CMOS RF Integrated Circuit
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Module - 03
Passive IC Components
Lecture -10
Wires

Welcome back to CMOS RF integrated circuits we are lecture 10 part of module 3; we were discussing passive components part of that we were discussing wires. So, far we have already covered inductors, capacitors, resistors all of this stuffs we have already covered; we were discussing wires. And, the as our discussion about wires we were first talking about this skin depth, skin effect and skin depth which we have mentioned before. So, if I have a wire and current is traveling through the wire at a frequency, at an angular frequency of omega then, the depth up to which the current is going to travel actually my statement is not really true. But that that is how I am going to phrase it to an engineer the depth up to which recurrent is going to travel; that is d is given by this formulation squared root of 2 by mu sigma omega.

So, as omega increases this depth decreases; why I said that this is an engineer speaking; and not physicist is because if you look at the real derivation. Then, at this depth the current density is about one- third or other 1 by e times the current density on the surface; that is what it really means. But to an engineer this is the depth at to which we can assumed that current is going to travel; after this we were basically we had started looking at the effective inductance per unit length of a wire. Now, by we are saying per unit length is going to become parallel even a short wire. But for now let us just try to figure out what is the inductance?

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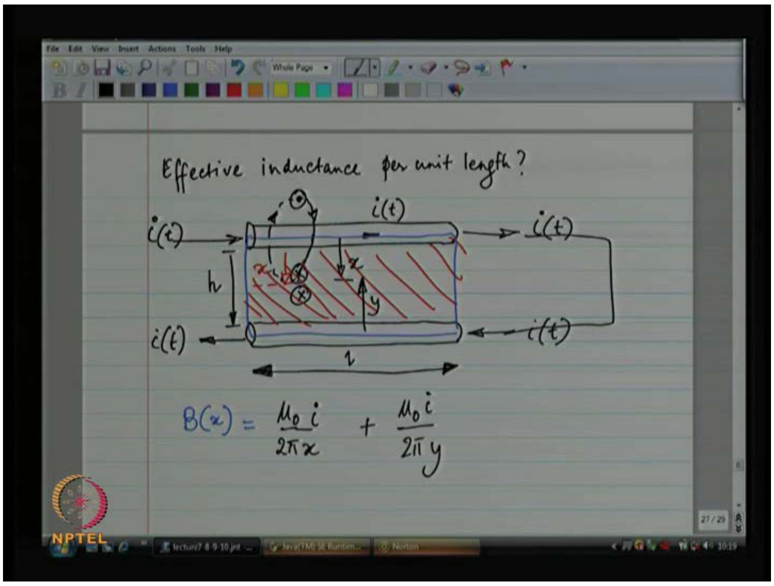

So, we have a wire which is carrying i and there is a return path for the current. So, there is another wire which is coming back its parallel to the original wire; this is the case situation that we are going to talk about. And, why we are talking about this as a special situation is because when I have a wire over a conducting ground plane. Then, an image of the wire is going to be beneath the ground plane.

So, this situation is gruntingly going to arise in our integrated circuits. So, that is why we are especially talking about this situation. Now, we are going to assume that this wire as a circular cross-section unfortunately on an integrated circuit; the wires do not have a circular cross-section they are going to have a rectangular cross-section. But this is an assumption because if I do a rectangular cross-section; the derivation is going to become unnecessarily complicated and I would rather not do that derivation.

So, this is what I am going to do. Now, at a distance x from a current i. So, I have a uniform current going through a wire current is high of t. So, at a distance x from this current; how do I measure the magnetic field, how do I find out the magnetic field? I create an amperian loop at a radius x around the wire. So, presumably the integral of the magnetic field along this loop is going to be I am sorry; the magnetic field is along this loop is going to be constant. So, the integral of the magnetic field is going to be magnetic field times 2 pi x. Now, this quantity is mu naught times i that amperes law. So, that quickly gives us result that magnetic field because of this forward current i at a distance x is mu naught i by 2 pi x. Now, this is not telling the anything about the return current; return current we are going to do it later.

So, I am going to assume that the return current is going to add some more magnetic field to the original right. So, if this is x and let us see this is y then the return current is going to create another portion. So, this is superposition at this used; superposition over here and this is correct this going to be the field. So, in any case I am not going to consider the return current right now; we are going to think about little later why a little later is because let me think about the flux because of this magnetic field.

So, let me try to compute the flux over this entire hatched area with red; that red marked portion let me try to compute the flux through that area. Now, length of this is L; and let us say the bottom wire is at a distance what you want to say h from the top wire. So, the gap between the bottom and the top wire is h meters. So, the flux is going to be the magnetic field integrated over the entire area ok.

