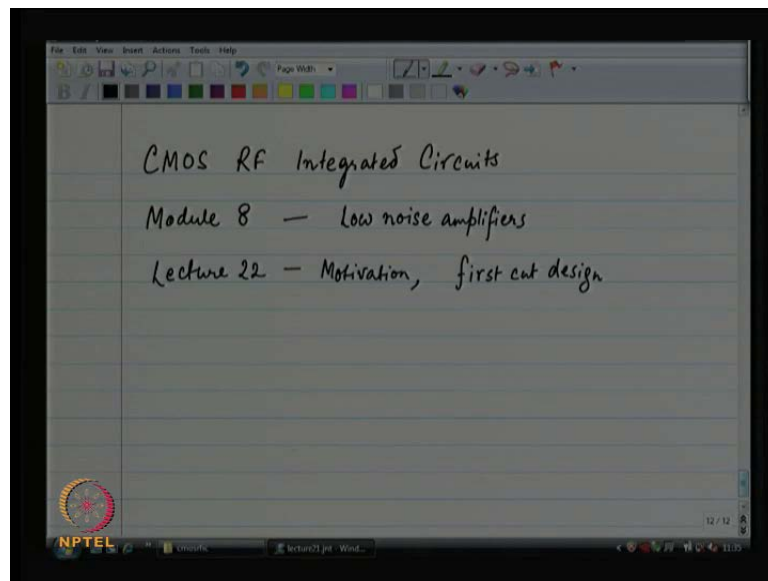


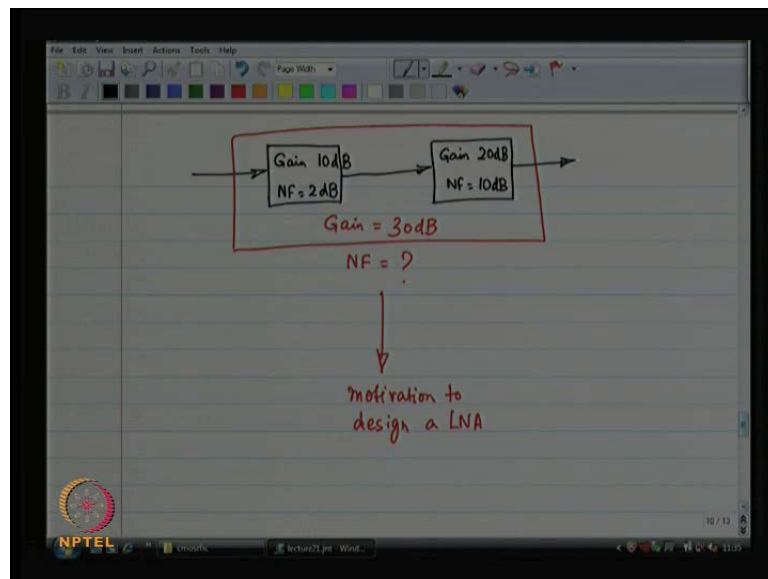
**CMOS RF Integrated Circuits**  
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**Module - 08**  
**Low Noise Amplifiers Design**  
**Lecture - 22**  
**Motivation, First Cut Design**

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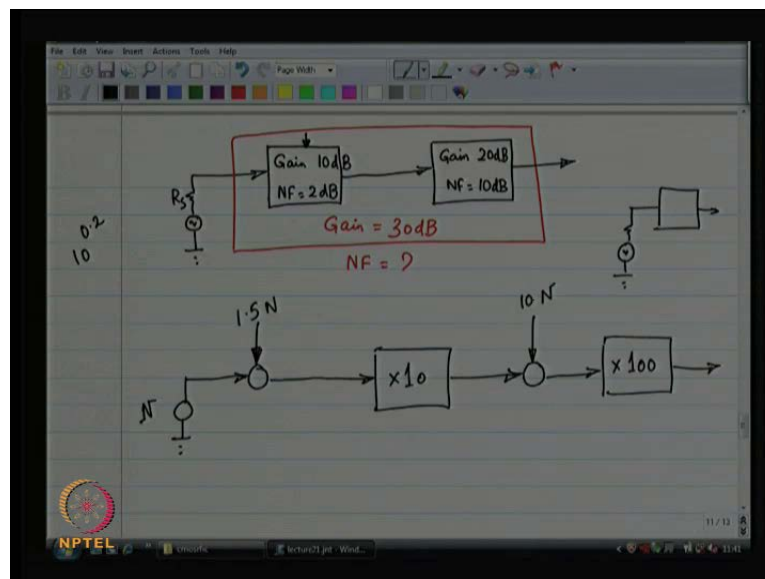
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Hello and welcome to CMOS RF integrated circuits. So, we were we had just about started talking about the 8 module low noise amplifiers and I promise that before we discuss low noise amplifiers we are going to try to motivate the problem and to motivate the problem.

