CMOS RF Integrated Circuits
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Module - 07
Noise
Lecture - 20
Various Noise Sources Sampled Noise on a Capacitor Noise in a MOS Device

Hello and welcome to CMOS RF integrated circuits; today we have started module7.

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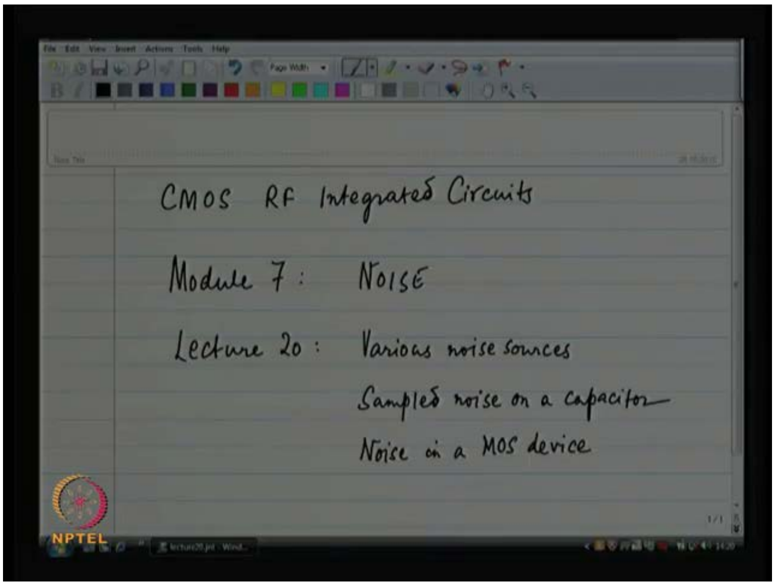

Actually, we started in the last class little bit; we are going to talk about various noise sources sampled noise on to a capacitor and noise that we see in a MOS device. So, in the last class; we had just about started this discussion.

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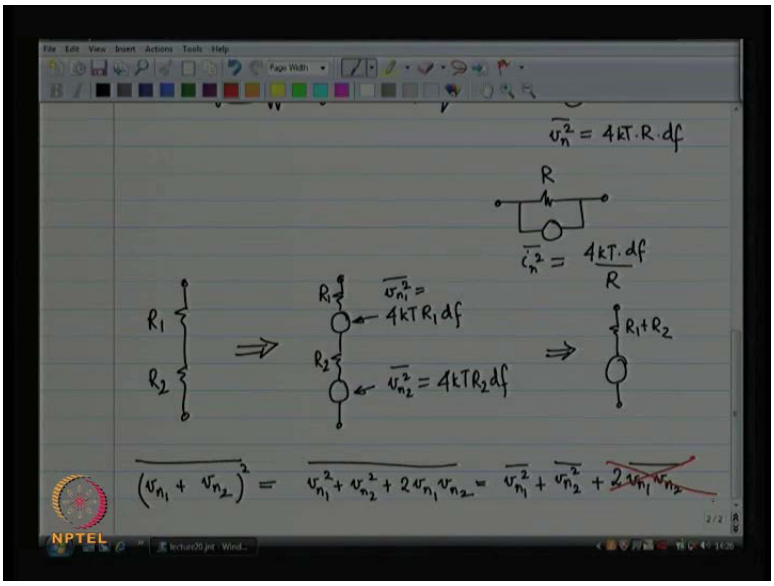

We saw that If we have a resistor R this is a source of noise and it can be modeled as a noiseless resistor R in series with a noise voltage; this noise voltage has a mean squared noise voltage of 4 k T times R, times d f where d f is the bandwidth which we are looking at frequency bandwidth. So, here k is the Boltzmann concept, T is the absolute temperature, in Kelvin's R is the resistance of the device and d f is the bandwidth. Now, we figured out where this noise is coming from; the origin of the noise is the facts that while the resistor are full of electrons that can move these electrons are vibrating.

Because of the temperature; they are thermally excited as you excite them more and more the electrons vibrate more and more; as a result these electrons are vibrating over resistance R. So, as they are vibrating more and more over the resistance R; they are creating a larger and larger voltage right. So, this is what we observed. So, this is the mechanism by which the noise is produced fine. And, I referred you to the work of Nyquist, Harry Nyquist done way back in 1929 or 1930 in which he proved this result ok.

