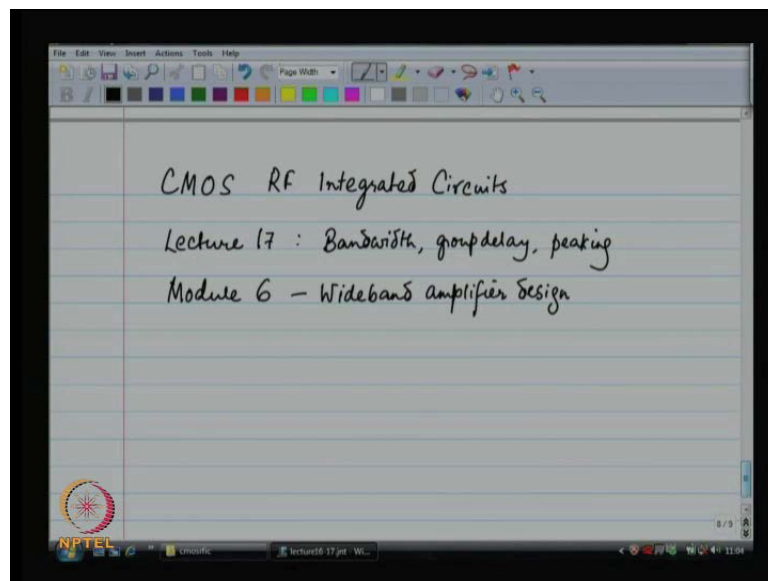


CMOS RF Integrated Circuits
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Module - 06
Wideband Amplifier Design
Lecture - 17
Bandwidth, Groupdelay, Peaking

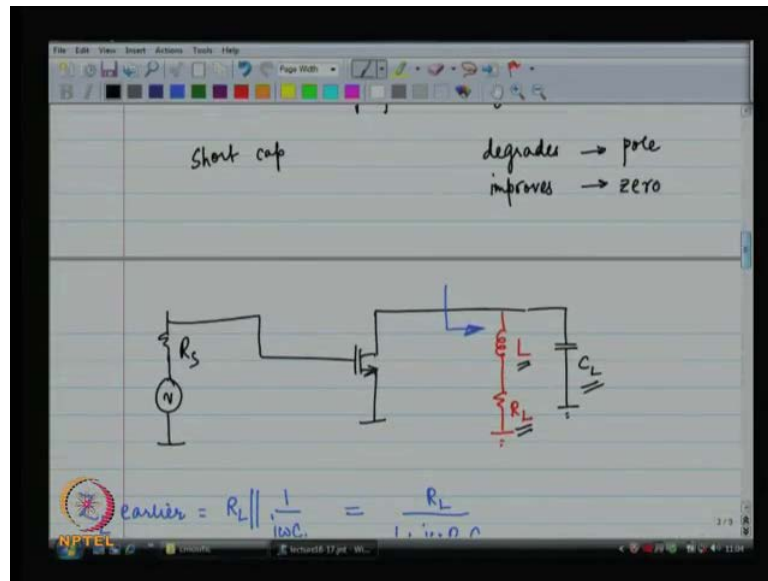
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Welcome back to CMOS RF integrated circuits.



Today is lecture 17 part of 6 module; we have been talking about wideband amplifier design. And, today we are going to look at trade of between bandwidth group delay and peaking.

