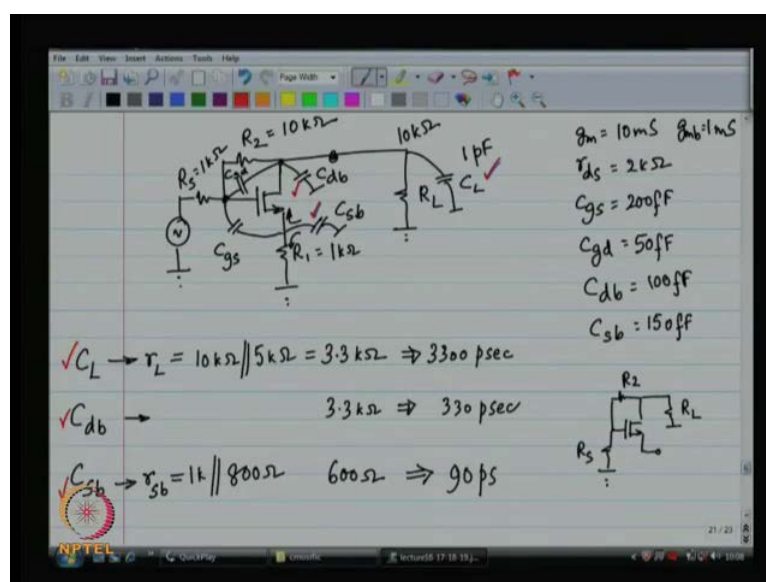


**CMOS RF Integrated Circuits**  
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**Module - 06**  
**Wideband Amplifier Design**  
**Lecture - 19**  
**Shunt Series Amplifiers (Contd.)**

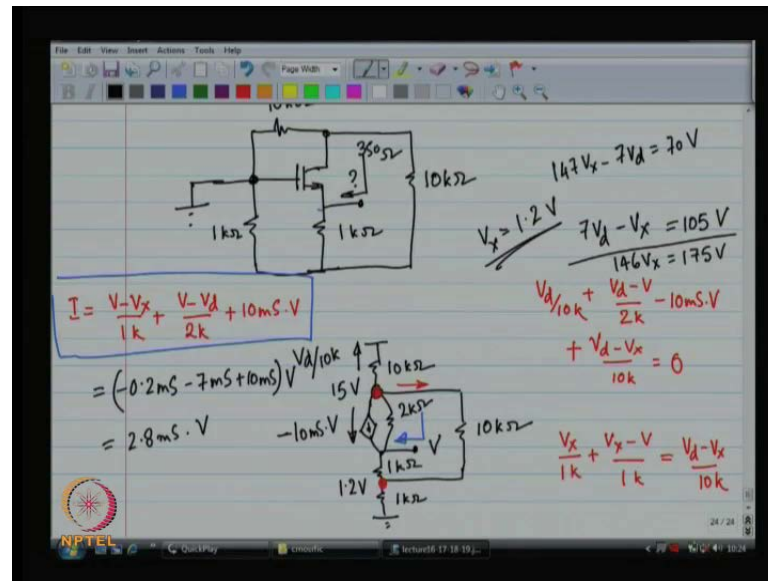
Hello and welcome back to CMOS RF integrated circuits. Today is the nineteenth lecture; we are going to finish what we were discussing in the previous lecture on shunt series amplifiers that is part of module 6 wideband amplifier design. And, then we are going to the next module that is the noise.

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So, what we were discussing in the previous class? We use the method of O C time constants to figure out, different time constants we had worked on few of the parasitic capacitors and the load capacitors. We had not really finished our work. So, we had worked on  $C_L$  over here and then, we had worked on  $C_{source}$  to body, we had worked on  $C_{drain}$  to body, which is very similar to  $C_L$  right. And, the 2 that remain are  $C_{gs}$  and  $C_{gd}$  these 2 remain and this I first want to accomplish today and then, we will move on to the next module.

