CMOS RF Integrated Circuit
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Module - 03
Passive IC Components
Lecture - 11
Transmission Lines

Welcome to CMOS RF integrated circuits. And, today's lecture is about transmission lines; as we were talking about in the previous class this is all part of the third module that is passive components. Now, in the previous class what we developed, we started with a piece of wire and developed a model for the wire. Now, we notice that the wires had a little bit of inductance per unit length, little bit of capacitance for unit length, some resistance per unit length, probably hope fully not, some conductance per unit length.

And, we cascaded these small, small units and made the complete wire and then, we were trying to do the business of analyzing. How a voltage or a current propagates through this wire and we came up with a couple of differential equations. 1 and 2 were the 2 partial differential equations that we started from, and we modified these partial differential equations. We came up with 3 and 4 I am sorry, we came up with these 2.

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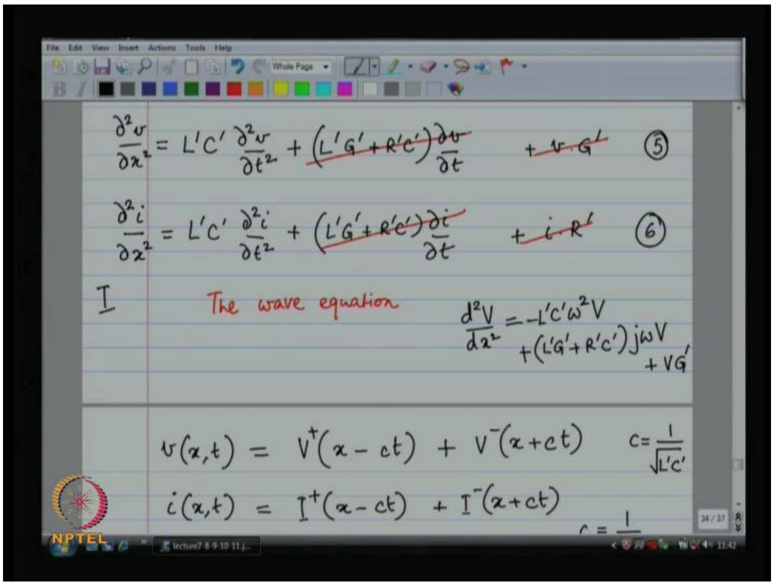

Let us call this 5 and 6 which are my final partial differential equations. Just that these 2 differential equations are not, are they are no longer coupled to each other, they are independent of each other now. So, I have got voltage, all voltages and I have got current all currents right. You have to remember that eventually the voltage and the current are in deep related, but not an issue right now. Then, we notice that the solution to this is basically the wave equation, you got a forward moving wave or backward moving wave. If I have the resistance and or the conductance to be non-zero then, it creates an attenuating wave. So, there is a way which attenuates overtime right. That is what we finally; we came up now. Just for this particular lecture, let us assume that R is 0 G is 0.

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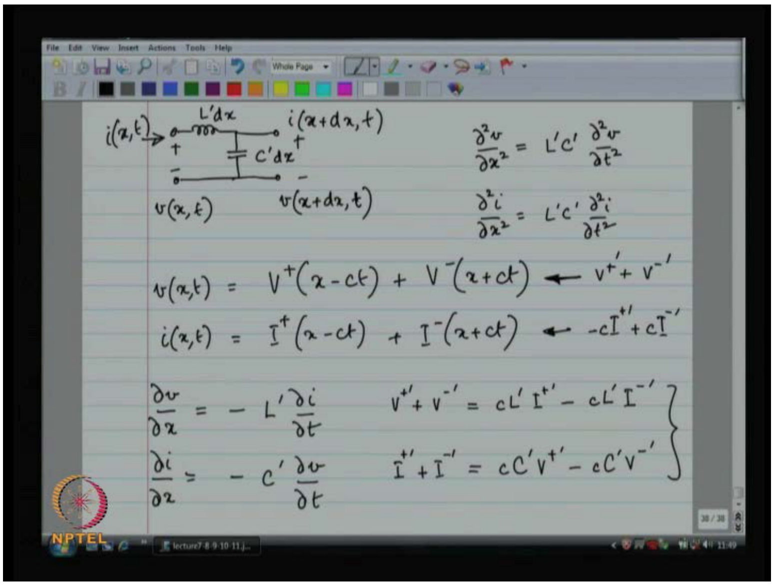

So, in that case my transmission line basically, looks like this. That is my basic transmission line and then you resolve those equations that we worked on yesterday and you end up with just these potions. Basically, i and v have the same relationships, as the function of x and a function of time. So, they have the same solution, the solution happens to be the wave equation, this is the wave equation the solution to the wave equation is something like this.

