Foundations of wavelets and multirate digital signal processing.
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Module-4.
Lecture -24.
Disqualification of ideal filter bank.

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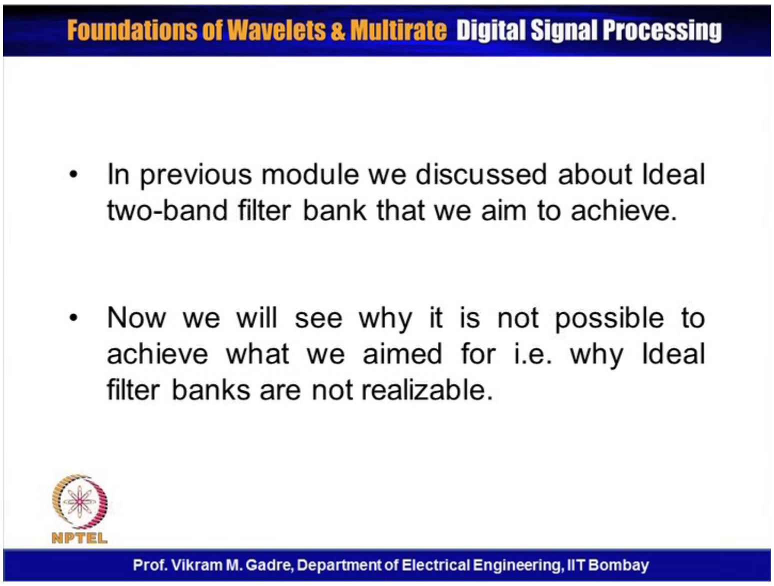

Now, you know without going too deep into each point, I would like once again to recapitulate, why the ideal filters cannot be attained. So, it has nothing to do with technology or the lack of computational power, there is something fundamentally troublesome about his ideal filters that make them unachievable or unattainable. Let me cite these points one by one for the sake of completeness and revision from a more basic course.

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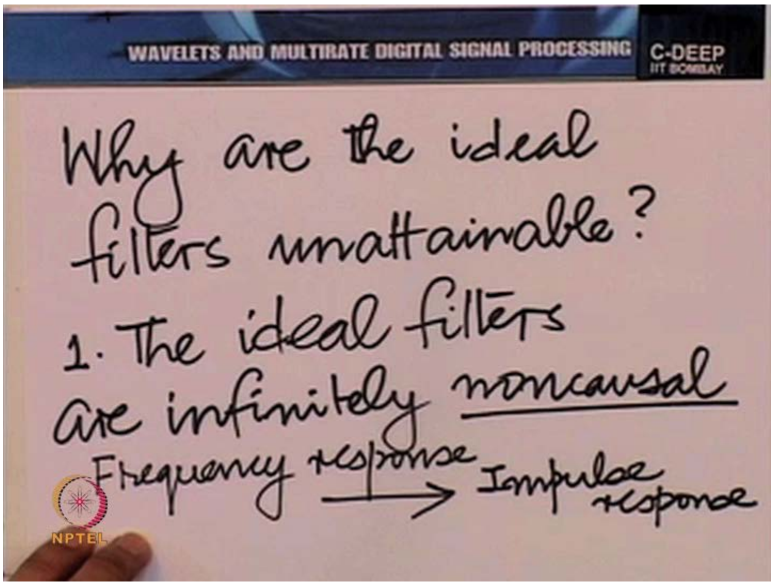

So, why are these ideal filters unattainable? Well, the 1st reason is that the ideal filters are infinitely noncausal. And this can be checked by constructing the impulse response, so from the frequency response, we can go to the impulse response. We also know how to do that.

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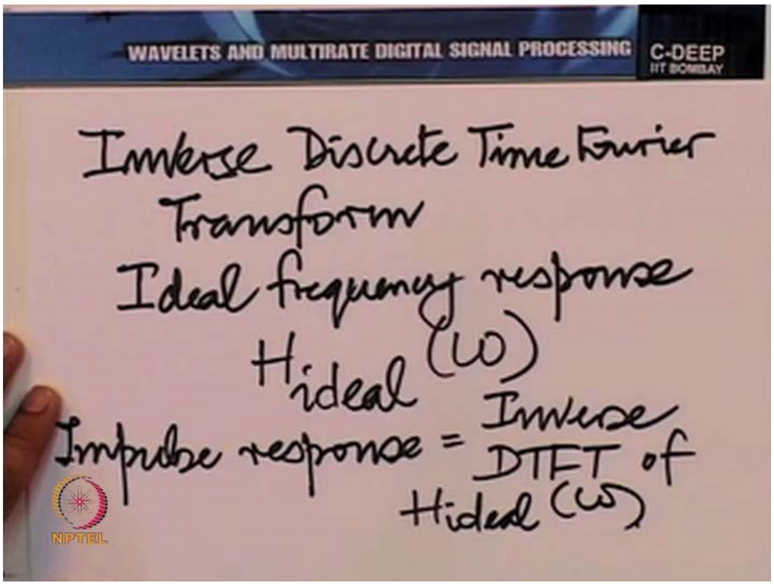