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So, C g s, C gate to source. So, this was our situation right I have nulled the input voltage source already. And, C g s is really the capacitance across the gate to the source junction right. So, what we have to do is we have to dereference it; we have to dereference the ground to make our lives easier. And, then we have to do the computation right, do you agree on that because; you know it is we are going to apply a floating voltage source and see what is the resistance it is hard to do that. So, instead we are going to dereference it, and I propose that we change our ground from what it already is. So, this was our ground node.

Now, I want to change it I propose that we change it to the gate. So, I propose that we change the ground node to the gate and then, all we have to do is look into the circuit from this particular point. So, this is the business that we are about to conduct, yes. So, as soon as we try doing that what we are going to find out is that now, my circuit and to redraw the circuit looks like this alright. This is how it is going to look like, and I need to compute the impedance looking into this particular node. Now, guess what, there is feedback going on over here, this is not going to really be in my formula, is it going to be in my formula? No, once again let me just quickly double check that what I have drawn is indeed correct, No, it cannot be correct, I am missing out some resistors. So, let us redraw this.

So, I have got the gate connected to ground that is fine. I need the drain to connect through 10 kilo ohms to ground; I need the drain to connect through 10 kilo ohms that is better, right. So, as you see; there is some feedback going over here, and the net result is not going to be very easy to analyze. It is not going to be very easy to analyze this, do we have something to start off from? Well, if you look at our previous computation, we had done some calculation as to a finding out resistance from some other point; this is what we had done. Now, does part of our new circuit look similar to this, can you identify? Not really it should look actually similar; maybe when I changed the ground things no longer are looking similar that is fine. What we are going to do is we need to analyze this. So, you pick some voltages, you write out the Kirchhoff law.

So, I have applied the voltage  $V$  at the gate at the source of the MOSFET, and the gate is grounded, I plan to find out what is the current coming through that voltage  $V$ . Now, the good thing is that  $g_m$  times  $V$   $g_m$  the current going through the  $g_m$  the voltage current source is very easy to find out, it is minus  $g_m V$ ,  $g_m$  is 10 mille Siemens or 11. There is a body also right  $g_m b$ . So, does not matter I will just try 10 mille Siemens times  $V$  that is the current going through the  $g_m$  alright. Now, once we have that settled; let us say that this particular node is at  $V_d$ . So, the current going upwards is  $V_d$  by 10 k. By the way, there are easier ways of analyzing this.

If you study the feedback theory, this is just a form of feedback and all you have to do is open the loop, break this particular loop over here, find out the loop gain and divide the resistance by that 1 plus the loop gain, and that is what you have to do. Which is fine, we are just going to brute force it, because; I do not want to teach you the feedback theory at this point of time, and we are going to move on with our lives fine. What are we doing over here? We are trying to solve Kirchhoff law at this particular node.

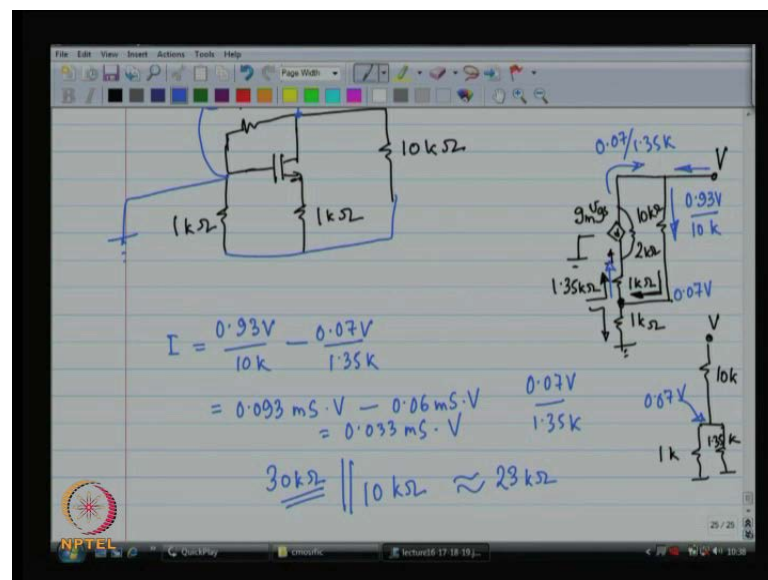
So, I have got  $V_d$  by 10 k plus  $V_d$  minus  $V$  by 2 k minus 10 mille Siemens times  $V$  right. And, that plus this particular current, let us say this is  $V_x$   $V_d$  minus  $V_x$  by 10 k is equal to 0; this is 1 of the equations that I have. And, the second equation is at this particular node right. So, we have got these 2 equations, 2 equations with 2 unknowns  $V_d$  and  $V_x$ , 2 simultaneous equations these need to be solved. And, you need to figure out what is  $V_d$ , what is  $V_x$  more importantly you need to figure out. What is the current coming in from  $V$ ? The current coming in from  $V$  is equal to this is what it is right. So, keep this in the background remember that this is our objective fine. In the meantime; let