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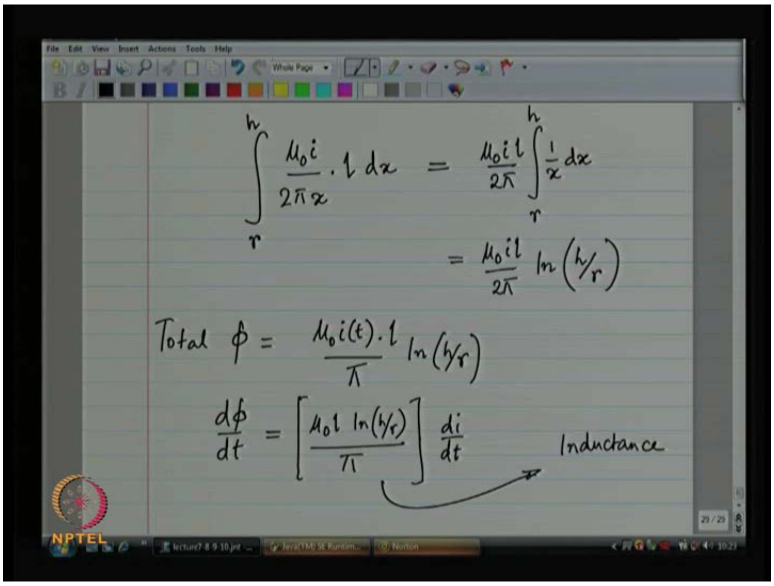

So, I am assuming that this wire as a radius of smaller. So, let us look at the area from r to h; r to r plus h I am sorry let us say this is h. So, this gives me this integral results in log integral of 1 by x is lawn of x. So, basically what I get is something like this. So, this is the total flux because of the top current; the forward current. Now, if I look at the return current; return current is also going to give me a similar integral. And, the result is basically going to be identical; the forward current is going to create some flux the return current is also going to create an identical flux. So, total flux is going to be twice of this.

So, that is mu naught I right and d phi by d t the rate of change of flux creates an e m f which is the. So, if I try to change the flux then an e m f is created which is equal to the rate of change of flux rather a back e m f is created; it resist the rate of change of flux which basically means that I get a potential drop across this wire which is equal to d phi by d t.

And, that is going to be equal to mu naught l lawn h by r by pi times d i by d t. Now, if I have an inductor of value l then the potential drop a cross the inductor is l times d i by d t. And, here I have got something across which there is a potential drop which is some constant times d i by d t. So, the inductance of this piece of wire is this and if I want to consider inductance per unit length; then it is the total inductance divided by the length.

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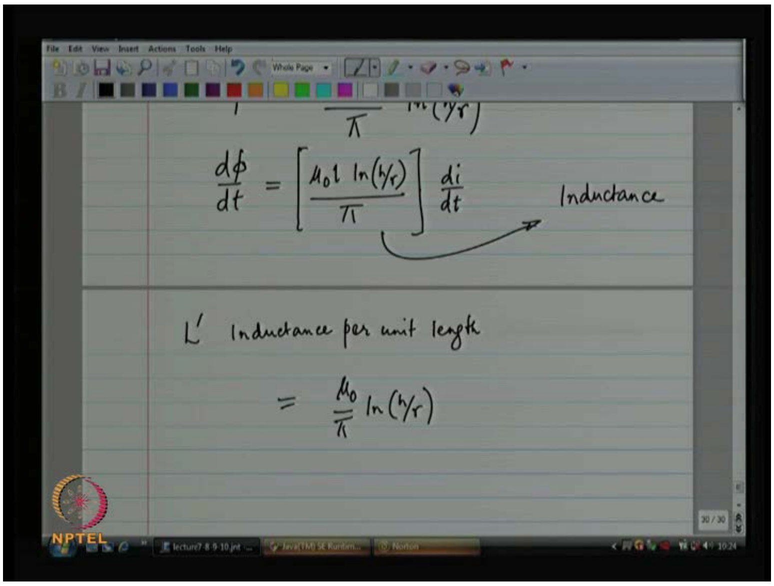

So there have been a lot of simplifying assumptions in this derivation. However, what I need you to understand from this is that even a wire has some inductance per unit length; longer the wire more the inductance right also more the separation between the wire and its return path more the inductance. So, things will change if you change the geometry. So, if instead of circular cross-section you go for a rectangular cross-section things will change; as an engineer I do not really care about the small changes I need some intuitive understanding of what is going on. And, this results gives me an intuitive understanding that is good enough; one more thing as you make your wire thinner and thinner inductance per unit length becomes more.