We can go to the impulse response by taking what is called the inverse discrete time Fourier transform. So, if you have the ideal frequency response, let us call it H ideal as a function of omega and the corresponding impulse response can be obtained by the inverse DTFT of this.

And how do you calculate the inverse DTFT, I leave this calculation for the class to do but I would like to put down the important steps here.

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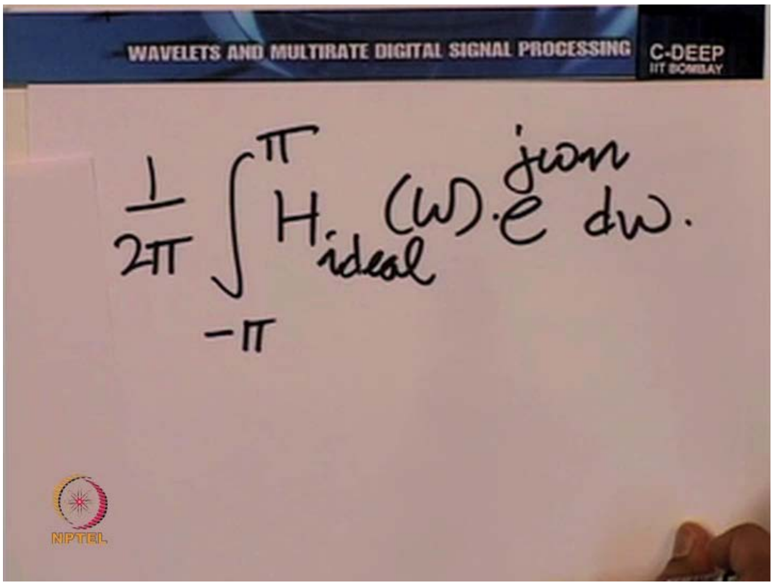

That is done as 1 by 2 pie integral from - pie to pie H ideal omega E raised to the power J Omega N D omega and in fact I leave it to the class to compute this for the ideal lowpass filter and the ideal high pass filter with a cut-off of pie by 2. I shall just put down the answer for the ideal lowpass filter.

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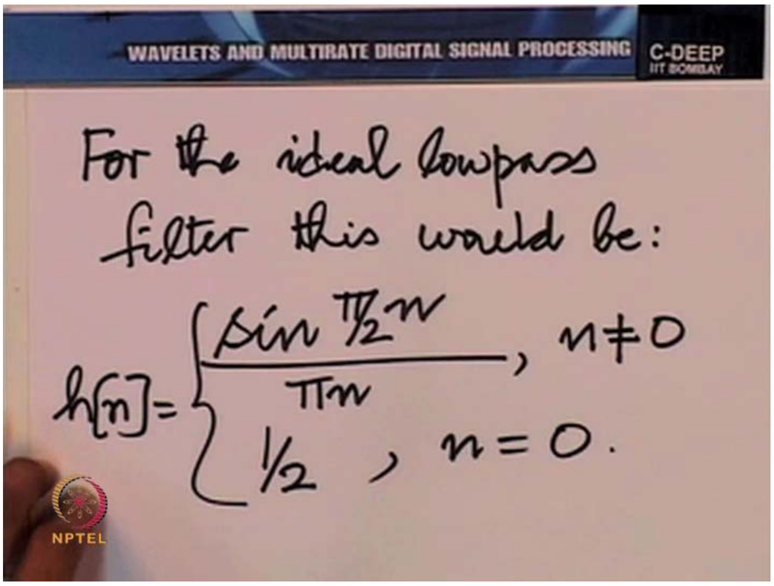

So, for the ideal lowpass filter, this would turn out for example to be sin pie by 2N divided by pie N wherever N is not 0.

