CMOS RF Integrated Circuit
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Module - 05
High-Frequency Amplifier Design
Lecture - 16
Method of Short Circuit Time Constant

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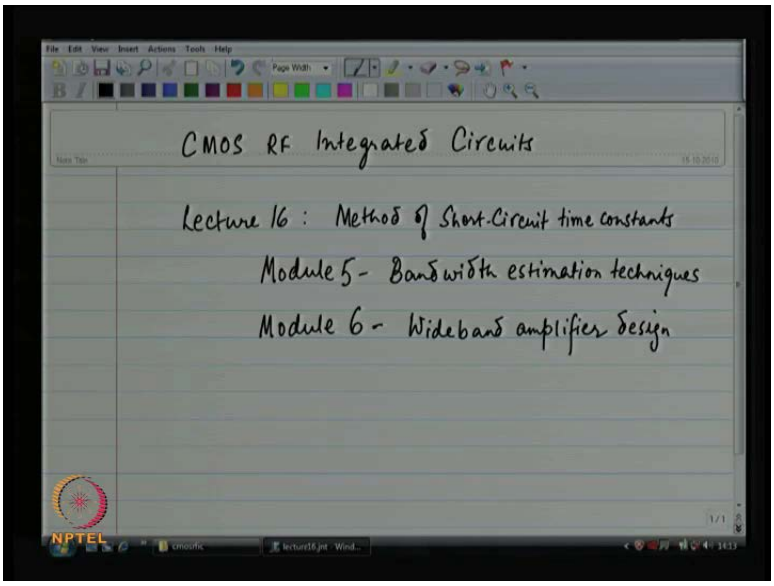

Welcome to CMOS RF integrated circuits; today is lecture 16 as part of lecture 16, we are going to finish with the fifth module that bandwidth estimation techniques as part of that I am going to discuss the method of short-circuit time constants. And, then hopefully time will permit and if time does permit we are going to start the new module called on a white band amplifier design. So, that is the plan for today, first am going to try to finish the old module and then, we shall start the new one. So, earlier in the previous lecture we were discussing the method of open circuit time constants.

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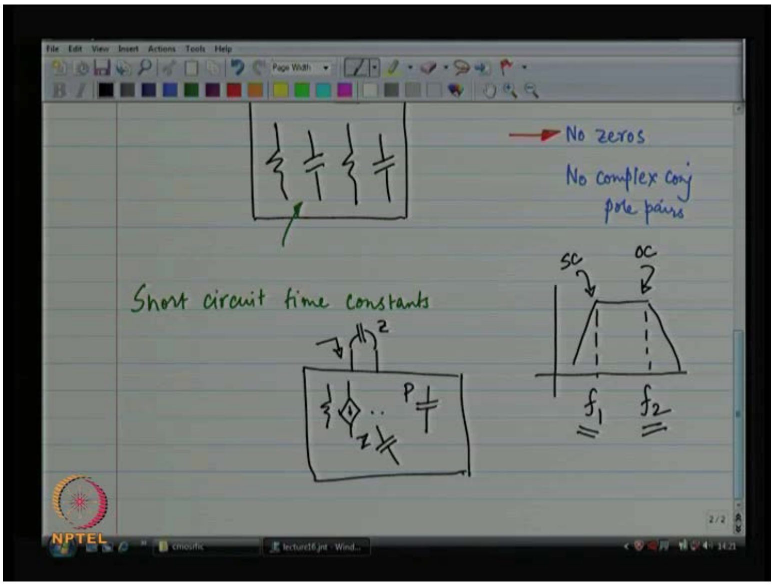

So, the method was basically you have a network full of resistors and capacitors; you have got an input and an output. So, step one; the question is what is the bandwidth of this network? This is what you want to find out. So, to do this, so what you first do is you pull out one capacitor out of the network. So, you have pulled out one capacitor out of the entire network. And, then the next step is that you kill all the other capacitors inside the network when I say kill all the other capacitors this is the method of open circuit time constants.

So, you open circuit all the other capacitors inside the network right. Then, you replace the capacitor that you have pulled out by a voltage source and find out the current that would generate. So, you replace these capacitors by a voltage source and find out the current that it draws. So, that voltage source divided by the current will give you the resistance.