So, thinner wires have more inductance, thicker wire have less inductance interesting all right. So, wire has inductance per unit length; does the wire have resistance per unit length? Yes, we already discussed that we even discussed our the cross-section. And, the skin effect and so on how the resistance is not really appear resistance; it keeps going up as a function of frequency etcetera what else, could a wire have maybe it has capacitance.

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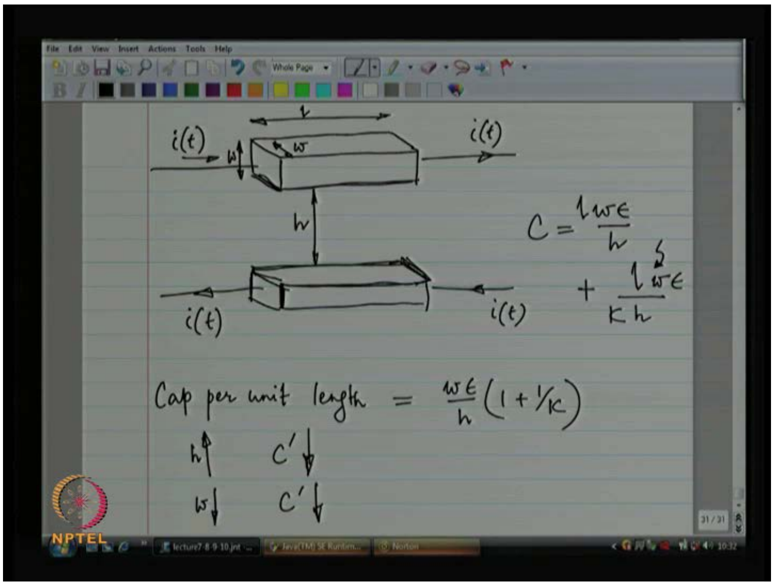

So, let us consider our wire. Now, for convenience I am not going to use a circular cross-section; I am going to use a rectangle cross-section. So, whenever what whatever is convenient I am going to do that right. So, let us say I have a wire carrying a current i of t and of course if you have a current going the current has to come back. And, is a return path of the wire; which is basically the mirror image of the wire on the ground plane something like this.

Now, this reminds you of parallel pair of plates more the separation more the fringing but even otherwise this is a parallel plate capacitor. And, if you just think about a parallel plate capacitor; the capacitance between the 2 plates is going to be equal to the area of the plate times the epsilon of the dielectric material divided by the gap between the 2 we said the gap plate size h right; this is what it is what is the area of the plate? Area of the plate is the length times the width or this is not taking into account the fringing; fringing is only going to add to this further. And, what is the fringing going to be proportional to the fringing is going to be proportional to the perimeter of the plate.

So, it is going to be proportional to the perimeter of the plate. And, it is also going to be proportional to the height of the plate. I hope you remember this I mean just look back a little bit on you will recall that the fringing capacitance is proportional to the perimeter which is l plus w. And, it is also proportional to the height I am sorry not that height the thickness. So, let us say the thickness is also w and it is inversely proportional to the distance between the 2; it is also proportional to the dielectric constant.

And, then we said that there is some fetch factor something some number of what there which we do not; which we need from computational export to figure out what of is fine. All I want is some understanding some feel for what is going on over here. Now, l plus w can be approximated as l right; w is small l is large; the this w is really the thickness of the plate. But if you considered that earlier we were talking about a circular wire which has both dimensions equal radius is r means the height is the thickness is the same as the width. So, that is why even though we are going for a rectangular cross-section; let me just assumed that the thickness is w and the width is w.

So, I just want to try to mean it the earlier computation; and that is why no other reason I chose that to be w. So, this is my capacitance and the capacitance per unit length is basically w time is epsilon by h divided by 1 plus 1 by this futz factor right; what is this tell me? So, look at the conclusions earlier and look at the conclusions now.

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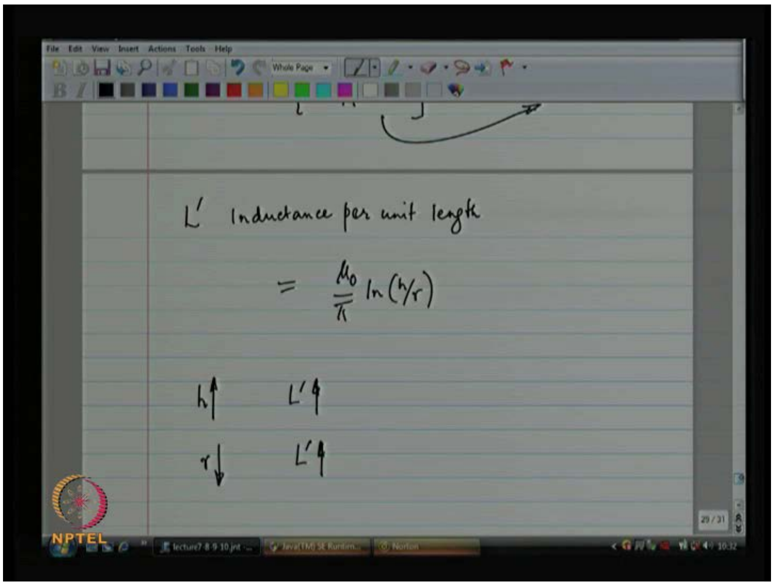