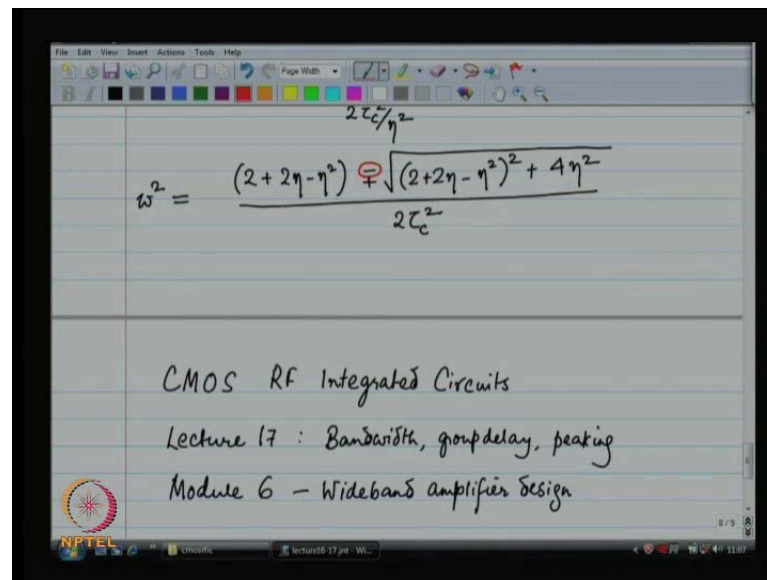
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Now, we are going to continue from where we had left off; in the previous class, in the previous lecture. So, in the previous lecture what we were really discussing is this particular network. So, instead of trying to drive a load which is just R and C parallel combination; why do not I tried to drive something that looks like what has been projected. So, L in series with R in shunt with the load capacitance. So, this is what we are trying to drive. And, we came up with the transfer function this was the transfer function; the Z_L earlier was the transfer function before and Z_L now is the transfer function now.

And, then we were really busy trying to predict, trying to find out what is the bandwidth. So, we came up with the rule for bandwidth; the rule for bandwidth is ω at which magnitude squared is equal to half and the magnitude squared is equal to half of the magnitude squared at d c right. So, just remember that. And, then we try to solve for ω instead ω_n ((Refer Time: 02:23)) a solving for ω^2 and we found that ω^2 has 2 roots given by the above 2.

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The image shows a handwritten equation on a presentation slide. The equation is:

$$\omega^2 = \frac{(2 + 2\eta - \eta^2) \pm \sqrt{(2 + 2\eta - \eta^2)^2 + 4\eta^2}}{2\tau_c^2}$$

Below the equation, the text reads:

CMOS RF Integrated Circuits
Lecture 17 : Bandwidth, group delay, peaking
Module 6 - Wideband amplifier design

The slide also features an NPTEL logo in the bottom left corner and a Windows taskbar at the bottom.

Now, what do you think? Let me simplify a little bit more on this. So, I am going to multiply the numerator by eta squared, denominator by eta squared. And, that will give me 2 plus 2 eta minus eta squared minus, plus, square root of 2 plus 2 eta minus eta squared whole squared plus 4 eta squared divided by 2 tau C squared. So, this was the expression for omega squared. Now, omega squared is a positive quantities cut to be a positive quantity; the 2 roots are both real numbers thankfully.

But otherwise if this quantity is so much, this quantity is going to be more than that; it is the same x squared plus something under root the whole thing. So, it is cut to be. So, the second term is more than the first term; which means that the minus result this one will give you a negative value, net will be negative. Now, omega squared, omega is a real number. So, omega squared has got to be positive.

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The image shows a digital whiteboard with handwritten mathematical equations. At the top, there is a small diagram of a system with a gain of $-3dB$ and a time constant τ_c . Below this, the magnitude of the transfer function is given as:

$$= \frac{1}{\tau_c^2} \left[(1 + \eta - \eta^2/2) + \sqrt{(1 + \eta - \eta^2/2)^2 + \eta^2} \right]$$

The expression in the brackets is labeled $f(\eta)$. Below this, the derivative of $f(\eta)$ with respect to η is calculated:

$$\frac{df(\eta)}{d\eta} = (1 - \eta) + \frac{1}{2\sqrt{(1 + \eta - \eta^2/2)^2 + \eta^2}} \cdot [2(1 + \eta - \eta^2/2)(1 - \eta) + 2\eta]$$

The whiteboard interface includes a menu bar at the top (File, Edit, View, Insert, Actions, Tools, Help) and a toolbar with various drawing tools. The NPTEL logo is visible in the bottom left corner, and the text "Lecture 17 pt. W..." is in the bottom right corner.