In other words you do not have to do any of this replacements, in other words what you need to do is you find out the impedance looking into the network from this 2 terminal points between these 2 terminals. What is the impedance that you see, once you have got that figured out then, R time C that resistance time C will give you the time constant corresponding to that particular open circuit time constant right, this was the method.

Now, we discussed that this works when our circuit does not have any zeros. It works in the absence of complex conjugate pairs of poles alright so, this is the general technique.

So, we did quite a few example of this, and we figured out how we can use this particular technique to our advantage, and how we can work wonders alright. Now, what the method of circuit time constant addresses, is this fact that open circuit time constant do not address, this method does not address any zeros.

So, the method of short circuit time constants addresses just this, it helps you figure out what are this zeros of the circuit. Now, to do this we have to be very careful you have to first know which are the capacitors responsible for poles, which are the capacitors responsible for zeros failing to do this is going to land you into trouble alright. So, we first need to know an advance which capacitors give us the poles which capacitors give us the zeros.

If we do know them then, the method of short-circuit time constants is as follows. All of the capacitors that are responsible for zeros have to be short circuited when you kill them. All of the capacitors that are responsible for poles have to be open circuited when you kill them alright. So, let us relook have got a network I have got a lot of resistors you know g ms etcetera, and have got some capacitors which give me poles I am writing P for them, and have got some capacitors which give me zeros I am writing Z for them.

Now, the method is as follows; you pull out a capacitor responsible for a zero. You pull that capacitor out, and you kill the other capacitor inside the circuit, when I say kill the kill the other capacitors, what you do is you open circuit all the poles capacitors, you short circuit all the 0 capacitors right. Now, what you need to find out is the time constant that you see looking in from this two terminals rather the resistance that you see looking in from those 2 terminals, resistance times the capacitance that will give you the time constant alright. You do this for all the zeros that you have, all the 0 contributing capacitors that you have, right.

And, that will give you 1 by that total time constant is going to give you the lower cut-off frequency of your circuit. So, the method of open circuit time constants is going to give me the higher cut-off frequency. The method of short circuit time constants is going to give me the lower cut-off frequency. In other words; if I pull out the capacitors responsible for the poles kill everything else, when you kill everything else you have to follow the same strategy. You shot out all the capacitors responsible for zeros, you open out all the capacitors responsible for poles right.

So, you pull out all the poles contributing capacitors, find out the sum of the time constant 1 by the total time constant will give you f 2. You pull out all the zero contributing capacitors, find out the total time constant 1 by that time constant will give you f 1. The proof of this is very similar to the proof of open circuit time constant the earlier strategy. So, the proof is very similar here, the assumption is that my transfer function has only zeros and no poles. So, very strange assumption and that is how it is. So, we are going to assume that we have got everything only zeros in our transfer function alright.

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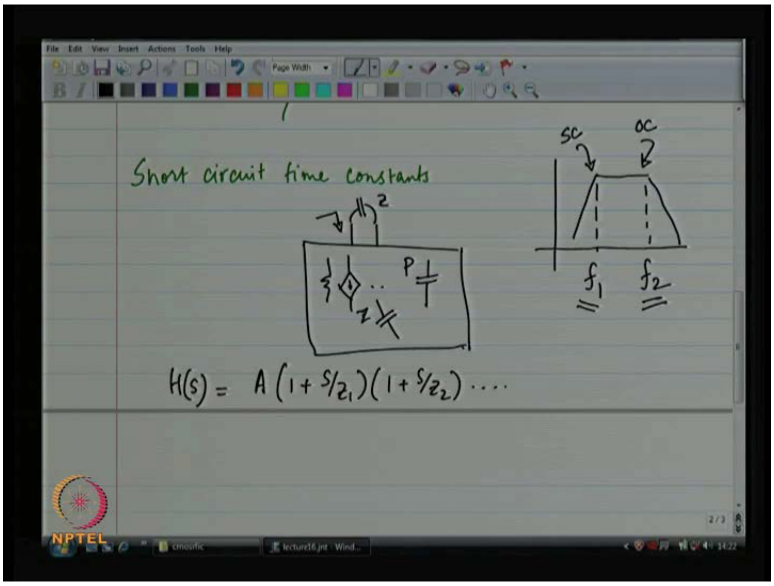

So, these are all the 0 contributing capacitors this is what it gives, you do not include the poles contributing capacitors in the short circuit time constant calculation. Similarly, you do not include the 0 contributing capacitors in the open circuit time constant right. So, this should solve some problems, what problems are it going to solve is going to solve the decoupling capacitors problems, right.

