CMOS RF Integrated Circuits
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Module - 05
High Frequency Amplifier Design
Lecture - 14
Bandwidth Estimation Using Open Circuit Time Constants

Welcome back to CMOS RF integrated circuits. Today we are going to start with the new module, bandwidth estimation techniques. Today's lecture we are going to discuss method of open circuit time constants. Basically in the earlier module we looked at the MOSFET. Now, we are going to start using the MOSFET and first of all we are planning to work at 1 gigahertz, 2 gigahertz, 5 gigahertz right.

So, we need to be able to estimate quickly the bandwidth of a given circuit. So, this module that is the basic intention at given a circuit look at it, do a few calculations, can you estimate the bandwidth that your circuit will perform at. Now, when we do this bandwidth estimation techniques first part, it assumes that the system is a low pass system, I have this basically, this technique has been developed and it is a very quick solution to a very difficult problem.

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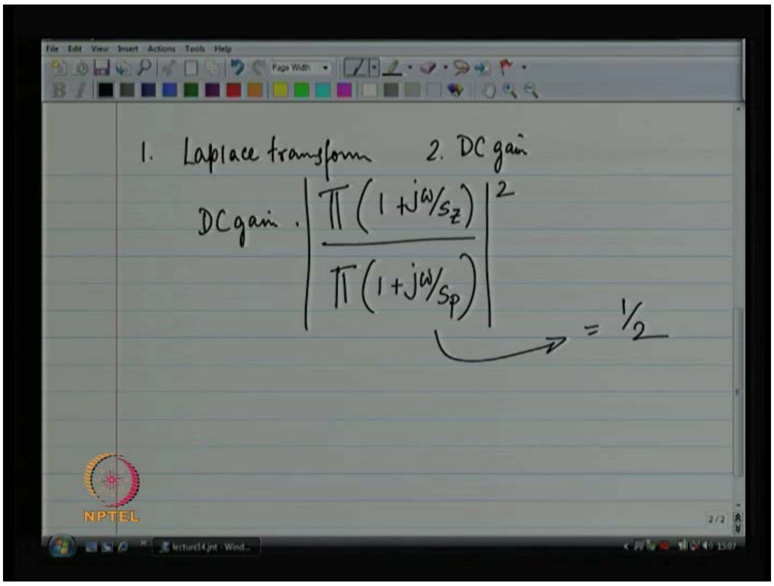

So, normally when you want estimate the bandwidth how do you do it? You have this complicated circuit, you first find out the Laplace transform. Find out the D C gain and then, you have D C gain times some poles and some zeros. This is how you are the Laplace transform looks like. And, now what you want to do is, to find out let us see the 3 db bandwidth. You want to find out omega, j omega such that this quantity is equal to half. We need to find out omega such that this entire quantity is equal to half. So, very complicated thing very complicated business.

And, to do this properly, you would post probably need a computer. You would have to solve some equations in many roots; many roots are involved in these equations and, we already have to put a lot of efforts. Now, as an alternative what we have got is a quick solution it turns out corrects very accurate. Also this method is called method of open circuit time constants.

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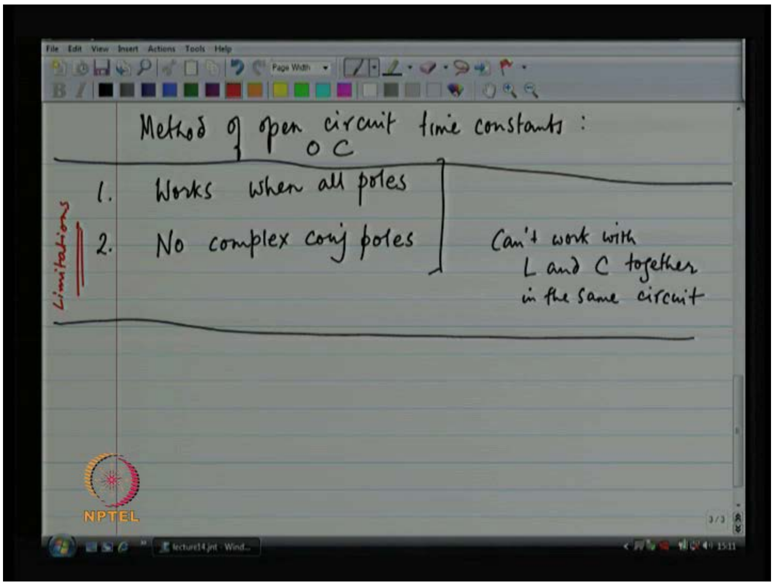

Now, before I start with the method of open circuit time constants, am going to tell you when it works, when it does not work. It works when the system as only poles, that is the basic most important limitations that we have got. When the system has only poles no zeros then, method of open circuit time constants works very well. Second thing is that all the poles are, have got to be real poles complex conjugate poles create problems. Complex conjugate poles can be worked with in those time open circuit time constants I call this O C time constants method.