So, this is the impulse response HN, that will be half for N equal to 0, I leave it as an exercise to verify this integral and I also leave it as an exercise for the class to calculate the ideal high pass filter impulse response.

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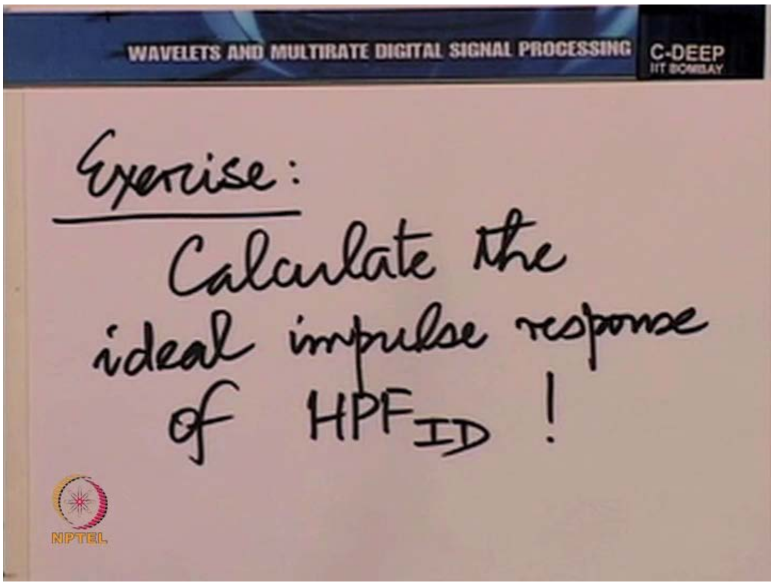

I leave it as an exercise, easy to do what one has a basic introduction to the inverse discrete time Fourier transform from a basic introduction to discrete systems. Anyways, the point was if we look at the impulse response, there are 3 things forbid 1 from realising this filter.

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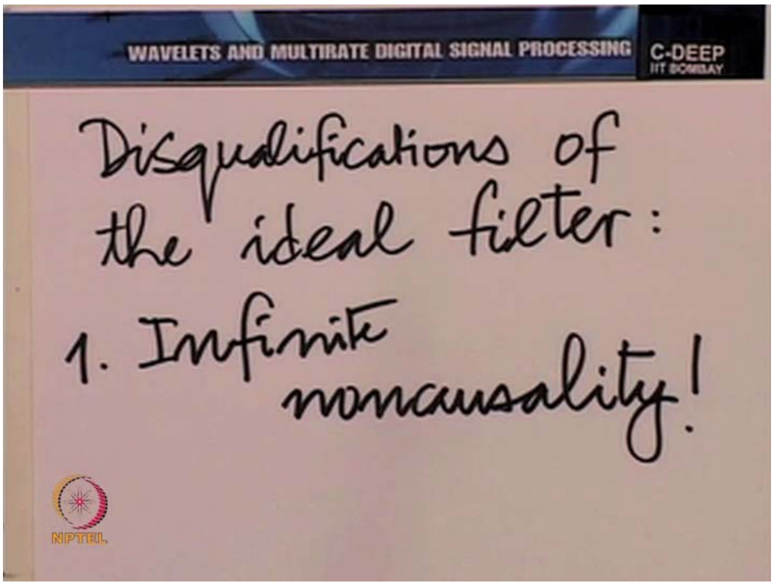

Coming back, the 1st thing was infinite non-causality, the disqualifications of the ideal filter. Point number 1, infinite non-causality. Now, when is a discrete linear shift invariant system causal? It is causal if the impulse response is 0 for all negative values of the integer index. So, HN is equal to 0 for all negative N is the requirement necessary and sufficient for causality. In this case, or for that matter in the case of any ideal filter, you could take either the high pass or the lowpass filter in this case, for both of them, you would notice, it is infinitely noncausal, meaning that if I were to delay the impulse response sequence by a few samples, infinite number of samples, you could never make it causal. So, it is infinitely noncausal.

A serious disqualification, which means if you wish to realise a causal filter bank, you cannot. So, you know it requires anticipatory behaviour, doing it on time. You need to use the future to work in the present, a strange situation to begin. You know, non-causality by itself is not always a disqualification, it is infinite noncausality which creates a problem. Infinite non-causality means one cannot make the filter causal by introducing some delay, that is what is a disqualification here. Disqualification number-one.