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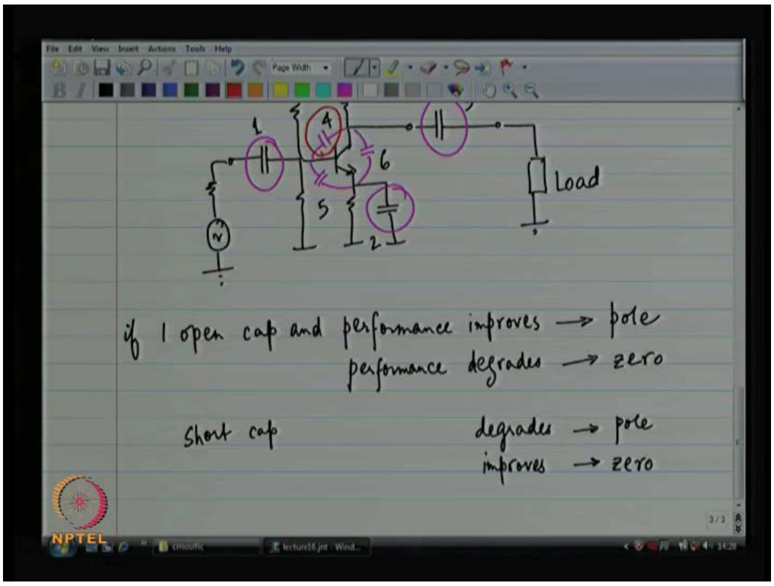

I mean, you all are familiar with this kind of a circuit, right. Now, typically what we do is these 2 capacitors actually, these 3 capacitors are supposed to be large capacitors. Remember, these are not parasitic once, these are the capacitors you intentionally put there, and you make them large. So, that at A C signal goes through at D C the signal is blocked right, that is the thing and behind putting those capacitors. These are the capacitors that contribute zeros and not poles in the system. If you say that these are large capacitors I am going to treat them with the O C time constant method then you will get ridiculous answers, as an you are killing your bandwidth every time you make the capacitor larger right. That is not the case; these capacitors affect the lower cut-off frequency.

So, the next obvious question is how do I know, which capacitors contribute poles and which capacitors are responsible for zeros. I mean there are so many capacitors here, this got to be some capacitance here, I do not know where else right. All kind of capacitors are there everywhere. And, sum of this you’re saying are contributing poles, some others are contributing zeros, how do how do I know which is which. How do I know which capacitors include my O C time constant, which to include for short circuit time constants, the answer to that is you have got to decide, and the way you decide is this. If you open the capacitors and see that you performance improves then it is got to be contributing a pole.

So let us look at it; let us call it 1 2 3 4 5 6. If I open out the capacitor number 1 the signal does not reach the base of the transistor. So, clearly performance is not going to be improved. So, it cannot be pole contributing capacitor. If I open capacitor number 2 then, the configuration becomes like an emitter follower gain reduces. So, therefore, the performance decreases. So, therefore; it cannot be a poles contributing capacitor. If I open capacitor number 3 then, once again signal does not reach the load and as a result it cannot be a poles contributing capacitor, right. If I open up capacitor number 5 then, life is better I have got less capacitance looking into the base of the device. So, therefore; the performance is going to increase. So, therefore; capacitance number 5 is going to create a pole.

