**Foundations of wavelets and multirate digital signal processing.**
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**Module-1.**
**Lecture -8.**
**Ideal 2 band filter bank.**

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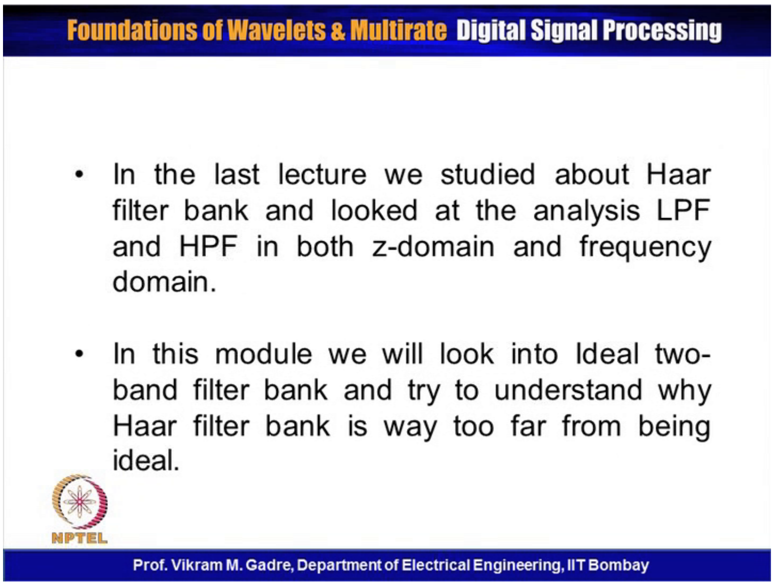


A very warm welcome to the 8th lecture on the subject of wavelets and multi-wave digital signal processing.

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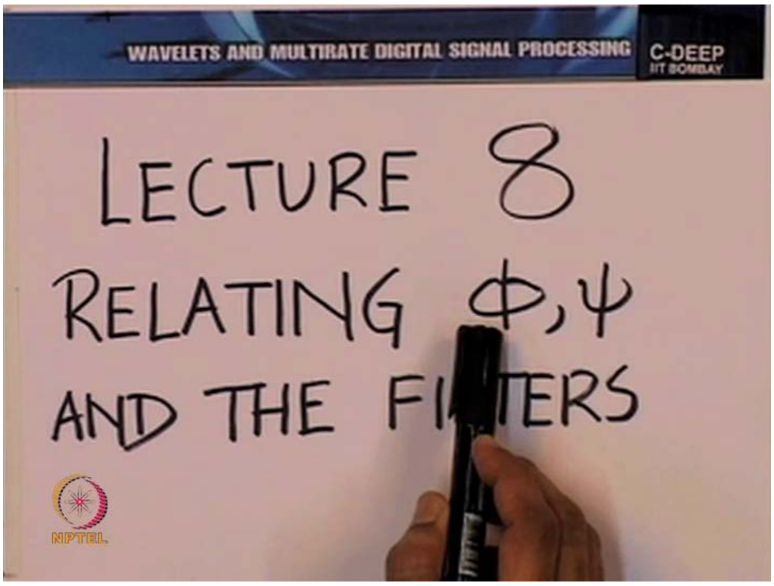


In this lecture, we shall build more intimately the connection between the filter banks that we talked about in the previous lecture and the underlying continuous time function, the scaling function Phi T and the wavelets Psi T. We suspect all the while that this connection exists, after all we built the filter banks out of the idea of multiresolution analysis with the Haar multiresolution analysis as an example. Before we go further, we must now make a few generalisations, which will help us then to build that connection more intimately.

So, let me quickly put on what we intend to do in today's lecture, we intend today to relate the scaling functions phi, the wavelets Psi and the filters that we talked about in the filter banks namely the analysis and the synthesis filter banks. Towards that objective, the 1st step is putting down a generic structure for the analysis and the synthesis filter banks. So, you see, let us consider the generic structure for the analysis filter bank 1st.

