Foundations of wavelets and multirate digital signal processing.
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Lecture -7.
Module-3.
Frequency response of Haar analysis filter bank.

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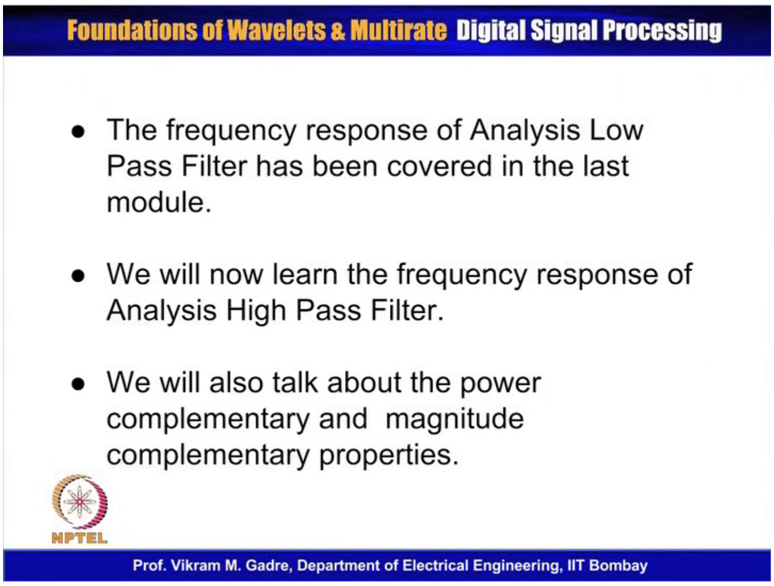

Anyway, so far so good, we have linear phase, we are not going badly. And if we look at the 2nd filter in the Haar filter bank, we shall have something similar. So, let us look at the 2nd filter.

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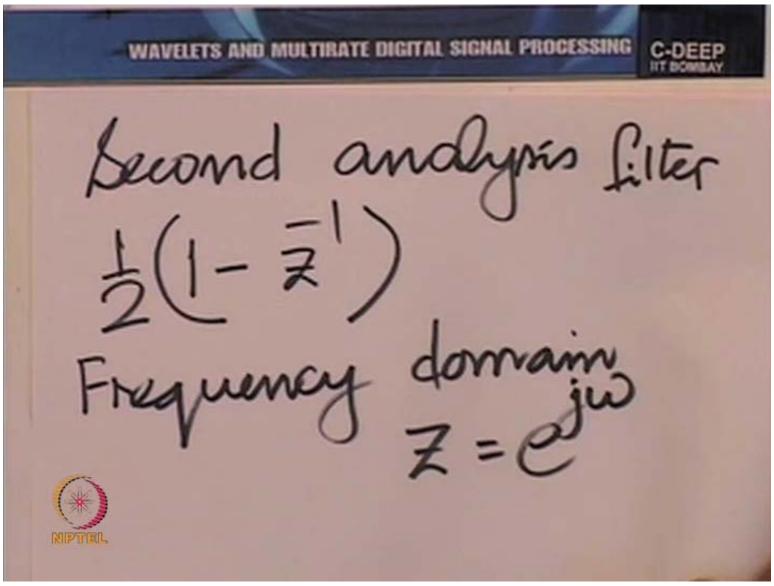

The 2nd analysis filter and that is of course 1 - Z inverse into half. Let us again find out how this filter looks in the frequency domain.