If I open up capacitor number 4 affect is grammatically reduced. So, therefore; it is a poles contributing capacitor. If I open up capacitor number 6 is basically load in the output you reduce the load performance is going to improve. So, therefore; capacitor number 6 is also creating a pole, right. So, this is how you decide which is contributing a pole, which is contributing a zero. Alternately you can also do the following, you can short the capacitors and if the performance degrades then, it is a pole. If performance improves is got to be a zero. You can do either of these 2 experiments to figure out, if that particular capacitor is responsible for a pole or for a zero.

Now, your home work problem is to figure out capacitor number 4 that is going to be home work problem, I would not give you the answer right away. Capacitor number 4 creates both a pole and a zero. So, how do you figure out what needs to be done, I would not give you the answer, and think about it try to work on it, and then we shall see. With this I am going to close the topic of open and short circuit time constants, bandwidth estimation. So, we have done enough, we know how to work with a circuit. So, given a circuit we can compute quickly and estimate if 3 db frequency, both the upper cut-off frequency and the lower cut-off frequency.

So, we know how to do that right now. Next we are going to move on to the next module that is white band amplifier design. We have learnt how to estimate bandwidth etcetera, but; of course, this techniques have a couple of very serious flaws. Flaw number 1 is that you cannot handle poles and zeros at the same time, right. And, flaw number 2 is you cannot handle a pair of complex conjugate poles or zeros for that matter, strange things are going to happen to track.

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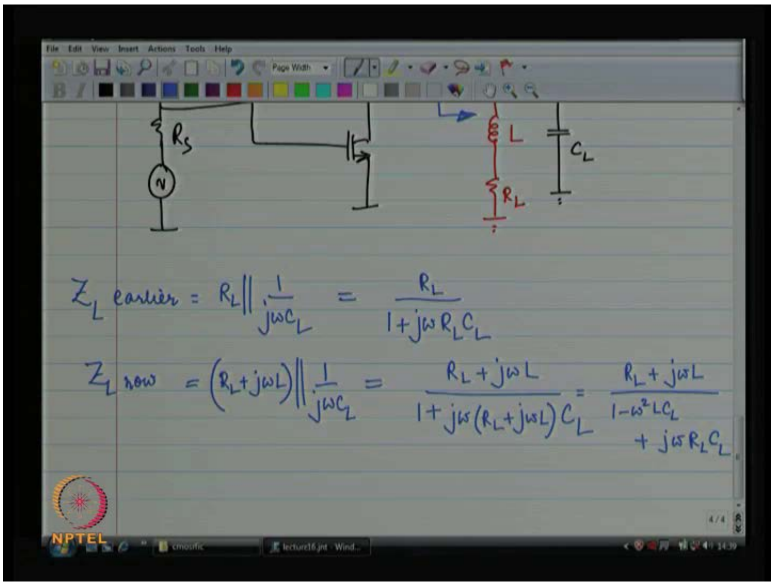

So, consequence of this is that you cannot handle circuits, which have both inductor and capacitors. Unfortunately that is how our life is going to be. So, we wanted to design a white band amplifier, and this is where we had started from right. This was our first cut circuit, we did the method of O C time constant, we figured out it as got some bandwidth what was it 75 mega hats with some rough numbers I gave you. Then, in the next step we improved it be added one more device, and it was improved then, in the next step we added another device it improved, even further. And, the final step we added even one more device and I got the bandwidth of what was it 250 mega watts or 280 mega watts or something like that reasonably good bandwidth.

So, I got something like 300 mega watts, I have forgotten the numbers right. How did we do this? We kept on adding devices and solving problems. So, wherever we had a problem with recognize the problem we added we added another device over there to fix the problem. So, that was R design strategy alright. Now, this works when we talk about integrated circuits, you can keep adding devices, because; devices are free on an integrated circuit can use as many devices as you want right. Might cost you a little bit of area, but; really I mean believe me it is nothing compared to what you are going to burn in terms of other accessory circuits. So, E S D protection and this and that is going to cost you much larger area penalties then, this extra 1 or 2 device.

So, 1 or 2 device is no big deal, digital circuit routinely used millions of transistors right. So, why cannot we, we can use as many devices is as you want to use. Now, earlier before integrated circuit made their headway, when products used to be built out of discrete components, lot of engineering had taken place, people sat and figured out how to make white band amplifiers even though they were not on an integrated circuit right. So, there in those times the philosophy was that every device I had is a penalty I have to pay for that device, I have to buy the device, I have to put the device over there, every active device is a penalty, costly. Passive devices I still have to buy it, but; it is not as expensive as the active device. Active devices are costly.