The earlier conclusions where when we are doing inductance the separation between the 2 wires increases, inductance increases; the radius of the wire decreases, inductance increases; what are the conclusions now? The separation if I increase the separation the capacitance per unit length decreases; if I decrease the radius of the wire by inductance increased where was that yes; inductance increased when I decrease the radius of the wire. If I decrease the width; width is comparable to radius because which was a squared cross-section.

So, if I decrease the capacitance is going to capacitance per unit length is also going to decrease. So, just the opposite effect as before right the precisely the opposite effect. So, when inductance increases capacitance decreases; when inductance decreases capacitance increases alright so far so, good. If I had chosen the same geometry for both as and if I had chosen both circular or both rectangular which I did not.

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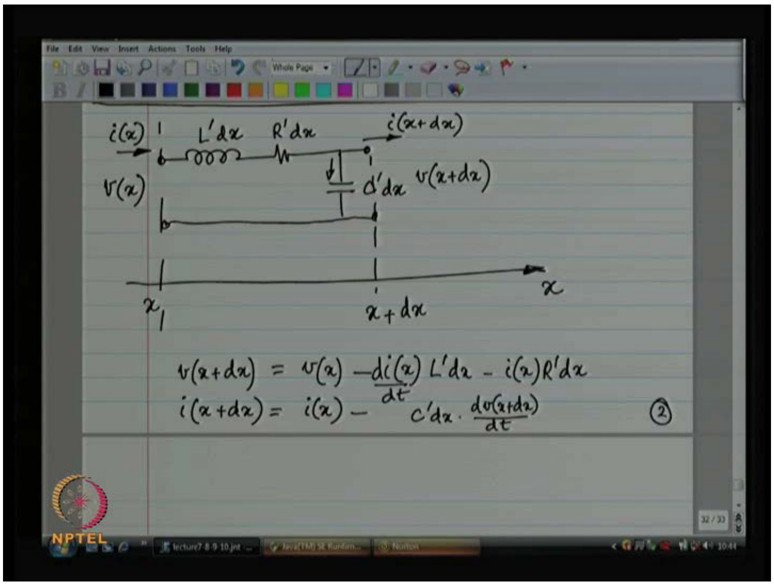

Because I wanted to simplify my computation but if I had chosen both geometries as the same. Then, you would find that L prime inductance per unit length times capacitance per unit length would be a constant unfortunately I cannot prove this. Because we did it differently; no when inductance decreases capacitance increases; when inductance increases capacitance decreases. So, it kind of gives you a sense feeling that L times c is probably going to be a constant it.

So, happens that it is constant for a given geometry is constant you have to do correct computations for both; we will come up the best results so far so good. So, we have figured out that when I say that we have got wire; wire is really something else it is not short-circuit it has some inductance per unit length it has some capacitance per unit length; it has some resistance per unit length if the dielectric material is no good.

Suppose the dielectric material was conductive little bit than there would be current also leaking out over here if the dielectric material was not a perfect isolator then there would be current leaking out. And, that would create a conductance per unit length as well right. So, our model for the wire looks like this; I am not being very is proficient with this.

So let us so this is a little bit of a section of a wire. Let us say I have got a wire of very small length d l; then this is the model for the wire. And, wire of huge length is basically cascade of these sections till I complete the entire length of the wire, right. So, this is my model for a wire; it is a cascade of lot of these sections each section has its own inductance, resistance, capacitance and conductance its so turns out that the conductance is mostly going to be equal to 0. For most dielectrics reasonable dielectrics is conductance is going to be equal to 0 unless you choose a bad dielectric material; you should not be having a conductance. For example, on an integrated circuit the dielectric material is silicon dioxide it is more or less perfect insulator. So, you can safely assume that g prime is 0.

So, one simplification at least g prime is 0 right but we still have to live with inductance per unit length, resistance per unit length and capacitance per unit length; this kind of a system is called a distributed system; the inductance is distributed. So, wire as certain inductance but this inductance is distributed all over the wire; it also has distributed resistance, it also has distributed capacitance right.