us just try to solve these 2 equations that we have got, let me multiply everything by 10 k to make life easier.

So, I will have  $V_d$  plus 5  $V_d$  that is 6  $V_d$  minus 5 times  $V$  minus 100 times  $V$  plus  $V_d$  again and minus  $V_x$  is equal to 0 that is your first equation. And, the second equation gives me; let us again multiply by 10 k to make our lives easy. So, 10 times  $V_x$  plus again plus 10 times  $V_x$  minus 10 times  $V$  is equal to  $V_d$  minus  $V_x$ , right. So, these are my 2 simultaneous equations, I need to solve these. So, the way solve this is high school arithmetic, and basically; and then I add which is going to give 146 times  $V_x$  is 175 v.

So,  $V_x$  is going to be about 175 divided by 146, this is something that I am not used to doing orally, but; anyway this is something like 7 by 6. So, 1.17, let me just try 1.2. And, next one  $V_d$  I can get from  $V_x$ . So, 7 times  $V_d$  minus 1.2 V. So,  $V_d$  is about 15 times V right. Then, I plug in all these numbers over here. So,  $V$  minus  $V_x$  is about is minus 0.2 V minus  $V_d$  is minus 14 which means; the resistance looking over there, is about 1 by 2.8 mille Siemens. So, that is about maybe something like 300 ohms, little more 350 ohms.

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So, all these calculations you have got to do carefully. Now,  $C_{gs}$  is 300 femtofarads, I have got 350 ohms over there. So, I should be getting something like 350 times 200. So, about 70 pico seconds, alright. And, lastly I am left with  $C_{gd}$ ;  $C_{gate}$  to drain now, gate to drain is also going to be similar. And, for this you are going to apply your voltage over

there across the gate and drain. Now, 1 nice thing about this is that the 10 k is going to come in shunt with everything else.

So, what we are going to do is once again we are going to dereference the voltage the ground, we are going to call the gate as my new ground. When I call the gate the new ground, I am going to redraw the circuit as this. Once again there is an element of feedback which is going to create some problems for me. But I do not fear these problems, I just redraw the circuit in this fashion, I apply my voltage and I want to find out what is the current coming in. Have I redrawn the circuit correctly? It looks like I have drawn the circuit correctly alright, this is the plan.

Now, replace the MOSFET with its model. The first thing is the 10 k comes in shunt with everything else. So, let us remove that particular story altogether. The next thing what is the impedance that I see when I look in upwards from this particular point? Let us make life little easier, what is the impedance that I see this is easy find out right. Is it easy to find out what is the impedance that you see when you look in upwards from that point? We have done it before, just now agreed, the same story.

So, looking in upwards from here we see about 350 ohms, looking in downwards from here, we see 1 k. So, what do expect looking in from here, I am sorry, you see 1.35 k over here. So, looking in here we saw 350 ohms, looking in to the source we saw 350 ohms. So, looking into this point we see 1 k plus 350 ohms. So, looking in to this junction, I see on 1 side 1 k on the other side 1.35 k right.

Therefore, I now have got a network, I have applied some V 10 k and it splits in to 2 pieces 1 k and 1.35 k. I need to find voltage over here; it is easy to find 1 k parallel 1.35 k is about 0.6 k. So, I have applied V it is a resistive divider 10 k and 0.6 k should give me something like, how much?  $V \cdot \frac{0.6}{10.6}$  or something like that no,  $V \cdot \frac{0.6}{10.6}$  or so, V times 0.07 or so alright.