You still can work with complex conjugate poles, pairs of poles, but it does not happen to give you very accurate results any more, somewhat lower accuracy like. So, these are the main to limitations. What does this mean? This means that you cannot work with inductors and capacitors at the same time. Since, our MOSFET has a lot of capacitors means; basically, means that as soon as I put in inductor over there I most probably would not be able to work with open circuit time constants any more, it is not go to give me most accurate result any more alright.

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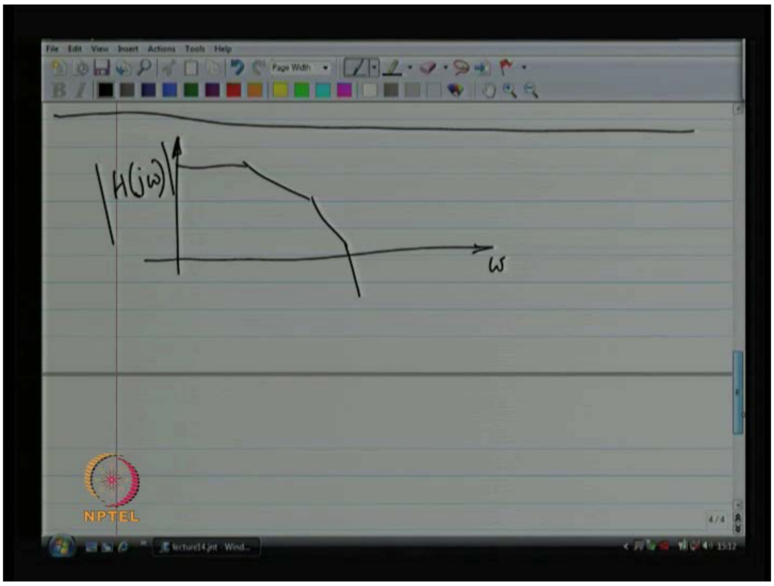

So, these are the limitations, plus points are of course, it is a very quick to forehand competitions, and it is very accurate usually if you abide by these rules, it is very accurate. Works when all poles, what does this mean? This means that you have got a low pass transfer function, everything is a pole. So, not even complex conjugate poles. So, Bode plot as got you look like this, something like this, right. This is how your Bode plot needs to look like for this method to work. Now, how does it what sir sorry, what do we do.

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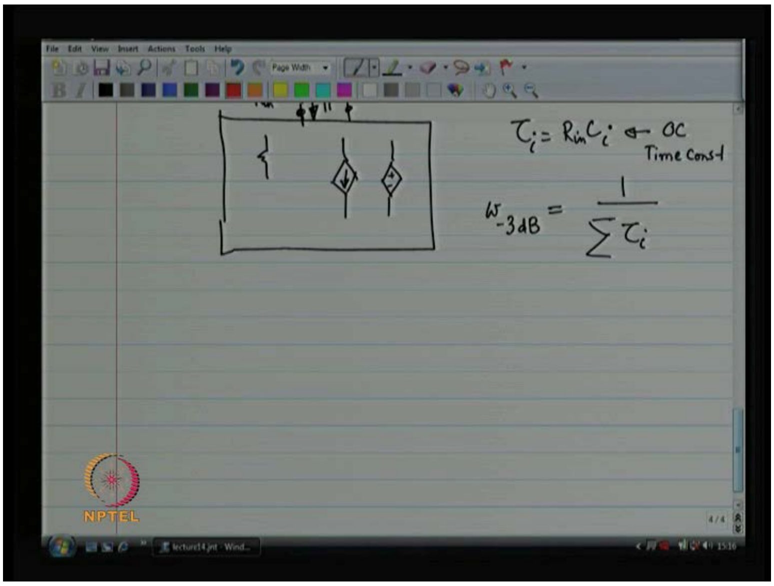

So, the method is this that if you got is a circuit with a lot of registers, capacitors, g m's, control voltage sources, control current sources, whatever you want right you have got this circuit with you. And, you want to find out the bandwidth of this circuit, bandwidth of the transfer function of the circuit. So, what you have got to do is, you got to pick out every capacitor suppose; I have got a capacitor, I have got a capacitor between these 2 nodes. So, you stretch it out of the network and remove the capacitor, remove all the other capacitor inside. And, then basically remove means; open circuit open circuit all the capacitors inside, open circuit the capacitor which are holding outside.