If the capacitance were not their then I could lump both the inductance and the resistance because all of these come in series and they add up nicely. But because there is the capacitance you cannot do that. So, how do we analyze a distributed system? Let us say that I have applied voltage v this is my x axis; at a distance x the voltage that is on that piece of wire is v and at a distance x plus let us call this d x for convenience instead of d l.

So, at a distance x plus d x I have got voltage v (x) plus d x also let us assume that the current going in is i (x) over here. And, the current coming out is i x plus d x. Now, this quantities are almost equal but let us develop some kind of an equation relating them. So, the first thing that you seen is v of x plus d x is equal to v (x) minus i (x) times L prime d x minus i (x) times R prime d x; this is the first important things to observe. And, the second important things to observe is i x plus d x is equal to v of x plus d x minus; so some portion of the current is i (x) minus oh I need see just mistakes.

So, the current through the capacitor is c d v by d t. So, that much current is leaking out through this capacitor. So, the current that is going out of the network is i x plus d x that is equal to i x minus whatever is the current through the capacitor; that is my equation number 2 and equation number 1 is the voltage drop. So, I have applied a voltage v x at the input side and on the output side its v (x) minus the drop across the inductor that is L d i by d t minus the drop across the resistor that is resistance times the current. So, that is what I have got. Now, if you look at these 2 equations they are partial. So, really v and i have to be functions of x as well as time; because you are doing derivative with respect to time. So, all of these derivatives are going to become partial derivatives. So, you now got a pair of partial derivative the partial differential equations in 2 variables x and t.

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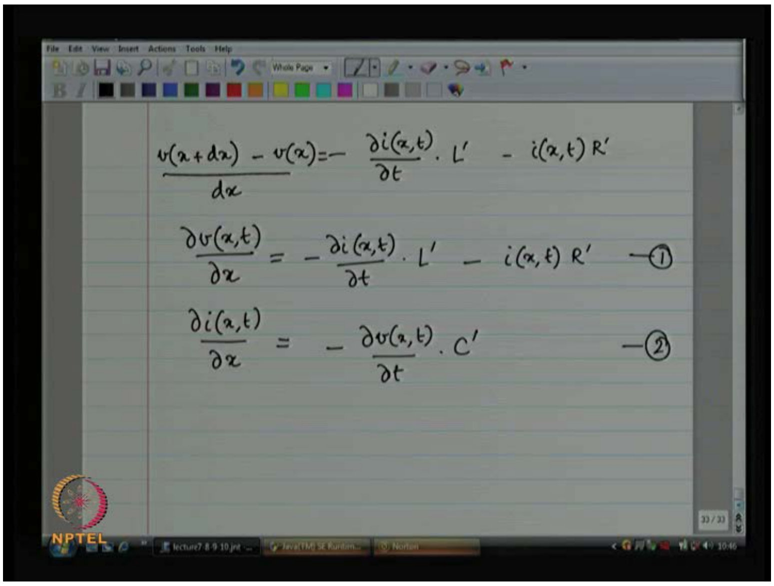

And, let us simplify little bit let us bring all the voltages together. So, I have got v of x plus d x minus v (x) is equal to this and let us divide the left hand side by d x. So, I have divide the right hand side d x as well. And, what I have got is this is what I have got equation 1. And, if I rewrite equation 2 similarly I am going to get dou i of (x, t) by dou x and that is going to be equal to minus dou v (x, t) by dou t times c prime. So, these are going to be the pair of partial differential equations that we have to worry about; there would have been forth term over here which would looks very similar. But of course I am assuming g prime is 0 does not matter you can let it be there in the equation set of equations ok.

So, now that we have this does this remind you of anything; this pair of equations does it remind you of anything may or may not remind you of anything. Let us first look at how can we simplified further; it should remind you of Maxwell’s equations right anyway. Let us first try to simplify this further. So, what I am going to do is I am going to take further second derivative; I am going to do further differentiation of equation number 1 with respect to x.

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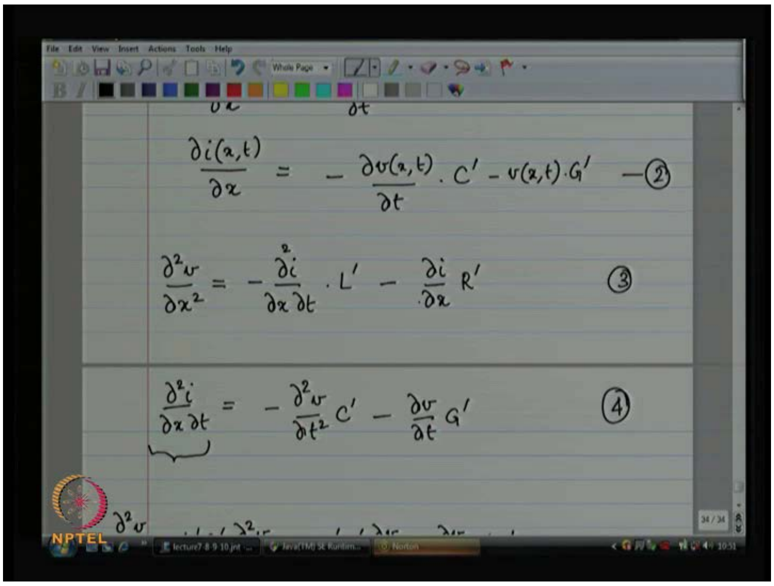