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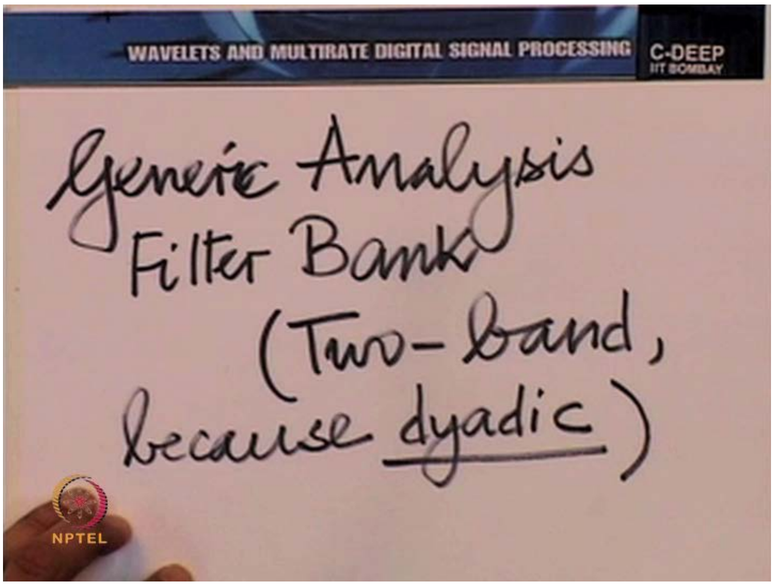


Incidentally, we should be talking about a 2 band filter bank here and that is because we are talking about dyadic MRAs here, recall that dyadic refers to powers of 2 changes, so what we are talking about is the generic structure 1st of a 2 band filter bank, 2 band analysis filter bank.

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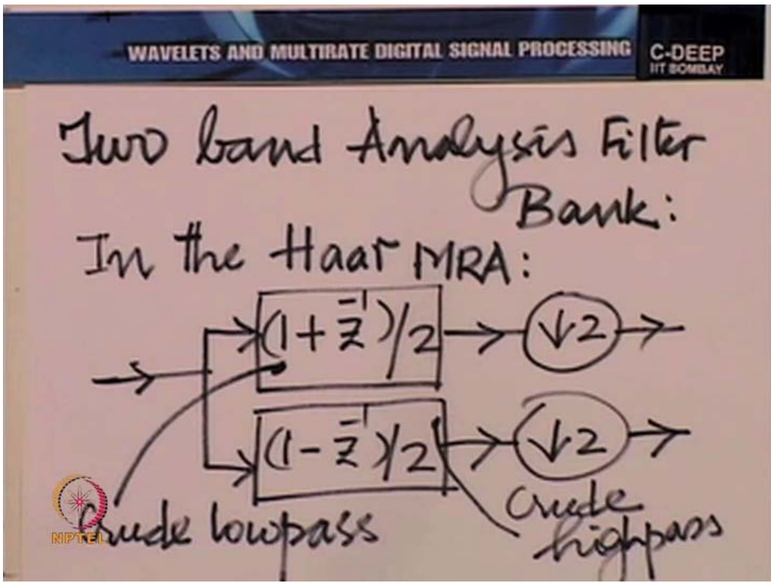


Now, I will start from the Haar, in the Haar, it looked like this, Haar MRA I mean, it looks like this, recall. This was the structure and we also analyse the frequency domain behaviour of these 2 filters, in fact this turned out to be a crude lowpass filter.

And this turned out to be a crude high pass filter, of course high pass and lowpass as understood in the discrete time sense. Now we also recall to other properties of these 2 filters that we had brought out the last time, one was what we call the magnitude complementarity, magnitude complementarity in the sense if we simply summed the filter system functions together, you got the identity system function one. The 2nd was what was called power complementarity, so if you summed the magnitude square, you got a constant.

A constant independent of omega I mean, which means if you took a sine wave and looked at the power of the sine wave emerging from each of the 2 branches, those powers would add up in a complimentary way for each frequency. Magnitude complementarity, power complementarity. Now, you know if we look that the Haar case, it was not very clear which idealisation we were moving towards. But then we had put down the idealisation the last time, so, let us now put down the complete idealisation.

We have a crude lowpass filter there, what would be the refined lowpass filter towards which we are moving in this analysis filter bank? Now, when we say towards which we are moving, what do we mean? Why should we move, why cannot we be content with the Haar multiresolution analysis? That is also a question that we need to answer. We will have to take the answers to these questions one by one. So, let us 1st answer the question what is the idealisation towards which I am trying to move.