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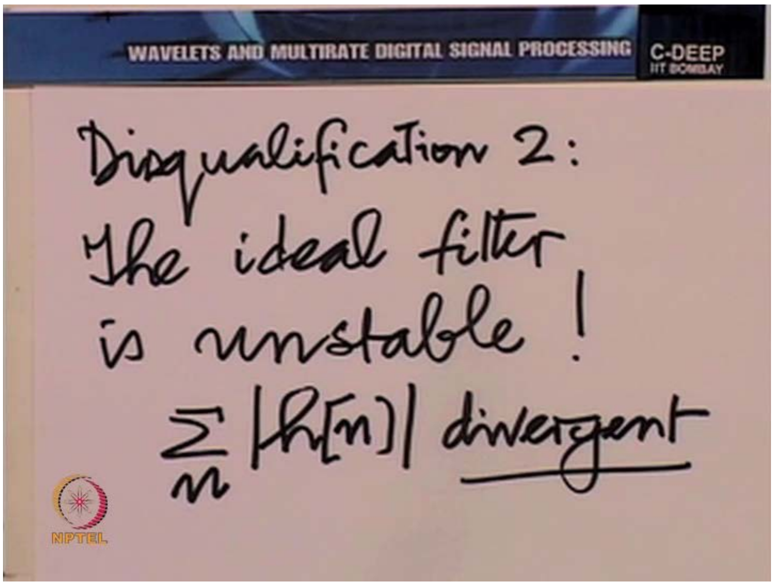

Disqualification number 2, the system is unstable, that is if you look at summation N mod HN, it is diverted, a terrible thing to happen. Now, this is also indicative of a subtle point, you know just because it is a filter, does not mean it has to be stable. What it means, you know the moment you say filter, the moment you state has a frequency response, what it means is that when you give a sinusoidal input, a bounded input of course, a sinusoidal input, the output is bounded, in fact the output is sinusoidal with the same frequency.

But that is to do with sinusoids, you could not very well have some other peculiar input where the output is unbound and that is troublesome. So, you know you could have a situation where you get a certain behaviour as far as sine wave goes but you are not guaranteed that the output will always remain within bound for a bounded input. A bounded input may not produce a bounded output., The system is unstable.

A discrete time system is stable if its impulse response is absolutely summable, of course we are talking about LSI Systems, linear shift invariant systems. So, this linear shift invariant system whose frequency response corresponds to the ideal lowpass filter by the cut of pie by 2 or any other cut-off or for that matter, an ideal high pass filter, all such systems are unstable which can be shown by showing that the impulse response is not absolutely some able, if you try and calculate the absolute sum of the impulse response, it would diverge.

Again I leave this as an exercise for you to show. Take the ideal impulse response of the lowpass filter for example with a cut-off of pie by 2 and try and show that its absolute sum is diverted, not very difficult, but an interesting exercise. Disqualification number 2, unstable. Disqualification number 3, a serious one too. The ideal filter is irrational.

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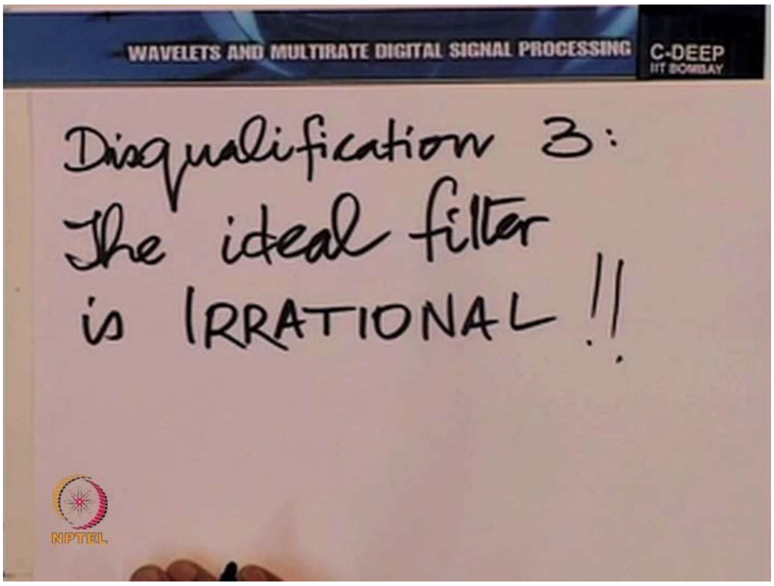

And what does this mean, now let me 1^{st} explain the meaning of irrational literally. You see, we say a filter is rational or linear shift invariant system is rational, if you look at the system function, that means that Z transform of the impulse response and find that can be expressed as a ratio of 2 finite series in Z, so numerator a finite series in Z, denominator a finite series in Z. If the system function of the LSI system could be expressed as a ratio of 2 finite series in Z, we say the system is rational.