And, that will give me dou squared v by dou x squared; on the right-hand side on the left and side. And, on the right hand side I am going to get dou i dou squared i by dou x dou t times L prime minus dou i by dou x times R prime. And, I am going to do another differentiation of equation number 2 with respect to t. If I do differentiation of equation number 2 with respect to t I get dou squared i by dou x dou t is equal to minus dou squared g dou t squared C prime minus dou v dou t G prime. Now, we are going to plug in the value of dou squared i by dou x dou t.

So, let us call this equation 3 and 4; 4 gives me relationship of dou squared i dou x dou t purely in terms of voltages; equation 2 gives me dou i by dou x purely in terms of voltages. And, equation 3 is something which is mixed up. So, I am going to replace each term in equation 3 and make everything purely in terms of voltages.

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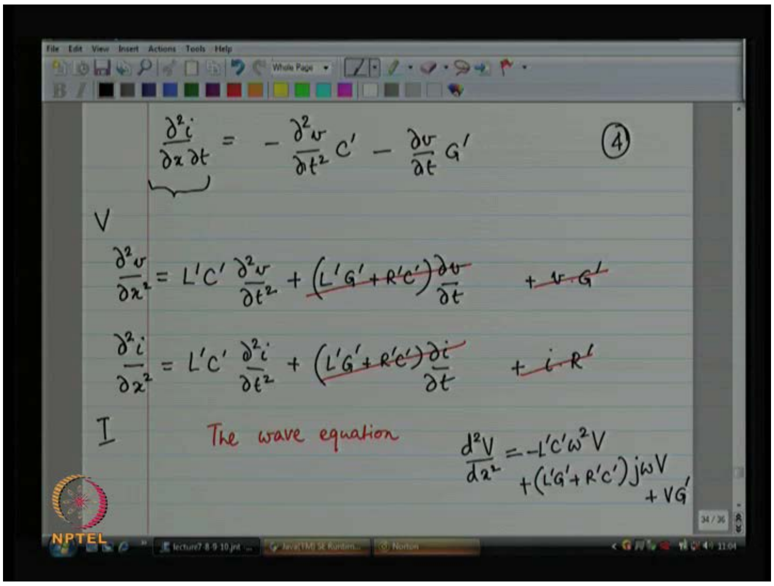

So, what I am going to get is something like this. So, this is my final differential equation; it is all in terms of voltages. And, only voltages but there are 2 variables x and t right. So, voltage as a function of x and time; distance and time and the entire equation is only in terms of voltages. Similarly, I can work out something for currents and what you are going to get is something like this; just going to write it out all in terms of currents.

So, we now have one equation for voltage as a function of x and time; one equation for current as a function of x and time; of course the voltage in the current are also related to each other you have to keep that in your mind at the back of your mind. But that we will use as a fact later on but all the voltages are in one equation, all the currents have in one equation. Now, does the do these equations remind you of anything?

So, let us say to start with; let us say that this resistance per unit length and the conductance per unit length are both equal to 0. Let us simplify matters and say that R prime and G prime are both equal to 0. So, when R prime and G prime are both equal to 0 and this term is missing, this term is missing because these 2 terms are also missing. Now, does this equations remind you of anything; both the equations are actually the same do these equations remind you of anything? Yes, they should remind you of the wave equation; if you go back to your engineering method, mathematics or I do not know what book you have studied before. Both of this equations are basically the wave equation there out of a textbook. And, the theory of differential equations say that if you know the solution to a differential equation and that is the solution, right. So, the solution to the wave equation is basically as follows there is.