So, we are basically going to be ignoring the negative root for omega squared. And, as a result omega squared is basically going to be $1 / (2\tau_c^2)$ times; this is my net result for omega squared which means that omega is a further square root of this fine. Now, suppose I want to maximize this, I want to maximize my bandwidth; this is the bandwidth, original bandwidth was $1 / \tau_c$ remember. So, the new bandwidth is something like of factor a more than $1 / (2\tau_c^2)$; bandwidth squared is a factor more than $1 / (2\tau_c^2)$. So, simplify a little bit more.

So, first of all when the bandwidth is going to be more than the original; whenever, this factors is more than 1 I am going to get it to be more than the original. And, when is it going to be greater than 1, when is it at what it eta is this equal to 1; we can solve for that eta. That is the eta beyond which you are not going to get bandwidth enhancement; we are only going to reduce your bandwidth, you are going to pay penalty in terms of a bandwidth.

So, we do not want that to happen fine. Next, what is the maximum bandwidth that is possible from this how do you find out maximum? You do it derivative; you put the derivative equal to 0; that will give you some kind of maxima or minima. Hopefully, the second derivative is also in the right direction you will get a maximum really; I mean as you change eta you are going to get some eta at which the bandwidth is going to be maximum.

So, can we find out the bandwidth at which the eta at which the bandwidth is maximum. So, let us do the derivative of this quantity; derivative with respect to eta. So, let me call this f of eta; derivative of something under square a square root, derivative of x power half is half of half times; x power minus half right. So, this is the derivative and let me simplify this a little bit because quite unwieldy.

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$$\frac{df(\eta)}{d\eta} = (1-\eta) + \frac{1}{2\sqrt{(1+\eta-\eta^2/2)+\eta^2}} \cdot [2(1+\eta-\eta^2/2)(1-\eta) + 2\eta]$$

$$0 = (1-\eta) + \frac{2+2\eta-3\eta^2+\eta^3}{2\sqrt{(1+\eta-\eta^2/2)^2+\eta^2}} \quad \eta = \sqrt{2} \quad (1-\sqrt{2}) + (\sqrt{2}-1) = 0$$

So, this is what I have got; and for maximum to happen this quantity has got to be equal to 0. Now, instead of going through the entire algebra; let me just tried to plug-in 1 number. So, let us tried to plug-in eta equal to square root of 2; I mean you can do it, you can solve this. But let me just first tried to plug this in to make like easier. So, if I do plug that in I get 1 minus root 2 plus 2 plus 2 root 2 minus 6 plus 2 root 2 divided by 2 times square root of 1 plus root 2 minus 1 the wholes squared plus 2.

So, this simplifies 1 plus square root of 2 minus 1 is basically square root of 2 whole squared is 2; 2 plus 2 is 4 square root of 4 is 2, 2 times 2 is 4. And, 2 root 2 plus 2 root 2 is 4 times root 2; and 2 minus 6 is minus 4. And, then you divide numerator and denominator by 4 and low and behold you end up with 0. So, instead of solving the complicated equation thankfully; I already knew the root and I plugged it in and I satisfied to you that that is indeed a root. So, when I do plug-in eta equal to root 2 I should be getting the maximum bandwidth; you can do the second derivative of this and

satisfy for yourself that the second derivative is indeed; the right sin second derivative has to be negative. So that you can satisfy for yourself.

And, so basically what I am trying to say over here is when you plug do plug-in eta equal to root 2; you get the maximum possible bandwidth. So, what does your transfer function look like when you plug-in eta equal to root 2 or rather where do the poles and 0 shop when eta is equal to root 2; you get the 2 poles to be at these 2 complex locations remember we did this in the previous class kind of bit this right.