Now, you can also do Thevenin and Norton equivalent transformation. And, the same resistance R can now be re-represented as a noiseless resistance R in shunt with a current source. So, this is a current source which has a mean square noise current that is equal to 4 k T times d f divided by R this also; we inferred from the previous results in the last class right. Then, what we did was; we put 2 resistors let us say R 1 and R 2 in series.

Now, when you put R 1 and R 2 in series; it is natural to combine the 2 into 1combined resistance plus R 2 this is what we would like to do.

Now, does this work in terms of noise as well. So, R 1 and R 2 in series that is basically noiseless R 1 in series with voltage force; and then noiseless R 2 in series with a voltage source. Now, the contribution of the first voltage source this 4 k T R 1 d f that is v n squared, v n f squared and v n 2 squared, mean squared is 4 k T R 2 delta f d f. And, when you add these 2 voltages what is going to happen; this can be represented as noiseless plus R 1 plus R 2 in series with a voltage source which is the sum of the 2 earlier voltage sources.

Now, the earlier 2 voltage sources were not really voltage sources; they were in terms of the square value right. So, the combined value we showed that since these 2 voltage sources in the mean squared sense are uncorrelated with respect to each other; they are random processors, uncorrelated to each other. So, therefore the square of the sum is equal to the sum of the squares. So, what I was trying to say in the last class. But I think we were really short at time is that if we take v n 1, v n 2 then the total squared of v n 1 plus v n plus 2 is the sum of v n 1, v n 1 squared mean of that v n 2 squared mean of that plus twice the cross correlation of v n 1 and v n 2.

Now, since these 2 noise sources are mutually independent of each other there are random processor is that do not have any bearing with each other. So, therefore the cross correlation of v n 1 and v n 2 is going to be equal to 0. And, therefore this entire term is not going to come into the picture; which means that the sum of the 2 mean squared of the sum of the 2 is equal to v n 1 squared plus v n 2 square; what this means is that now I can model this as 4 k T R 1 d f plus 4 k T R 2 d f which is equal to 4 k T R 1plus R 2 d f. And, that fits perfectly well with my intuitive answer that if I put 2 resistor series R1 and R 2; then the combination of the 2 should be equal to R 1 plus R 2 in terms of noise these are also the statement is also true in terms of noise.

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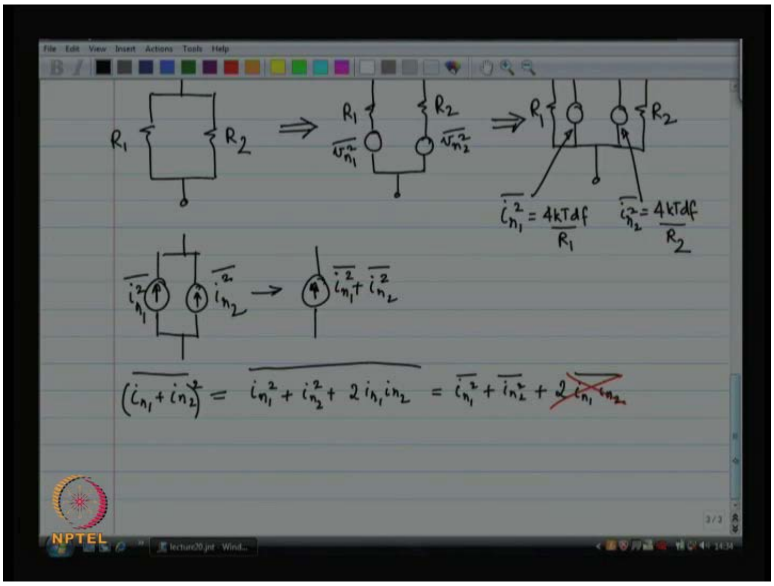

So, the next thing that we have to work on is 2 resistors in parallel. So, if I have got 2 resistors R 1 and R 2 in parallel; I will replace these 2 resistors R 1 and R 2 by resistors in series with noise voltages. I know the values of v n 1 squared, v n 2 squared means of these 2 I know. Now, unfortunately this is a difficult problem to solve. But life will be easier if I do the Thevenin into Norton equivalent transformation; in which case I am going to have a current source in parallel to the resistor.

