

## Foundations of Wavelets and Multirate Digital Signal Processing

Professor Vikram M. Gadre

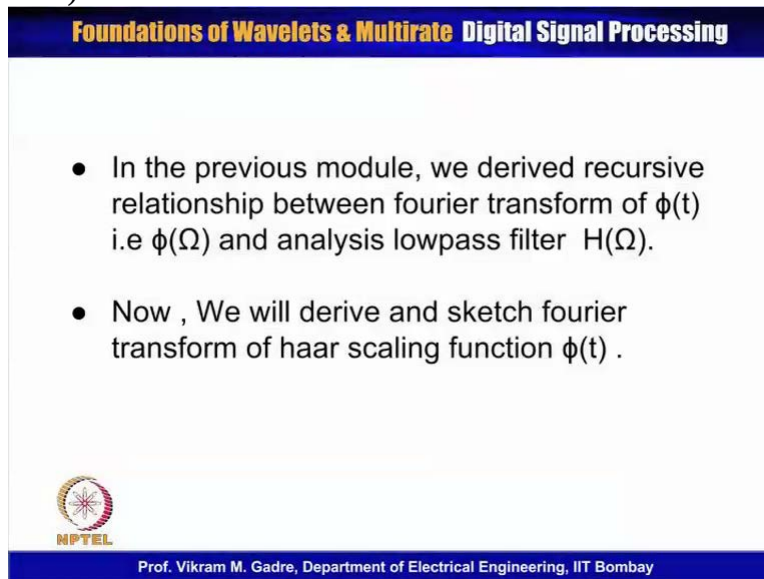
Department of Electrical Engineering  
Indian Institute of Technology Bombay

Module 5

Lecture No 28

Fourier Transform of Scaling Function

(Refer Slide Time: 0:16)



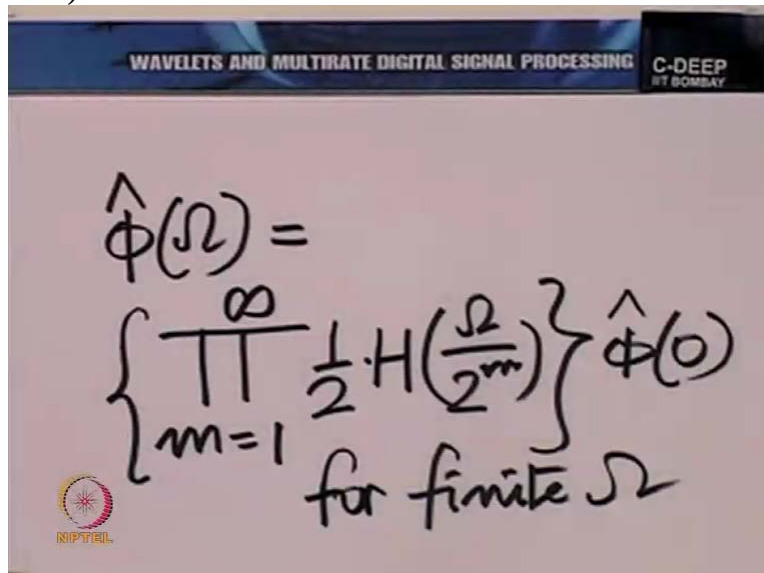
**Foundations of Wavelets & Multirate Digital Signal Processing**

- In the previous module, we derived recursive relationship between fourier transform of  $\phi(t)$  i.e  $\phi(\Omega)$  and analysis lowpass filter  $H(\Omega)$ .
- Now , We will derive and sketch fourier transform of haar scaling function  $\phi(t)$  .

 NPTEL

Prof. Vikram M. Gadre, Department of Electrical Engineering, IIT Bombay

(Refer Slide Time: 0:19)



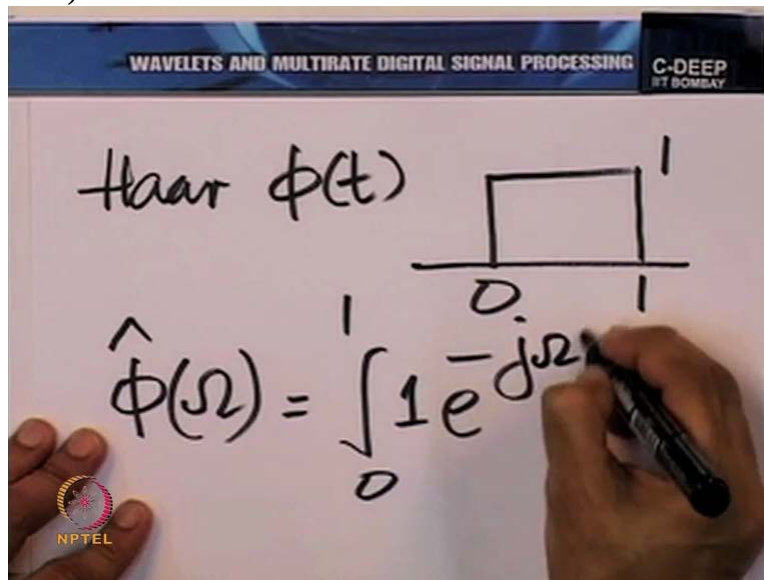
**WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING** C-DEEP IIT BOMBAY

