**Foundations of Wavelets and Multirate Digital Signal Processing**
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**Module 5**
**Lecture No 31**
**Conclusive Remarks and Future Prospects**

A warm welcome to this conclusive session of this massive open online course on wavelets and multirate signal processing. In this way will have a quick conclusion and looking ahead on what we would do in a subsequent larger massive open online course. By 1st requesting all my friends here, my teaching associates who have helped me create this material, so I'm going to ask them to take just a minute each and summarise what they learned in the specific lecture that they dealt with. Let me begin with Jayawardhan. Jayawardhan you can start.
Professor-student conversation starts.

Student: hello everyone, I am Jayawardhan. So 1st of all in this course, we started with the need of simultaneous time frequency localisation. We also looked at the idea of natural domain of a function. Specifically we looked at images and how they can be represented at a particular resolution and how do we move from one resolution to a higher resolution using direct wavelets. Then, this was the 1st time an haar wavelet was introduced to capture the incremental information in going from one resolution to another. Next, we looked at the L2 norm and the LP norm in general LP norms of functions and the need for finite L2 norm for piece wise constant representation of functions.

Professor: Very good, Jayawardhan. Now, who would like to take over from him? Who comes next? Yes, so Pulkit is going to talk about the next part of the course.

Student: Hello everyone, I am Pulkit Singh. 1st of all we looked at how to represent a function using the piece wise linear constant representation. Then we went on to locate the hierarchical order of the ladder of subspaces which Sir had taught us in the lecture. Then we went on ahead and studied the concept of a scaling function in the case of a haar wavelet and its applications over there introduced in the further lectures.

Professor: Very nice, Pulkit. Who would like to take over to talk about the next part? Saurabh. Saurabh is now going to take over.

Student: Hi, I am Saurabh. The most interesting part I ah learnt in this course was how to ah see functions as a sequence. So, what I dealt with was to deal with functions as a sequence and then use them, use properties of sequences on them, like trying to find out norms and inner products for…

Professor: So what you are saying is you brought an equivalence between functions and sequences and that made it interesting because of because there are a lot of things about sequences which are easier to deal with than functions inherently. Yes, please go on.

Student: So that we can extend it to higher dimensions using ah like we can go in higher dimensions using sequences and the next thing we dealt with was touching upon Percival's theorem. Yes, this was all.

Professor: Very nice, Saurabh. So, who would like to take over from there? What was next? Yes, Shivam is going to tell me what came next.

Student: So as Saurabh Singh was saying that in this course, we studied the equivalence between functions and sequences, so what I feel is that this subject of wavelet is very closely related to the mathematical subject of what we study in basic mathematics what is known as linear algebra. I also try to add some of the additional information regarding some concepts on linear algebra like direct some and also we saw that when we introduce when we are able to define the concept of in their product on a particular set which is a special set in itself like a vector space, then we can go ahead and define you know equalities like Percival's theorem and so on and we saw what is the importance of doing so. So this was the part of the course which dealt with the functions. Now, to implement these functions, we went ahead and studied a system which is known as a filter bank. So a filter bank we studied that how mainly a filter bank.

Professor: Now Shivam, you should tell us what a filter bank is. What is meant by a filter bank?

Student: So filter bank is basically a system, discrete time system which takes some input and gives you some output. So, an input which is given to a filter bank is broken down into many components.

Professor: It has multiple you have to be careful, it has multiple outputs.

Student: Yes sir.

Professor: It has one input, it has multiple outputs or that multiple inputs and one output. There are 2 possibilities. Either the analysis filter or…

Student: Yes.

Professor: Go on, yes.

Student: This part of the course was very interesting to me and I would like to you know Sir, ask you that from where we go ahead and in the next module of this course, you know you in the 1st lecture you mentioned the concept of uncertainty. Sir, you can just tell a few lines.

Professor: Yes, you know if I ask very good. So now you know, this whole course is just the beginning of your study of wavelets and multirate systems. Where we go from here is of course, you have just sort of been introduced to what a filter bank is, how you could construct a filter bank which could not be iterated to lead to scaling function or a wavelet and then how a multiresolution analysis can we keep it out of it. But you know what are the challenges? For example, why can not we be happy with the haar multiresolution analysis? After all, the haar has all the nice and easy properties to understand but the problem is we have not at all looked at the frequency domain here. The whole idea, as I told you in the beginning, was to deal with the 2 domains at once. So the frequency domain has actually dealt been dealt with very poorly by the haar wavelet or the haar MRE and this is something that we would now need to go on to understand if we wish to carry this subject further. What is the basic limitation if any which allows you to deal with the time and frequency domains together? How can you work best around these limitations that are there and what is the design skill required to build multiresolution analysis while keeping this limitation in mind? That is essentially what is meant by the uncertainty principle. In fact, in a subsequent massive open online course where we would develop this subject much further, we would like to go into these issues. You know what are the basic limitations which come in when you want to deal with both the domains together? How do you address those limitations? How do you build systems around those even if there are those limitations? And then, how do you apply systems that deal with time and frequency domains together in practical applications? So this is where we would be. And, I'm very happy that all my teaching associates have closely interacted with all of you and built material for this course. We

do hope all of you enjoyed this course and we would be very happy if you ask us questions or if you suggest how we should continue to engage with all of you beyond this in a larger massive open online course. Maybe, we can have one concluding remark from my teaching associate, Abhinav.

Teaching associate: My name is Abhinav and what I dealt with was extension of concept of Z transform to the discrete time frequency domain and the concept of ROC. So we also had a look on and on the frequency response of analysis lowpass as well as the high pass filter.

Professor: Good. Thank you Abhinav. This is where we left off and in fact this is what would lead us to the next part of a massive open online course. Right? So is with that then, we come to the end of this discussion session. And we do hope once again that you enjoyed what we presented before you and we look forward to engaging with all of you in future as well. Thank you so much.

Student-Professor conversation ends.

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