This a forward moving wave and there is a backward moving wave. V plus is my forward moving wave, V minus is my backward moving wave. Alright, what about the current? The current is also going to have a very similar relationships, but; this I plus and I minus basically are different from V plus and V minus, it can be anything. What is the relationship between the voltage and the current? That was our intermediate equation or original equation that we discarded, you remember equation 1 and 2. We discarded equations 1 and 2 early on, because they were coupled to each other right. But this is what is going to tell us the relationship between V and I.

So, these 2 relationships need to be added in, into my expressions for V and I and dou v by dou x, if I look at this as my solution V. dou v by dou x is going to become equal to the derivative of V plus, plus the derivative of V minus, right. What about dou i by d t? Dou i by dou t is going to be minus c times the derivative of I plus, plus c times the derivative of I minus.

So, the derivative of V plus is going to be equal to minus L prime correct, this is what happens when I substitute everything. What about the other version? The other version is very similar. So, I get I plus, plus I minus is going to be equal to c times, right. Now, I have got 2 equations my unknowns are my unknowns are I plus and I minus. Let us say I know V plus and V minus my unknowns are I plus and I minus. Can I figure out the values of I plus and I minus from these 2 equations. So, surely you can right.

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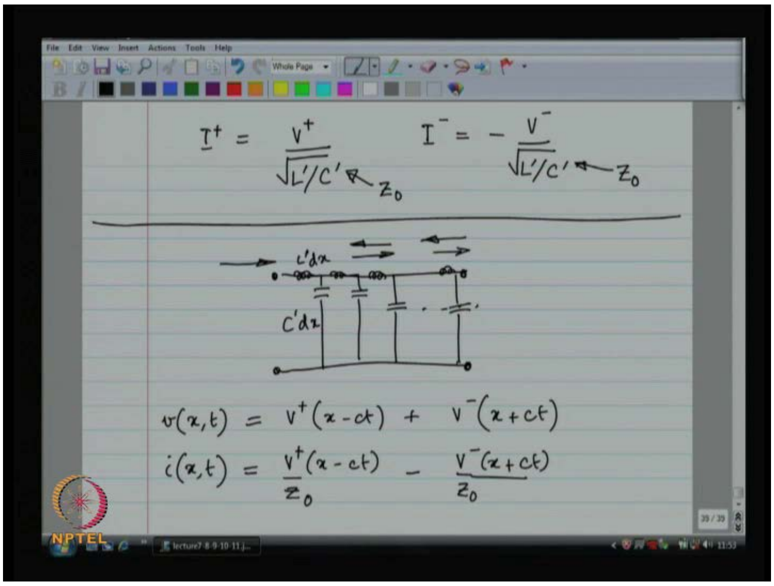

These are the 2 independent equations you are going to get the values of I plus and I minus and it turns out that I plus is going to be equal to V plus times. So, this is what it turns out satisfies both of my equations I plus and I minus, if I choose these as my values then, it is going to satisfy everything. Now of course, you remember that c is equal to this quantity, this is we did in this previous class right. So, this basically gives us the relationships between the current waves and voltage waves.

So, what I have got so far. I have got a wire, it is not really one wire, it is a pair of wires. And, what is going to happen is that if I launch a wave on to the wire then, this wave is going to all the way portion of it is going to reflects back. So, at any given point of time at any given point in space on the wire on the x-axis in general there are going to be 2 ways; one going in the forward direction, one going in the backward direction. So, that is the physical interpretation of the whole story.

So, I got a forward moving wave backward moving wave. And, these forward and backward moving waves are both voltage and current waves. The voltage forward moving wave is equal to the current forward moving wave times square root of L by C that is Z naught, remember. And, the current backwards moving wave is negative of the voltage backward moving wave divided by Z naught. So, this is what I have here, V plus and V minus are functions of x plus c t, and x minus c t. So, this is my general solution to the transmission line problem of course, this is a special version where, I have ignored the presence of resistance and conductance.