We wanted to solve a numerical first a numerical which was basically this this is what we wanted to solve we have got 2 blocks first on is has a gain of 10 d b noise figure of 2 d b next 1 has a gain of 20 d b noise figure of 10 d b alright. Now, we know that the combine gain is 10 d b plus 20 d b that is 30 d b is the combine gain of the system now what is the noise figure of this was the first motivating problem that I need to solve and once I solved this we should be able to understand why I would like to design a low noise amplifier at all and also where low noise amplifier should be etcetera. So, let us now attempt to solve this problem. So, to do this first of all you have to recall what is the definition of your noise figure.

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The definition of the noise figure was suppose I have got a source over here . So, the source has its noise right the source inseparable from its noise you are talking about in antenna. Antenna has an impedance and that impedance is going to create some noise. So, we cannot avoid it now what is the noise contributed by the amplifier itself in relation to how much noise is coming from the source. So, that extra noise is 10 d b more than the noise of the source. So, I can modeled this as follows. So, I have got some noise

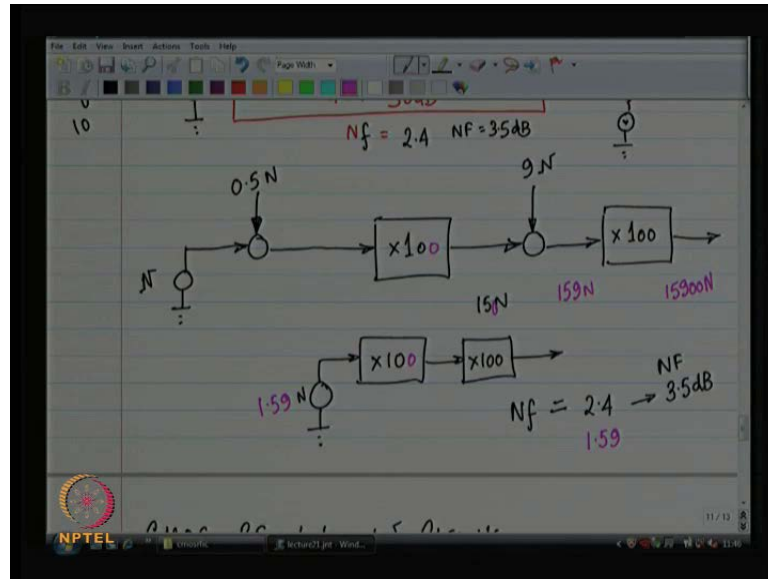
of the source and I will have some extra noise coming in over here which is 10 dB more than noise of source I am sorry 2 dB more than noise of the source that is the noise figure that will go through a gain of 10 dB alright

Now, we have to think about the noise figure of the second building block over here now the second building block has a noise figure of 10 dB what does that mean what does it mean when I said that the second building block has a noise figure of 10 dB if I put a source over there then the second building block can be modeled as something some noise coming from the source some noise coming from itself which is 10 dB larger than the noise of the source right and this is going to go through a gain of twenty dB and its coming up.

So, I want to measure or I want to estimate the combine noise figure of everything taken together right alright now let say lets now instead of noise figure lets convert these into noise factors noise figure is a just a dB representation of the noise factor. So, 2 dB is how much. So, let say that the noise of the source is  $n$ . So, I have got  $N$  as a noise of the source this is really a noise power. So,  $v N^2$  squared voltage square mean square voltage that is what we are talking about 2 dB more than that is how much. So, you have to divide by 10. So, 10 to the power 0.2 any ideas how much this is.

This will about a factor of 1 and a half alright every dB is of the factor of 1.25. So, 1.5 times  $N$  is coming there 10 dB is really again of 10 twenty dB is a gain of 100 alright 10 dB more than the noise of source is 10 times the noise of the source alright. So, this is what we have got. Now, what you think does it translate to at the output of my system what is the noise. So, I have got noise  $N$  plus 1.5  $N$  that is 2.5  $N$  goes through a factor of ten. So, I have got 25  $N$  over here I adding 10 times  $n$