Each individual active transistor is a price penalty, can I do something with passive instant can I improve the bandwidth of this amplifier with passives. So, this question was explored had been explored long back and people came up with some rather nice solutions, that are still interesting. Even though we are on an integrated circuit, even though, we are allowed to use as many transistor as you possible transistor is free, even with that philosophy even then, there are some beautiful things about the older philosophy. Where, transistor were expensive, passives were not as expensive, there are some nice things about that. And, we should explode those strategies as well.

So, coming back what can I do to this circuit without using more transistors, more MOSFET s. I am not allowed to use more MOSFET s, what else can I do to this circuit to make it half a wider bandwidth. So, this is the question what can you do to this circuit to make it have a wider bandwidth. Now, the answer that came up was this, you know R L and C L were anyway split in my older topology as well, and with the extra devices had split up R L and C L. So, I am giving myself that freedom.

So, the first suggestion is that why do not I add an inductor over here, what does this do. So, let us take a look, we are going to compute the impedance looking into the load network. Earlier the load network was R L in shunt with C L, and Z L earlier was basically, R L in shunt with 1 by j omega C L alright. In other words this is equal to R L by 1 plus j omega R L C L. So, this was my load earlier, what is it now? So, now it is R L plus j omega L in shunt with1 by j omega C L, which is equal to R L plus j omega L divided by something like this. And, you can simplify this a little bit, can be simplified little bit in this fashion. So, what have we got? We have now got arrived at a system which has a zero and 2 poles. So, earlier our system had just one pole now, Z L has a zero and 2 poles alright.

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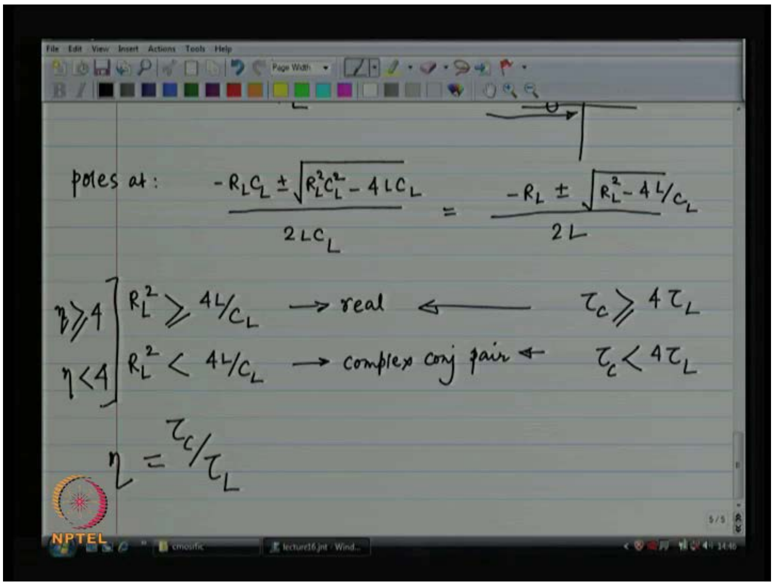

Where is the zero, what is omega such that the result is zero, omega should be equal to minus R L by L j omega. What is j omega? such that no to make am sorry, this is fine if you replace j omega with minus R L by L then, you get the numerator to be equal to zero, and you get nothing at the output. So, if you plot on the s-plane, if you plot the zero the zero is over here at minus R L by L. As you increase L, the zero moves closer and closer to the j omega axis, fine. Next, where are the poles? So, I just rewrite this 1 plus S times R L C L plus S squared times L C L that is my denominator polynomial.

So, what should S be such that the denominator is equal to zero. Please, let me know if I am making too many mistakes over here, I doubt, there are too many mistakes alright. This is my, these are the 2 locations of the poles. Now, if R L squared is more than 4 times L by C L then, the locations of the 2 poles are the real axis. If R L squared is less than 4 times L by C L then, there are going to become a pair of complex conjugate poles right. So, I am going to rewrite this condition in a fashion that suits me. How I am going to write it is as follows I am going to say R L times C L as got to be 4 L by R L for it to be real.

