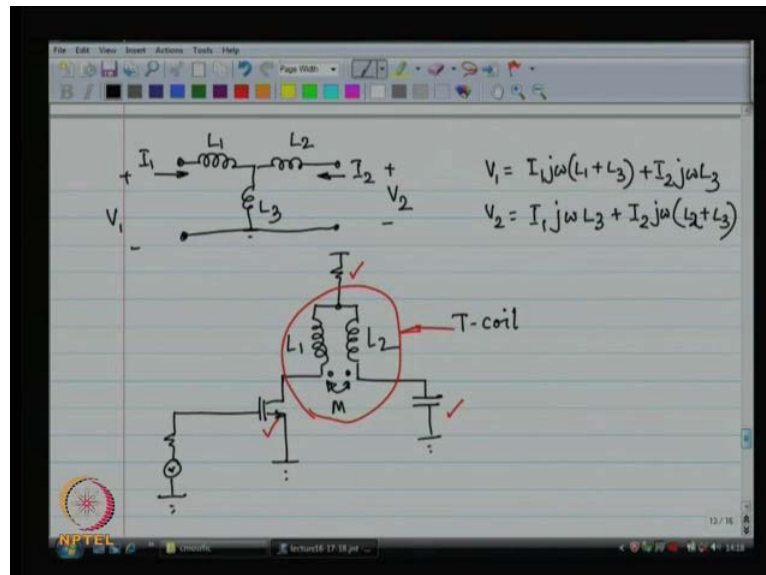


**CMOS RF Integrated Circuits**  
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**Module - 06**  
**Wideband Amplifier Design**  
**Lecture - 18**  
**Shunt Series Amplifiers**

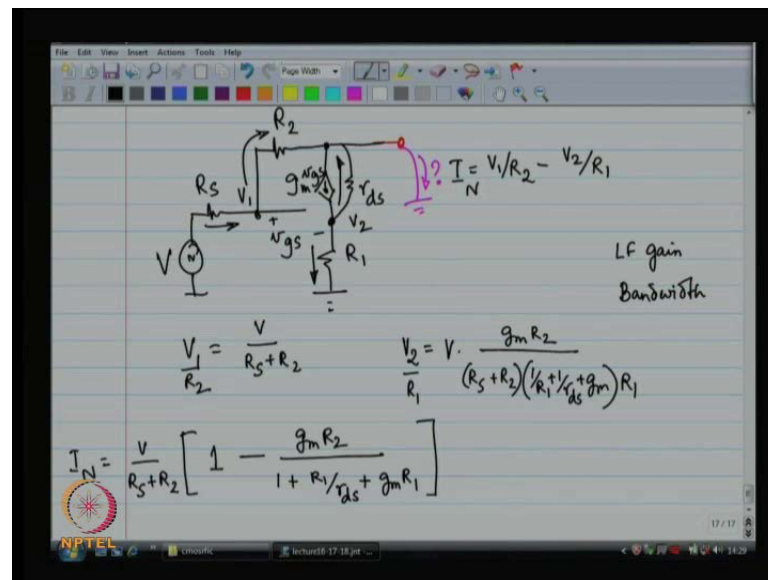
Welcome back to CMOS RF integrated circuits; as part of module 6 that is wideband amplifier design that is what we were doing. Today, we are going to discuss the shunt series amplifier.

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In the past we have studied we have basically modified the load put in inductor over there; basically created a zero and a pair of conjugate pole pairs. And, have tried improving the bandwidth; we came up with I basically told you that 3 inductors put together will work even better than the one inductor in the solo inductor; that particular circuit is called that a set of 3 inductors is the T coil. So, that is also used a lot of times. And, then today we are going to discuss the shunt series amplifier.

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Now, the shunt series amplifier is basically you have got source degeneration as well as something across the gate and drain. So, in the past I called that  $R_2$  and the bottom one  $R_1$ . So, that is fine. Now, this is basically my plan; and what do you think about this we want to first find out the low frequency gain. Then, we want to estimate the band width right; these are the 2 things that we have to do. Now, low frequency gain we are going to find out in a moment.