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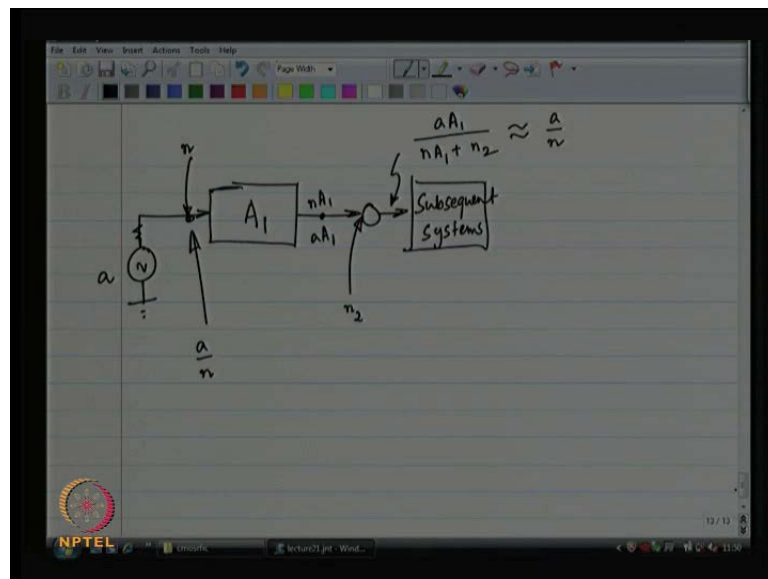
So, I have got 35 N over here 35 N goes through a factor of 100. So, I have 3500 times N over here, but input referred I have gone through a factor of 1 thousand. So, it is as if I had applied 3.5 N over here and gone through my gain of 10 and my gain of 100. So, these 2 systems are equivalent. So, if I have a system which with a noise input of 3.5 times N goes through a gain of 1000 I will get 3500 N output and similarly ah. So, basically these 1 systems are equivalent that is that is it right

So, the input referred noise is 3.5 times N now your noise figure is the total input referred noise divided by then noise of the source. So, you've got a 3.5 times N divided by N which is factor of 3.5 we made a mistake we made a couple of mistakes. In fact, sorry noise figure of 2 d b what does this mean that the input referred noise of the first stage is 1.5 times n. So, 1.5 times N is the input referred noise which means 0.5 N needs to be added N is already coming from the source alright this is mistake number 1 mistake number 2 is similarly this should be nine times N alright

So, with these mistakes taken in account we will have 15 N over here 24 N. So, I felt that the numbers went unreasonably large. So, I went back and fixed the mistake. So, I will get 2400 N over here and as a result this is equivalent to 2.4 times N 2.4 times N is a factor of what. So, what is the noise figure the noise factor the noise factor is 2.4 and if you want to express it in terms of the decibels then you do  $10 \log$  of 2.4 and that will give you something like 3 and half d b ok.

So, this whole thing will have a noise factor of 2 point four. So, it is very interesting I started off with 2 blocks first block noise figure of 2 d b next block noise figure of 10 d b the noise figures do not add I do not get 2 plus 10 equal to twelve d b of noise figure. So, this is a observation number one. So, why is this happening. So, to understand that think of the gain of the first stage suppose I had a gain of the 20 d b in the first stage suppose this first 200 d b instead of 10 d b alright what would happen this new number would become 100 this would become 100 I would get 150 N over here.

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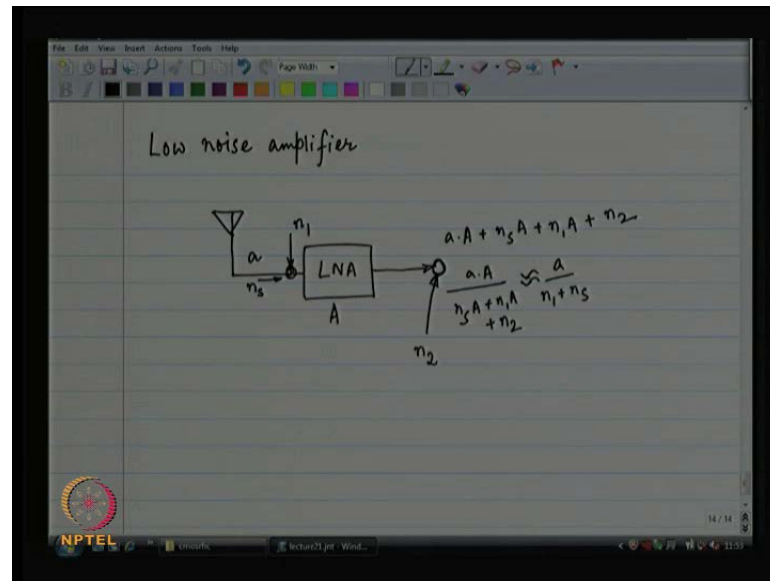
So, as a result I would get 159 N over here and I would get 15900 N over there and that would be equivalent to 1.59 times N into 100 into 100. So, my noise factor would then reduce to 1.59 right. So, if I change the gain of the first stage if I increase the gain of the first stage then the overall noise factor is actually dropping. So, what I am saying over here what I am trying to say So, you've got a system you've got a system with a certain amount of gain this particular system has its own noise alright now whatever noise comes after this because of the subsequent systems already see an amplified signal. So, think of this. So, i've got a signal coming in let say the signal has an amplitude of small a alright let say the input referred noise for this is. So, much let say small N alright. So, let say the input referred noise for the first amplifier is small N amplitude of the voltage coming into the first amplifier is small a.