And, similarly; R L times C L as got to be less than 4 L by R L for it to be complex conjugate pair, this is my rebranding. I am going to further rebrand this, and say R L times C L is really the time constant from R L and C L, and L by R L is the time constant from L and R L. So we are going to rebrand this same result am going to rebrand it, and call this R L times C L has tau C. And, I am going to call L by R L has tau L, L for inductor, and C for capacitor alright. Further what this means is that if I write tau C by tau L, if I call this something what you want to call it eta.

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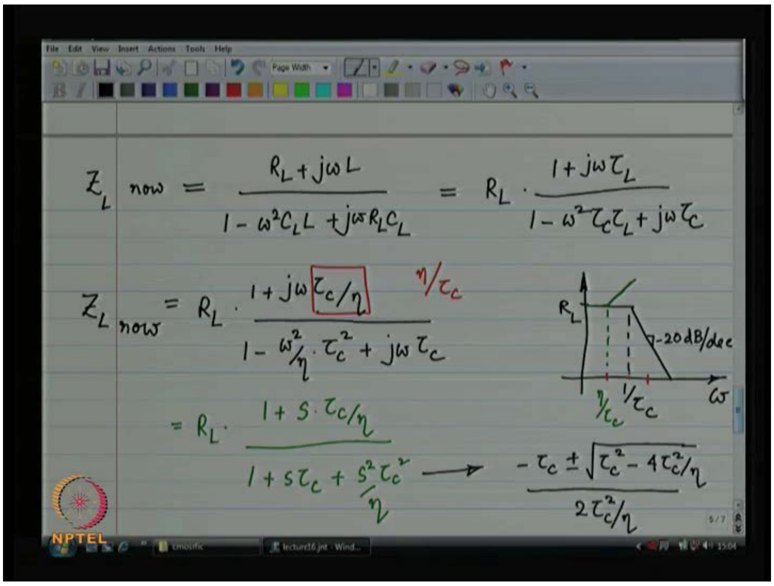

Let us call it eta alright. Let us call, let us define eta as tau C by tau L. And, this will basically give me that condition number 1 basically means that eta should be greater than equal to 4, condition 2 eta has got to be less than 4. So, let us rewrite our, so I am going to rewrite this in this fashion, instead of talking about L and R L and so on and so forth. Let me just take R L common outside, numerator is going to be 1 plus j omega times L by R L, L by R L is really tau L, and in the denominator I have got 1 minus omega squared times C L times L. C L times L is also equal to C L times R L times L by R L. So, this can be recast as tau C times tau L. And, then the third term is plus j omega tau C. So, I have neatened it up a little bit. And, further what I am going to do is I do not like too many time constants, I am just going to use this eta variable and brand everything in terms of eta.

So, everything is in the term of tau C and eta. I, tau L is equal to tau C by eta. So, this is my load network. Now, if you examine the load network earlier, it used to look like this, at D C the value was equal to R L and then, at the frequency of one by tau C it starts dropping at 20 db per decade. And, of course, the gain is gm times or rather, I forgotten the output impedance over here does not really matter; gain is g m times the output impedance in shunt with a this load right. So, that is also going to exhibit similar behavior, the net gain is also going to exhibit similar behavior fine. Now, let us examine what is the new load impedance? The new load impedance has got a zero, where is the zero? Zero is that eta by tau c.

So this is one by tau C, and if eta is a more than 1 then the zero comes for the down if eta is less than one then, it comes over here. So, let us say it comes earlier, in that case you will be off to a flung start and start off from zero, at zero frequency impedance is equal to R L. And, then at eta by tau C suppose eta is less than 0, less than 1 am sorry. You fly off at plus 20 db per decade then, you wait for your poles. Where are the poles? If eta is less than 1 then, what have we got here, if eta is less than half and what are we got? We have got a whole squared no, if eta is equal to 1 then, we have got a pair of complex conjugate roots right. At 1 minus, one of those cube roots of 3 right, you remember if eta is equal to 1 then, you have got something is 1 of those cube root of 1, am sorry.