But before that I just want to say something about the bandwidth; how are we going to find out the bandwidth of this circuit; can we use the method of open circuit time constant? Yes, we can use the method of open circuit time constants there is no problem; why? Because there are no inductors; because there are no inductors there is most probably not going to be pair of complex conjugate poles also; I am going to assume that there are no zeros, there is going to be a zero; I am going to assume that there are no zeros. So, that will give me an estimated bandwidth right.

So, this is basically the idea, fine. So, I can definitely use the method of o c time constants to find out the bandwidth; I know how to use it, what to do with it. So, before that let me try to find out the low frequency gain; how do you find out the gain of a circuit? You use the Norton equivalent model right; you want to model your circuit this is my circuit as a current source in shunt with an output resistance.

So, if I can model this as this Norton current source and then output resistance. Then, I can find out the gain very easily right; the gain is  $I_N$  output voltages  $I_N$  times  $R_{out}$  in parallel with  $R_L$  fine. So, to find the gain what we do is; to find the Norton equivalent current we use the what experiment, what experiment do we do to find out the Norton equivalent current? We do the short circuit experiment; we short the output to ground and see how much current is flowing here that will tell me what is the value of  $I_N$ ? If you do it here then  $I_N$  will go through the short circuit.

So, that is why short circuit experiment right just a reminder. So, we are going to do the short circuit experiment. So, as soon as I do the short circuit experiment I can throw out  $R_L$ ; if there is any capacitor at the output some lump capacitor to round that also gets thrown out etcetera, etcetera; mosfet is going to be replaced by a  $g_m$  and then  $r_{ds}$ . If you also want to incorporate the body effect then it is effectively going to be  $g_m$  plus  $g_{mb}$ , fine.

So, this is my situation all right. So, let us say that the voltage at this point is  $v_1$ ; the voltage at this point is  $v_2$ . And, then I have got to write out two Kirchhoff's equations, Kirchhoff's current equations. So, I have got  $V$  minus  $v_1$  by  $R_S$  that is the current over here that should be equal to  $v_1$  minus 0 by  $R_2$ .

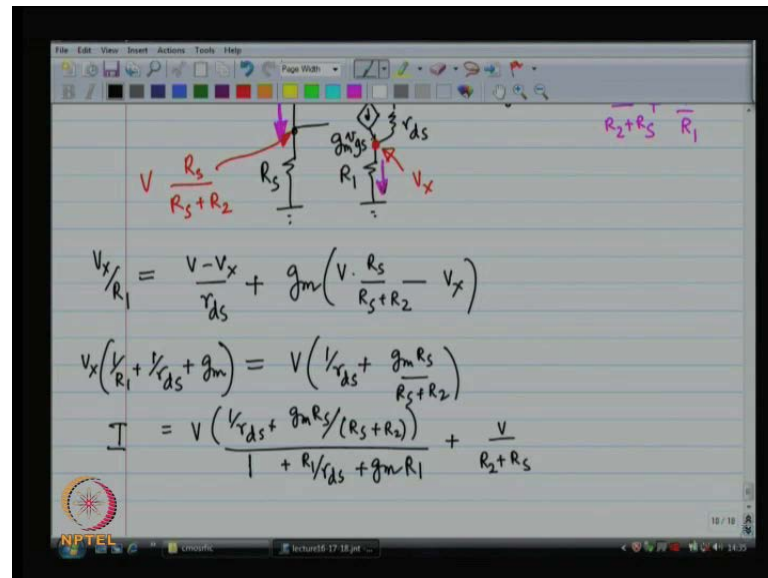
So, what is  $v_1$  fine in terms of  $V$ ? So, in terms of  $v$  I already know what is  $v_1$ . Now, we need to do a second Kirchhoff's equation for the node  $v$ . So, for  $v_2$  what I have got is as follows  $v_2$  by  $R_1$  that is the current going downwards  $v_2$  by  $r_{ds}$  is the current going through  $r_{ds}$ . And, that should be equal to  $g_m$  times  $V_{gs}$ ; what is  $V_{gs}$ ?  $V_{gs}$  is  $V$  minus  $V_2$ , fine.

So, basically you collect all the terms with  $v_2$  when you connect collect all the terms with  $v_2$  this is what you get, right. So, these are my 2 node voltages but am I interested in the node voltages; not really what I am interested in are all the is the short circuit current right. So, the short circuit current is going to be how much? It is going to be  $v_1$  by  $R_2$ ; that current is going to go through the short circuit right the current through  $v_2$  by  $R_1$  also is going to come from the short circuit; it is coming from the short circuit coming out of the short circuit.