And, you measure the resistance looking inside. Now, there are no capacitors in this, presumably there are no inductor in this circuit. So, you will only see a resistance you would not see anything which is capacitance or inductance, you will see a resistance. So, what is the resistance here, you measure that resistance. The capacitor that you removed that it is C, you measure this R in so, R in times C gives your time constant. You do the same thing for all the different capacitors in your network and tabulate your results. Now, the claim is that the 3 db bandwidth is the reciprocal of the sum of all of these time constants.

So, these are called the open circuit time constants. You open out all the capacitors and measure the time constants for one of the capacitors. You do the same thing for everyone some all of the open circuit time constants 1 by of that should give you the 3 db bandwidth in radians per second. The result surprisingly, it is very need very quick computation the result that you get out of this is very quick very accurate, because it ignores zeros that may or may not have been there. Usually the result is an underestimate zeros are only going to increase your 3 db bandwidth presence of zeros will only flatten out, if you got is 0 over here you got something like this, you got a 0 at this point.

So, zeros are only going to increase your bandwidth. So, as a result the method of open circuit time constants is usually an underestimate of what could have get right. Now, what I am going to do brought my calculate or over here, and I am going to do an example step-by-step work it out. So, first I am going to do an example step-by-step work it out then, my plan is to prove this method to some extent. And, then will do further and further examples.

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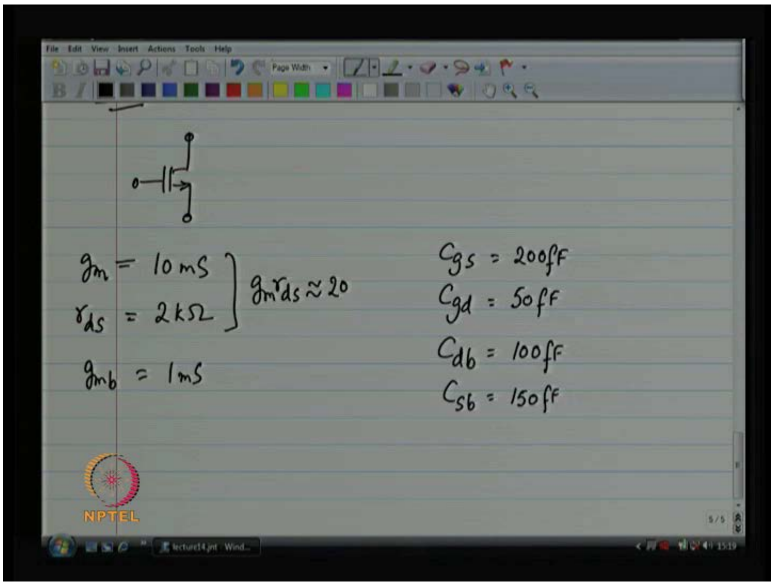

Suppose, I have MOSFET device, and the following are the parameter of my MOSFET device. I have pushed current through the MOSFET device such that g m is 10 mille Siemens r d s is 2 kilo ohms, someone notice that g m times are r d s typically a number around 20. This is fairly true over all technology known's some of, only gets worse does not get better. So, at modern technology notes 90 nanometers, 45 nanometers this number gm times are r d s actually much less than, 20. You might get something like 10, 8 something like that that is probably as high as you can get it. Little bit older you will get actually 20. So, g m r d s, I am giving you these 2 g m b let say it is 1 mille Siemens.

These are my D C parameters, let us forget about R G for now, get resistance let us forget about it. Then I have got my capacitances; let us say C g s is 200 femtofarads, C g d is 50 femtofarads, C drain to body is 100 femtofarads, and C source to body is 150 femtofarads. Let start with these numbers while, source to body less than drain, more than drain body capacitors, have put C source to body as 150 femtofarads drain to body as 100 femtofarads. Why source to body capacitance more? Because; the drain potential is higher so, drain to body is more reverse by accent source to body. So, junction is wider which means drain to body capacitance will be lesser, source to body capacitance will be more, respect to each other. So, these are the numbers, we are going to work with for this exercise.