So, once I know this, my life should be a little bit easier. And, the next thing is what is the current that is going upwards over here, I have point 0.07 times V the impedance that is see looking in to that particular node is 0.35 k. So, current going upwards is going to be 0.07 volts divided by 10.35 k right. What is the current this way? Current this way is V minus 0.07 V divided by 10 k.

So, that is about 0.0, right. So, the net current coming in over here, is something like this that the net current coming in over there. And, then I need to do the mathematics, the calculation and find out how much is the current exactly. And, that is going to give me the resistance. This mathematics, this calculation is beyond my mental arithmetic capabilities.

So, I am going to pause over here, and I am just going to assume that you have found out the correct number, right. So, that is basically going to give you something for  $C_g d$  over here. I have not computed this because it is too difficult for me to do it orally, you will have to do it, and you will have to work out exactly how much is the time constant that you see.

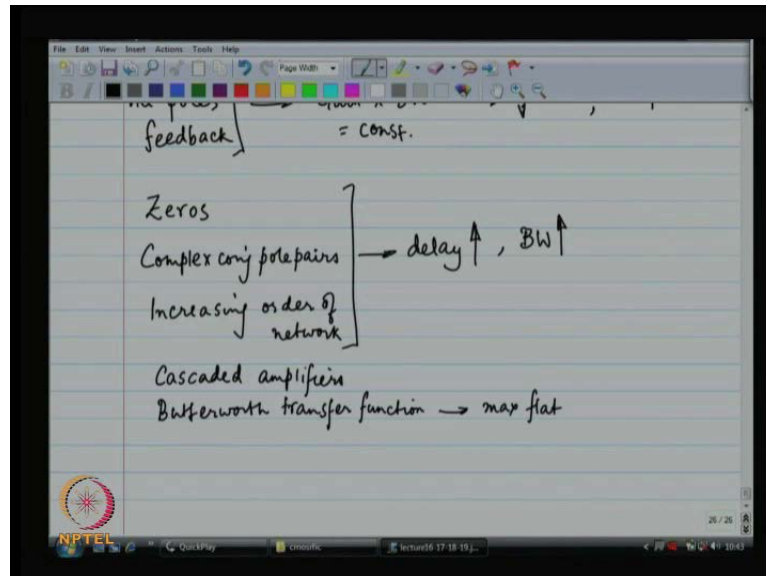
Now, do you think it is going to be large small in kilo ohms or in ohms 100 of ohms, what do you think it is going to be. So,  $0.93 \times 10^4$  is something like this. And the other one is probably going to be something like. So, you are going to see a large resistance over there that is what you anticipate 30 kilo ohms or so. Now, 30 kilo ohms in shunt with you already have got a 10 kilo ohms over there, 30 kilo ohms in shunt with 10 kilo ohms is about how much 23 kilo ohms or so. So, what we are going to see over here is 50 femtofarads with 23 kilo ohms is going to give me a substantial number 50 femto.

So, 100 femto times about 12 kilo ohms, 11 kilo ohms. 100 femto times 11 kilo ohms is 100 pico times 11. So, it is about 1100 picoseconds. So, this is still going to be a problem for us. Now, in addition to this, do you also think there is also going to be a zero here? Yes, there is going to be a zero. We unfortunately have not been able to estimate the location of the 0 with our method of open circuit time constants. So, the 0 is going to enhance the bandwidth, all of these other time constants these are all small,  $C_{db}$  is also not very large. What is going to affect you are the load capacitance and  $C_g d$ .

Now, you can fix both of these  $C_g d$  is usually fixed by a cascade structure remember. So, we can put a cascade over here, same topology and try to fix the  $C_g d$  load can be fixed with a buffer. So, you can work on these things and make a real good white band amplifier alright. These are basically methods of making white band amplifiers, what we have let me try to summarize, what we have seen so far? Is that you can trade off a few things to get more bandwidth; number 1 you have all heard of the gain bandwidth products. So, you can trade off gain to get more bandwidth right. However, the game

bandwidth product is really, when you have got all pole transfer function and then you apply feedback and so on, and so forth.