So, this is what I am really interested in. So,  $V$  minus  $v_2$  by  $R_S$  plus  $R_2$  that is one thing sorted out and  $v_2$  by  $R_1$  is this particular quantity. And, therefore what I have

really got  $I_N$  is equal to  $v$  by  $R_S$  plus  $R_2$  times 1 minus this is what I have got alright you agree? So, that is your Norton equivalent current fine; that is experiment number 1 done; experiment number 2 is a the output impedance.

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So, this experiment I am yet to do all right. So, what we are going to do the over here is as follows; we are going to replace the mosfet by  $g_m$  and  $r_{ds}$ . And, to find the output impedance you have to kill the input voltage; null the input voltage all right so far so good. So, if this is the case and you have applied a voltage  $V$  over here you want to find out what is the current?

So, first of all there is going to be a current through this pathway and how much is that current that current is  $V$  by  $R_2$  plus  $R_S$ . So, just keep this in mind  $V$  by  $R_2$  plus  $R_S$  is already established in that pathway. And, as a result of that the voltage at this particular node is  $R_S$  by  $R_S$  plus  $R_2$  times  $V$  it is a voltage divider. So, this current has already been established all right.

Then, what is going to happen to this particular node? Let us say that this node has a voltage  $V_x$  right I do not exactly know what is the value of  $V_x$ . So, let me right the Kirchhoff's node equation for that particular node  $V_x$ . So,  $V_x$  by  $R_1$  that is the current going down wards that is equal to  $V$  minus  $V_x$  by  $r_{ds}$  plus  $g_m$  times  $V_g S$ ; where  $V_g S$  is equal to this quantity; and then once again I collect all the  $V_x$  terms together.

So, when you collect all the  $V \times$  terms together this is what you are left with fine. So, what does that mean; first of all what is my net current? My net current is the current through the through this old pathway plus the current through  $R_1$ ; because this eventually has to come from here right. So, my net current is this and that is one of the currents and the other current I have got is this particular quantity. Now, what are we going to do; what is the impedance that you see? Impedance is  $V$  by  $I$  which basically means that I need to do a little bit more of simplification otherwise it is not really tenable.

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The image shows a digital whiteboard with handwritten mathematical derivations. The derivations are as follows:

$$R_{out} = \frac{(1 + R_1/r_{ds} + g_m R_1) (R_2 + R_S)}{1 + R_1/r_{ds} + g_m R_1 + g_m R_S + \frac{R_2 + R_S}{r_{ds}}}$$

$$I_N = \frac{V}{R_S + R_2} \cdot \frac{1 + R_1/r_{ds} + g_m R_1 - g_m R_2}{1 + R_1/r_{ds} + g_m R_1}$$

$$Gain = \left[ \frac{1 + R_1/r_{ds} + g_m R_1 - g_m R_2}{1 + R_1/r_{ds} + g_m R_1 + g_m R_S + \frac{R_2 + R_S}{r_{ds}}} \right] \approx \frac{(R_1 - R_2)}{(R_1 + R_S)}$$

Red annotations on the whiteboard include:  $R_2$  with an arrow pointing to  $2R_1 + R_S$ , and a checkmark next to the denominator of the  $I_N$  equation.

Which means that my output resistance is the inverse of this complicated quantity. Now, whenever you do something like this where you can make mistakes very easily you should always do sanity checks. So, what is a sanity check over here; first of all what happens first of all look at the dimensions what are the dimensions of the numerator? So, in the numerator the first piece over here should have dimensions of unity. So, dimensions are unity in all the 3 terms fine; second term has dimensions of resistance. So, the numerator has dimensions of resistance, denominator should have dimensions of unity because the net result should be a dimension of resistance. So, check does the denominator have dimension of unity; unity, unity, unity no dimensions right.

So, this is a first cut check to see if your result was indeed correct; earlier what we had done not an equivalent current it is a current; the dimension should be in amperes. So, I

have got volts over here divided by resistance. So, I have got amperes over there inside the brackets I should have no dimensions. So, I have got 1 which is good then numerator is no dimensions, denominator is also no dimensions.