$$\hat{\phi}(\Omega) = \left\{ \prod_{m=1}^{\infty} \frac{1}{2} H\left(\frac{\Omega}{2^m}\right) \right\} \hat{\phi}(0)$$

for finite  $\Omega$

 NPTEL

(Refer Slide Time: 0:24)

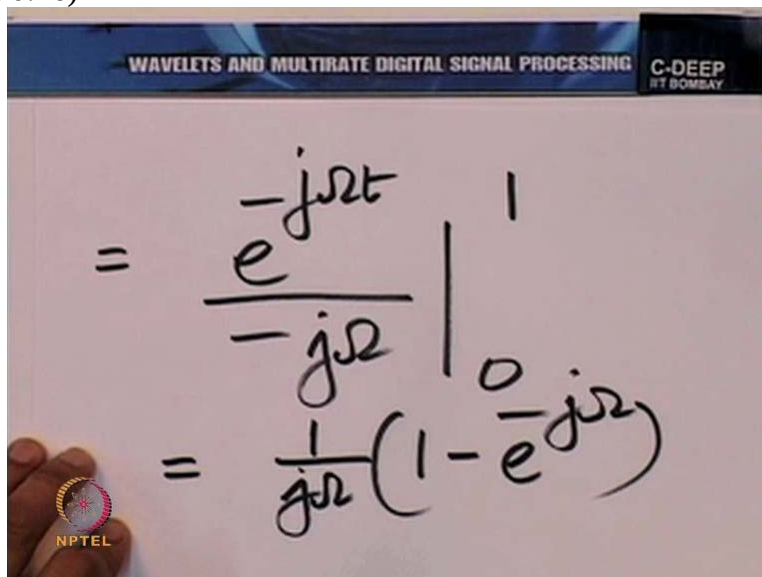


The slide shows the Haar wavelet function  $\phi(t)$  as a rectangular pulse from 0 to 1 with a height of 1. Below it, the Fourier transform is written as an integral:  $\hat{\phi}(\omega) = \int_0^1 1 e^{-j\omega t} dt$ . The slide header is "WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING" and "C-DEEP IIT BOMBAY". An NPTEL logo is in the bottom left.

$$\text{Haar } \phi(t) \quad \begin{array}{c} 1 \\ \hline 0 \quad 1 \end{array}$$
$$\hat{\phi}(\omega) = \int_0^1 1 e^{-j\omega t} dt$$

Let us look at the Fourier transform of  $\phi_T$  in the Haar context. Accentuate is this and the Fourier transform is easy to calculate.

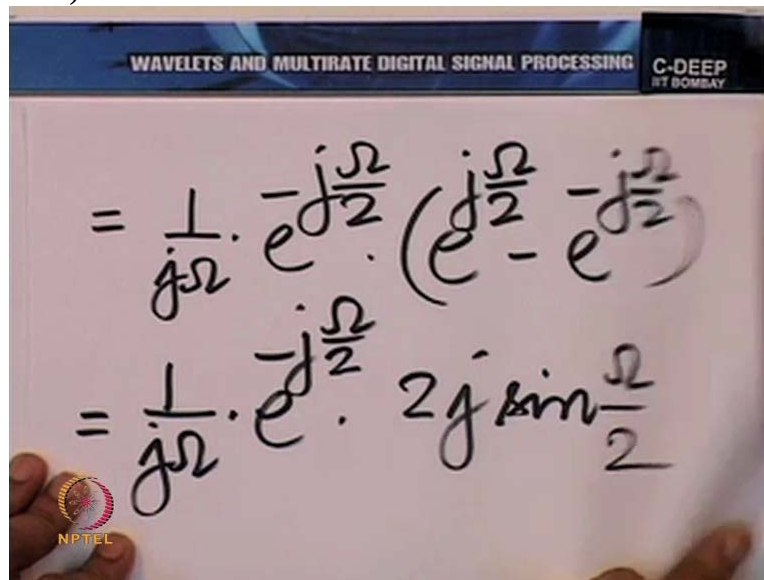
(Refer Slide Time: 0:46)



The slide shows the evaluation of the Fourier transform integral:  $= \frac{e^{-j\omega t}}{-j\omega} \Big|_0^1 = \frac{1}{j\omega} (1 - e^{-j\omega})$ . The slide header is "WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING" and "C-DEEP IIT BOMBAY". An NPTEL logo is in the bottom left.