So, of course when we talk about rational system, we are automatically talking about linear shift invariant systems, it is only linear shift invariant system which could be rational or irrational and rational or irrational refers to system who have a system function, linear shift invariant systems who have a system function. Which means the Z transform of the impulse response exists in some non-null region of convergence.

Now, the Z transform of the impulse response of the linear shift invariant system could either be rational, which means it is a ratio of 2 finite series in that or it could be rational, in which case it is not. Let me give you an example of an irrational system function.

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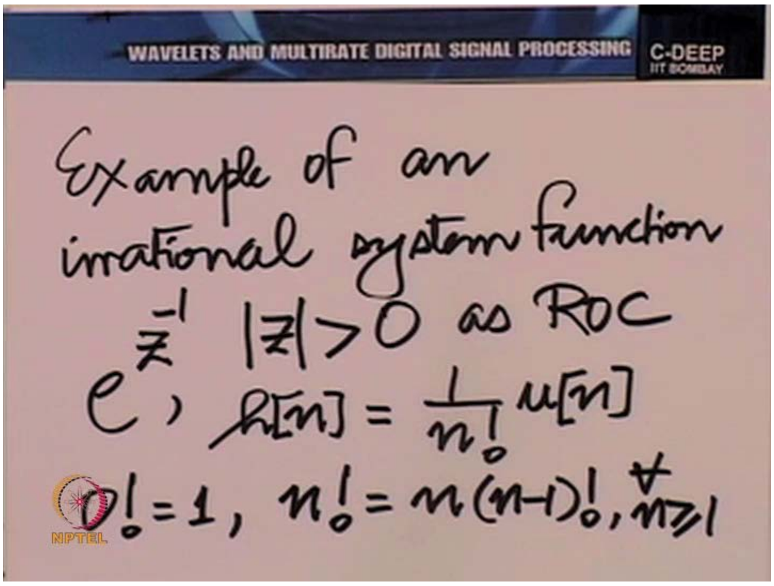

The example of an irrational system function could be E raised to the power of Z inverse with mod Z greater than 0 as the region of convergence. And of course the corresponding impulse response can easily be seen to be 1 by N factorial UN where N factorial is defined by, 0 factorial is one and N factorial is equal to N times N -1 factorial recursively for N greater than equal to1.

Interestingly, this system is stable, so it is not that all irrational systems are unstable but irrational systems have a fundamental problem, irrational systems are unrealisable at least today, we do not know any neat way of realising irrational systems. Rational systems can be realised with a finite amount of resource. What do I mean by resource? Adders, multipliers and delays these are the basic resources of a discrete time realisation. A rational system can be realised with finite resource.

And irrational system in principle requires an infinite amount of resource to realise. Now, you know, this is all falling in place, the non-causality, infinite non-causality and instability came as a bit of a surprise but this 3rd disqualification is not quite a surprise, what we are saying in effect is that if you want an ideal filter, be willing to put an infinite resources to get it. So, you know this is the whole, I mean I would say actually irony of discrete time system design.

I keep mentioning this, whether it is a basic course or an advanced course like this, the irony of many design problems is that you know which ideal you are striving towards and you also know that you cannot achieve that ideal with finite resources. But you know, what keeps engineers and mathematicians and scientists and what have you active all the time that you also know that you can go arbitrarily close to the ideal, provided you are willing to invest more and more resources, and there are many ways of doing it.

There are different ways of investing resources and going closer to the ideal. Perhaps some paths take you closer to the ideal faster, at least in a certain range of resources and some paths slower and again there are compromises. If you go faster in one sense, you may go slower in the another sense. Oh, nature drives the engineers, scientists and mathematicians to no end. Anyway, that was just a philosophical diversion, coming back to this problem, the ideal 2 band filter bank is unrealisable but we can go tantalisingly close as we desire.

So, you can build a 2 band filter bank arbitrarily close to the ideal if you are willing to invest more and more resources and over the past 15 to 20 years, people have come out with so many different designs, now why have people sought these different designs, again that question needs to be answered but then now we will 1st answer the easier of the 2 questions. Suppose I do happen to design different 2 band filter banks.

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