So, all is well; this is vocalised a first cut check to whether your expression is correct or not. If you make a mistake here then that is it you again rewind and work it backwards. Next sanity check would be to see what happens when  $R_1$  is equal to 0; what should happen when  $R_1$  equal to 0 or let us say let us suppose  $R_1$  is equal to 0;  $R_2$  is infinitely large and  $R_S$  is equal to 0. If  $R_1$  is equal to 0,  $R_2$  is infinitely large then the output resistance should be equal to  $r_d S$ ;  $R_1$  equal to 0,  $R_2$  is infinitely large.

So, this is 0, 0;  $R_2$  is infinitely large. So, the numerator is infinitely large; denominator 1 plus 0, 0  $g_m$  times  $R_S$  plus  $R_2$  plus  $R_S$  by  $r_d s$ . So, that is approximately equal to  $R_2$  by  $r_d s$ . So, you have got  $R_2$  by  $r_d S$  in the denominator and you have got  $R_2$  in the numerator. So, your net is equal to  $r_d s$ . So, this is another check; third check would be let us say  $R_2$  is infinitely large; let us have  $R_1$  no problem, let us say  $R_2$  is infinitely large. Then, my system boils down to it should have a resistance of  $r_d S$  plus  $R_1$  plus  $g_m r_d S R_1$ ; this is my formula remember the formula right yeah.

So, supposing  $R_2$  is infinitely large does it boil down to this formula let us check;  $R_2$  is infinitely the large. So,  $R_2$  plus  $R_S$  is approximately  $R_2$ ; again  $R_2$  is infinitely large. So, the entire denominator is basically going to be  $R_2$  by  $r_d s$ . So, my denominator is  $R_2$  by  $r_d S$ , numerator is  $R_2$  times 1 plus  $R_1$  by  $r_d S$  plus  $g_m R_1$ . So,  $R_2$  and  $R_2$  cancel out with each other 1 by  $r_d S$  really is times  $r_d S$ . And, as a result you get  $r_d S$  plus  $R_1$  plus  $g_m r_d S R_1$  which is your formula right.

So, these are all sanity checks that you can perform; what happens when  $R_2$  is 0 what happens when  $R_2$  is 0?  $R_1$  is also 0; I should see  $R_S$  in shunt with 1 over  $g_m$  in shunt with  $r_d S$ , right. So, you do all this sanity checks just to you do not need to do all of them; you can do a couple and double check make certain for yourself that what you have done is indeed correct. So, these sanity checks are usually very helpful when you do complicated algebraic; you solve complicated algebraic equations all right. So, this is my  $R_{out}$  and this is my not an equivalent current.

So, therefore the voltage at the output you convert the Norton equivalent current to a thevenin equivalent voltage right. Let us say  $R_L$  is infinitely large let us not bother about

$R_L$ . So, if you do not bother about  $R_L$  then you just multiply  $I_N$  and  $R_{out}$  and that is your voltage at the output divide by  $V$  that will give you the gain. So, therefore the gain over here is  $I_N$  times  $R_{out}$  divided by  $V$  right. So,  $R_S$  plus  $R_2$  cancels out 1 plus this complicated term cancels out all you are left with are these 2 terms. So, this is the net gain of the circuit ok. Now, let us do a couple of approximations let us say that  $r_{ds}$  is very large; that is my first approximation. So, if  $r_{ds}$  is very large; then what we are going to get is the following.

And, then the second approximation that I am going to make is that  $g_m$  is also very large in which case 1 is small compared to  $g_m$  times something; which means you can really ignore the 1 and  $g_m$  will cancel out with the  $g_m$ . So, you will basically get  $R_1$  minus  $R_2$  divided by  $R_1$  plus  $R_S$  this is the gain of the circuit all right. Now, is that something you like; first of all do you want gain to be more than 1 or less than 1; do you want gain to be positive or negative? You want gain the magnitude of gain to be more than 1 right; you do not mind if the gain is negative. In fact, if you have a single transistor something then you expect the gain to be negative; you are looking at the voltage at the drain you expect the gain to be negative; you do not expect it to be positive. So, now we are talking. So, in that case what is this what is the story over here should  $R_2$  be larger than  $R_1$ ;  $R_2$  smaller than  $R_1$  want the gain to be negative and you want it to be more than one.