Now, when I have 2 currents in shunt I can combine the 2 currents I can add them up. If I have i n 1 and i n 2 in shunt with each other; then, I can combine them to be 1 current source which is a value i n 1 plus i n 2. Now, so i n 1 plus i n 2 squared mean of that is equal to the same mathematics as before i n 1 squared plus i n 2 squared plus 2 i n 1, i n 2; combined mean of everything which is equal to i n 1 squared mean of that plus i n 2 squared mean of that plus twice times i n 1, i n 2 mean of this. Now, the first term i n 1 squared we know what it is, second term i n 2 squared we know what it is; the third term i n 1, i n 2 mean of that is 0.

Because i n 1 and i n 2 are 2 independent random processors; they have nothing to do with each other. So, the cross correlation of i n 1 and i n 2 is equal to 0. So, this third term is equal to 0 which means that I have got 2 current sources in parallel one as R m S of i n 1 has mean squared of i n 1 squared, second one has mean square of i n 2 squared; they are independent of each other with respect to each other. Then, the sums of these 2 currents have a mean squared which is equal to the sum of the 2 individual mean squared noises.

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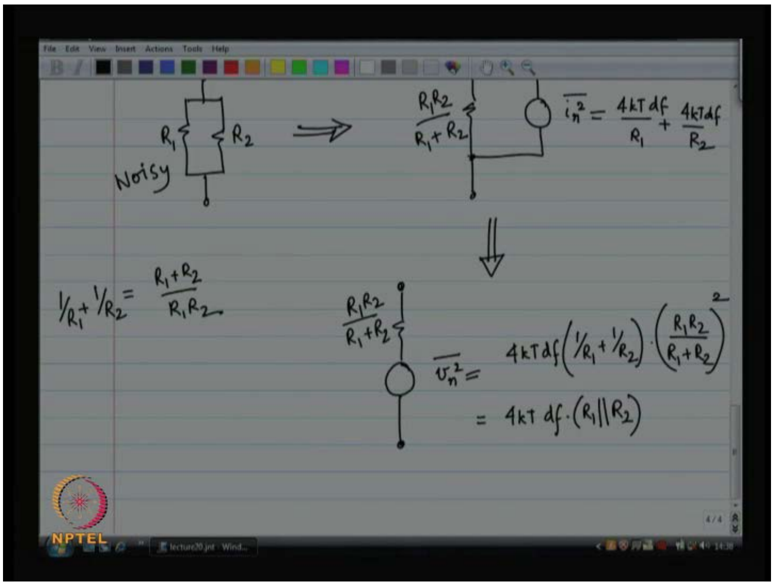

So, what that means is the following that R 1 noisy, R 2 noisy combined can be equivalently represented as R 1 noiseless in shunt with R 2 noiseless in shunt with the current source which has a mean squared current that is equal to i n 1 squared plus i n 2 squared mean of each; which is equal to 4 k T d f by R 1 plus 4 k T by d f by R 2; this is the story so far. Now, of course I can combine the 2 resistors to be 1 a value R 1, R 2 by R 1plus R 2. And, if I do so then I can also transform this current source into it is thevenin equivalent. So, if I transform this current source into it is thevenin equivalent; then, what I am going to get is this resistance R 1 and R 2 by R 1 plus R 2; in series with a voltage source where v n squared is equal to i n squared times the square of the resistance all right.

Now, I hope you have figured about by now that 1 by R 1 plus 1 by R 2 is really equal to R 1 by R 2 by R 1 R 2 right. So, this is going to give me 4 k T d f times R 1, R 2 by R 1 plus R 2; which is the result that you expect. So, if you lump the 2 resistors together R 1 and R 2 combined to form a new resistance R 1, R 2 by R 1 plus R 2; then, the noise of the new resistance is surprise surprise 4 k T times R 1, R 2 by R 1 plus R 2 times d f right. So, if you put 2 resistors in parallel then in terms of resistance value you get R 1 parallel R 2, in terms of noise you get the noise contribution of R1 parallel R 2 you do not have to work with R 1 and R 2 individually. So, this is nice. So, series and parallel combination of resistors box; the problem is that once you move beyond resistors nothing is nice anymore all right. So, once you have capacitors, inductors etcetera life becomes miserable probably.