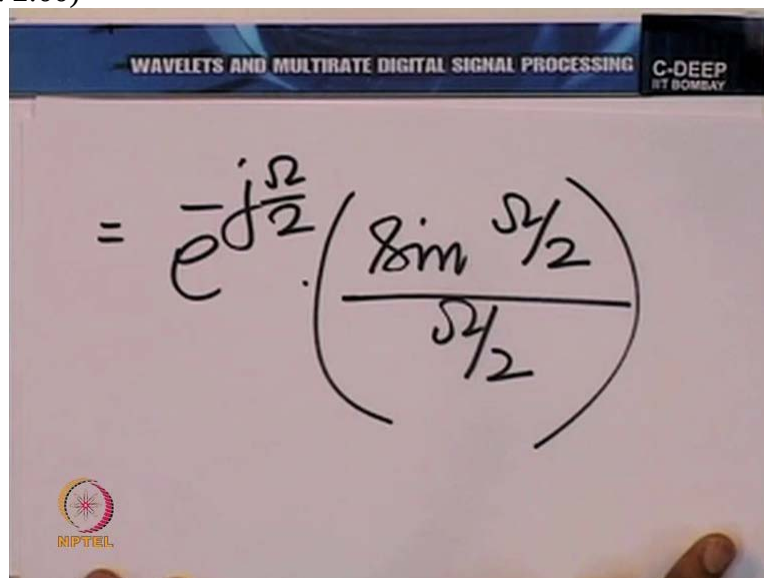
$$= \frac{e^{-j\omega t}}{-j\omega} \Big|_0^1$$
$$= \frac{1}{j\omega} (1 - e^{-j\omega})$$

(Refer Slide Time: 1:21)


$$= \frac{1}{j\Omega} \cdot e^{-j\frac{\Omega}{2}} (e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}})$$
$$= \frac{1}{j\Omega} \cdot e^{-j\frac{\Omega}{2}} \cdot 2j \sin \frac{\Omega}{2}$$

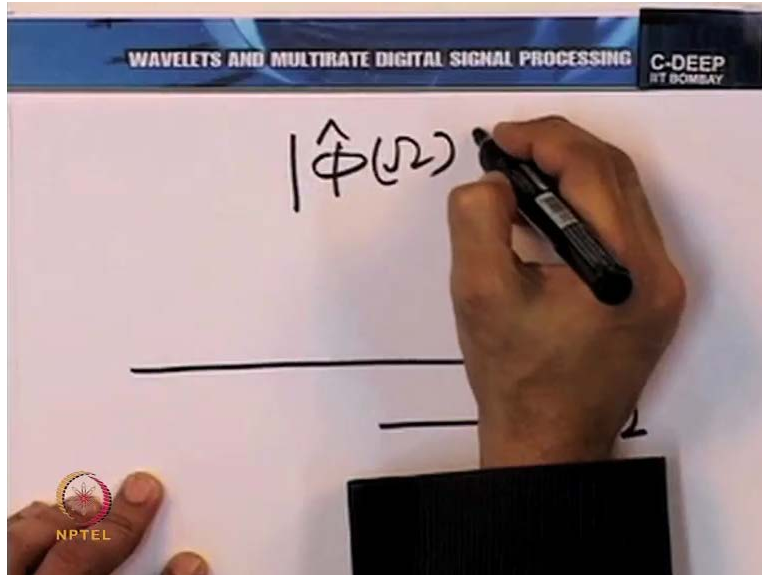
Now we can simplify this using the standard trick of taking out  $e^{-j\Omega/2}$  term. And then we noted that we have a sign hidden there.

(Refer Slide Time: 2:00)


$$= e^{-j\frac{\Omega}{2}} \cdot \left( \frac{\sin \frac{\Omega}{2}}{\frac{\Omega}{2}} \right)$$

And doing away with the  $J$ s we have. Now as you can see, this Fourier transform goes towards 0 as capital  $\Omega$  goes towards infinity. So the Fourier transform vanishes as capital  $\Omega$  goes towards infinity. And in fact became even sketch the magnitude to get a feel of this.