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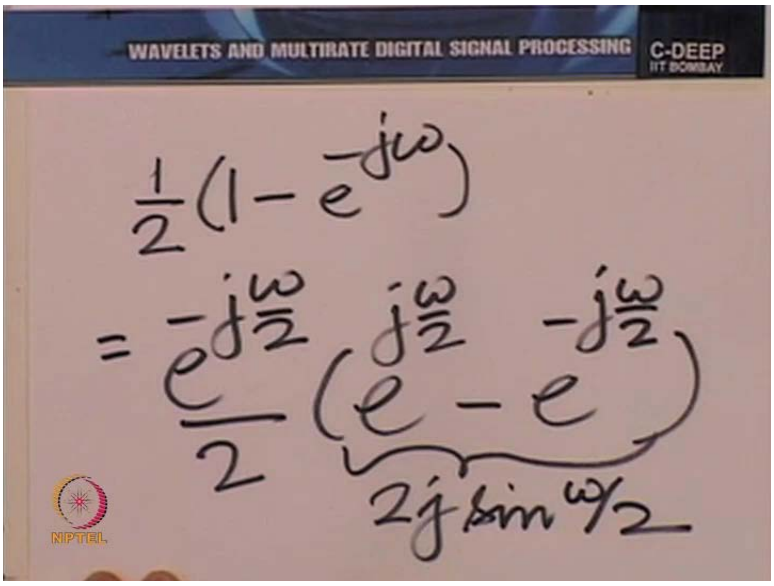

The frequency domain would show it as Z equal to E raised to the power J Omega, whereupon we have 1 - E raised to the power - J Omega by2 and we play the same trick, we take E raised to the power - J Omega by2 common and we have E raised to the power J Omega by2 - E raised to the power - J Omega by2.

And once again one can recognise this is essentially 2 J times sin omega by2. So, now I can simplify this, put it all together.

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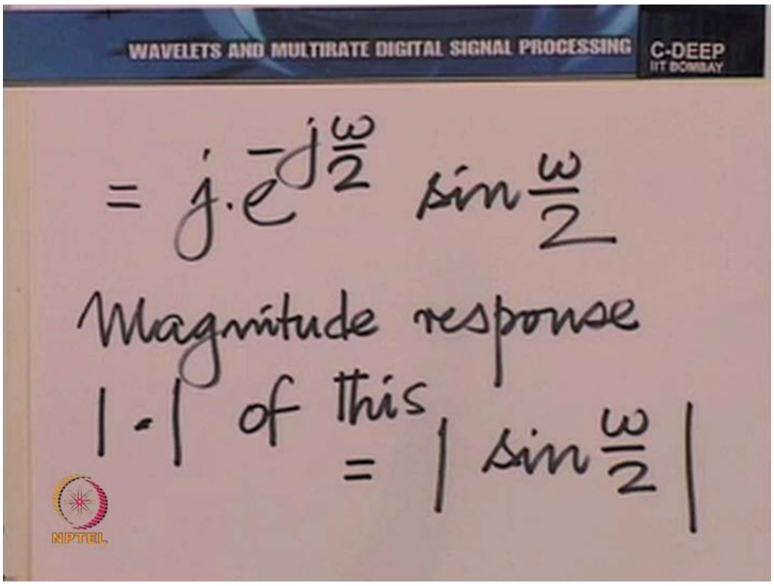

Once again I look at the magnitude response 1st, the magnitude response is a magnitude of this and that is easily seen to be mod sin omega by2. Let us sketch the magnitude response as a function of omega.

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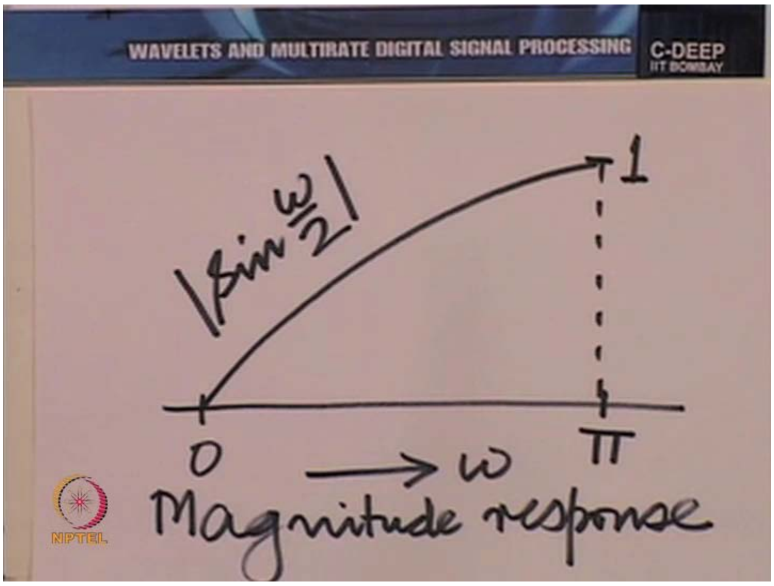

Again we will sketch it only between 0 and pie for the reasons that I have just explained. So, it will have an appearance something like this.