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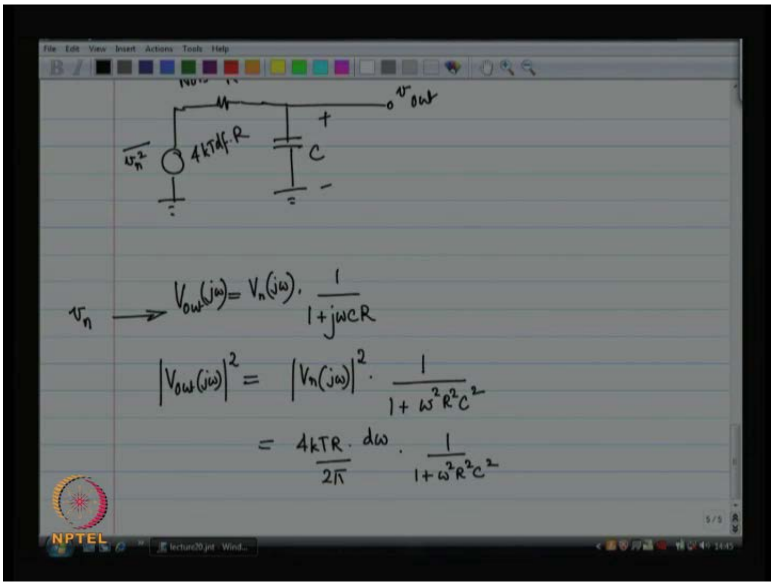

So, what we are going to do now next step; we are going to examine an R C network all right. Now, the R C network the resistor is the only thing that is noisy; the capacitor is noiseless right. Now, the question that I would like to ask you is as follows; suppose, you have got an R C network and suppose you apply the input across the R C and you take the output just across the C what lines will you see at the output? This is the question all right. So, to do this we have to replace the resistor it is equivalent model which has a noiseless resistor and the noise voltage source and then we have to work our way.

So, let us see what can be done. So, we are going to replace the resistor with noiseless resistor in series with a voltage source; a voltage source has a mean squared noise voltage of 4 k T d f times R all right. Let us say that I can apply my input I will see some output then what I am going to do is I am going to null the input I am going to use the super position. Basically, null the inputs apply the noise see what noise comes at the output; the result will be the sum of these 2.

So, for the purposes of noise we can as well consider that the output is equal to 0. Because we are going to do the super position; as far as the noise is concerned. So, let me connect the input to 0 volts; and I need to find out what is the voltage across the capacitor that is going to give me the output. So, I have got a voltage source. So, let me move the voltage source the other side to make life easier this is what I have got. And, I want to find out what is the voltage across the capacitor.

So, if I apply a voltage V across here instead of V n squared. Let us say we apply V n; if I apply V n then what is the voltage that is see across the capacitor. I see the voltage across the capacitor has a Laplace transform that is equal to V n of S times R by 1 plus S C R am I right. No, I am not right 1 by 1 plus S C R this is correct. So, the voltage across the capacitor has a Laplace transform that is like this all right. Let us not write Laplace transform; let us convert j omega.

Now, the next step for us is to take the magnitude of this all right. And, V n of j omega magnitude squared is what? V n of j omega magnitude squared of it is really equal to 4 k T R times d f; d f can be translated in terms of d omega how do you translate d f ? So, this is what I have got over a little bit of bandwidth, over a small frequency d omega small angular frequency d omega if I look; I am going to get so much noise. Now, the noise at one frequency is not related to the noise at any other frequency these are independent of each other. As a result if I integrate the noise, if I want to find out the total noise you really have to find out, you really have to take the noise voltage squared at each of these individual frequencies and add the total; that is how we are going to do it right.