So, you will get these 2 points, you will get all 3 coefficients one, one and one. If you draw the root locus plot, and what you are going to find is that has you change eta the locations of the poles change in this fashion. The traverse an arc with centre right over there that is what you are going to find, as for example; as eta tends to 0, if eta is equal to 0, you do not get I mean your 0 is a really becoming very important. You have got to multiply everything or by the eta, the term in the middle the term with S. So, you have got to rewrite this as. If eta approaches 0 then, what you are going to get is basically 2 poles on the j omega axis. That is what you will end up with, is that right? Might not be correct, if eta is equal to 0 then, I just get one pole at 1 by tau C. I will have to check on that.

What we are basically doing is as you tweak this value of eta, the location of the poles changes. What you going to find out is that at eta equal to 4, you have got the 2 poles to be on top of each other on the real axis. Exactly at eta equal to 4, you are going to get 2 poles on top of each other, 2 real poles on top of each other. So, are else. So, basically you will get minus R L by 2 L as the 2 poles. The entire quantity we need the square root is going to become equal to 0 at eta equal to 4. Eta less than 4, we are going to get complex conjugate pair of poles, eta more than 4, we are going to have real poles.

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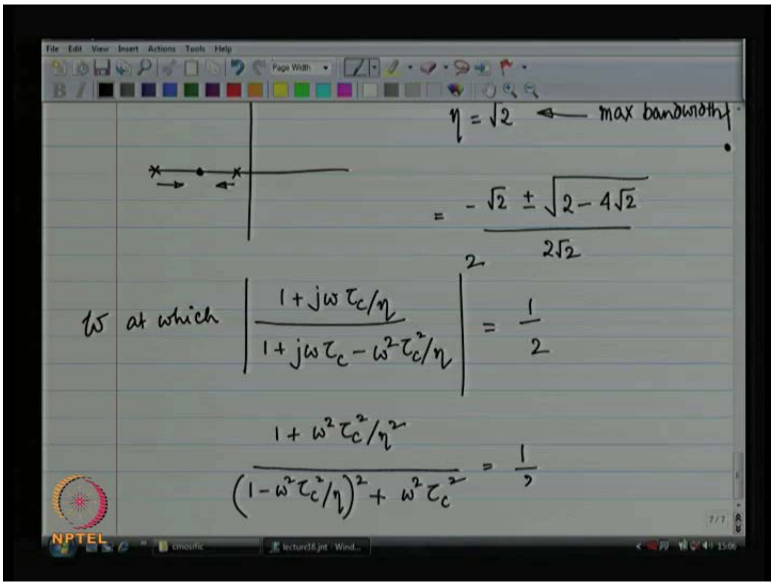

So, if you remember your control theory; what this leads to or what happens when you change the locations of the poles, and what happens when you have got 2 poles, the poles move and then, from one point they start arcing out in the form of a complex conjugate pair. What happens? Do you remember, first of all the root locus plot I think have made a mistake in the root locus plot. It does not go in this fashion; I think this is what it is. Correct me if I am wrong, the 2 poles are at where are the locations of the 2 poles? These got to be mistakes here, is this correct? These are the locations of the 2 poles and they do move seemed to move in the form of an arc towards the j omega axis, alright.

It does not matter, what matters really to me is the value of eta itself. If I make eta more than 4 then, you see that the quantity under the square root is positive and as a result you get 2 real roots. If I make eta less than 4, then the quantity under the square root is no longer positive and I get a pair of complex conjugate roots. As I make eta lesser and lesser this quantity becomes larger and larger in the imaginary value.

Now, once we understand this more or less, let us plug in some numbers. Let s say eta equal to root 2; let start with eta equal to root 2, when eta is equal to root 2 what I get for the pair of complex conjugate poles is minus square root of 2 plus minus square root of 2 4 times root 2. So, what is the square root of 2 minus 4 times root 2, this is definitely going to be less than 0. If it is less than 0, then you are going to get a pair of complex poles, alright. one can show that at this particular value of eta you get the maximum bandwidth. What is the bandwidth? The bandwidth is the frequency at which this impedance becomes one by root 2 times the magnitude of Z L now becomes equal to 1 by root 2 times R L, alright.

