Foundations of Wavelets and Multirate Digital Signal Processing

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Lecture - 1
Module - 3
Haar Wavelet

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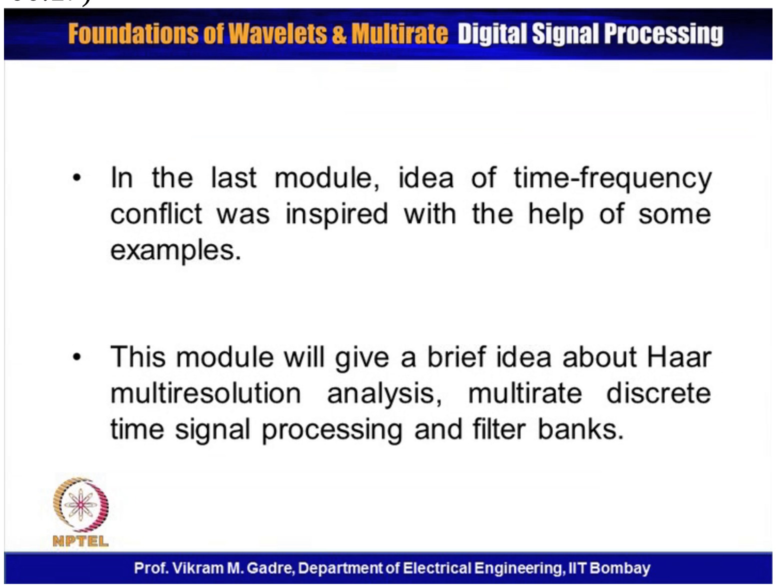

So we are going to build up the whole idea of wavelets by starting from what is called the Haar Multi Resolution analysis. Haar incidentally is a name of a mathematician. Call him a mathematician, call him a scientist what you will, but one of the beautiful things that this gentleman proposed was what is called the dual of the idea of Fourier analysis. What do we do in ferial analysis?

We allow even discontinuous wave forms, we allow non smooth wave forms and we convert them into a sum or a linear combination of extremely smooth functions, namely the sign waves. Haar said can we do exactly the dual. Can we in principle take smooth functions and convert them into a linear combination of effectively jagged of discontinuous functions. Why on earth would one like to do something like that?

Again lets deflect for a minute, a few years before this might have seem silly to do, but today it is not. What are you doing when you are doing digital communication? You are transmitting audio, you are transmitting pictures and you are doing all this actually with a large level of discontinuity. How does one record digital audio? One firstly samples, so one takes values of the audio signal at different points in time. One digitizes them, and one then records those digital values as a stream of bits.

All of these are highly discontinuous operations. You are forcibly introducing discontinuity in time and on top of that you are introducing discontinuity in amplitude by quantization. So wanting to represent the beautiful smooth audio in terms of very discontinuous bit streams is very-very beneficial to digital communication. And in fact none of complain when we have a good audio digital recording, sometimes we even say that digital recording is better than an analogue recording that we had in the past.

So going from smooth to non-smooth has its place in modern communication and signal crossing. And when Haar proposed that one should look at the whole philosophy and the whole principle of being able to go from smooth to non-smooth, perhaps he was looking into the future, when this would be absolutely essential. What we are going to do in the very first few lectures immediately following this is to look at one whole angle of wavelets and multirate digital processing based on the principles that Haar propounded.

So we are going to look at what is called the 'Haar Multi Resolution Analysis'. And in fact if we understand the Haar Multi Resolution Analysis in depth, we actually end up understanding many principles of wavelength. Many of the essentials of multirate processing specifically, what is called 2 band processing very well. So we shall draw upon the Haar Multi Resolution analysis to understand some of the basic concepts that underlie this course and of course build upon them further later on.

From the Haar we shall slightly progress to better multi resolution analysis. From the better in what sense we understand and there are many such different families from this better multi resolution analysis one of them being what is called as the Dobash family. Dobash again is the name of a mathematician scientist who proposed that family of multi resolution analysis. As I said at a certain point in the course immediately following this we shall then look at the uncertainty principle.

Fundamentally and in terms of its implications. From there we shall move to the continuous wavelets transform. So in the Haar multi resolution analysis we have a certain discretization in the variables associated with the wavelets transform. Later on we shall go to what is called the continuous wavelets transform where the variables, the independent variables are associated with the wavelength transform all become continuous.

Following that we shall look at some of the generalizations of the ideas that we have build up earlier in this course. And towards the last phase of the course we shall look in depth at some of the important applications to which wavelets at multiple digital processing provide great advantages.

Now, I would like to spend a little while in this lecture on building up in parallel. Some of the developments that took place, to introduce the subject of multi rate digital processing. What is multirate? What rate are we talking about here and why do we need to talk about multi rate? Why is it connected with wavelets? Let's go back to the audio example or may be let's first go to the biomedical example.