So, you've got a signal to noise ratio of  $A/N$  at the input of the first stage now what happens is you have a gain of the first stage. So, let us call this  $A_1$  right this gain applies to both the signal as well as the noise. So, you get  $A_1$  times  $A$  you also get  $N$  times  $A_1$ . So, the noise is already amplified by the first stage now to this amplified noise you are [- you] adding some more noise let say  $N_2$  you already got amplified noise if you add a little more noise to it it is almost as if you're adding nothing because the noise is already amplified the signal is also amplified by the same factor right.

So, as a result the overall signal to noise ratio is not going to degrade much. So, the signal to noise ratio over here is  $A/N$  I am sorry  $A_1$  times  $A$  divided by  $N$  times  $A_1$  plus  $N_2$   $N_2$  is a small quantity over here the signal to noise ratio is  $A/N$ . So, you can show that if  $A_1$  is large if you gain of the first stage is large then this is going to be approximately equal to  $A/N$  right.

So, this is a something this relationship whatever if you do this mathematically if you do it with variables as opposed to numbers like we did you come up with a formulation that formulation is called the Friis equation I am not bothering to develop that formulation it is not important at this point of time what is important at this time is to understand that if the gain of the first stage is large then only the noise of the first stage is important the noise of the subsequent stages is irrelevant to me because you know what I am adding only small quantities of noise and a first stage already has amplified the noise of the first stage right. So, the subsequent systems behave as if they are not producing any noise. So, I can ignore the noise produced by the subsequent systems I just make a very good first stage. So, that is why we want make something called a low noise amplifier we are going to make this a beautiful amplifier

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That does not add much noise ok. But amplifies and what we are going to do is we are going to put this low noise amplifier right after the antenna. So, your antenna is capturing some signal signal has some noise because of the antenna because of everything else the low noise amplifier is adding very little noise compare to what is already there right. So, the low noise amplifier is adding very little noise of its own, but it is amplifying everything. So, there is a noise in the source there is signal that is called the signal as a its amplifying everything by a large number let us call it capital a. So, at the output of the low noise amplifier I am going to see a times a plus N S times a plus N 1 times a this is the signal that I am.

This is the total thing that I am going to see at output of which the signal to noise ratio is a times a divided by N S times a plus N 1 times a which is equal to a by N 1 plus N S right. So, what I am trying to do over here is I am trying to make sure that a is large a is a large number. So, that if I add further noise over here because of my subsequent systems that further noise becomes irrelevant to me. So, if I add N 2 over here then the total noise over there is going to be this plus N 2 and this plus N 2 is not going to make a dent in my equation because a is large.

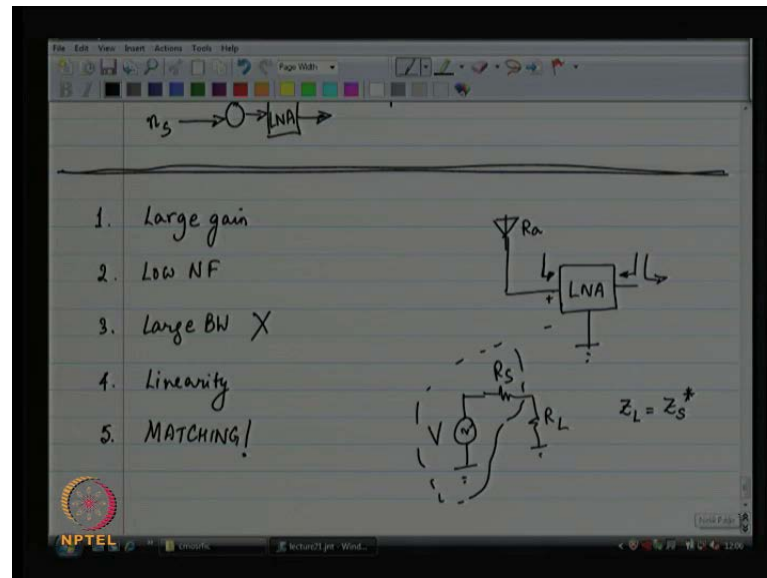
So, this is my plan. So, that is why I want to make a very good amplifier right upfront right next to the antenna it should be a low noise amplifier what happens if amplifier does not amplify what happens if amplifier instead of amplifying let say it makes the

signal smaller if it makes a signal smaller suppose the gain of this first stage instead of 10 d b or twenty d b its minus 10 d b suppose; that means, instead of multiplying by 100 I divide by ten. So, the overall gain is multiplication of 100 now my numbers are going to change right now i've got a 1 point 5 N over here that is going to be divided by 10 instead of multiplied.