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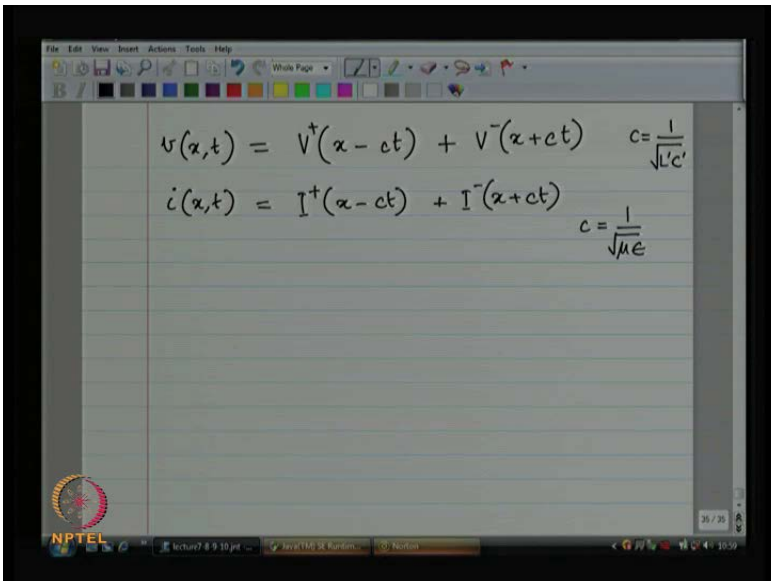

So, v (x, t) is going to be a function let us call it v plus this is function; I am sorry I am going to write it as v I will write it as c; where c is equal to 1 by squared root of L prime C prime. So, C is kind of similar to the speed of light its basically the speed of the wave propagating over the wire. So, if you do your computations correctly then L prime and C prime should give you the speed of the wave. And, the speed of the wave is going to be very related to the speed of light; typically this is what you are going to get as the speed of the wave which kind of tells you that if you happen to know the inductance per unit length; you can easily compute the capacitance per unit length without doing detailed derivations at anyway.

We get 2 different geometries and then we got and then we did some hand waving. And, then we said that really they are going to be to things are going to be identically the product of L and C is going to be constant etcetera we just hand waved or wave through anyway. So, this is actually the reason why the product is going to be a constant it is the speed of light on that particular material.

So, there is a lot of physics behind it we just hand waved or way. And, we said one is inductance per unit length, one is a capacitance per unit length really what is happening is and electromagnetic wave is propagating over the piece of wire. And, the speed of that wave is unknown quantity; the speed of that wave is 1 by squared root of mu times epsilon all of these are actually coming from Maxwell's equations. And, we are reformulating Maxwell in our own convenient way in terms of voltages and currents. Maxwell wrote as equations in terms of electric and magnetic fields right; the electric field integral of the electric field is going to give you the voltage; what about magnetic field, how does it relate to the current? A current is going to give you a magnetic field at a distance. So, they are kind of related to each other; however we have spun it in our own way.

So this is the solution to the wave equation v plus indicates of for forward moving wave, v minus indicates a backward moving wave; these waves are moving at velocity c meter per second. Similarly, i (x, t) is also going to give you very similar solution. Now, this v plus and this v minus these are functions of x and t and that is really the relationships of how it is a function of x and t? Its really function of x minus c t function of x plus c t; that is what it means alright you can plug these back into the original equations.

And, you will get your desired results; you will see that they work. Our equations are somewhat different we assumed that over here that G prime and R prime are 0 right G prime and R prime create a loss in the wire. When the wire is completely loss less an electromagnetic wave can propagate over the wire without any attenuation. If the wire is loss c then the electromagnetic wave is going to get attenuated as it propagates over the wire.

So, all this velocity etcetera they are going to be fudged up; once we plug-in nonzero conductance and resistance per unit length; hope fully the conductance per unit length is anyway equal to 0. So, for silicon G prime is 0. So, how are you going to solve this? Now, I already give you a hint with the electromagnetic wave; when I talk about electromagnetic waves they are typically sinusoids or a sum of sinusoids right; it is easy to think about sinusoidal waves anyway form, any shape complicated shape.

Let us say pulse or a squared or whatever you want can be dismantled into a lot of sinusoids at a lot of different frequencies it is possible. Now, I am going to use that fact and I am going to assume that maybe sinusoids is going to fit may bin. Let us see if it this. So, now we are going to start talking about phasers. So, let us see v is the phrase of V is the phaser for voltage, capital I is the phaser for current. So, these 2 equations are going to be modified as d squared V by d x squared that is equation number 1 is equal to L prime C prime what is what is the second derivative of the phaser of the phaser V?

What is the second derivative? So, if I got V cos omega t or let say I have got V time e power j omega t; then the second derivative is going to give me omega squared time the original is it omega squared or minus omega squared minus omega squared. So, cosine to sin and then sin to cosine again. So, that is one negative or you can think of j omega times j omega is minus omega squared. And, the second term will give you so what is the first derivative of phaser V its j omega times V. And, the third term is V times G prime. And, similarly you can work out the equation for the current is going to be a very similar equation. So, let me rewrite this equation.