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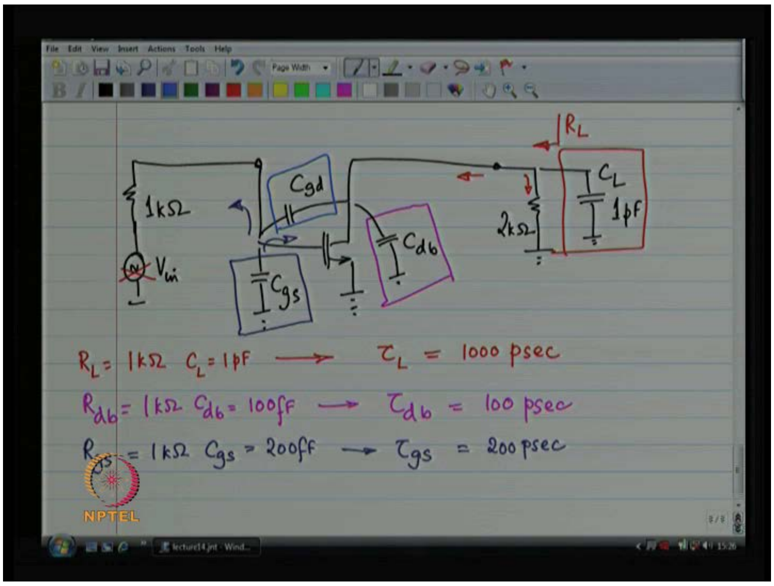

Let us say that I have got have voltage source which is inseparable from its source impedance. That is my source, and I am planning to drive this into my load, my load is 2 kilo ohms in shunt with 1 pico farad. And, I am planning to design this amplifier. So, as the first cut to the amplified that is going to be my first example; I am going to do this, this is my plan. And, I want to estimate the bandwidth of this amplifier. So, let us put all the capacitors in place and then, 1 by 1 we are going to pull out the capacitors and find out the time constants for each of these.

So, what are the capacitors that we have? We have got C g s, we have got C g d, C source to body is irrelevant because sources already at ground. We have got C drain to body, and we have got the load capacitors, and the MOSFET in addition we have g m, we have got r d s and g m d is between body which is grounded, and source which is grounded. So, the current going through g m d is going to be zero.((Refer Time: 17:17)) So, first step let us pull out the capacitor, first of all you let us pull out the capacitor C L and we want to measure the resistance when we pull out C L. So, C L is outside and we want to measure the resistance looking in over here with respect to ground, but of course, and what else all? The other capacitors are nulled, which means all the other capacitors inside or removed.

So, what do I see when I look in naturally you are also going to null V in, if nulled dot V in, what I see? I see first of all I see 2 kilo ohm looking down words, and looking into the MOSFET I see r d s of the MOSFET, right C g d is not there, C g s is not there. So, whatever is connected to the gate is really irrelevant. So, I basically see r d s of the MOSFET in parallel with the 2 kilo ohm that is at the load. And, my r d s happens to be also equal to 2 kilo ohm. So, R L is 1 kilo ohm, which means that the open circuit time constant is 1 kilo ohm time's 1 pico farad which happens to be 1 nanosecond. I am going to write it in terms of picoseconds so, 1000 picoseconds.

Alright, done fine, next we are going to work with C db. C db is very similar to C L basically; you see R L do not see anything else. What is the value of C db? C db is 100 femtofarads that was quick. Next, we are going to work with C g s, you remove all the other capacitors, you remove C g s, and find out what is the resistance that C g s is going to see. On 1 side C g s is going to see 1 kilo ohm, on the other side C g s is going to see the gate of the MOSFET which as infinite resistance.

So, basically it is going C 1 kilo ohm in parallel with infinitely large resistance. So, basically C is 1 kilo ohm. Now, what have we got left over here, we have got C g d left. So, that is going to be the last one, I kept it for the last, because it also little tricky to do this one. Why is it tricky? Because it is no longer with respect to ground, all the other capacitors are with respect to ground.

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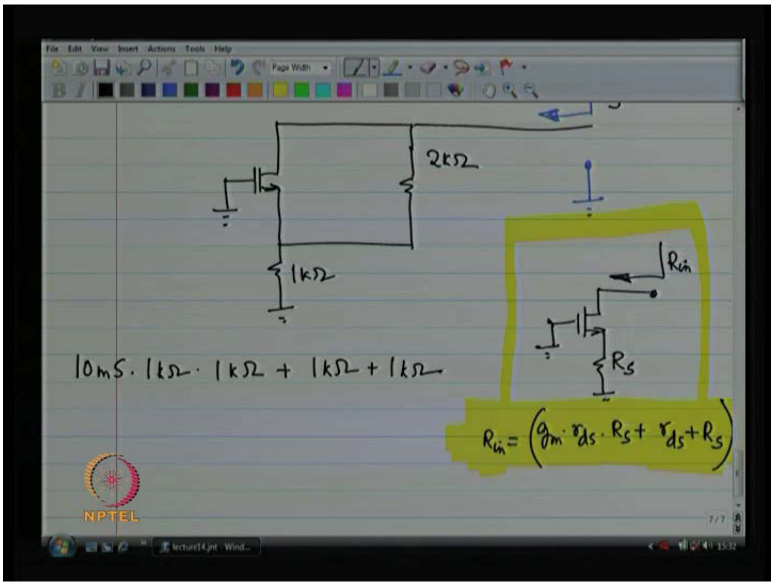

So, this is what we have got, let me rewrite it, this is what we have got, and I need to find out the resistance seen between these 2 terminals. So, I apply of voltage source between these 2 terminals and see, what is the current going through? Now, before I do that I would like to do something, I would like to redraw my circuit with the change in the ground terminal.