So, I am going to get 0.15 N over here now nine N is going to be very significant compare to 1.15 N. So, I am going to get 9.15 N over here that multiplied by 100 is going to be 915 times N 915 times N divided a factor of 10 means that my input was 91.5 N wow. So, your noise factor is now going to become equal to ninety 1.5 which is terrible right. So, this is what happen if I attenuated it instead of amplified right this this is really a very dramatic number you've got ninety 1 as opposed to 1.5 or 1.59. So, your noise factor is now 91.5 which means the noise figure is almost equal to 20 d b right

So, this is a horrible design. So, make sure that the first stage after your antenna is the low noise amplifier it is very important also the first stage should not attenuate as soon as you attenuate in the first stage your entire noise characteristic is going to depend on that alright subsequent stages noise is going to look larger at the input alright. So, this is my motivation to build a low noise amplifier a low noise amplifier does not reduce the noise that is coming from the signal it is not possible to reduce the noise the signal already has some noise in it you cannot get rid of that noise what the low noise amplifier is going to do it is that it is going to make sure that compare to the noise in the signal it is not going to add much more noise by itself that is all I can do.

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So, the signal at the input of the low noise amplifier already has noise what I am going to make sure is that the extra noise that I add because I have a low noise amplifier because I have an amplifier I am going to have some extra noise right that extra noise is equal to or smaller than nor not a large fraction of the noise existing in the signal. So,  $N_1$  and  $N_S$  should be comparable probably  $N_1$  should be lesser than  $N_S$   $N_1$  should be as small as possible basically this is what a low noise means right.

So, with this as the background motivation we are going to start talking about strategies to make a low noise amplifier. So, before we start with the strategy to make the low noise amplifier what are all the desirable characteristic of the low noise amplifier number 1 you should have a large gain number 2 you should have a low referred noise or rather low noise figure anything else anything else that comes to your mind.

You're making an amplifier here amplifier will made up of mosfet bla bla mosfet should have a finite  $F T$  you remember we talked about  $F T$ . So, you're saying large bandwidth is it. Large bandwidth is not really an issue can I make a tuned low noise amplifier if I know my signal is coming as 2 gigahertz I am going to make sure that around 2 gigahertz I have a low noise amplifier at d c I do not have anything. So, strategy.

So, large bandwidth is not an issue I will just try to down in and also point it out that this is not an issue linearity is an issue you do not want your low noise amplifier to introduce distortion in the signal what else anything else that comes to your mind. Think about all

the topics that we have studied in this. So, far matching right the input impedance of the low noise amplifier should be matched to the impedance of the source to be impedance of the antenna. So, the input impedance you've got a low noise amplifier that looks like this the input impedance of the low noise amplifier should be matched to the resistance of the antenna why

Two reasons 1 is to conserve the shape of the signal and the second is to do maximum power transfer why would want to do maximum power transfer because if you do maximum power transfer then you are also going to get the maximum possible gain from your low noise amplifier am I right. If you do maximum power transfer then you will get the maximum possible gain at the output. So, the if you do maximum power transfer then you will get the maximum possible voltage at the input of the low noise amplifier which means that you are going to get the maximum possible output voltage of output of your low total gain of the system will be maximum right if you have maximum possible input voltage

So, that is why you want to do maximum power transfer now all these happens when you do matching when you match the impedance of the antenna to the impedance of the low noise amplifier alright what about the output presumably the output also I need to have matching. So, that you can effectively get the maximum possible gain. So, you want to do maximum power transfer as far as the output is concerned to right. So, as far as the output concerned you want to do maximum power transfer here also.

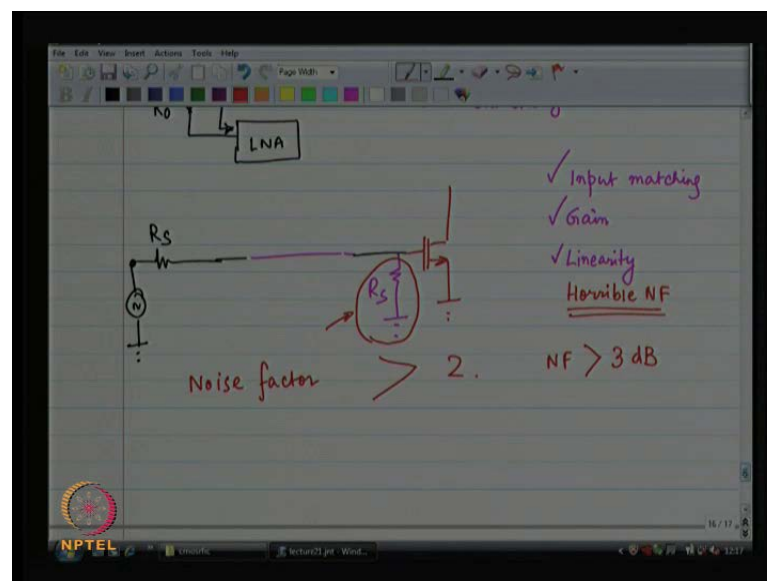
So, both sides you need to do matching also to conserve the shape of the signal you want to do matching over there you can is is that right what was the maximum power transfer theorem the maximum power transfer theorem was if you have control if you do not have control over the source resistance. So,  $r_s$  is part of  $v$  and you cannot do nothing about it alright.