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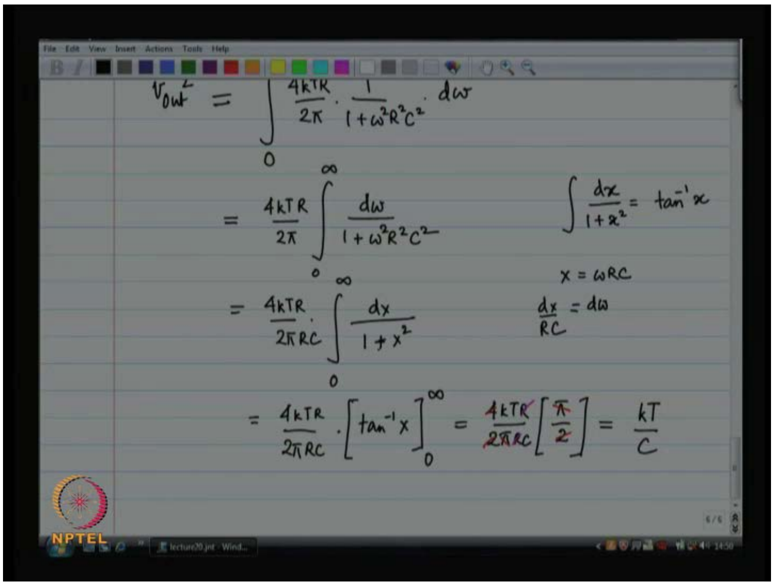

So, the total noise voltage squared at the output, output noise voltage squared is equal to integral of 4 k T R by 2 pi divided by 1 plus omega squared, R squared ,c squared times d omega going from what to what; travelling from frequencies starting from 0 all the way to infinity. So, omega is going to range over this entire bandwidth. Now, we have to work on this integral; this integral is actually not very difficult. And, if you look up your table of integrals then what you are going to find is that the integral, the omega I am sorry, integral d x by 1 plus x squared is equal to tan inverse of x that is what you are going to find all right.

So, let us try to put that into that form. So, to put that in that form I have to replace omega R C by x. So, if I make this substitution then d omega will be d x by R C. And, what that means is that this is what it means as far as we are concerned. And, then I use my integral from wherever you have studied integration all right. Now, as x tends to infinity tan inverse of x is basically equal to pi by 2; x tends to 0 tan inverse of x is 0 all right. And, now you do the simplification and all of the 2 will go, pi will go, R will cancel out all you will are going to be left out with is k T over is C; surprise, surprise what did we just do, what have we done over here; we started with a resistor and a capacitor.

Before we started with a resistor and a capacitor I gave you the entire story that where the noise is coming from; noise is coming from the resistor all lossy elements are noisy, lossless elements are also noiseless. So, the capacitor which is not consuming any power is also not going to contribute in terms of noise right. So, we started from this was our starting point. And, we took a normal R C network you put resistor capacitor like I have drawn over here. And, the question was how much noise I see at the output. Now, when we talk about a resistor the noise that you see over a given bandwidth d f is 4 k T R times d f.

I am interested in finding out how much noise right. This is some sort of R C network presumably it is low pass structure does not matter; at the output I am going to see noise frequencies. And, if I add up all the noise all at all the different frequencies that amounts to working this integral out, I work out this integral; integral was not real but not very difficult either. And, I found out that the total noise ;the total all the noise that you see the total frequencies when you look at mean square noise voltage at the output of this R C network then that is equal to k T by C. So, mystery number 1 is it is no longer proportional to R; mystery number 2 is now it is inversely proportional to C.

So, the resistor contributed the noise, the capacitor was noiseless. Now, the result is showing that if I make that if I put a larger capacitor there I will get a lesser noise; it has nothing to do with the resistor at all what is going on. So, you are probably all confused right. So, we started by saying that the resistor is noisy. Now, I am showing you that the total noise decreases for a low pass R C structure total noise decreases if I increase the value of the capacitor; what is the story?

Next thing to observe is that no d f anymore all right; earlier the total noise for the resistor over a given bandwidth is 4 k T R d f that is the mean square noise voltage over a given bandwidth delta f. In our case, in this case the noise voltage square, the mean squared noise voltage at the output is over the entire range of all frequencies; we are no longer saying let us restrict ourselves to his bandwidth; over all frequencies mean squared noise voltage k T by C period it is a beautiful result, it is a classic result. And, the conclusions can be enormous.