So,  $R_2$  has got to be much much larger than  $R_1$  right; not only larger than  $R_1$ ,  $R_2$  has got to be  $R_1$  minus  $R_2$  I am sorry  $R_2$  minus  $R_1$  has to be more than  $R_1$  plus  $R_S$ ; to give you any decent gain more than 1; which means that  $R_2$  has got to be more than twice  $R_1$  plus  $R_S$  to give you any gain which is more than 1 all right.

So, now that we have some idea of what is going on over here; we have some idea we have this is an accurate expression at low frequencies it has  $g_m$  in it, it has  $r_{ds}$ . So, even for non ideal mosfet model good decent mosfet model at low frequency you can work it out. If you want to incorporate body effect all you have to do is substitute  $g_m$  by  $g_m$  plus  $g_{mb}$  right; remember to do that. And, you have got a very accurate low frequency equation description of the circuit. The next thing is what about an approximate idea of what is going on; approximate idea is when  $g_m$  is large when  $r_{ds}$  is large. And, then that is the situation then what I have got is a gain which is

approximately equal to  $R_1 - R_2$  by  $R_1 + R_S$ . Let us take a few numbers and see for ourselves how approximate this approximate business is.

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$$\text{Gain} = \frac{1 + R_1/r_{ds} + g_m R_1 - g_m R_2}{1 + R_1/r_{ds} + g_m R_1 + g_m R_S + \frac{R_2 + R_S}{r_{ds}}} \approx \frac{(R_1 - R_2)}{(R_1 + R_S)}$$

$$\begin{array}{l} R_1 = 1 \text{ k}\Omega \quad R_2 = 10 \text{ k}\Omega \quad g_m = 10 \text{ mS} \\ R_S = 1 \text{ k}\Omega \quad r_{ds} = 20 \text{ k}\Omega \end{array} \quad \approx -4.5$$

$$\frac{1 + 0.05 + 10 - 100}{1 + 0.05 + 10 + 10 + 0.55} = \frac{-89}{21.6} \approx -4.2$$

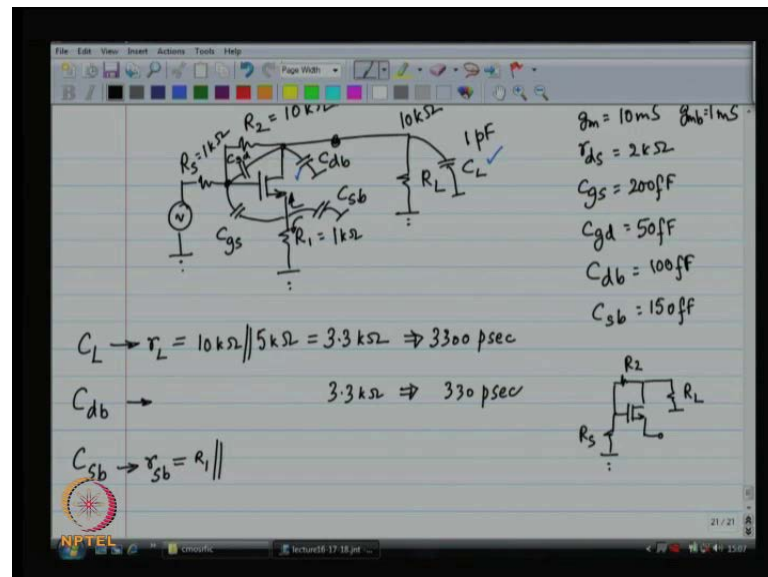
Let us say  $R_1$  is 1 kilo ohm,  $R_2$  is  $R_S$  is also 1 kilo ohm;  $R_2$  has got to be more than 3 kilo ohms let us say  $R_2$  is 10 kilo ohms. And, let us say  $g_m$  is 10 milli Siemens  $r_{ds}$  is 2 kilo ohms. So, let us say these are the numbers that we are talking about. So, in that case the approximate result gives me  $R_1 - R_2$  that is minus 9 divided by  $R_1 + R_S$  2. So, I get about minus 4 and a half gain; which is not bad or 12, 13 db right. But if I plug in the numbers of  $g_m$   $r_{ds}$  etcetera I get  $1 + R_1/r_{ds}$  is about half  $g_m$  times  $R_1$  is 10 minus  $g_m$  times  $R_2$  that is 100 divided by  $1 + R_1/r_{ds}$  is 0.5 plus  $g_m$  times  $R_1$  is 10 plus  $g_m$  times  $R_S$  is another 10 plus  $R_2 + R_S$ ;  $R_2$  is 10 kilo ohms,  $R_S$  is 1 kilo ohms.