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Gain bandwidth product; gain goes down, bandwidth goes up. This is really when I have got all poles, when I have got feedback around the system etcetera. The other strategy that we have got is by inserting zeros, strategically inserting zeros by using complex conjugate pole pairs by increasing the order of the network. All of these cost me in terms of delay; delay is something which I can tolerate. If I reduce the gain my system becomes less useful, if I increase the delay then there is no harm.

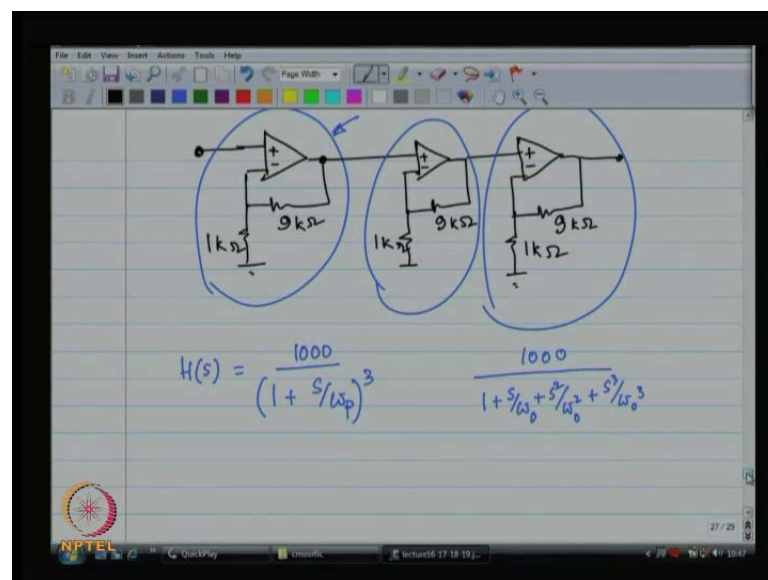
So, I can increase the delay, and I can get bandwidth in ((Refer Time: 34:15)) of some more latency in my circuit. So, this technique is used a lot of times all of the earlier techniques by placing an inductor coil etcetera where, basically to emphasize this factor queue allow for the response to be delayed, but; it is going to be sharper. So, instead of getting a slow response starting now, I will get a sharp faster response staring a little later, this is basically the idea. There are ultra wideband amplifiers which use a cascaded stage of a lot of transistors, lot of stages, and these particular techniques are used a lot of times in microwave circuits etcetera.

These particular also play with the delay. So, you basically say let me get my signal a little later, but; I want a sharp signal with large bandwidth. So, these are general techniques that are used the other technique that we did not discuss is under the

principles of wideband circuits. Is when you cascade amplifiers, you can try to follow the butter worth transfer function, instead of just cascading and getting the same pole location every time. So, this we did not discuss; I am not going to go into that. So, this could be your extra reading.

So, it is like this suppose; if I want a gain of 1000, I can get a gain of 1000 by cascading 3 amplifiers, each of gain 10 right. Each amplifier is going to give me its own pole, its own delay. Now, what you have to do is something like this. So, suppose you want a gain of 1000, let us say you have got some more amps with you, I mean the op amp is just to make life easy, you can replace the op amp with your own transistor circuit over here, it is just illustrative.