So, there is no limit to what the value of the resistor is the value of the resistor can be anything it can be 0, it can be 1 ohm, it can be hundred ohms it can be 1 giga ohm, it can be an open circuit, it can be tending to infinitely large number of ohms, 100 giga ohms; it does not matter the integrator is always equal to the total noise is always equal to k T by

C right. So, it does not matter what the value of that resistor is; it may be there, it may not be there can be open circuit, it can be infinitely large resistor. I will still get the same value for the total noise voltage mean squared noise voltage over the entire bandwidth; that is still going to be equal to k T by C ok.

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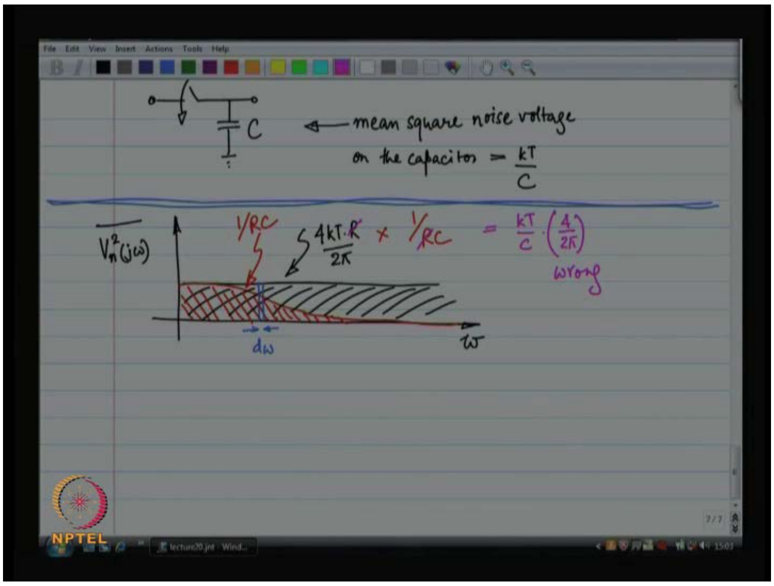

So, if you have switch the sampling of a voltage on to a capacitor during the sample phase the switch is on; when the switch is on it looks like the R C network right. And, then suddenly switch off the switch you remove the switch; in which case the switch is now a resistor which is very very large it is also fine. The noise that is deposited on the capacitor is k T by C the R M S value of the noise not R M S, the mean squared value of the noise voltage that you have deposited on to the capacitor is k T by C. It is coming from the switch but it is value has nothing to do with the switch. The resistance of the switch is irrelevant; it is coming from the switch true its final value is independent of the resistance of the switch right.

So, this is a very sweeping result all you have to do is make your capacitor larger; the noise voltage that you will see is smaller is going to be smaller. Now, before we jump too much with this k T over C business how is this happening? What is going on over here? So, it is like this when you look the power spectral density of a noise voltage it is uniform. Let us not draw the function of f let us say a function of omega; if I draw the power spectral density of noise; then, it is uniform it has a value of 4 k T times R this is the noise of a resistor. And, then you say let us look at this band of width d f d omega. Because this was omega in which case it is no longer it is 4 k T R by 2 pi; no 4 k T R by 2 pi all right.

So, this is the story we have got as far as the resistor is concerned. Now, this noise is passing through a low pass filter. And, a low pass filter has a cut off frequency of 1 by R C t that is the fun part. Now, when you pass this noise through a low pass filter what you get is this; this is what comes out of the filter only this portion comes out of the filter. Earlier, if you had done the integration from 0 to infinity; you would have gotten infinitely large amount of noise. But now if you do an integration from 0 to infinity; it is going to be finite. Because the tail has been removed I mean it has been cut off; filter cuts off high frequency noise it is a low pass field all right.