So, before I redraw my circuit; let me connect all the grounds together. And, now what I want to do is I want to call this particular note as my ground, see ground if any you can reference to any particular terminals. So, ground is just word for the reference potential. So, instead of choosing the sources the reference potential, why do not I choose the gate as the reference potential? Can always do that and as soon as I do that I am going to now I will have to redraw my network a little bit to make it look convenient right.

So, this is how I am going to redraw my network right. So, what is the impedance when I look in over here? Now, you remember how to do this, well if you have done your analog circuits and remember, you would probably remember this configuration, which is basically identical to this. And, input impedance looking in from the drain and I have got R S sitting on the source is basically g m times r d s times R S plus r d s plus R S that is basically the precise expression for the resistance looking the drain. If you do not remember this, I suggest that you memorize this; this is a very useful formula, very useful formula.

So I suggest that you memorize this situation and this formula is going to come up again and again and again. So, it is better that you just memorize the expression. Anyway so, what is going to be my resistance looking into the network over here? So, what is the R g d going to be equal to here, is basically identical to what have indicated, there is that r d s is no longer just r d s is r d s in parallel with 2 kilo ohm that is all right. So, my r d s is 2 kilo ohm in parallel with another 2 kilo ohm is basically 1 kilo ohm. So, what you are going to see is gm which is 10 mille seaman times r d s which is now 1 kilo ohm it is 2 kilo ohm parallel with 2 kilo ohm times the resistance attached to the source which is another 1 kilo ohm plus another 1 kilo ohm that is r d s plus the resistance attached to the source which is another 1 kilo ohm.

So, 10 mille seaman's times 1 kilo ohm happens to be a factor of 10 times 1 kilo ohm that is 10 kilo ohm, plus 1 kilo ohm plus 1 kilo ohm. So, this is equal to 12 kilo ohms. So, tau for the gate to drain capacitance is 12 kilo ohms times 50 femtofarads and that should give me 600 picoseconds. And, then my next step is to add up all of these tau's if I add up all of these tau's then, my total tau is 1900 picoseconds, which basically tells me that my 3 db frequency the bandwidth is 1 by 1900 picoseconds and radiance per second that is my quantity. So, that is 526 mega radiance per second divided by 2 pi and that will give me 85 megahertz, alright.

So, it turns out if you do the simulation of this with these particular numbers the result you will get is fairly closed. It is called be a little more than this instead of 84 maybe you will get 90, 85 to 90 something like that. This is basically the method of open circuit time constants, I did not spend much time on this even other tutorial as an example workout step-by-step I worked out every step of this it took me about 12 minutes or so, to work this out. So, it is fairly straightforward as you get more and more practice, you will be able to do it faster and faster. And, you basically get the bandwidth of your circuit. Now, next step before I proceed for that next step is to proof this.

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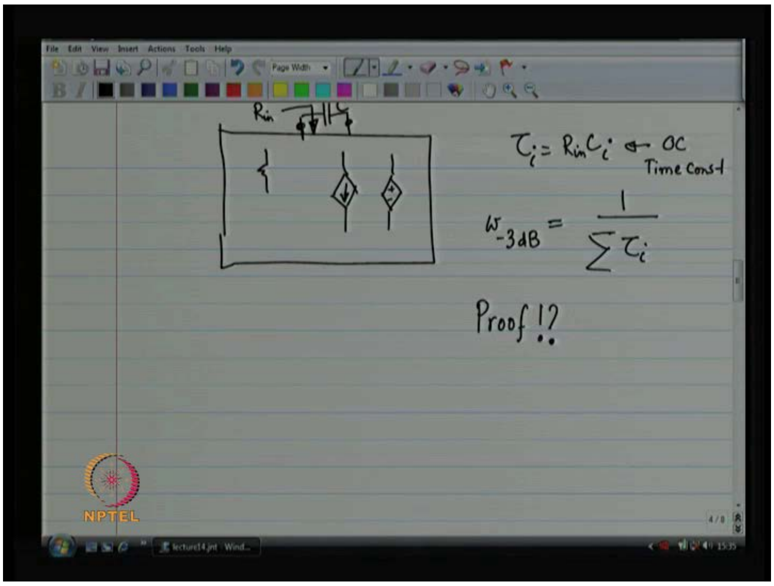

How are you going to proof this? I mean; we cannot just say so, that so, happens that it is fairly accurate and that is the proof no, that is not the proof. The proof is this that if you got a lot of capacitors then, each independent capacitor creates a pole. And, you can express your system remember we are talking about all pole system.