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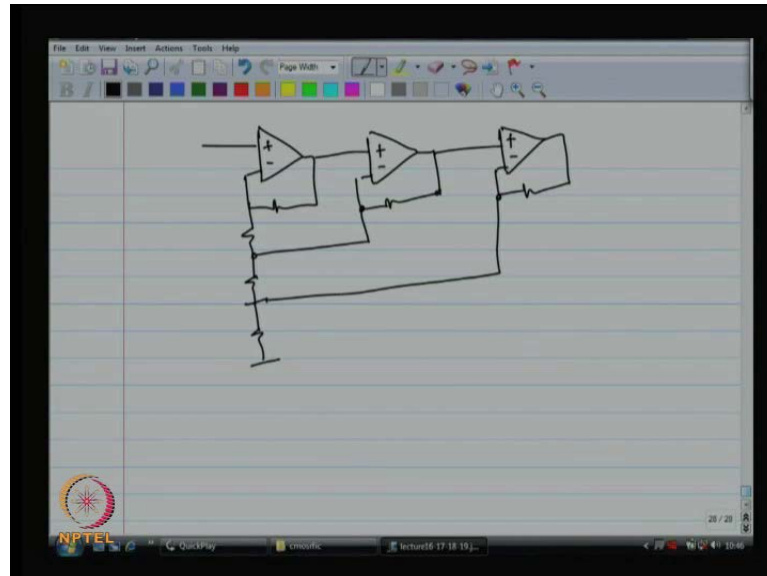
So, this is 1 way of getting a gain of 1000 right, you put 3 successive amplifiers each of gain 10. Now, this is 1 way of doing it, the other way could involve some of the thing a little more complicated. So, this is what is going to give is as follows; this is going to give one pole at a certain location, this is going to give a pole at the same location, this is going to give a third pole at the same location.

So, your net transfer function  $H(s)$  will basically be 1000 divided by 1 plus  $S$  by  $\omega$  whole cubed, alright. So, instead of this can you arrange these 3 amplifying stages in a way that you get your pole locations differently? Can you arrange your 3 amplifying agents in a way that you get this kind of a transfer function? So, this is the butter worth



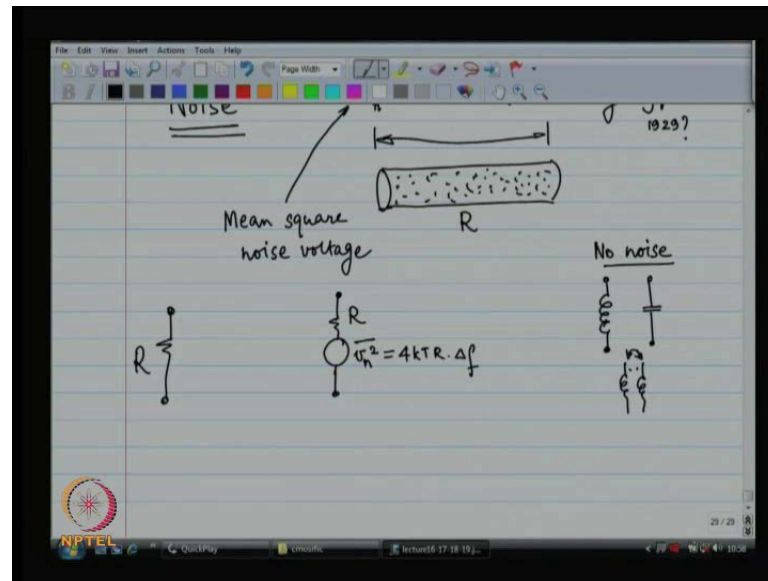
transfer function, it can be done. So, you say that I no longer want to a simple cascade of 3 amplifiers like that I would like something more complicated, why not something like this.

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This also could be made something looking like that looks like this, could also be designed. And, this could also be designed to gain of 1000 and this will probably have more bandwidth than what you have got over there. Now, you would not have the same location of the poles anymore right. So, one can do all these things to get more bandwidth. Now, you replace your op amp that I have drawn over here, with your favorite circuit your favorite amplifying circuit etcetera. So, basically a wide banding technique is when you cascade amplifiers instead of having repeated poles at the same place try to find out a strategy where you can have butter worth polynomial in the denominator. That will give you maximally flat bandwidth, the largest possible bandwidth alright. So, on this note I am going to end this discussion.

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And, now we are going to move to the next topic, next module that is noise. And, the module after this we are going to start discussing the real stuff that is the low noise amplifiers. So, this is the prelude to that. Noise; now, first of all what is noise? Noise is a random process which is because when you have resistor, and when you have, let us say when you have got current moving through a resistor, it is really a lot of electrons in this material that are moving through, and the electrons are jostling with each other, hustling and bustling and pushing each other. And, they are actually moving in random directions with temperature. The overall direction is in the direction of the current.