When the resistance and conductance are present like in the previous lecture we discussed; we will use the phasor notation because it is better to use the phasor is going to be easy for us when we use the phasor notation. We are going to be assuming that all of these are some of sinusoids; we are going to break up any signal into its composition in terms of sinusoids. And, we are going to treat each sinusoid 1 at a time right each sinusoid is going to create a forward and backward moving wave which is slowly being attenuated by this factor sigma. That was my solution in the previous class; we did this, I think we did this quite thoroughly in the previous class.

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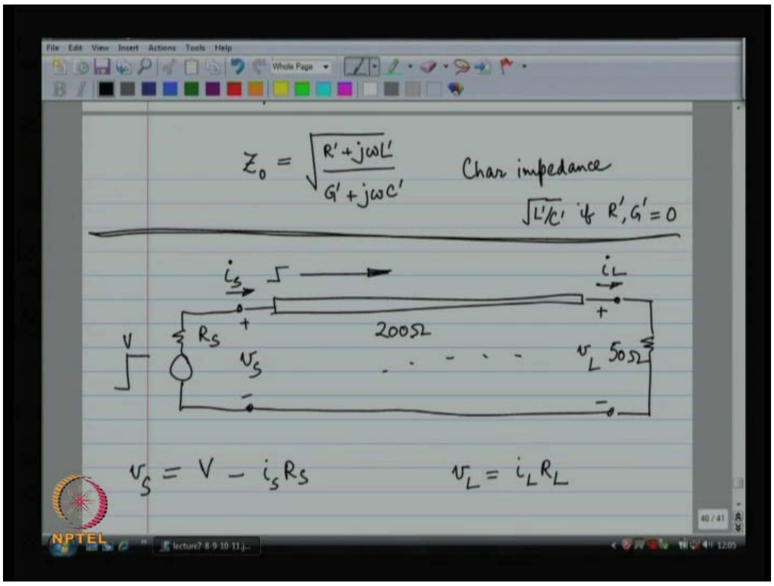

So, they are also you basically are going to end up with more or less the same solution. Just that now, Z naught is going to be a complex number. So, in general Z naught you can show that Z naught is going to be equal to square root of this quantity. So, you pick the frequency Z naught you have to compute your Z naught for that particular frequency. Otherwise, there is not much to be changed in the set of equations is just that. Now, V plus V minus these are going to be phasors that we are going to talk about, V and I are also going to be phasors that we are going to talk about. So, they already have e part j omega t built into them, they are already sinusoids. So, if I have. So, a sinusoid voltage if I launch a sinusoid voltage, you know that portion of sinusoid voltage is going to reflect back from the load etcetera.

Now, this derivation, this set of derivations I needed to do to relate back to what we jumpstarted earlier, how did we jump start in the earlier, in our introduction we had jumped started on this business of reflections. So, right what did we say? We said that I have a wire, is a transmission line of characteristics impedance Z naught. Now, you see what the Z naught? Z naught is not a resistance this is called the characteristics impedance. Z naught is mathematical entity, it is not a resistance at all. If you have R and G equal to 0 then, Z naught is going to be square root of L by C, L primed by C prime right, in any case the real part of Z naught can be a approximated to this quantity. As any engineer you do the backup envelope competition, and you can approximate the real part of Z naught to this particular quantity fine.

So, what is going to happen, I apply, let us say a step from 0 to 1 volt. Let us say my source resistance is 50 ohms, and let us say Z naught is 200 ohms, and let us say my load is 50 ohms, some random numbers right. What is going to happen? first what needs to happen is you need Kirchhoff's of law to be satisfied everywhere. So, I need Kirchhoff's of laws here, I need Kirchhoff's of law here, right. So, Kirchhoff's of law is going to tell me that the potential on the sources of side transmission line V S is going to be equal to 1 volt, let us say V volts minus i s stands R s this is what Kirchhoff's of law is going to tell me at the source side.

Remember, Kirchhoff's of law is always satisfied no matter how hard try, it is very difficult not to satisfy of Kirchhoff's of law. In fact, if a physics person claims to you that Maxwell has an extra turn which makes Kirchhoff's of law not being satisfied, say there is still called displacement current. So, if you call it displacement current that extra turn in Maxwell equations Kirchhoff's of law is once again going to be satisfied.