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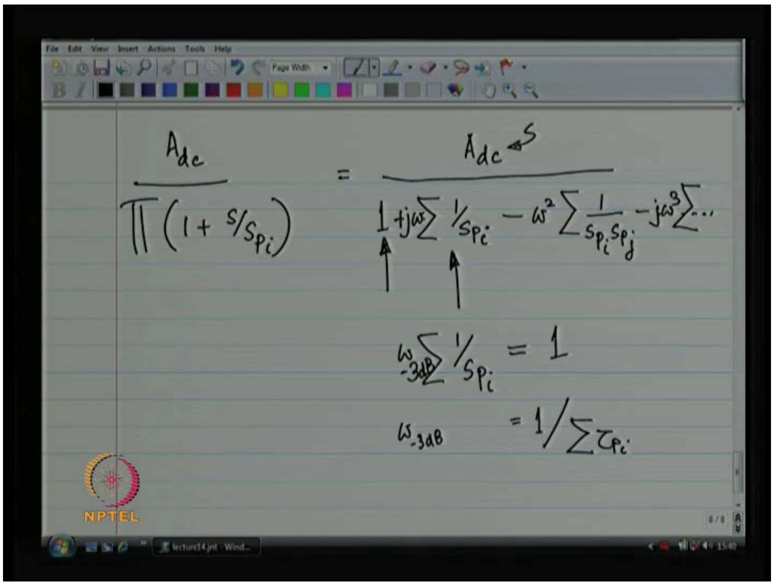

So, your system is basically your D C gain divided by the product of all of the poles, right. Let us see each pole is coming from each capacitor. So, n capacitors n poles this is not really true, but; let us see that what happens, n independent capacitor do get give me n poles. What is an independent capacitor? In our example circuit they have got C L and C db parallel to each other they are not dependent, you can club them into 1 whole thing. And, that will give me 1 pole right. So, that is why I said independent capacitor, we have got all of these poles, and if you simplify this then, basically you get one as all of the as the D C term. And, for the first order term you get s no, you get sigma S time 1 by S P i.

The second order term that is s squared something like this etcetera. Now, at the 3 db frequency, before the 3 db frequency arrives which term is important which terms are not important. So, before the 3 db frequency arrives my gain is approximately going to be equal to A d c. So, 1 is always there everything else is small while, everything else small because S is really j omega.

So, as you increase omega the contribution from the first order term increases. So, before the 3 db frequency arrives contribution from the first order term is not even there right. That is basically the idea. So, at the 3 db frequency it is got to be. So, if omega is small omega square is even smaller, omega cubed is even smaller right. So, the higher order terms are even smaller then the first order term right.

The first order term is the largest at the 3 db frequency, therefore; the contribution of the first order term is all that matters. Which means; that the 3 db frequency is the frequency at which the contribution from the first order term is equal to the contribution of the zeroth order term. Which means; that that 3 db frequency is such that condition I wrote down here, 1 by S p i happens to be tau p i right, which means that the 3 db frequency is 1 by that total of all of the time constants.

Now, if so, happens that whether I club to parallel capacitors or not, it does not matter. The time constant for 2 parallel capacitors is going to be the same. Time constant of C L is something, time constant of C db is something time constant of C L parallel C db is going to be the sum of the other 2, earlier 2. So, it does not really matter whether, I lump together into 1 parallel capacitance or not, which is why our technique basically worked. So, first I said that let us forget about all the independent capacitor business. And, now I went back and side that here, you can have all the capacitor independent of each other alright. So, this basically is a hand waving proof of my open circuit time constants technique.

The next step is going to be to continue with my example. So, we started with this as an example, and we continue and try to make a better amplifier. So, what you see in over here, the method of open circuit time constants; is that you can basically see for yourself which capacitor is contributing how much time constant, which capacitor is responsible for slowing down my system.

So, which capacitor over here, is the culprit why is my 3 db frequency only 84 megahertz, why cannot it make it 1 gigahertz, who is responsible? C L right, it looks like C L is contributing 1000 picoseconds, all the others are much less in terms of their contributions. So, if this design needs to be fixed, I need to fix the time constant corresponding to C L.