So, with temperature as you increase temperature all of these electrons, let us say there is no current, let us say you have a static situation; as soon as you have a material which is filled up with electrons, all of these electrons are really jumping around based on how much energy they have. This is statistical thermodynamics, they have energy because the temperature is high or whatever there is a non-zero temperature. So, they all have energy they are all really moving around in 3 dimensions right. You can do you statistical thermodynamics and figure out all the 3 dimensions etcetera. What are the different energies in all of these 3 dimensions?

So, all of these electrons are moving around. Now, as the electrons are moving around, what is really going to happen is a potential is going to be developed across this particular element, this element is a piece of resistor. As electrons are moving around

over the resistor, they are going to develop a potential. So, it can be shown; that if this resistor has a resistance of  $R$ , if this resistance is 0 then, as the electrons are moving around even though they have a lot of energy in them. They can move around the resistance is 0 potential developed across the resistor is zero. So, it is really not of much consequence; however, if the resistance is non-zero you have got some resistance over there, they have got to develop a potential across that particular element.

So, it can be shown; that the potential that they develop  $4 k T$  times  $R$  times  $\Delta f$  this power spectral density. I am sorry, this power spectral density of the voltage that is developed across the resistor alright; it is not very clear right. Once again electrons are jumping around, the voltage that they will develop. The mean squared voltage over a little bit of bandwidth frequency is so much. It is proportional to temperature, it is proportional to resistance which we do expect, and it is proportional to the Boltzmann constant.

And, you can work this out using statistical thermodynamics, there is 1929 or earlier, I think it is 1929 paper written by gentleman named Nyquist proving this result. You have heard of Nyquist right. You are all electrical engineers you should hear about Nyquist, Nyquist sampling rate, Nyquist theorem, Nyquist method of stability etcetera, Nyquist plots. So, this gentleman figured out has proved in 1929 or so, paper I am not sure of this. It is as early as that the power that the mean squared noise voltage that you are going to see across this resistor over a given band of frequencies is  $4 k T R$  times  $\Delta f$ , where  $\Delta f$  is the band of frequencies.

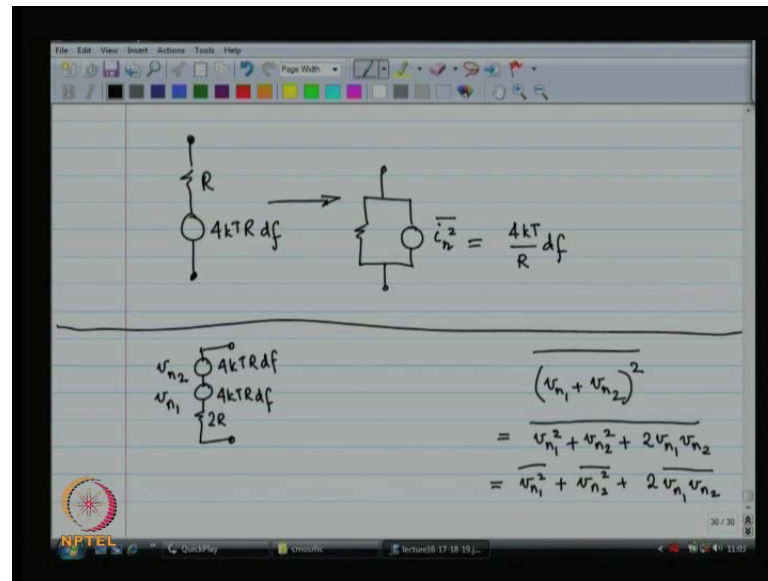
So, a resistor is really is going to be modeled by a resistor in series with a voltage source, noise voltage source. So this is going to be the model of the resistor, across the resistor you can see that you are going to see that you are getting noise voltage. Now, this noise voltage does not have a sign, it cannot possibly have a sign. You cannot only have a positive voltage or negative voltage, mean noise voltage should be 0, mean squared noise voltage is so much. So, we usually do not put this plus and minus symbol over here, we sometimes like to put a star lot of people have lot of different conventions as to how they denote this as a noise voltage source. Lot of people just put circle over there, put nothing inside and to show that it is a noise voltage.