So, let me put down 1st the actual frequency responses once again and then the ideal frequency responses towards which we are trying to move.

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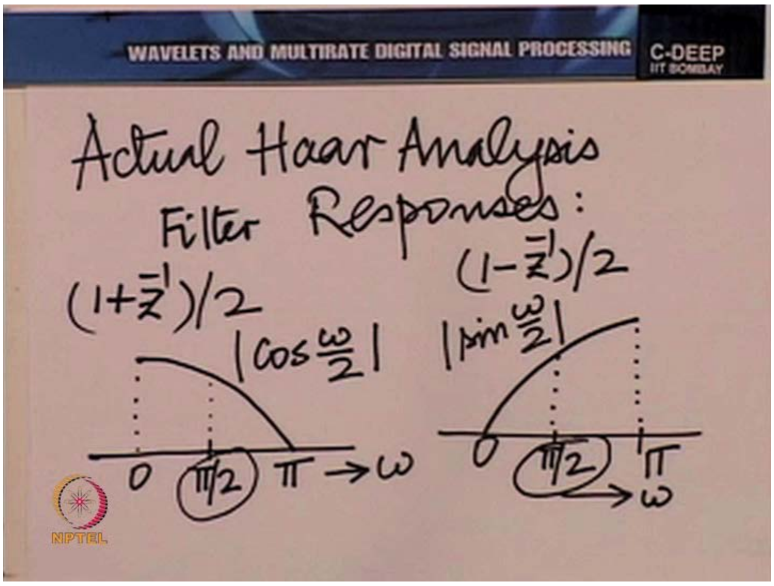


So, the actual Haar analysis filter responses, the 1 + Z inverse by2 filter had a response that looks like this, essentially more cos omega by2 between 0 and pie. Of course we call that there is periodicity and the 1 - Z inverse by2 filter has essentially a response that looked like mod sin omega by2.

And you will recall that there was magnitude and our complementarity here, in fact cos square + sin square is 1 and therefore there is power complementarity and if you just add up the system functions together 1+ Z inverse by2 + 1-Z inverse by2, you get 1 and that is magnitude complementarity. Now, you know, if you look at these 2 responses and if you mark them around the centre, the Centre is pie by 2, so if you mark them around the centre, you see a certain symmetry in these responses are about the Centre.

That gives us the hint where were moving towards in the ideal sense. Ideally, we are trying to make there is a lowpass filter with pie by 2 as the cut-off. And again we are trying to make this a high pass filter again with pie by 2 at the cut-off.

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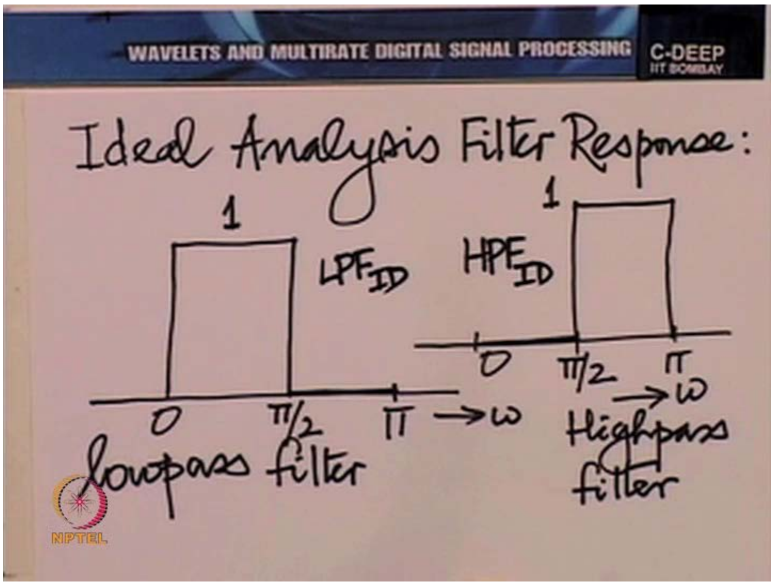


So, let us put down the ideal analysis filter responses towards which we intend to move. So, the lowpass filter should have a response that looks like this between 0 and pie. It must be 1 between 0 and pie by 2 and 0 between pie by 2 and pie and this entire pattern must be mirrored between - pie and 0 and of course then repeated the whole pattern between - pie and pie and then repeated at every multiple of 2 pie, all this is of course naturally from the properties of a discrete time Fourier transform or a discrete frequency response.