You agree. So, I need to see low impedance, when I talk about C L from C L I need to C L low impedance that is basically what is going to fixed my design right. So, what is your guess, what how do you create a low impedance? If you looking to the drain of a MOSFET, you see high impedance, if you look into the source of a MOSFET you should see low impedance right. So, you have to had a source follower over here, that is what is called do get you the low impedance that you want.

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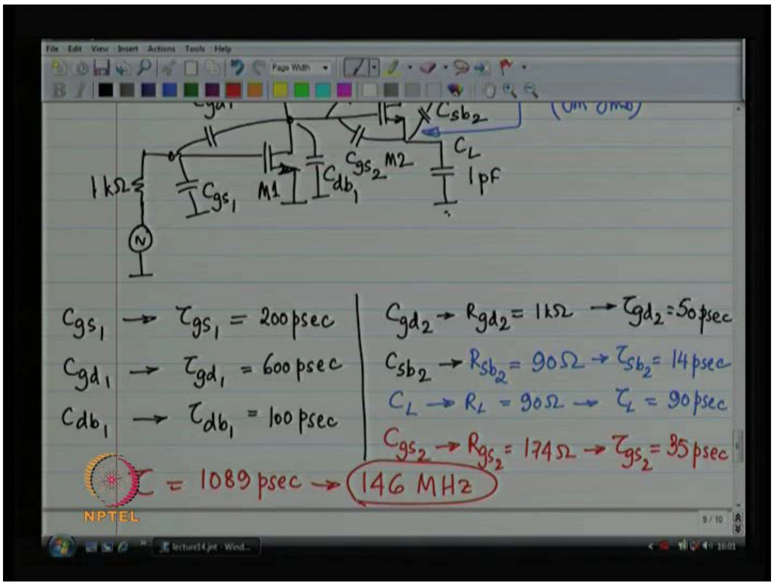

So, let us modify or design, let us say that I split the load into the resistive in the capacitive component, just for doing it. And, let us say I place the 2 kilo ohms over here that was my load resistance and the 1 pico farad over here. So and basically buffering I am putting a source follower to drive the 1 pico farad load capacities. So, this is my new plan, new gain plan. And, I now have to work out what is the bandwidth of my new circuit. So, do work out the bandwidth of the new circuit we need to place all the different capacitors.

So, let us number my MOSFET, let us call this M 1, second one is M 2 and I have got C g s 1, I have got C g d 1, I have got C drain to body 1 then, I have got C gate to drain of the second one, C gate to source of the first of the second one, C source to body of the second one, what about C drain to body of the second one? It is irrelevant, because both sides of that capacitor are at ground. Now, got C L which is 1 pico farad so, this is my situation right. What do I do? I am going to take each and every capacitor pulled out of the circuit and find out the resistance looking in.

So, to start with let me start with C g s, C g s 1. So, there no change from my previous calculation right, when I pull out C g s 1 remember C g d 1 is already not really there, the resistance I see still 1 kilo ohms. So, there no change from before which basically means that has got 200 picoseconds. So, this is done then, C g d 1 is there a change in C g d as for C g d 1 is concerned, when I added the new transistor, get thinks change as for as C g d 1 is concerned, as for a C g d is concerned. It things change still have the 2 kilo ohm over there. So, the resistance that I see is 2 kilo ohm, I have got what I use to have and then I have got a new MOSFET over there.

Looking into the new MOSFET; obviously, I see infinitely large resistance. So, it makes no difference right. So, nothing is going to change C db 1 anything else changed? Nothing else changed. That is why; I splitting into pieces right. Now, we have a few new once, have got C G D 2 is really between that gate of the second MOSFET and ground. So, I pull it out, I apply a voltage over there, what resistance do I see? The same resistance as the drain to bulk of the first MOSFET, which is 2 kilo ohms in shunt with r d s of the first MOSFET, which happens to be equal to 1 kilo ohm total, is 1 kilo ohm. So, I basically see 50 picoseconds.

Then, next one is C source to body 2; what do I see or even the load, what is the resistance that the load is going to see? As far as the load is concerned, it is basically going to see 1 by g m of M 2 is that all, what about g m b, s basically going to see 1 g m plus g m b. So, looking in from here you see 1 by gm plus g m b this combination right.

Body is at ground, gate is also at ground source is wiggling. So, basically see 1 by g m plus g m b, gm and g m b come in parallel each other. So, how much is 1 by gm plus g m b my, the gm that I chooses 10 mille Siemens, g m b is 1 mille seamen's. So, 1 by of 11 mille Siemens is about 90 ohms am sorry, 90 ohms. So, C s b source to body was 150 femtofarads right, 150 femtofarads is 90 ohms and as a result I get 14 picoseconds.