In the biomedical example we said we would have quicker parts in the response and slower parts in the response. The slower parts of the response are likely to last for a longer region in time. The quicker parts of the response are likely to last for smaller regions in time. So here other than the concept of being able to localize in a certain region of time and of course correspondingly on frequency.

There is also a distinction between what kind of localization is required for higher frequencies and lower frequencies. If we spend a little bit of thought and time in understanding these 2 kinds of components, we'll realize that most of the time when we are talking about the slower parts of the response or lower frequencies we are talking about compromising on what is called time resolution. So I bring in the idea of resolution here.

Resolution means the ability to resolve, the ability to be able to identify the specific components. So for example frequency resolutions relates to being able to identify specific frequency components. And going further and being a little more pin pointed, when I am talking about frequency resolution what I am saying in effect is… Suppose I have 2 sign waves whose frequency start coming closer and closer together, over what region of time do I need to observe them so I can actually identify the 2 frequencies separately.

How can I resolve the 2 frequencies? How much can I narrow down on the frequency axis? Now what we are talking about is not so much how much we can narrow down but how much we need to. When we talk about higher frequency content or things that vary quickly, it is often though not always the case that we are willing there to compromise on frequency resolution, but we want time resolution. So things that take place quickly and are transient short lived demand time resolution.

And things that occupy the lower frequency ranges which last for a long time demand frequency resolution. So very often it is true that when one goes down on the frequency axis one demands more frequency resolution, the ability to resolve frequency is more accurately as opposed to time resolution. The ability to resolve which time segment it occurs. And when one goes to higher frequencies where one normally demands more time resolution and lesser frequency resolution.

One is asking for how closely one can identify 2 segments or 2 parts of the wave forms which vary quickly, that means one is trying is narrow down on the time axis and in doing so one must compromise on the frequency axis. So this is what brings us to the idea of multirate processing. You see it means that when I talk about bands of higher frequencies, I must use smaller sampling rates in a discrete time processing system.

When I am talking about lower frequency ranges, I must use larger time sampling points or sampling intervals. Why must I do so to be most efficient in the processing operation? When I am talking about lower frequencies so in an evoke potential wave form, if I am trying to look at the slower components I should not unnecessarily sample too frequently. It only increases my data burden and does not offer me anything special.

On the other hand when I am analyzing the quicker components, it is inadequate to use a low sampling rate. I would be doing injustice to the components. For those of us who might be exposed to the concepts of sampling elaises. If I am not faithful in my sampling rate of the quicker components I would introduce elaises. I would introduce purious effects which I don’t want. So on in all we recognize that it is not a good idea to be using the same sampling rate for all frequency components.

So unlike a basic course on discrete time signal processing where we assume all frequencies are at the same sampling rate, here we need to deal with the same frequency rates that are effectively at different sampling rates in the same system. That means we also need to deal with systems that operate with different sampling rates and that is why we talk about multirate discrete time signal processing.

Now at a conceptual level we understand at a very well why there is a close relationship between multi rate discrete time signal processing, the idea of uncertainty, the requirement of the resolution. And if we go further than when we do multi rate discrete time signal processing, we also bring in a new concept of filter banks verses filters. As I said if we go back to the bio medical example there is the effect or there is the desire to separate components.

So when I wish to separate components, naturally I wish to have all different operators all at once. So I need a system of filters which not only have certain individual characteristics but which also have collective characteristics. So I need to be able to analyze and then synthesize, and all this with localization included. This is what we mean by a filter bank.

So a bank of filters as opposed to a single filter in discrete time signal processing refers to a set of filters which either have a common input or a common point of output or a summation output. This concept of a bank of filters in fact 2 banks of filters an analysis filter bank and a synthesis filter bank, taken together is very central to multi rate discrete time signal processing. We shall be looking at that concept in great depth.

So we shall be building up the idea of a 2 band filter bank in reasonably great depth in this course. The concept of a 2 band filter bank is of great importance in being able to construct wavelets. In fact we shall see even from the Haar multi resolution example that there is an intimate relationship between the wavelet or the Haar wavelet and the 2 band Haar filter bank.

So much so that if I construct a properly designed 2 band filter bank I also construct a multi resolution analysis that goes with it so to speak. All this is very exciting. And what we intend to do in the lectures that follow from here is to take these concepts one by one. So in the next few lectures we intend to talk about the Haar multi resolution analysis, to build up certain basic ideas from it.

With that then we come to the end of this first introductory lecture on the subject of wavelets and multirate digital processing and proceed there with- in the next lecture to talk about the Har multi resolution analysis. Thank you!