In other words the frequency at which this particular quantity 1 plus j omega tau C by eta divided by 1 plus j omega tau C minus omega squared tau C squared by eta. So, the frequency at which the modulus of this is equal to 1 by root 2. Can we work this out, is possible to work it out. Modulus of numerator divided by denominator is the modulus of the numerator divided by the modulus of the denominator. Modulus of the numerator is 1 plus omega squared tau C squared by eta squared, this is modulus squared. And, the modulus of the denominator is 1 minus omega squared tau C squared by eta the whole squared plus omega squared tau C squared, and am saying let this is got to be equal to half the figure out what my bandwidth is.

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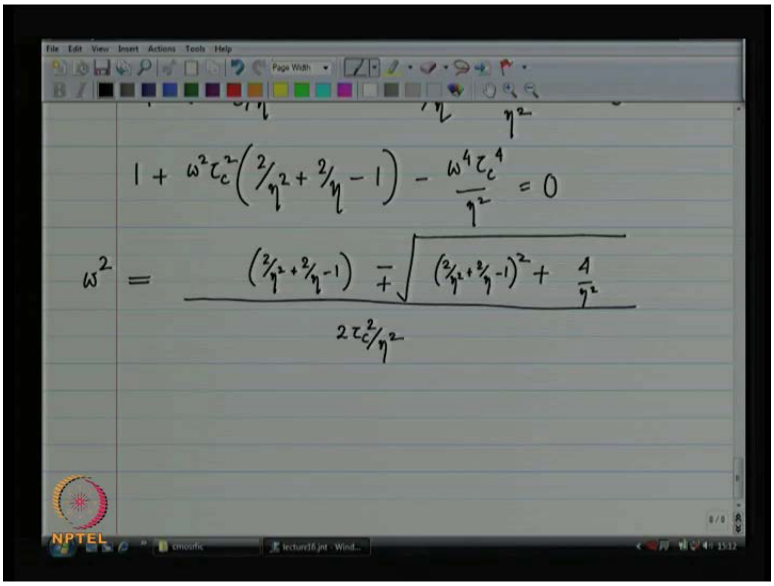

So, let me do little bit more of reshuffling, and am going to get 2 plus twice omega squared tau C squared by eta squared is equal to 1 minus twice omega squared tau C squared by eta, plus omega to the 4 tau C to the 4 by eta squared, plus omega tau C squared. These to quantities have got to be equal and then, you solve for omega to get the bandwidth. So, let us remove and then, let us simplifies I have got 1 plus omega tau C squared times 2 by eta squared plus 2 by eta minus 1 minus omega power 4 tau C power

4 divided by eta squared equal to 0. And, little bit more of mathematics and that will give me my bandwidth, I now have to solve for omega right.

So, I have got the quadratic so, let me recast my quadratic to with appropriate signs. So, I am going to put plus over here, minus over here, and minus over here and actually does not matter. Let us solve this for omega and that should give me my bandwidth. Can I solve for omega or omega squared. So, quadratic it is a quadratic and omega squared. So, in the first step I will just have to solve for omega squared. Hence, going to give me 2 roots for omega squared, which one are you going to take? So, it is good question, let us hang on a little bit.

So, omega squared is going to be given by how do you solve the quadratic equation here, famous formula. So, this is going to give me 2 roots. Now, thankfully both of these roots are going to be real quantities no, complex quantities involved looks like no complex quantities are involved. In this you really goof up no, you cannot just goof up over there is no complex quantities, at all both are going to be real quantities, alright. And, little bit more simplification is warranted let us cancel out tau C squared tau C power 4.

And let us get rid of this minus sign right. And, this is basically how we are going to compute omega squared, and from this we are going to compute the bandwidth. So, we use this result in the next class, and proceed from here. You are going to see nice things when eta is equal to 1 by eta equal to root 2 so on and so forth. So, let us stop here, and will proceed from here in the next class.

Thank you.