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$$2\sqrt{(1+\eta-\eta^2/2)^2 + \eta^2}$$

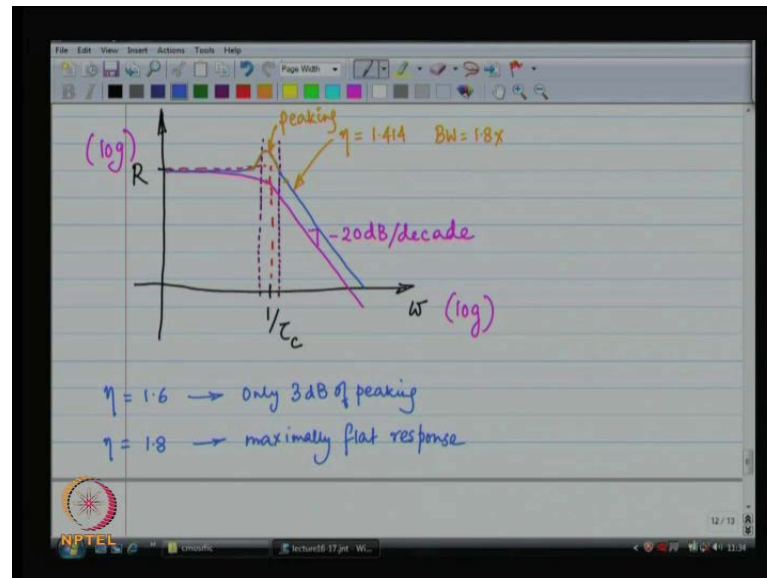
@ $\eta = \sqrt{2}$, $Z_{L_{\text{new}}} = R_L \cdot \frac{1 + s \cdot \tau_c / \sqrt{2}}{1 + s \tau_c + \frac{s^2 \tau_c^2}{\sqrt{2}}}$

Zero at $-\sqrt{2}/\tau_c$

Poles at: $\frac{-\tau_c \pm \sqrt{\tau_c^2 - 4\tau_c^2/\sqrt{2}}}{2\tau_c/\sqrt{2}} = \frac{-1 \pm \sqrt{1 - 4/\sqrt{2}}}{\sqrt{2} \tau_c}$

And, we have got a 0; at what frequency? zero in the left half plane. So, these are the locations of the poles which are not the same as is fine. So, let me simplify this; I am going to divide numerator and denominator by tau C. And, then what I am going to further do which is fine.

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So, this is what we have really got; somehow this is not the same as the previous result why is that? I would believe that there was a mistake in the previous result this should not have been two-times root 2, this should have been 2 times tau C; which I have seen also I inadvertently substituted by root 2 we would not catch it So, anyway so, these are the locations of the 2 poles. Now, 2 is definitely less than 4 times root 2 which means that the quantity under the square root is negative; which means that the locations of the 2 poles are imaginary.

So, let me take out j over here and instead of doing 2 minus 4 root 2 I am going to do 4 root 2 minus 2 divided the whole thing by 2; you could also think of it in this fashion all right. So, these are the locations of the 2 poles it is so happens that when you actually plot the magnitude response of this function. The earlier function if I plot the magnitude response of the earlier function it was something like this right. This was the magnitude response of the earlier function; of course you are you have a log scale here and un-log scale here; we like doing things in the log scale.

Now, what has happened is you got a 0, where is your 0; you have a 0 at a frequency which is a little more than 1 by tau C, root 2 times more than 1 by tau c. So, you have got a 0 frequency right over here where are the 2 poles; the 2 poles will show their effect at a frequency which is root 2 times less than 1 by tau C. So, the 2 poles are going to show their effect somewhere over here; in the bode plot the real part is what matters right. So,

this is the story. So, what is going to happen is that what else happens when there are pair of complex conjugate poles; when there are a pair of complex conjugate poles what happens is, it is going to be peaking.

So, somehow this is going to be the effective transfer function. So, because of the 2 poles I am going to get minus 40 dB per decade fine. Because the poles are a complex conjugate pair I am going to get substantial amount of peaking. And, then in addition to that a little bit after because of the 0 I am going to get plus 40, plus 20 dB per decade; as a result of which my net loop is going to be minus 20 dB per decade. So, this is the complete story this is how the transfer function is going to look like.