(Refer Slide Time: 2:53)



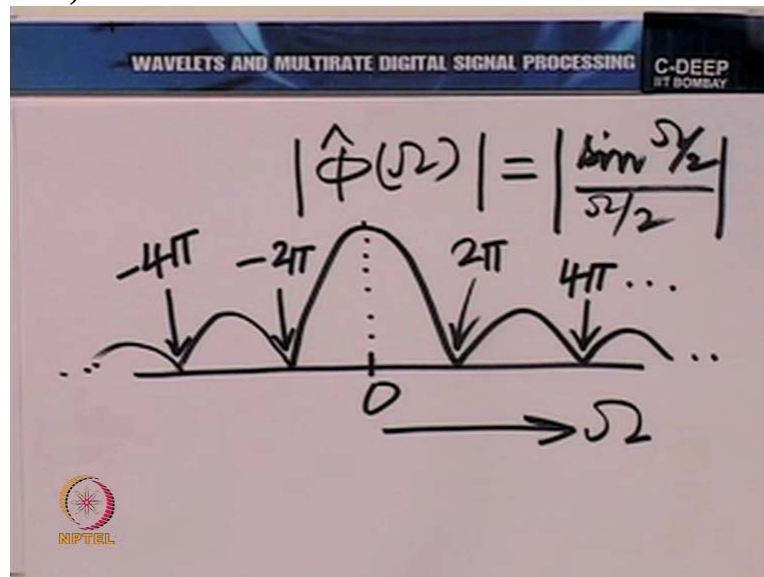
The magnitude of phi cap omega looks like this.

(Refer Slide Time: 3:05)

$$= e^{-j\frac{\Omega}{2}} \cdot \left( \frac{\sin \frac{\Omega}{2}}{\frac{\Omega}{2}} \right)$$

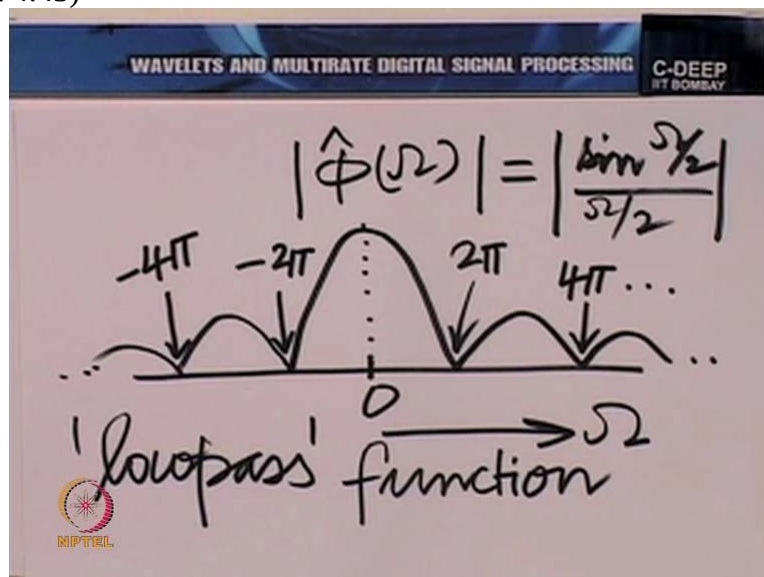
You see, at omega equal to 0, you will notice that one can use Lobita's rule and show that this is equal to 1 in magnitude.

(Refer Slide Time: 3:16)



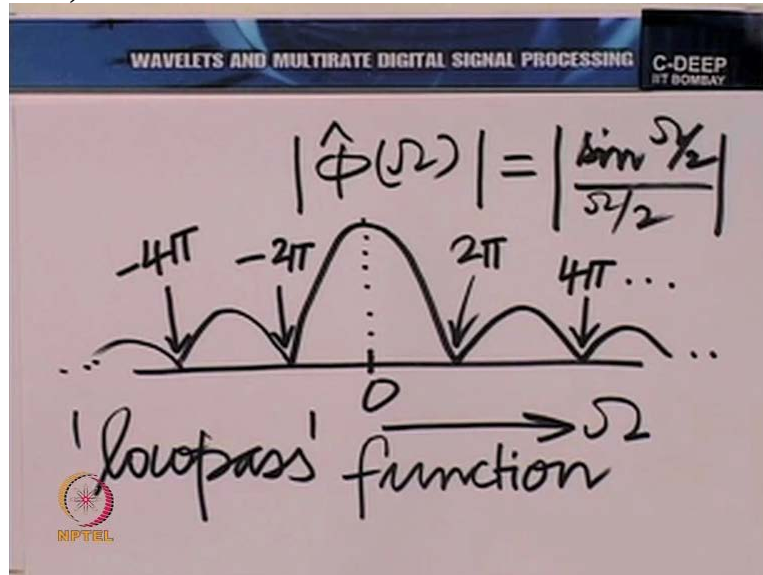
So it has a pattern like this. Of course we know where this comes. This will come at  $\omega$  by 2 equal to  $\pi$  or at  $2\pi$ . This would come at  $4\pi$  and so on. This is how the magnitude looks. Now you know, this very clearly shows that  $\psi$  has a concentration around 0 frequency. So this is interesting. We begin from the low pass filter, we construct a dilation equation, a recursive direction equation starting with a low pass filter and we get a low pass function.  $\phi_T$  is essentially a low pass function. A low pass function means it is predominant in the frequency domain around capital  $\omega$  equal to 0.