Alright, then you write that it is a noise voltage source. So, now if this resistance is 0, if this resistance happens to be 0, do you think there is going to be any noise voltage? No. So, suppose I have got an inductor, and ideal inductor which does not have any resistance. The noise voltage developed across the inductor is going to be equal to 0. Similarly, I have got a capacitor the noise voltage developed across a capacitor is going to be equal to 0. Now, you can show from this that wherever you have got lossy elements, lossy elements are elements that consume power that convert electrical energy to heat energy those are lossy elements. Lossy elements will create noise, elements that do not have any loss in them, will be noiseless this is just an extension of this right. If you have got a diode for example; if electrons jiggle around inside the diode then, they will develop a potential.

If you have got a MOSFET channel of MOSFET electrons are jiggling around the channel or let us say electrons are jiggling on the gate then, they develop a potential across the gate source once they develop a potential across the gate source it creates a noise, noise in the current. So, all of these are noisy elements inductors and capacitors are noiseless elements. Inductors and capacitors are also lossless elements; all the others are lossy elements.

You can also put mutual inductors in this group of lossless, noiseless elements. So, if you make a circuit which is passive and lossless then it is also going to be noiseless, it is not going to add any noise of its own, alright. Next thing to observe that as you decrease the temperature, the noise is going to decrease at 0 Kelvin the resistor is noiseless, but; at 0 Kelvin no electrons move at all in fact. So, things are very different at 0 Kelvin so, we cannot really talk about that temperature.

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Next observation; we have got the resistor modeled as a resistor in series with noise voltage source. Let us say  $v_m$  is the noise voltage at any given point of time. This can be modeled as a resistor in shunt with a noise current source, Kelvin into Norton conversion. And, if the mean square noise voltage is  $V_n$  squared then, the mean squared noise current is going to be  $v_n$  squared by  $R$  squared. If the mean squared noise is  $4kTRdf$ . So, this is the alternate model for a resistor, you can have a noise current that is going it is really modeling the electrons moving around.

Once again, if there is no resistance over there, if the resistance is 0 and even when the electrons are moving around it does not matter because it is a short circuit. So, all of this current is going to go through this loop. So, it does not matter; even if the current is very large does not really matter, alright. So, this is what we have got so far. These are the different noise module; this is the module for a resistor. Next step is what happens when you put 2 resistors in series, you should see, you should be able to replace 2 resistors  $R$  in series with 1 resistor which is really  $2R$  alright.

Now, 2 resistors are in series can be modeled when they are noisy they are going to be modeled as 2 noise voltages series with  $2R$ , but; what you have got to remember is that these voltages are mean squared noise voltages. Then, what you have got to remember is that these noises are random processes, when 2 mean squared noise voltages come in series with each other, it is not the same as series voltages. So, let us say this is  $v_{n1}$ , this

is  $v_n^2$  then, really what is the voltage? The voltage I have is  $v_{n1}$  plus  $v_{n2}$ . So, the mean square noise voltage is going to be this, right.

Now, noise voltage number 1 is  $4kTR\Delta f$ , noise voltage  $v_{n2}$  square is also  $4kTR\Delta f$ , but;  $v_{n1} v_{n2}$  mean of that is going to be equal to 0, because; these are 2 random processes which are uncorrelated with each other. So, when you do the mean of the product it is really going to be equal to zero. So, this term will not be present and so, what you are going to get is that total noise is going to be  $v_{n1}^2$  plus  $v_{n2}^2$ , which is exactly the result that you want, alright. We are going to carry on from this point in the next class.

So, we are discussing noise, and we are discussing ways to work around with noise, work with noise, model noise etcetera. So, far we have talked about the resistor which contributes  $4kTR\Delta f$  as the mean squared noise voltage. And, we also have discussed that only lossy elements contribute in terms of noise. Everything else is noiseless, inductors, capacitors, mutual inductors. So, let us stop here, and we will proceed from here in our next lecture.

Thank you.