This is going to be 1 here, this is mod sin omega by2. Now, for the phase response, now please remember last time we had a convenient situation, we had 2 terms, one of them contributed no phase and the other one contributed the phase. So, we were comfortably put.

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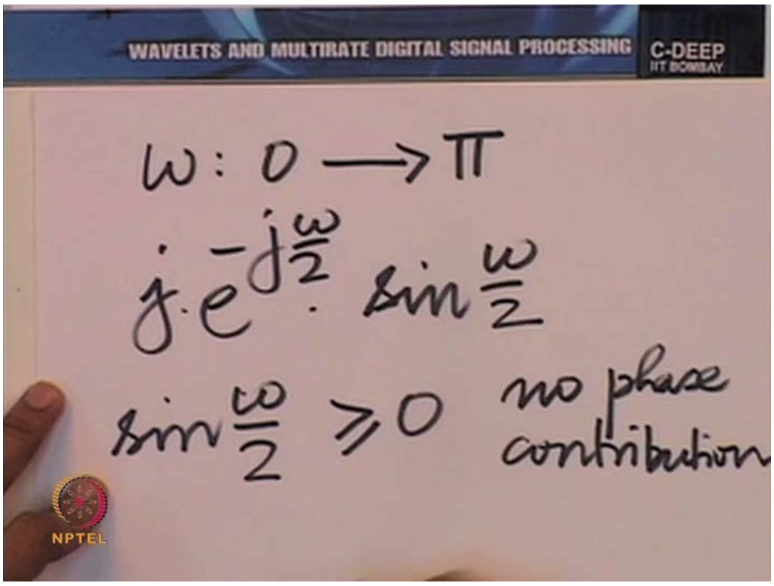

This time we will have to be a little careful, so you know let us make life easy by 1st looking at only 0 to pie, remember we are going to have conjugate symmetry, so let us consider omega from 0 to pie.

And let us look at the frequency response expression, JE raised to the power - J Omega by2 times sin omega by2. Sin omega by2 is nonnegative, so no phase contribution here. However, both of this term and this term have a phase contribution. In fact the phase contribution is 90° or pie by 2 from here and - omega by2 from here.

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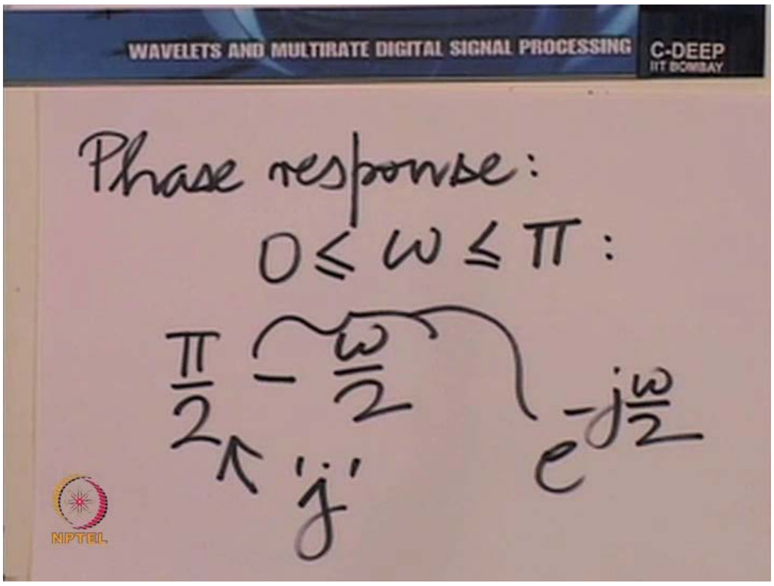

So, overall the phase contribution or the phase response is, I mean only between 0 and pie, pie by 2 and - omega by2, this is contributing essentially J in the expression and this part is coming from E raised to the power - J Omega by 2 in the expression.