So, anyway Kirchhoff's of law it is very hard not to satisfy Kirchhoff's of law. And, even inside the transmission line Kirchhoff's of law is going to be satisfied, all that you need to realize is that the translation line is not a wire. There are paths leakage paths all over the place. So, the portion of current can come out. So, i s is not going to be equal to i L and that is not a evolution of Kirchhoff's of law.

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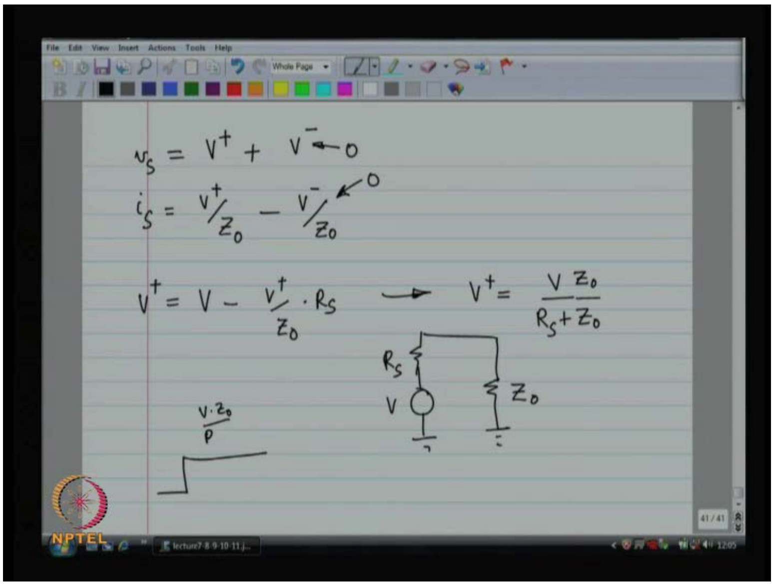

We have got these 2 are basic starting points; first that the voltage across the load is i L times R L, voltage V S is going to be V minus i s times R s right. Then, we realize that i s first of all V S at the beginning of the transmission line is going to be equal to a forward moving wave plus a backward moving wave ok. And similarly, i s is going to be a forward moving wave plus a backward moving wave right. So, if you understand this then, suppose this is a pulse wave form, what is the backward moving wave initially? You just applied the pulse everything is static before this there is no knowledge to the circuit that you are going to apply the pulse in future right.

So, nothing is coming backwards, you are about to apply the plus nothing is backwards. So, there is no backward wave initially, when you launch your wave there is no backward moving wave you just launch it. So, V minus initially is 0 right. So, you only have a forward moving wave in that case you have got V S is equal to V plus that is V plus is equal to V minus i S which is V plus by V Z naught times R s. So, what is V plus going to be equal to? Verifying this yourself, alright. So, you basically have 2 put all V plus together and solve the simple equation and you will get V plus equal to V times Z naught by R s plus Z naught.

So, almost as if you have got a voltage source V with R s and you are launching it onto resistor of value Z naught, just that it is not really a register. So, be very careful here it is not a real a register it is a wire launching it on to the wire, but; initially the wire is looking like its characteristics impedance, because there is no backward moving wave. So, that initial instant of time there is no backward moving wave. So, this is what you get. So, now I know the value of V plus.

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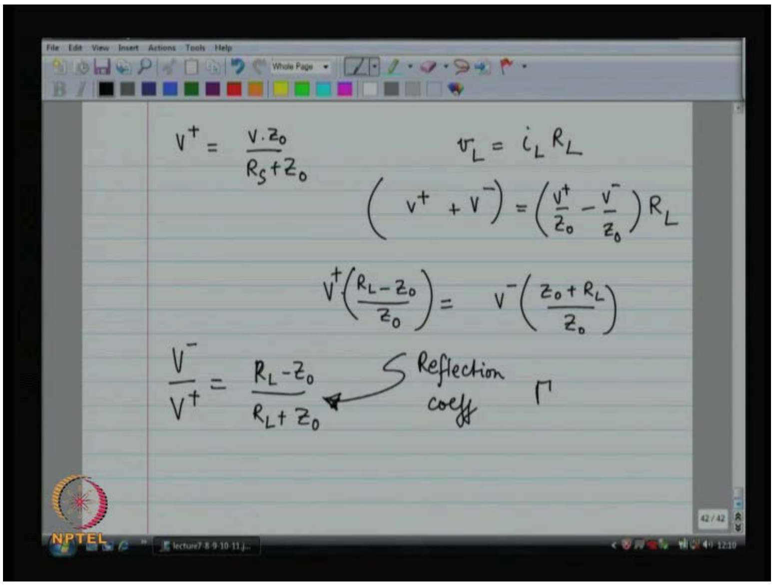