So, we have got 11 kilo ohms divided by  $r_{ds}$ . So, 11 kilo ohms divided by 2 kilo ohms is about 5.5. So, what I have got over here is 1.5, 11.5 minus 100. So, about minus 88.5 in the numerator and in the denominator I have got 27. So, I have got a gain which is 88 and a half divided by 27 that is about 3.3. So, I have got lesser again but ballpark I get a feeling for what the numbers are going to look like? So, instead of a  $r_{ds}$  as 2 k if I had chosen  $r_{ds}$  as 20 k right. Suppose  $r_{ds}$  was 20 k then what you would have got is 0.05 over here, 0.05 and 0.55. And, that would mean something like this which leads to again which is more closer to 4 and a half.



So, you can do these you can play a little bit over there. So, this is what we have got. So, this is our shunt series amplifier; we can get gains out of this by choosing appropriate values of  $R_2$  and  $R_1$  etcetera and in relation to  $R_s$ . But of course and now the next question is what is its bandwidth?

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And, to do to work out the bandwidth what we have to do is we have to imply the method of open circuit time constants; I did not even consider a load  $R_L$ . You basically have to put  $R_L$  in shunt with whatever output resistance you have got or what you can do is you can split the gain that you have got over your  $R_{out}$  and your  $R_L$ . And, you can find out the net gain; that is also fine.

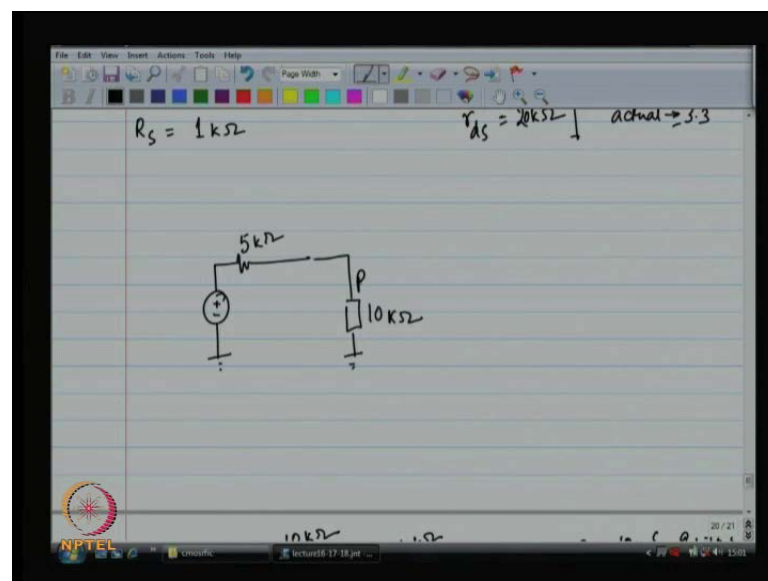
So, for now I assumed earlier assumed that  $R_L$  is very very large; you need not make that assumption fine. Let us pick up some sample numbers. Let us stick to the numbers that we have got right  $R_1$ ; 1 k let us stick to these numbers alright. And,  $g_m$  is 10 milli Siemens,  $r_{ds}$  is 2 kilo ohms this. Let us also pick some numbers we had used earlier  $C_{gs}$  is 200 femto farad,  $C_{gd}$  is 50 femto farad,  $C_{db}$  is 100 femto farad and  $C_{sb}$  is 150 femto farad; these were numbers that we were using in for as well  $g_{mb}$  is 1 more milli Siemens.