Similarly, load also sees 90 ohms. So, 1 pico farad I see 90 ohms. So, I basically see 90 picoseconds. Beautiful from 1000 picoseconds have brought it down to 90picoseconds with addition of a few more. What else I am missing over here, C g s 2 alright. Now, this is going to be difficult because it is no longer referenced to a ground potential.

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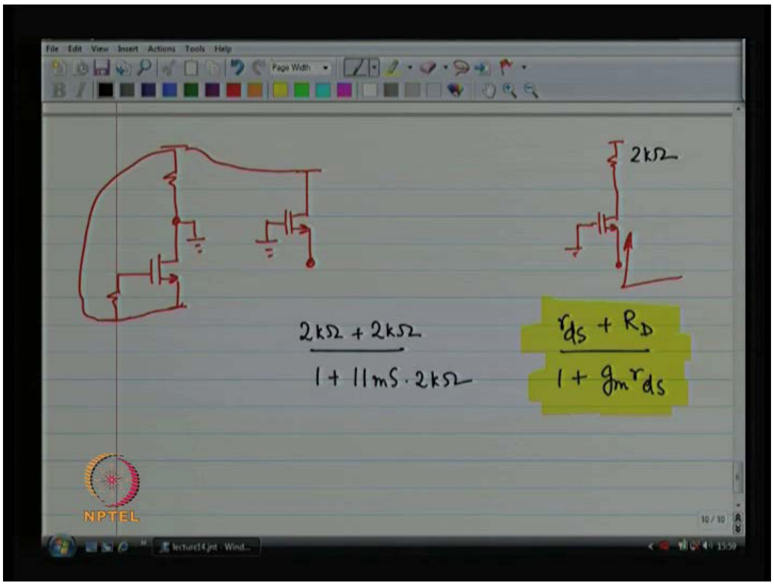

So, let us do this separately. So, it is like this, you remove all the capacitors, touch those to nodes with a multi meter, and find out what is the resistance that you see, that what you want to do. Now, to do that nicely, what I would prefer to do is to de reference one of the voltages, one of the notes and make that the new reference. So, instead of referencing everything the ground let me hook up all the grounds to gather right. And, instead of calling the old one my ground, I am going to call this my new ground right. So, now let me rearrange my circuit, my new circuit looks like this, that is how my new circuit looks.

I want to find out what is the impedance what is the resistance looking into the source. Now, to do that what are these values by the way this was 1 kilo ohm, this was 2 kilo ohm. Now, to do that first let me try to find out what is the resistance looking in this way, as the first step. What you see when your look in to the MOSFET from the source, this is the register over there. What you think, you are going to see. So, you replace the MOSFET with its g m, is there any current going into the 1 kilo ohm resistor no, there is no current going into the 1 kilo ohm resistor. So, therefore; that drop across the 1 kilo ohm resistor is zero so, therefore; the current through the current source also going to be 0, because V g s is 0 right. So, as soon as V g s is 0 the current going into the MOSFET is 0.

So, the impedance looking in over here is infinitely large. So, basically you need not even worry about all of this is as if is not there fine. Now, I just have this, and if you go back to your analog circuits the impedance looking into the source. When you have got the drain connected through a resistor to ground, gate is also connected to ground; this comes from a template it is basically look like this, r d s plus R D divided by 1 plus g m r d s right. This is also an important formula; I think you would be wise to memorize this. You be very wise if you memorize this important formula.

So, going by this simple important formula what we have is r d s is 2 kilo ohm, R D is also 2 kilo ohm divided by 1 plus g m is about 10, forgot g m b, g m b comes in shunt with g m. So, I have got 11 mille Siemens times, r d s is 2 kilo ohms right. And, this basically the denominator gives me a something like 2 times 11 is 22. So, denominator gives me a factor of 23, numerator is 4 kilo ohms. So, basically the resistance that I see is 4 kilo ohms divided by 23 which is 173 ohms. So 174, I am sorry. So, as far as that time constant is concerned I have got 174 ohms, I have got a get to source capacitance of to 200 femtofarads.

So, I have got 35 picoseconds. So, look at my new design I sacrificed my 1000 picoseconds instead now, I have got 4 different capacitors 50, 14, 90, 35 have; obviously, done better my total tau now is 900 picoseconds from earlier plus 189 picoseconds. So, I have got 1089 picoseconds. So, that gives me bandwidth of 146 megahertz right. So, we have done a fantastic job with this, I am going to stop, I hope the example kind of gave you an insight of how to work with open circuit time constants and what to do about that alright.

Thank you.