So, V plus is a pulse of value V by Z naught I am sorry, this is the value of V plus it is a pulse and it is travelling along the wire in the forward direction. And, similarly; there is a current wave also which is travelling along the wire in the forward direction. Now, what is going to happen? It is going to hit the load now on the load. The voltage is equal to the current times the resistance, the load resistance this is coming directly from Kirchhoff's of law. This voltage at the load is going to be in general equal to V plus, plus V minus forward moving wave plus a backward moving wave. And, the current is going to be equal to the sum of forward moving current in a backward moving current, agreed? I know the valley of V plus already.

What should the value of V minus V? So, it is a simple matter of doing a couple of transpositions and. So, what you are going to find out is V times Z naught minus R L by R S plus Z naught right, something of this fashion. So, what does this tell you, what is V plus, what is V minus by V plus? if a wave of size V plus hits you, it is going to create or reveres moving wave of size V minus.

What is the value of V minus? It is going to be V plus times R L minus Z naught by R L plus Z naught. So, this quantity is the reflection coefficient, also known gamma. We use this formula before earlier on in the first lecture we use this formula and jump started the whole discussion right; however, this is the way it comes out. So, now, this V minus wave goes and hits the source at again right.

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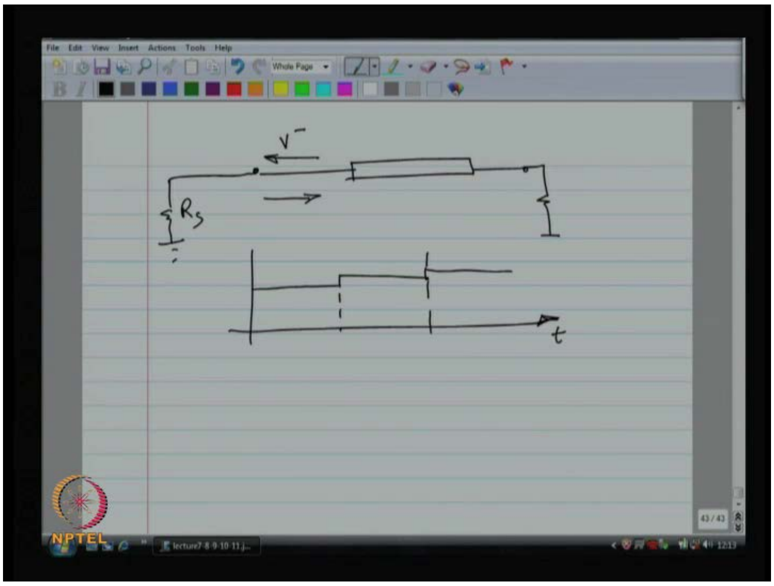

When I hit, the with source V minus V plus is already their right, V plus was the result of this source, of the voltage source, when I created the voltage source, when I applied the voltage source at first it created a positive moving wave fount called V plus that voltage is still around. So, V minus adds onto that voltage. The new wave which is moving backwards adds onto the old wave, the old wave still present over there. So, if you use a superposition, you can use superposition, if you do superposition then, you can ignore the fact that V plus was there, and you can ignore fact that I launched a wave to start with. And, you can just consider the fact that you have got V minus a wave is coming backward it hits R S.

And, then what is going to happen? It is the same situation as before the 1 we just before the computed you are going to get a new forward moving wave which is going to add on the old 1. So, you will get another reflection over there, it is going to add on old 1 you are going to get a new V plus, extra beat of V plus you are going to get. So, eventually the waveform V plus is going to look like this, you will get lot of steps in V plus. So, you have got as a function of time if I just look at the value of V plus initially, V plus is something, and then after that double that transit V plus is going to change a little bit. After 4 times the transit time V plus is going to change a little bit.