(Refer Slide Time: 4:43)



Low pass function of course is an informal term. You may always argue that after all it does have bands at higher frequencies too. But the point is, its prominent bands are around 0 and the farther you go away from 0, the more the spectral magnitude drops off. In that sense, it is low pass. In fact just as we try to build the idea of ideality in a filter bank, you also would like to build the idea of ideality in this  $\phi T$ . The ideal  $\phi T$  is actually the ideal low pass function. So low pass function which is like a brick wall.

(Refer Slide Time: 5:34)



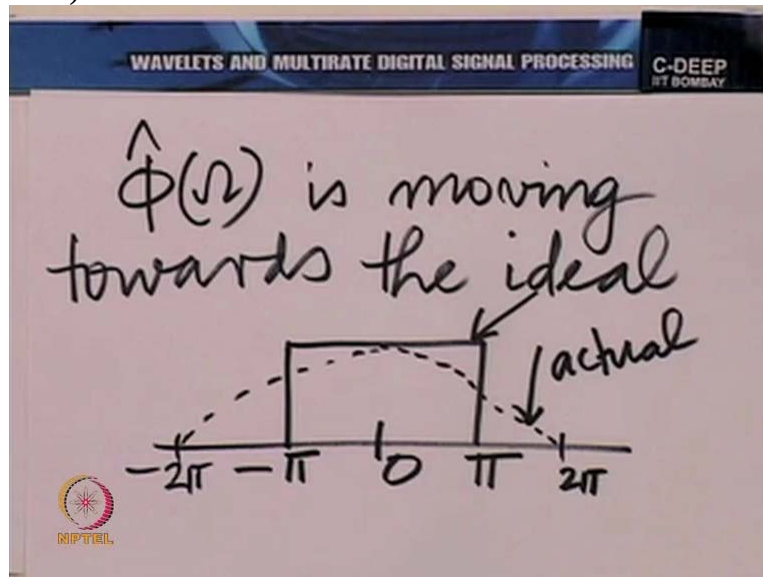
So in some sense,  $\phi T$  in the spectral domain or  $\phi \hat{\omega}$  for that matter is moving towards the ideal. You know this is where we are moving and where we are is somewhere here. Very far away from it of course. Not surprising. After all that is what we saw in the Haar multiresolution analysis. Ideal here, actual there. Now once again, there is the same conflict that drives the engineer, the scientist, or the mathematician. We know that we want to go towards the recall ideal. But we also know that the recall ideal is unattainable for various reasons. The reasons are similar to what I talked about last time for the unattainability of the idealism in a filter bank. So we would need to repeat them once again here. However, what we will now do is to see what is the relationship with whether ideal or practical, what is the relationship between  $\phi \hat{\omega}$  and the discrete time Fourier transform of the filter bank low pass filter in terms of construction?

So in other words, we have written down a dilation equation in the frequency domain but we need to translate that dilation equation into a constructive step. How do we construct  $\phi T$  given the low pass filter impulse response? In a way, if we do that, we have answered the question,



how is the design of a filter bank related to the design of a multiresolution analysis? So let us do that. Towards our objective, let me put before you once again that infinite product here.

(Refer Slide Time: 7:57)



So you know, now you also understand why  $\phi$  cap zero should not be 0.  $\phi$  cap zero must be nonzero. In fact,  $\phi$  cap zero is very often the maximum value of the magnitude of  $\phi$  cap  $\omega$  because of the low pass character. So we have seen this lowpass character in the Haar context. Now, we shall assume it to be true of most multiresolution analysis and proceed from there. So this is just a constant, a nonzero constant to be vary but what we need to identify is this.

Audio ends here