So, the maximum power transfer theorem was that if  $r_L$  is equal to  $r_s$  conjugate the general case if  $R_L$  is equal to  $R_S$  then I am going to get maximum power [tra] transfer alright and maximum gain as a result. So, this happens only when I say that  $r_s$  is part of the voltage source that we are saying, but when I am talking about the  $I_N$  a output here I have direct control over what the  $R_S$  is. So,  $R_S$  equal to 0 will give me the best gain right if I can make a  $r_s$  as small as possible then I can get highest possible gain given a

load. So, in that sense it is not really clear that you have to do output matching over here output matching will be necessary to conserve the shape of the signal in any case right also the optimum I mean you can keep decreasing the output impedance of the LNA your gain is marginal it keep increasing, but really do you want to spend. So, much in decreasing the output impedance of the LNA to 0 probably do not want to spend. So, much marginal benefit that you get is probably not worthy.

So, in that case the optimum point would be to set your output impedance of the LNA equal to the input impedance of the next stage or the conjugate of the input impedance of the next stage. So, that is why you want to do output matching alright. So, we unfortunately are going to start with matching alright we are not go by the serial number when we are going to discuss the low noise amplifier we are going to start with matching how can we match the input impedance of our amplifier to the antenna to the resistance of the antenna.

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So, the antenna is a resistive or mostly resistive alright I want the input impedance of the low noise amplifier to be equal to  $R_0$  at the chosen frequency where the low noise amplifier is going to operate how can I do this. First of all if you think of what you study in your matching earlier now when you back we did matching probably initial part of this course we had a matching network this matching network was made up of inductors and capacitors that was it maybe mutual inductors, but mostly inductors in capacitors and

what I was doing was if I have a resistance over here then I would I was able to match it to an arbitrary source resistors alright

So, I have to choose which what kind of whether the impedance transformation is going to upwards or downwards and accordingly I would have to either choose  $\Gamma$  the  $\Gamma$  shape in a certain fashion right. So, I would either have to have a series  $\Gamma$  and  $r \Gamma$  or I would have to have a shunt  $L$  and  $R L$  and accordingly I would figure out what the matching network needs to be now the key over here that I want to want you to understand is that the load that you're delivering power to has got to be resistive.

If you do not have a resistive load and you cannot possibly do matching if this load over here is not resistive at all if it is resistive resistor in shunt with a capacitor I can do it I can do matching I basically need to input and inductor shunt with the capacitor to tune out the capacitor right resistor in series with capacitor I can do it, but if it is only a capacitor is it possible to do it. Can I do maximum power transfer on to a capacitor no because a capacitor by definition is not going to consume any power at all right. So, it is not possible to do matching when the load is a pure capacitor or a pure inductor there has to be some resistive component in the load now our load over here I mean what are you trying to drive over here what we are trying to drive is a gate of the mosfet probably probably the gate of the mosfet what does the gate of the mosfet look like looks like a capacitor.

Is there any resistance there's probably some gate resistance you are right there's probably some little bit some gate resistance associated with the pole silicone right; however, mostly it is a capacitor that you are trying to drive it is conceivable to do maximum power transfer on to that gate resistance and then proceed, but let say to first order there is no gate resistance to your first order gate resistance is zero the input impedance of the mosfet is a pure capacitor.

What do you do in that case how are you going to tackle this how are going to do matching it is not possible to do matching in under such a scenario right you need something that dissipates power otherwise you haven't got a maximum voltage at the input you haven't got the maximum voltage at the input and you're not going to achieve the maximum possible gain from your circuit.

So, how we are going to tackle this problem I need suggestions. So, the first suggestion is the obvious let put a resistor over there right this is gone to shut out my matching issue I do not even need a matching network over there or even if I do put a matching network it is just a tune out the c g s of the capacitance must capacitors right just to get rid of all that I probably need to put an inductor in stant etcetera fine.

So, let say flee ignore the capacitance contributed by the mosfet by the gate of the mosfet c g s. c g d etcetera etcetera. So, all that has been taken care of by the matching network. So, i've got R S and presumably you are gone to put another R S over here and you are going to get beautiful matching and this should work right. So, under your list of objectives your lets call this the zeroth cut design ok.