V plus itself is going to change, if you think about it right. if you do not think about superposition, if you do think about superposition, when we every time you are getting the gamma times the old wave, but; then you have to add up all the past history. So, that is why; we have a reflection coefficient on the sources side, reflection coefficient on the source side you just replace R L by R S. And, you will get R S minus Z naught by R S plus Z naught. This is going to be the reflection coefficient on the sources side.

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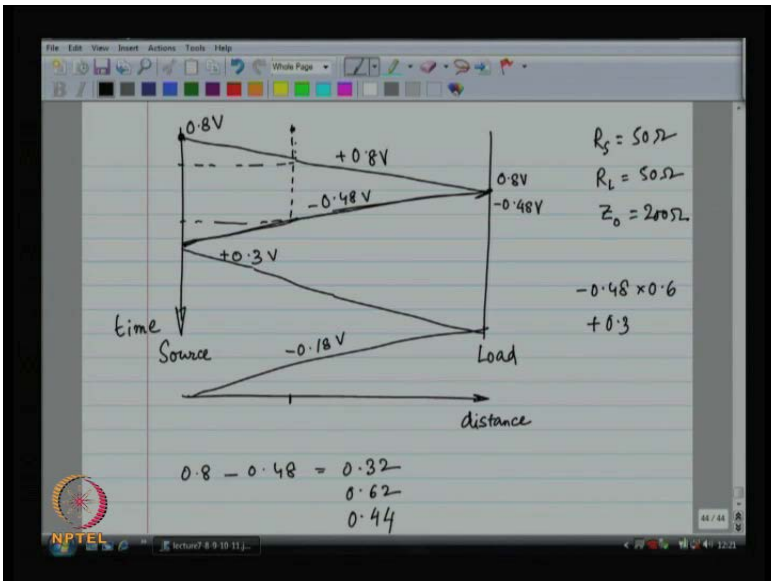

So, how are we going to tackle this final example; suppose, it takes me 1 second to travel from the source to the load, suppose R S is 50 ohms, R L is 50 ohms, suppose Z naught is 200 ohms, this is what we started with. And, suppose I launched the 1 volte wave. So, when I launch a 1 volte wave, this 1 volte wave seas a resistance of 50 ohms, it seas characteristics impedance of 200 ohms.

So, what is the portion that reaches the wire is four-fifth of 1 volte. So, you basically launching V plus of value pointed 0.8 volts. Now, this is how we draw it, this V plus wave V plus propagates over the wire it hits the load after it transit time whatever, it is 1 second. 1 second later it hits the load, 0.8 volts hits the load and then, portion of it reflects back gamma times 0.8 reflects back. So, instantaneously you get V minus launched right, and the value of that V minus is going to be what is your gamma? Gamma is minus 0.6.

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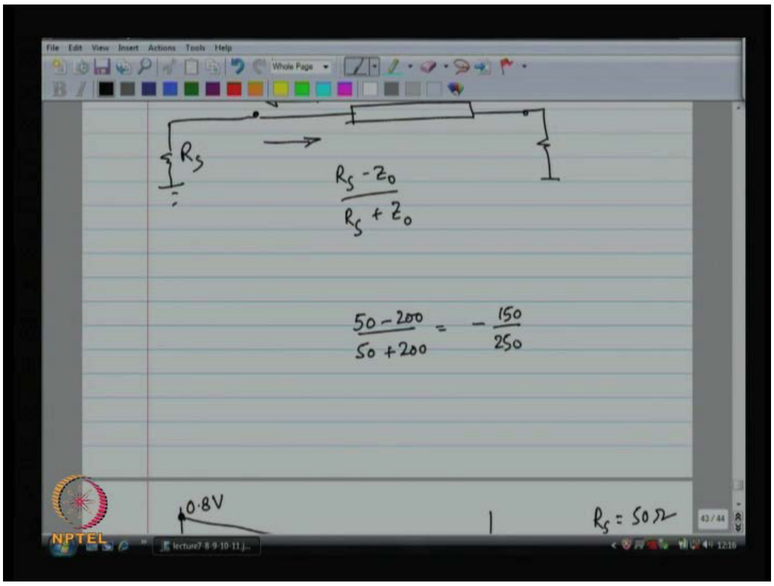

So, instantaneously minus 0.6 times 0.8. So, minus 0.48 volts gets launched backwards. And, this is your V minus which is propagating all over the wire. So, at any point of time if you want to compute the voltage at any distance, let us say you want to figure out the value of the voltage at this point on the wire. So, you go over there and you keep adding up these voltages, and that is how the voltages going to change. So, all of these are super posed on each other. So, at this time it 0 volts then, at this time it is 0.8 volts then, at this time it is 0.8 plus minus 0.48 volts which is 0. 3 volts right and so on.