Now, if I integrate this noise then I am going o get k T by C; as the total mean squared noise voltage over all frequencies all right. So, what you are really doing is you have got this 4 k T R by 2 pi this is the power spectral density. But at the same time the cut off frequency of this filter is 1 by R C; and that is what is instrumental in removing the R from your expression. So, if you multiply 4 k T R by 2 pi times 1 by R C; that is the portion that is coming through the other portion is not coming through. If you make that as a first order approximation then what you end up with is R cancels with R and you get k T by C times 4 by 2 pi; it is close to the answer.

But not exactly the answer because you have not done the integration properly; you have just done a super approximate deal over here you have said that only frequencies below 1 by R C pass through frequencies above 1 by R c do not pass through. So, that is what you have said. And, as a result you have got a little you have got a wrong result over here; the proportionalities are still the same.

So, this is what is wrong; you should be getting 1 over there not 1 by 2 pi 4 by 2 pi is less than 1. But the k T over C part is correct. And, that is kind of giving you intuitive feeling that the total noise is no longer going to be proportional to R instead it is going to be inversely proportional to the value of c. Why? Because the cut off frequency is 1 by R C. If I increase R, I double the value of R the cut off frequency is going to go down by 2. So, lesser frequencies will pass through. So, the total noise I will see at the output is going to be lesser all right. So, this is basically the idea. So, now how are we going to do our analysis; if we have a lot of resistors.

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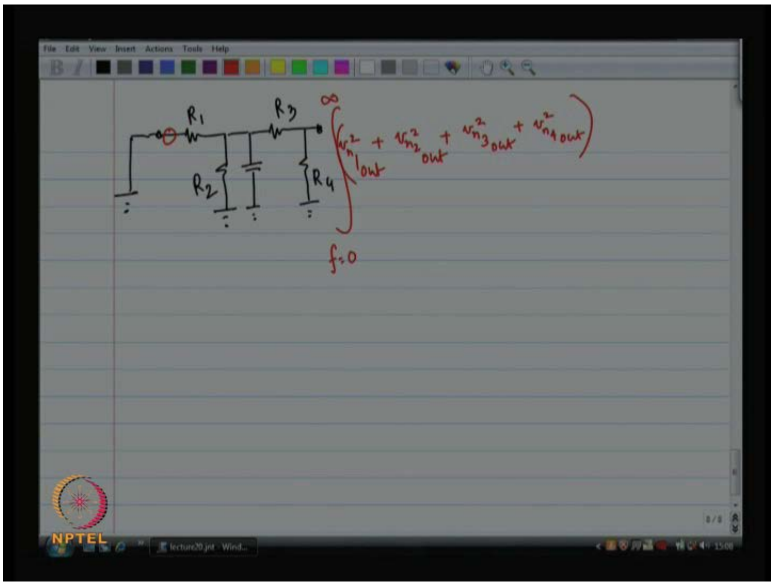

So, suppose I have got a network which is full of resistors and capacitors let the inductors also; does not matter. Let us say that the resistors are the noise sources then all you have to do is we have to consider each noise source in it is own merit; find out what is the noise voltage? Because of each individual noise source what is the mean squared noise source as a function of frequency; add up all of these, add up all the individual you are doing super position of a lot of different sources. So, noise source number 1; what is the noise voltage at the output because of noise source number 1 as a function of frequency mean square; mean squared noise voltage and output find that out. Do the same for noise source number 2, do the same for noise source number 3.

And, then you add everything up and then you integrate everything from 0 to infinitely large frequencies that will give you the total mean squared noise voltage over all bandwidth for all frequencies for the network that was started. So, let us just do a quick example, let us say that this is what I have got R 1, R 2, R 3, R 4; I do not know what this is. But let us just say this is what I have got right. What are you going to do; you are first going to find out what is the contribution of R 1? First of all input is 0; then, you say you replace R 1 with it is equivalent noise model you assume R 1 is noisy everything else is noiseless; you put your noise source over here. Noise source has a value of 4 k T

R 1 times d f find out what is V n square the output because of 1 noise source over there right. It is going to go through the transfer function of the entire socket.