Now, do you want this kind of a transfer function? The answer is maybe but not really I want flat pass band. So, my 3 dB frequency has increased substantially; I agree this is probably my new 3 dB frequency which has dramatically improved. But I like a flat pass band how much was the 3 dB frequency improvement? We did not really compute that did we? Let us quickly. So, $1 + \sqrt{2} - 1$ is really square root of 2, square root of 2 squared plus square root of 2 squared is 2 plus 2 which is 4 root of 4 is 2 2 plus square root of 2 is something like 3 point 4.

So, your bandwidth actually increased by about 3.4 times a bandwidth squared increased by 3.4 times. So, bandwidth increased by square root of 3.4 is about 1.8 yeah about 1.8. So, bandwidth improved by about 1.8 times. So, I got a bandwidth enhancement of by 1.8 times $1/\tau C$; however, I ended up with substantial amount of peaking. So, this phenomenon is called peaking; why did this happen? This happen because I was over aggressive because of the pair of complex conjugate poles being rather close to the $j\omega$ axis. So, that is why this happened.

And, the way this can be avoided is by pulling the poles away from the $j\omega$ axis; in other words you reduce your aggressiveness. So, in other words let me increase the value of η and I will get something better. So, one can show that if I reduce η , if I increase the value of η to let us say 1.8 or so, I am sorry 1.6 or so then, I get about only 3 dB of peaking.

If I increase the value of η even further remember what is the maximum η that you would like to; you can figure out the value of η for which you will get any bandwidth enhancement by comparing this quantity f_0/n to 1; it should be more than 1 for any

kind of bandwidth enhancement fine in that number is about 4 all right. So, η equal to 1.8 gives maximally flat; what is maximally flat? Maximally flat is like butter words, butter words are response is maximally flat.

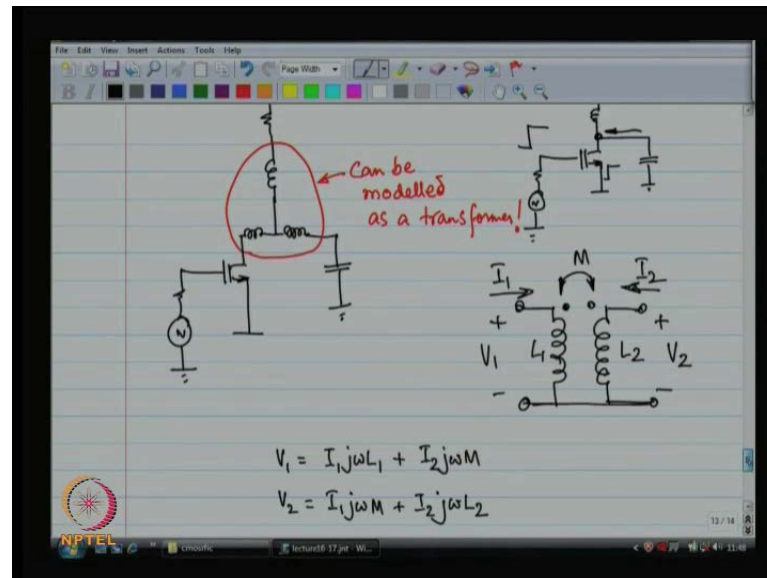
So, when the denominator polynomial magnitude squared is of the form $1 + \omega^4$. In case of second-order this is the magnitude squared of the denominator polynomial that will give me maximally flat or current magnitude squared is looks like something looks like this magnitude squared fine. And, then η equal to 1.9 gives you a constant group delay linear phase. So, all of these different conditions are nice things you want as higher bandwidth as possible. So, you would like η to be as close to 1.4 as possible.

Then, you say that when I design η to be 1.4 I get good bandwidth yes, I get really good bandwidth; but I get horrible peaking. So, you say that let me reduce my peaking. So, you sacrifice your bandwidth a little bit and you get only 3 dB of peaking; you increase the value of η to 1.6. Now, you get a different bandwidth enhancement; your earlier bandwidth enhancement was 1.8 times. Now, it is going to be something lesser than that is what you anticipate you always pay in terms of something; then, you say that no I did not really want 3 dB I wanted you maximally flat response.