So, this minus 0.48 volts hits the source and once again gamma is minus 0.6. So, minus 0.48 times minus 0.6 is going to be how much is it? Is about plus 0.3 or so, it is about plus 0.3 or so. I do not know the exact that value. So, I will now, have another forward moving wave which is plus 0.3 volts. Now, this 0.3 volts is going to hit the load and it is again going to reflect back 0.3 times minus 0.6. So, you get minus 0.2 volts going backwards, and so on and so forth. This keeps going happening back and forth back and forth and finally, if you look as times tends to infinity.

What should be the voltage at the load? Commonsense as times t tends to infinity, study state all the capacitors are open circuits, all the inductors are short-circuits. So, the wire is really a short-circuit what does that mean; that means, that you have got R S you have got R L right, you should be getting the appropriate voltage division, you should be getting the right voltage at the output as the time tends to infinity. So, your calculations should go towards that always, your calculations should eventually settle, unless you have done something dramatic. If your resistance is negative or in some dramatic mistake is there, your calculation should always settle to the final desired value and.

So, as you see; you got 0.8 to start with the target was 0.5 to reach the load. You transmitted 1volt, 50 ohms was the source resistance, 50 ohms was the load resistance. So, he expected 0.5 volts to reach the load. It is started with 0.8 then, after some time you got 0.8 minus 0.48. So, 0.32 then after some time you got a further plus 0.3. So, you got 0.62, after some more time you got a further minus 0.18. So, that adds up to 0.44 you see we are slowly zooming towards the correct value which is 0.5 as time progresses. Now, the more the mismatch between the source load and the Z naught the longer it is going to take for this to settle down.

And, you also see that you transmitted a pulse instead of receiving a pulse bought after some time, what you received? You first got 0.32 volts at the load and then it jump to 0.62 volts then, it came down to 0.44 volts then, presumably it is going to point something closer to 0.5 and so on. This is what you are going to end up with right. So, therefore; it is important just for purpose of by fidelity, you transmit of pulse you should receive a pulse.

So, just for that purpose it is important for us to make sure that the source on the load matched to each other. And, this is where the whole topic of matching is going to arise, and we have already discussed L match, phi match all of those different matching networks. We have already discussed note that those are matching act given a frequency here, we are talking about brood brand plus. So, this is never really going to work, even in spite of the L match etcetera this brood brand plus is never really going to be faithfully reproduced, unless I really have registers over there. So, what we have got so for in our discussion.

We have kind of understood that not kind of, we went in throw detail we solved all the equations we understood the transmission line, we understood the fact that what is going on is because the transmission line is distributed network. There are leakage paths, if I launch current I from the source I is not going to reach the load; some of it is going to leak out through the capacitors through the conductance’s. Similarly, if I put a voltage V across the transmission line on the source side, I am not necessarily going to see V on the load side some of that voltage is going to drop across the inductors or across the registers. That is number one that is what we saw.

Then, we solved the equations for the transmission line we took a special case when L, when have I only got inductances in capacities this is last less situation. The lastly situation is the just a little worse than this, that also we can solve unfortunately we can solve that only when we talk about sinusoids. When I launch a sinusoid, I can solve it. When launch something more complicated than sinusoid then, I will have to break up the more complicated signal into some of the different sinusoids, some could also be an integral.

So, I have to do the Fourier transform of the sinusoids to get it representation in terms of sinusoids. And, then for each sinusoid I have to work out what happens? Right, for each sinusoid in general the reflection I am sorry, the characteristics impedance is going to be different. Where did I write that? Yes, for each frequency characteristic impedance is different. So, for each frequency the reflection coefficient is also going to be different.

So, this is all bad news; at the end of the day you need a computer to figure out everything. So, if you are situation involves some kind of some of sinusoids then, I suggest that you just stimulated and be done with it. We have good computational techniques that will work out everything for you. If it is just a sinusoid, we can understand with the help of equations what is going to happen. So, we end up basically with the same equations as the last less case; where, are those equations? So, we end up the basically with the same equations, let me just write the amount again.