Then, do it for resistor number 2, then do it for resistor number 3, and finally, do it for resistance number 4. And, then you have to integrate all of these presumably you have already got d f inside your expressions; you have to integrate all of these over all frequencies that will give you the total mean squared noise voltage over all frequencies. Sometimes, these frequencies blow up, these integrals blow up to infinitely large in those cases you have to restrict yourself and say that I would not go from 0 to infinity. Because that will give me a bad result it is basically going to tell me I have infinitely large amount of noise right. And, you will restrict yourself to frequencies that you are interested in. So, typically that is what we do right.

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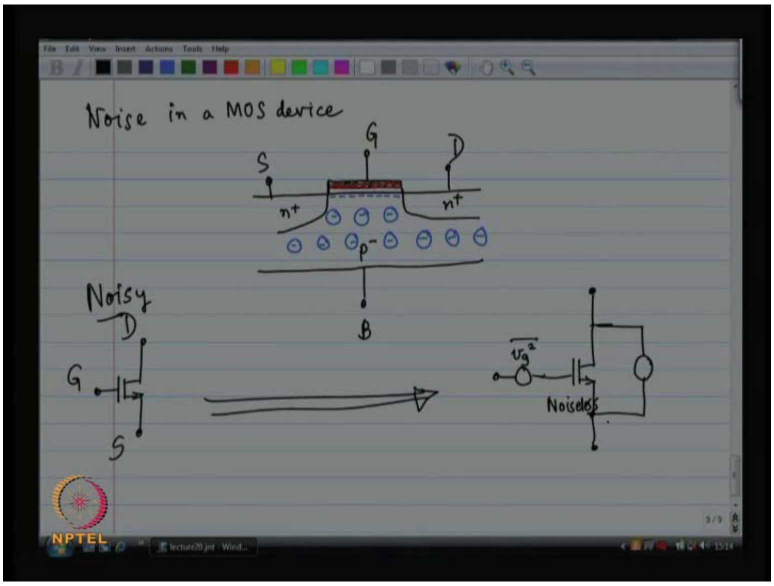

So, I am going to end this part of the story over here and move on to the next part is noise in a mosfet. So, think about noise in a mosfet what are the different sources of noise? So, first you make the gate oxide and then the poly on top of the gate oxide and then you implant the drain and the source. So, this is your mosfet. So, now what can cause noise in a mosfet? So, the first thing that comes to mind is the fact that the gate itself is made out of poly material. So, this gate is poly material and poly material is really used to make resistors as well. So, that means that the gate is resistive and therefore it is going to contribute noise agreed.

I will tell the gate is a distributed register right; if you have made a contact at the center the gate is a distributed register; the charge is going to distribute yourself each part of the gate sees it is own capacitance and charge develops all over the gate. So, it is really a distributed resistance you can lump it in to an equivalent series resistance with capacitance; you can lump it into that model that will give you some noise. So, the gate will give you some gate noise fine what else. So, what else can contribute to noise in a mosfet.

So, when you apply our voltage between gate and source let us say source drain body everything is connected to 0 volts; you apply a voltage between gate and 0 volts. So, gate is let us say at 5 volts much above the threshold voltage then what is going to happen? A channel is going to develop right beneath the oxide layer of the gilt, right beneath the oxide layer of the surface a channel is going to develop. First of all the bulk of the mosfet is going to get depleted of all the holes that were there. Now, considering that you have only the lightly doped bulk of the mosfet. There were not too many holes to be depleted in the first place that is fine life is like that.

So, there are these fixed charges over the bulk of the mosfet these are the places where the whole have gone away from right. Now, besides these fixed charges right underneath the gate layer of inversion is formed. Now, what do you do with this layer of inversion? What does it remind you of? What about these electrons? These electrons are moving because of temperature they are vibrating are not they it is exactly like a wire you have formed a wire; in a wire the electrons are moving around because of temperature, because they have energy to move around. So, as long as they have energy to move around they are going to move around; they are going to bounce against each other etcetera.

And, as these electrons move around they are going to develop a potential you could think of it that way or you could convert it into Norton model also. And, say that there is an additional noise current in shunt with the channel agreed. So, if this is a noisy mosfet. So, the mode for a noisy mosfet is first of all you have got some gate noise and then you have got the noiseless mosfet. And, in shunt with the channel of the noiseless mosfet there is.