So really you should be plugging in  $g_m$  plus  $g_{mb}$  over here does not matter right. And, then we are going to play with the open method of open circuit time constants. So, to do that we need to identify all the capacitors that we have got; the capacitors that we have

got are these I could not label C gate to drain; these are all the capacitors. Let us say that  $R_L$  and  $C_L$  what you want to put  $R_L$  and  $C_L$  as; what you want  $R_L$  and  $C_L$  to be? Let us say  $R_L$  and  $C_L$  are 10 kilo ohm and 1 Pico farad all right just some numbers over here; what about what would happen to  $R_{out}$ , what is  $R_{out}$  over here with my numbers  $R_1$  is 1 k,  $R_2$  is 10 k. So,  $R_{out}$  would be 1 plus  $R_1$  by  $r_{ds}$  is about 0.5 plus  $g_m$  times  $R_1$  is factor of 10 times  $g_m$  was 10 milli Siemens  $R_1$  was 1 kilo ohm;  $R_2$  plus  $R_S$  is 11 k yeah; this whole thing is to be divided by 1 plus  $R_1$  by  $r_{ds}$  is 0.5 plus  $g_m$  times  $R_1$  is 10 gm times  $R_S$  is also 10.

And,  $R_2$  plus  $R_S$  by  $r_{ds}$  is a factor of 5 and a half,  $R_2$  is 10 kilo ohms,  $R_S$  is 1 kilo ohms. So, 11 kilo ohms by 2 kilo ohms that is 5.5. So, I have got my denominator as 27 right. So, 11.5 times 11 k divided by 27 how much is this; how much you think it is? It is about 5 k little less than 5 kilo ohms; do not want to do the full mathematics it is about 4.8 k. Let us put a round number 5 k. Now, I am trying to drive a load which is 10 k ok.

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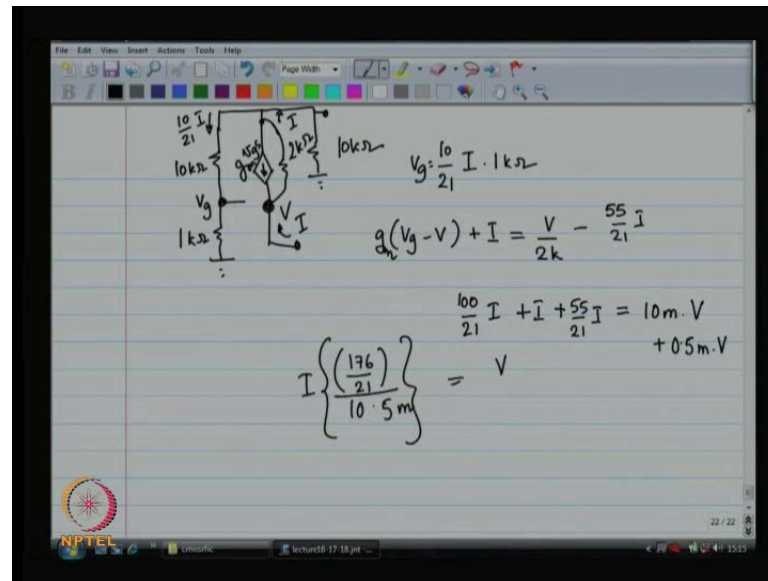
And, my output looks like a voltage source a d c it looks like a voltage source value about 3.3 times  $V_{in}$ . So, that means that two- third of 3.3 is actually going to be transferred to the output; which means that my net gain is actually only going to be only about 2.2 times the input; net output voltage is only 2.2 times the input all right minus. So, keep this in mind right; please keep that in mind that is; let us now proceed with the method of open circuit time constants.

So let us do them one by one. Let us first start with  $C_L$ ; when I talk about  $C_L$  the resistance that  $C_L$  is going to see is 10 kilo ohms in shunt with the output resistance of the whole thing; which was about 5 kilo ohms, remember. So,  $R_L$  is really something like 3.3 kilo ohms all right. So, net what I get over here is 1 Pico the from time constant I get for the load capacitor is 1 Pico farads times 3.3 kilo ohms that is about 3.3 Nano seconds. Now, obviously this is huge right and with this kind of a design you are not going to get anywhere all right; this kind of a design you are not going to get anywhere.