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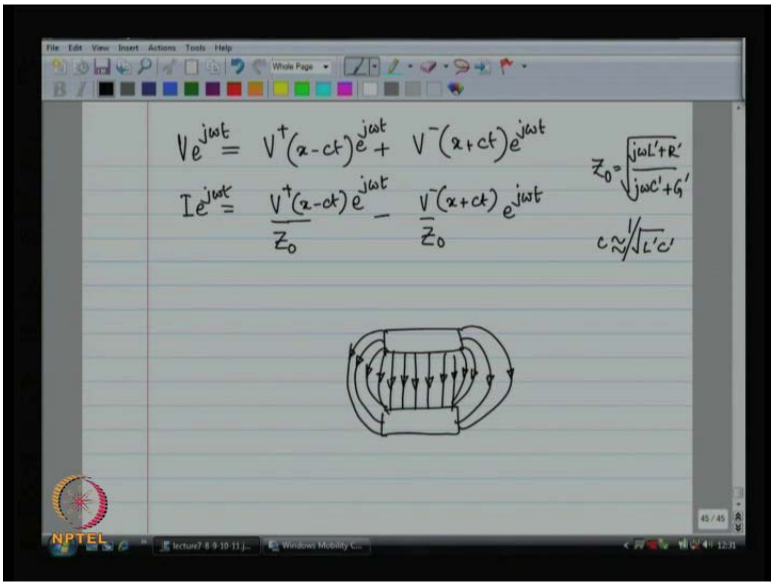

So, the phasor V is now going to be, and the phasor I is now going to be. However; when I say phasor it really means; that I need to multiply this by a factor of e power j omega t. So, add a frequency omega, this is what is going to happen. And, hear Z naught is that complicated expression something like this. c is not exactly going to be the speed of light, but; almost the same quantity, it is going to be almost the speed of light not exactly. So, C can still approximated has the speed of light.

So, what will means for us? This will basically mean that as I progress over space, I am also going to get attenuation that is the only difference. So, what we did in the previous class; let me just remind you, this was my final solution to the equation. And, we said that we can say that voltages e power sigma plus j omega x times e power j omega t. And, then I find out the values of sigma and j omega by taking the appropriate square root and then I build up my function. So, what is basically going to happen is that as you progress over space you are going to get attenuation.

So, start off with sinusoids on the load side, as you progress over the transmission line you are going get attenuation, because there is a resistance as far as voltages concerned there is a resistance. If you think about current then, there is conductance, which provides a leakage path and as a result of magnitude of the current is going to decrease. So, we are going to summaries the entire module with this. In this model we first started with registers then, we worked on capacitors, and then we worked on inductors, after that we work on transmission lines or wires rather.

So, under registers we just discuss the units ohms per square was the unit, and if a surface resistivity is given to you, you can figure out by counting the number of squires in the track. What is the resistance of the track? That is as far as resistance goes it is easy. Capacitance; capacitance is very easy to think of all you need is to parallel plates. So, you got to plates you have got the capacitors. Now, a epsilon by d, this age old formula that you probably remember from high school days. It is a very good formula, it usually works, but keep in mind that this formula was built for infinite plats, infinitely large plates.

So, when you restrict the size of plates which is usually the case then, you not only have to account for a epsilon by d, you also have to accounts for fringe effects between the 2 plates. This fringing is going to happen along the perimeter of the plate, also the ticker plate the more flinching. So, these are my field lines. So, thicker the plates more flinging, fringing is going to happen along the parameter. So, as a result if I want to just look at the fringe capacitances then, the fringe capacitance is proportional to the parameter it is also proportional to the thickness of the plates.

Then, we talked about inductors, we talked about different ways to make capacitors then, we talked about inductors. What where the different non idealities in conductors, core losses then, we talked about first we talked about copper losses then we talked about core losses or eddy current losses. We figured out, how the eddy currents are pulling? And, what is the exact mechanism then, we also talked about parasitic capacitance as a result of which are going to get of finite resonant frequency for the inductor beyond which inductor will start looking like a capacitor. Then, we further went and studied weirs, we talked about skin depth then, we talked about weirs having inductance capacitance resistance. And, then we developed a model for transmission lines. So, with this I am going to close this chapter. And, in the next class we are going to talk about new things.

Thank you.