So, I am just showing the region between 0 and pie and what happens between - pie and 0 and then at around every multiple of 2 pie follows naturally. So, this is the lowpass filter response, similarly the high pass filter response which I will draw here. The high pass filter needs to have a response of 1 between pie by 2 and pie and of course whatever is between 0 and pie is mirrored between - pie and 0, the response is 0 between 0 and pie by 2 and then of course periodically repeated and every multiple of 2 pie.

Let us call this LPF ideal and high pass filter H PF ideal, these are frequency responses of the analysis filters towards which we desired to move. Let us now put down the ideal frequency responses of all the filters, analysis and synthesis in a 2 band filter bank. So, where are we moving, what direction are we moving? So, the ideal 2 band filter bank would have the following structure.

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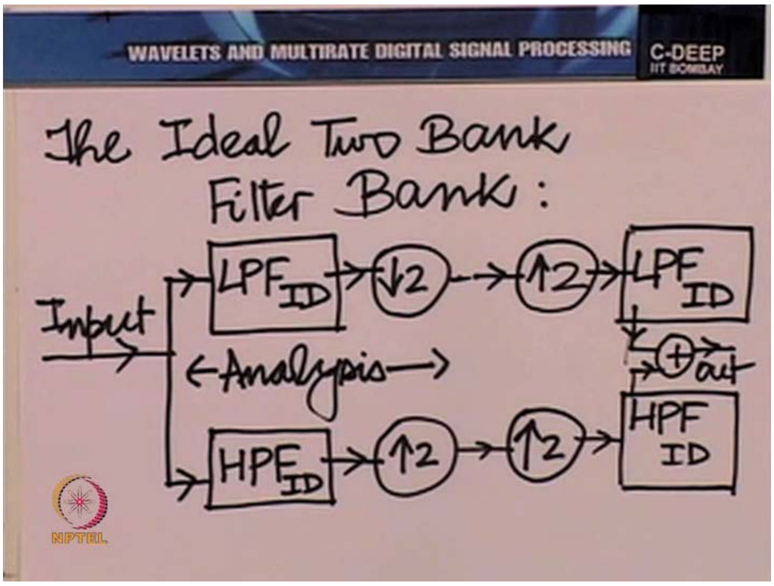


The input sequence here being fed to LPF ID ideal here, HPF ideal there, downsampled by2 subsequently, this is the analysis portion, followed by upsampled by2, again lowpass filter ideal here, high pass filter ideal here and the outputs from this are to be added to produce the overall output.

So, here I have just marked the outputs to fit into the drawing, the outputs from this and the output from this are added together and produce the overall output. This is the ideal 2 band filter bank. Notice, in the ideal 2 band filter bank, the analysis filters and synthesis filter are identical, there is no difference, in fact on one of the branches, both of them are ideal lowpass filters with a cut-off of pie by 2. The $2^{nd}$ branch, both of them are ideal high pass filters, discrete time filters with a cut-off of pie but once again.

So, you know, you see complementarity there in some sense and in fact if you look at it, these are obviously magnitude and power complimentary. If you take the frequency responses and items together, they of course add up to a constant, if you take the squares of the frequency responses and add up the squares, even add up to a constant trivially because there is no region of overlap absent. So, in fact that is where we are moving towards. Now, it is a moot point so far as I said why we need to move any farther from where we are? Why do we need to work harder than what we do for the Haar wavelet, are we lacking something in the Haar wavelet?

Well, of course one thing that you can see we are lacking is the distance from the ideal filter, we are far from the ideal. If you look at frequency responses of the Haar's analysis side and therefore of course the synthesis side, I left it as an exercise to calculate the frequency responses for the synthesis side, they are almost the same. We are, I mean we are very far from the ideal, so that is of course clear. But why we need to move farther from the Haar can be answered in many ways, we shall take up this answer slowly part by part.

But before we take up that answer, what we need to do is now to go the other way, we came from continuous time to discrete time, now slowly we want to see if my design of a multiresolution analysis relates to the design of the 2 band filter bank that I have just drawn here. So, you see, we have ideal filters here and we have written down the frequency responses of the ideal filters but in practice the ideal filters can never be attained.

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