So, again you pay a little bit when you say that no I did not even want maximum maximally flat response; I wanted linear phase why would you want linear phase? Linear phase means constant group delay means whatever the frequency is that frequency goes through the circuit with a certain constant delay; it is a very desirable property of a circuit within the bandwidth.

So, within the bandwidth constant group delay you increase your η even further which means you reduce your bandwidth enhancement a little bit more and so on and so forth. So, of course the lesser the value of η is the better the bandwidth performance. And, in 1.4 is the absolute maximum bandwidth you can get and then you keep trading of right. So, this is the nice circuit it has been used for years and years together.

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What was the circuit like? All we are doing right now is we are playing with the load; let me draw this in a little different fashion and all right. Now, can I do even better than this turns out that you can do even better than this; if you put an inductor over here. And, basically resonate out the capacitor; you can do even better than that when you put an inductor over there. Now, why would this perform is better let us quickly go back to our original was this right; this was the very first circuit. So, when you apply a pulse at the input; suppose, you apply a step at the input initially current was flowing through the device. Now, the current has to no; I am sorry, initially no current was flowing through the device now current will has to be established.

So, the recurrent also will follow a step, should follow a step. So, the current through the device is also going to follow a step but the voltage at the output you want that to follow a negative step. So, initially the voltage was the power supply. Now, you would want the voltage to go down this is the basic idea; this is this should be the ideal step response. Now, because there is a capacitor over there this is not really going to happen. And, even when the current goes through a step response because the load is an r c the capacitor is going to charge up slowly. So, this is what you are going to achieve out of this first cut.

Now, what I do is I say that forget the first cut; let us now talk about the second cut way of put an inductor over here. Now, what is the property of an inductor? An inductor likes to keep the current through it a constant this is the most in this is a capacitor likes the

voltage across it to be a constant; an inductor likes the current through it to be a constant right. Now, what is going to happen is that initial current through the inductor is 0. So, the inductor is going to say I would like the current through me to remain 0; I do not want the current to change.

So, when the current goes through a step response; current through the most device goes through a step response was going to happen is all of this current is initially going to come out of the capacitor. What was it earlier? When the inductor was not that there what was it; the voltage here was ready. So, some of it as the voltage volts start dropping current would come through the resistor; more and more current would come through the resistor. Now, the inductor is blocking current through the resistor completely all the current has to come from the capacitors which means that the capacitors is going to discharge faster all right this is the step.

Now, let us look at the next step; in the next step what we are going to say let us get out of this. So, the current goes through a step. Now, imagine that the mosfet even though the current is really going through a step there are parasitic capacitance is here; significant parasitic capacitance are present. Now, in that happen when the current is going through a step; the current through the inductors will would like to be 0 inductors like current through them to be conserved, magnetic fields should be constant energy is in the form of magnetic energy.

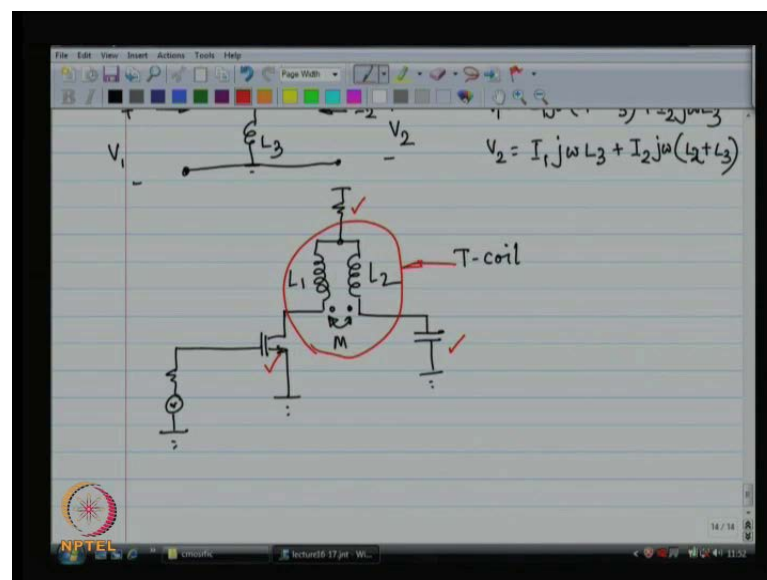
So, the inductors are not going to allow any current to flow through them; which means that the capacitor which was pre charged to power supply this parasitic capacitor is going to discharge. So, first the parasitic capacitor discharges. Now, the parasitic capacitors have discharged. Now, the inductors do not have a choice all the current now has to come through the inductor. And, now the same story starts again; now this inductor is going to carry all the current. And, the inductor on the top still says that no, I do not want any current to come from me; which means, that all the charge on the load capacitor is discharged right.