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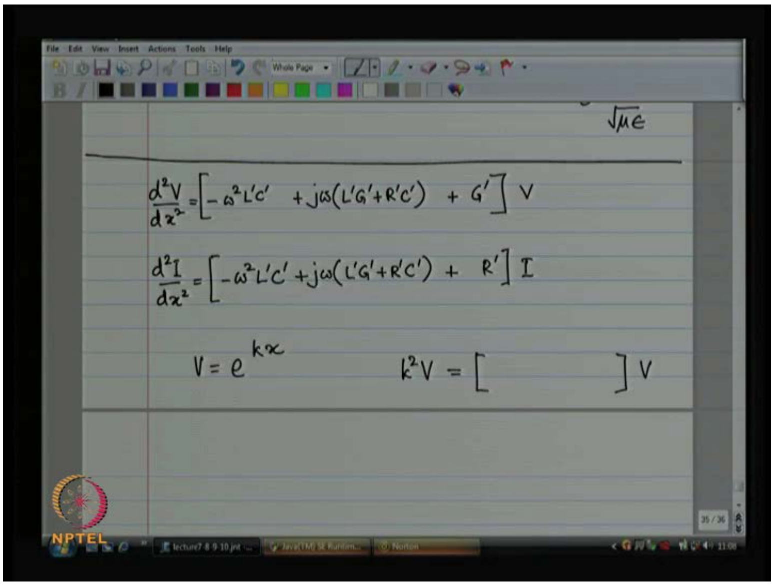

Similarly, the current equation is I am just going to replace all the inductance by capacitances; all the resistances by conductances and by see what is are right? We have got 2 beautiful equations; what is the solution to this dou squared V by dou x squared is something times V. So, what should V look like V should look like an exponential right; I do not know what this case? And, you plug it back in V equal to e power k x. So, dou squared V by dou x squared is going to be k squared times V and that is whatever is inside the brackets times V.

So, really k should be equal to the square root of the quantity inside. Now, the square root of the quantity inside it could be the positive squared or it could be the negative square root; you always have 2 squared root. First of all it is a complex quantity there is j omega times something right. If the resistance was 0 if the conductance was 0; then it would not have been complex, it could have been purely real, it could have been a negative quantity in fact real and negative. So, this square root of a real and negative number is plus or minus j times something; square root of complex quantity is going to be what? Square root of the magnitude times exponential of plus or minus j times the phase. So, we need to understand that there are 2 solutions for k; one is the positive root the other is the negative root right. So, what is the voltage finally going to be equal to?

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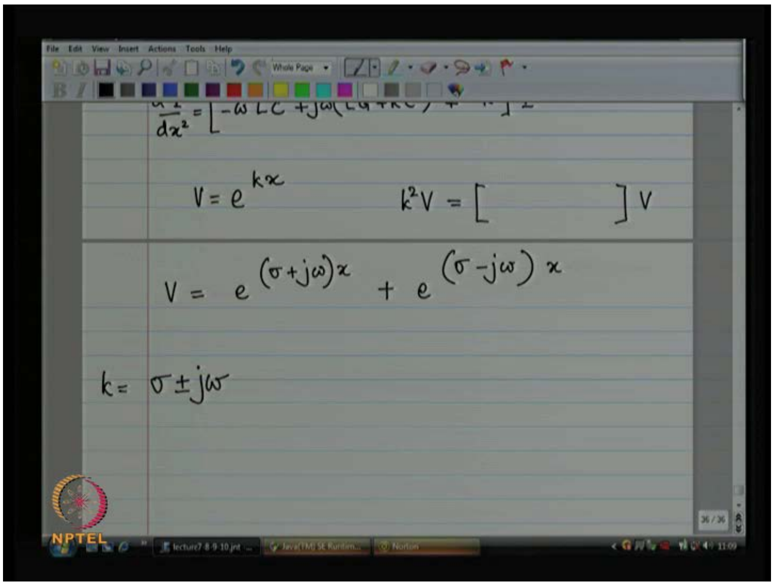

Finally, the voltage is going to be equal to e power the positive square root of these thing times x plus e power the negative square root of whatever is in the bracket times x. And, is positive and the negative square root these are both complex numbers, agreed? So, these are both complex numbers means there is going to be real part width, there is going to be an imaginary part width. So, the positive and the negative square roots; let us say these are sigma plus minus j omega.

So, I have got; so these 2 are basically going to give me forward moving wave and a backward moving wave; sigma we will be able to figure out that sigma is going to be a negative quantity. And, as a result we are really going to get an attenuation over space. So, basically the intuivation like this you get a forward moving wave, you get a backward moving wave; the net voltage is a some of the forward and the backward moving wave attenuated in terms of distance because of the loss. So, we shall stop here and we will carry on from the next class.

Thank you.