So, your zeroth cut design I haven't design the complete thing, but input matching I have achieved. So, presumably I will work on the design further and I will get good gain I will get I will do whatever needs to be done to get a good linearity etcetera, but a low noise amplifier needs to have a low noise figure what is the noise figure of this what is the minimum noise figure offered by this low noise amplifier.

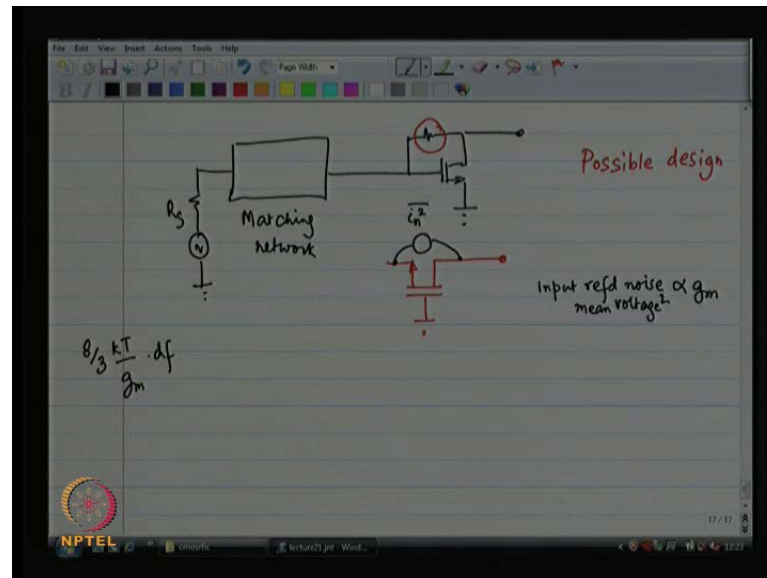
So, I want to compute the noise figure of their particular low noise amplifier do not think about the mosfet etcetera etcetera those are only going to add to the noise figure further just the fact that I have got r s over here means that my noise figure noise factor is how much. So, whatever noise is coming from the antenna r s is contributing equal amount of noise this particular r s is contributing equal amount of noise. So, my noise factor should be at least 2 on top of it there is mosfet which has its own noise I did not even bother about the mosfet right.

So, just I looking at the circuit you can say that this has a noise factor at least of 2 which means a noise figure of 3 d b now in the word of the low noise amplifier a noise figure of 3 d b is not competitive every point 1 d b of noise figure counts alright and if you produce a low noise amplifier of noise figure 3 d b people are going to laugh at you right every point of 1 d b of noise figure is very very important.

So, this is just not good enough this not gone to make production your boss is go to laugh at you if you make the circuit right you want noise figure to be something good number is 1 d b 1 point 2 d b 1 point 5 d b 2 d b right 3 d b is a bad design and that 2 3 d b without even considering what is what else is there. So, the mosfet did not even comment

to the picture I am already at 3 dB. So, once I put the mosfet there it is going to go up 4 dB 5 dB something like that this is going to be right. So, this kind of low noise amplifier is not competitive because of a horrible noise figure. So, my zeroth cut design where I did matching, but I did nothing else did not really work I cannot work as a low noise amplifier ok.

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So, then we backed to square 1 I have got a system I am willing to put a matching network in between and I need an amplifier, but input impedance of the amplifier should have a resistive component I like the gate of the mosfet to be an amplifier, but the input impedance of that is not resistive at all its kept. In fact, it is capacitive. So, what do I do. So, one thought is that maybe we are making of mistake by putting a gate over there maybe the gate is not necessary we know other structures of the amplifiers we do not need to put the gate all the time ya why do not we do the common gate amplifier can we do the common gate amplifier instead of what we were doing earlier this is feasible. In fact, this can be done.