In the third step what we are going to do is we are going to add another inductor over there. And, what this inductor is going to do; it is going to say an on a little bit this inductor is also going to block out current from this capacitor initially. And, then when the current starts flowing through the second inductor which one is it going to choose; is

it going to choose to come through the inductor capacitor series or inductor resistor series it is going to choose to come through the inductor capacitor series. Because at the right frequency the inductor in the capacitor together look like short-circuit. So, that is what we have achieved over here right.

So, this particular circuit gives really good performance; combination of these 3 inductors could also be modeled as a transformer. Now, this is where you have to ask questions why is this transformer, how does this remind you of a transformer; does it remind you of a transformer, mutual inductance might have forgotten, mutual inductance right the drop across the inductor is I want times ωL drop of them. So, and what about here ok.

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So, these 2 looks similar, these 2 sets of equations look similar; which means that I can model 1 is the other. So, as long as your mutual inductance is less than L_1 plus L_3 is less than the self inductance is of each you can model that is usually the case the mutual inductance is going to be less than the self inductance. Because the coupling is ideally at best equal to 1; the coupling is usually going to be less than 1. So, that basically means that when I have these 3 inductors together I can make this instead of making 3 inductors separately; I can make it has a set of 2 mutually coupled inductors. So, this is called the T-coil something like this is called the T-coil; the T-coil is use routinely.

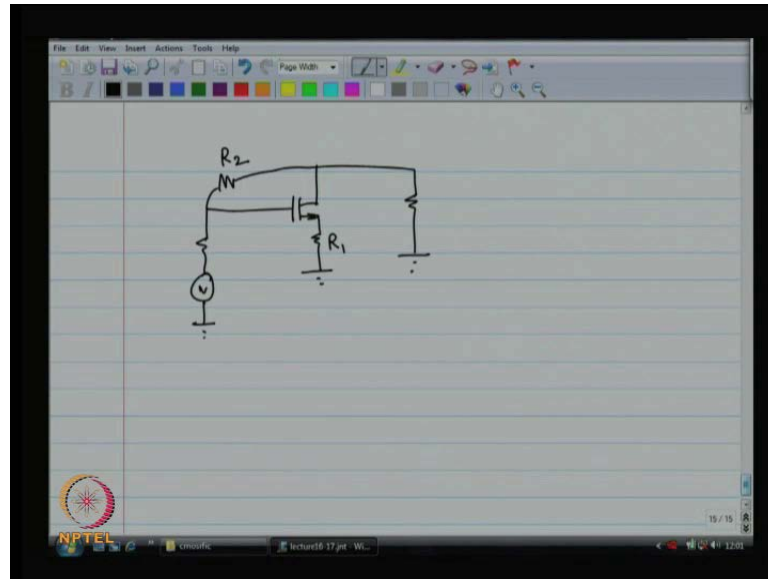
So, you can insert a T-coil into your. So, there is nothing else right; there is your old circuit, your first cut circuit which had your mosfet resistor capacitor load. And, what you do is you insert this T-coil this is called the T-coil. And, certainly your bandwidth has in the supposed to be improved dramatically; if you make the right t coil over there all right. So, these are different techniques of making wideband amplifiers; these have been used time and again I am going to discuss one more which is using the principle of feedback. So, so far what I have be seen; we have seen that we can in instead of having regular R C circuit; we can introduce some whole and 0, complex pair of complex conjugate poles and 0 right. And, we can try improving the bandwidth over here as you progress towards the T-coil you include more and more poles and 0; I mean these are more complicated systems; the order is much more alright.