So, what you really have to do is you have to use a bigger transistor, smaller resistors all over the place. So, that the output resistance of your circuit is much lesser. So, if I choose 10 times larger devices, 10 times larger conductances then presumably I will get better numbers for the load capacitance a better ((Refer Time: 47:01)) right. So, that is a design point that you have to do. So, instead of choosing  $R_L$ ; so much I am sorry instead of choosing  $R_1$  as 1 kilo ohm,  $R_2$  as 10 kilo ohms why did not you choose  $R_1$  as 100 ohms,  $R_2$  as 1 kilo ohm,  $R_S$  as 100 ohms right.

So, this and  $g_m$  why did not you choose it as 100 milli Siemens and 200; I am sorry 2 kilo ohms  $r_{ds}$ . So, we could have done that fine let us proceed with what we have got; what about  $C_{db}$ ?  $C_{db}$  also sees the same resistance  $C_{db}$  is about 100 femto farad. So, I do not see that much. So, I have got 2 capacitors out of the way; next  $C_{source}$  to body; what is the resistance that you see looking in upwards into the source of the mosfet; looking downwards you see  $R_1$  looking upwards what do you see? First of all what is my circuit? My circuit is like this; that is my circuit over there; what do you think you are going to see? I am going to see a small resistance, large resistance. First of all whenever you look into the source you see a small resistance. So, if you see kilo ohms its most probably not right; if you see 100 of ohms may be you have done a good job the computation.

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So, let us try something like this is what we have got and of course you replace the mosfet with  $g_m$  and  $r_d$ s. So, you have applied  $V$  at the source and you want to find out what is the current going into the source that is the question? So, suppose the current going into the sources is  $I$ . So, what is the current coming out of the drain; it is going still going to be  $I$  all right. Now, this  $I$  is going to split into 2 parts; one side is going to the gate towards the gate, one side is going to go into the load; one side is going to go towards the source the other going to go towards the load; what fraction will go towards the source? Towards the source you see more resistance than the drain toward the load.

So, you will see less current right. So, 10 by 201 times the current  $I$  will go towards the source; agreed? Now, this is 10 by 21  $I$  coming and what is the drop across 1 kilo ohm this times 1 kilo ohm is the voltage that is on the gate all right. So, what is  $V_g$ s?  $V_g$ s is  $V_g$  minus  $V$ . So, next question is what is  $V$ ? So, the current through the  $g_m$  is  $g_m$  times  $V_g$  minus  $V$  right and that current plus  $I$  is the current through  $r_d$ S. And, what is the current through  $r_d$ S? The current through  $r_d$ S is the voltage at the output node the voltage over here minus I am sorry  $V$  minus the voltage at the output node divided by 2 k.

And, the voltage at the output node is 10 by 21  $I$  times 11 k; that is the voltage at the output node divided by 2 k. So, I basically done a Kirchhoff's node equation at this particular point. And, that is going to give me what is the relationship between  $I$  and  $V$ .

So, then I put all the V terms together. all the I terms together V g really is something which is proportional to I.

So,  $g_m$  times  $g_m$  is rarely 10 milli Siemens times 1 k 10 milli Siemens times 1 kilo ohms times 10 by 21 I. So, 10 milli Siemens times 1 k is the factor of 10. So, I have got hundred by 21 I minus  $g_m$  times V and then I have got I over here and I have got 55 by 21 I over here; and I have got half a milli Siemens times V on the other side right. So, that basically tells me that I times 155 and 21 is 176 by 21; hope I am doing my arithmetic correctly.

So, the voltage that you have got is something like this; which means that the resistance that you see looking in at that particular node is this quantity 176 by 21 is about 8.5; now little less than 8.5. Let us say 8.3 or so and milli in the denominator means k in the numerator. So, 8.3 kilo ohms divided by 10 and a half its about 800 ohms. So, we said that we will see a low impedance we have got a low impedance.

So, we are going to stopover here  $R_1$  we had to put in parallel. So, 1 k in shunt with 800 ohms whatever that is that is going to be about 600 ohms or so. And, 600 ohms 150 femto farads probably give me 30 Pico seconds; no 90 Pico seconds alright. So, we are going to stop here we have got a couple of more capacitors to work on; actually those are the 2 important ones and we are going to wrap it up after that. So, so far we have been discussing the shunt series amplifier and we will see its benefits shortly.

Thank you.