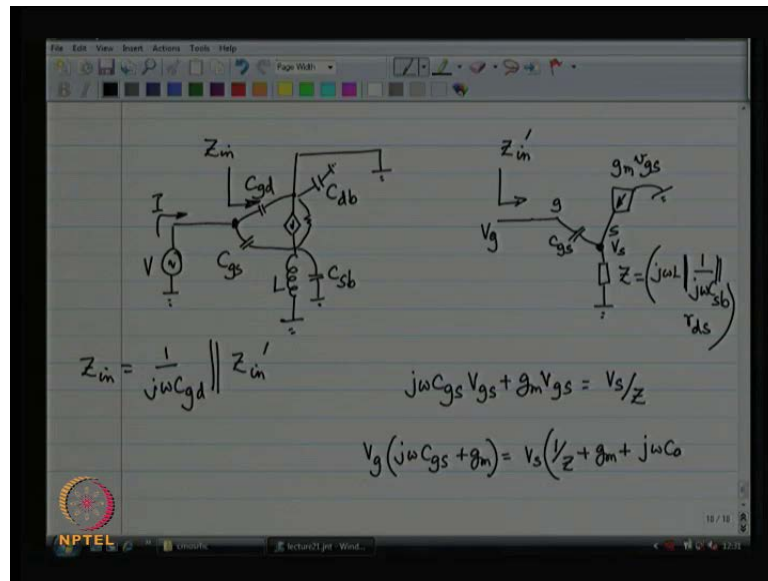
Next possibility is why are you only why are you thinking that the input is only going to be connected to the gate what about my beautiful shunt series amplifier this could have a resistive component as its input impedance. So, this is definitely a strategy. In fact, to get maximum gain you want to avoid this resistance all together this is definitely a strategy and people make these amplifiers as low noise amplifiers you can do your matching.

The problem with this kind of design is that you are using a resistor over here and a resistor contributes noise creates noise alright and in general a resistor is something which is avoidable if you are talking about the low noise amplifier. So, you would like would not rather put a resistor over there. So, this is a possible design it is used. In fact, in a lot of low noise amplifiers oh specially at the input of the fossils scope this is what is done or if you are talking about ah active props this is what is done right.

However the problem over here is that the low noise could have been lowed if the resistor was avoided alright this is. So, far. So, good are we proceeding systematically. So, this is also a possible design there are problems with this also the problem with this is that the channel noise which comes in start with the channel the channel noise is also proportional to  $g_m$  of the device this particular channel noise is going to go into the source resistance directly right with means that your total noise input referred noise is going to become proportional to  $g_m$  this is not something which is desirable this not something very desirable if I increase  $g_m$  you saw right in the previous lecture you saw that when gate referred noise of the mosfet was  $\sqrt{3 k T}$  divided by  $g_m$  what does this mean this means that if I make a device with a large  $g_m$  the noise gate referred noise is actually going to drop channel current noise is increasing, but it as if I applied a lower voltage lower noise voltage at the gate

So, it is not desirable to have something which is proportional to  $g_m$ ; that means, that at high frequency when I need large  $g_m$  I will get a lot of noise from the mosfet itself. So, far. So, good now. So, after all this what we do what do we do what is the what is the acceptable design. So, anything is acceptable as long as the noise figure is low it is the catch word.

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Now, 1 of the interesting strategy is that people use is they put hand inductor and they generate the source. So, what I am going to do as a first cut is I am going to try to find out the input impedance of this particular structure alright. So, let us try to find the input impedance of this to work out input impedance of this what you have to do you have to apply a voltage source and you have to see what is the current going in and you have to replace the mosfet with its small signal model alright these are all the different components.

So, I am going to redraw this [no audio] alright. So, this is my redrawing of that c d b becomes irrelevant as a result and next thing I am going to do is I am going to lump l c s b and r d s into 1 combined element called z which is the parallel combination of all these and you keep this thing in your scratchpad. So, this is z alright. So, this is a significantly simplified drawing of my system I have applied b volts over here. So, c g d is basically just going come in parallel with everything else. So, if I find the input impedance without c g d the final [-in] input impedance will be c g d instant with whatever I have found. So, c g d also is not particularly important alright. So, this is what we going to do

This is the system we are going to analyze for now . So, I have applied v over here and then you have to work out a krichhoff's [-node bold] node equation let say this is v s let say I have applied v g this is. So, therefore, if I work on krichhoff's node equation I get j

$\omega C_g s$  times  $V_g s$  that is the current going through  $C_g s$  plus  $G_m$  times  $V_g s$  and that is equal to  $Z V_s$  by  $Z$  ok. So, we are going to solve this equation and find out  $V_s$  right. So, you put  $V_s$  together all  $V_g$  together. So, you have got  $V_g$  times  $j\omega C_g s$  plus  $G_m$  is equal to  $V_s$  times  $1$  by  $Z$  minus  $G_m$  minus no plus  $G_m$  plus  $j\omega C_g s$  and as a result  $V_s$  is  $V_g$  times  $j\omega C_g s$  plus  $G_m$  divided by  $j\omega C_g s$  plus  $G_m$  plus  $1$  by  $Z$  now we have found out what is  $V_s$  and therefore, we can find out the input current right alright we are going to stop here.

Next class we are going to continue from this point we will actually had to leaving at the rather interesting juncture it will be good if you just solve it you'll see that something interesting pops out and try to see what can be done further. So, I am going to leave that leave this on this note that you continue with this analysis try to figure out what pops out and then in the next class we are going to give I am going to give my interpretation of what is popping out alright. So, with this I am going to close this lecture.

Thanks.