So, you basically go to higher order as you go to higher order what happens? Suppose, you make a linear group delay, what is going to happen is the group delay going to be more or less? In original the group delay is going to become more than the original right. So, what you have really played with over here as you go to higher and higher order what you have played with is the total group delay; you have made the total group delay larger. And, by sacrificing some delay you have got more bandwidth.

So, bandwidth the circuit is faster. But the delay is more; what that really means is the things are coming faster; things are going out at the same speed at which things are coming. But the delay, the latency of the circuit is a little bit more typically this is acceptable most cases this is quite acceptable; the delay is that we are talking about anyway are very very small.

So, the next speed of the circuit increases but the delay you trade of. So, with by trading of delay, by increasing the order of the circuit, by trading of delay; you achieve more and more bandwidth. So, the other trade of is the again you wide of gain bandwidth products; if I somehow decrease the gain I can increase the bandwidth right. So, that can be done by using principles of feedback.

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So, this is a very common circuit. And, what I propose over here is let us put feedback this might not be sufficient, you might need something like this as well. So, this is called shunt series feedback. So, you have put R in series and R in shunt to an amplifier. So, the shunt series amplifier also can improve speed dramatically by playing with the gain right. So, what happens; you increase the bandwidth by a factors which is the loop gain of the system, look gain enough the system. So, shunt series feedback is also routinely used in lot of different scenarios these are used as wideband amplifiers.

In fact, these are used at the input of oscilloscopes routinely; oscilloscopes have to be wideband instruments when you buy an oscilloscope. Let us say its 1 gigahertz oscilloscopes; it should be able to process all frequencies starting from 0 all the way to 1 gigahertz. So, that is what a wide band amplifier terms; 1 gigahertz oscilloscopes does not process only signals at 1 gigahertz, it has to process all frequencies between 0 and 1 gigahertz right.

So, it is a wideband system, and these are this particular shunt series amplifier. For example, is used lot of times at the input as the input front ends for an oscilloscope alright. So, we are going to summarize over here, what we have seen over the last couple of classes are basic techniques of improving the bandwidth of making wideband amplifiers.

So, wideband amplifiers what we did was we said that yes. We know bandwidth estimation techniques, and we know how to make better and better amplifiers using more and more devices. But these techniques have evolved over time when discrete even discrete devices were used and as a result each and every device was expensive. So, each and every device was used with caution and as a result you ended up with single device amplifiers with good bandwidth. So, we explore this particular arena, and we saw but; that by the process of increasing the order of the load instead of a first order load, if I have a much higher order load I pay in terms of delay. But I can get better performance in terms of bandwidth.

So, that is what we saw first we put 1 inductor over there, then we put 2 inductors, and then we put 3 inductors, and as I put more and more inductors the system became more and more complex. I did not attempt to analyze these systems, I just hand wave my way through I said qualitatively what happens you really have to set circuit simulator or you have to set with pen and paper rill hard with probably, some kind of software like mathematic or MathCAD or something which can solve algebraic equations. And, you have to work your way through the mathematics, and you will indeed see that you get significant bandwidth performance improvement. We also replace the 3 inductors with pair of mutually coupled inductors that is not difficult to build on an integrated circuit.

3 inductors is costly, more area, mutually coupled pair of inductors is much less costly you can build using the same area 1 inductor on top of the other or just double the area of 1 inductor on the side of other, that reduces the coupling. So, 1 inductor on top of the other is preferred when it is when you want a mutually coupled pair of inductors. So, that is significantly low in terms of area penalty and you get the same net result. So, that is called a t coil. And, then we did not really discuss the shunt series amplifier this I am going to discuss in the next class, but; kind of introduced to you that maybe you can pay in terms of the gain and improve your bandwidth, it is done a lot of times. Thanks and will take it up from here in the next class.

Thank you.