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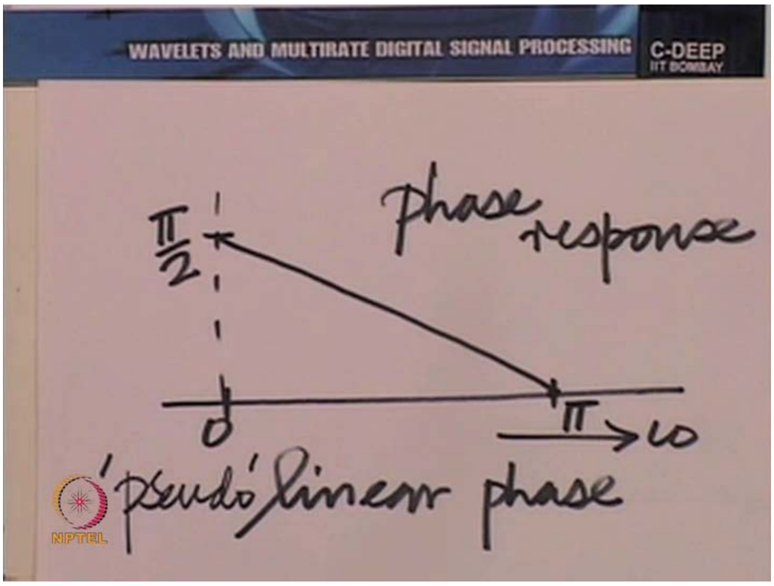

Let us sketch this response, I mean the phase response. So, of course that omega equal to 0, it is going to be pie by 2, at omega equal to pie, it is going to be 0, this is the situation. Now, once again we have linear phase, well, almost linear, not quite linear, if it was strictly linear phase, this would have been a straight line indeed but a straight line passing through the origin. So, it is not really linear phase, this is called pseudo-linear phase, seemingly linear phase. In fact for completeness, let us draw the magnitude and the phase response all the way from - pie to pie for both of these filters now for the sake of completeness.

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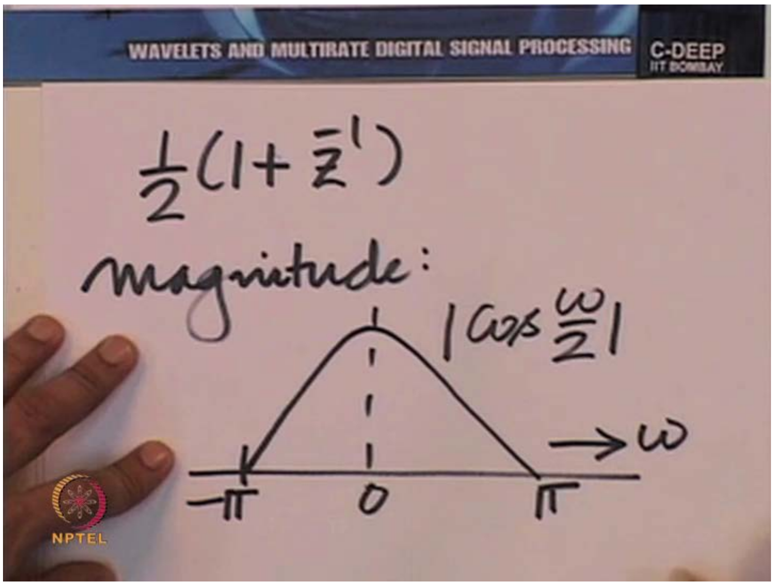

So, the situation is for the filter, half 1 + Z inverse, the overall magnitude response should look like this between - pie and pie I mean, essentially mod cos omega by2 and the phase.

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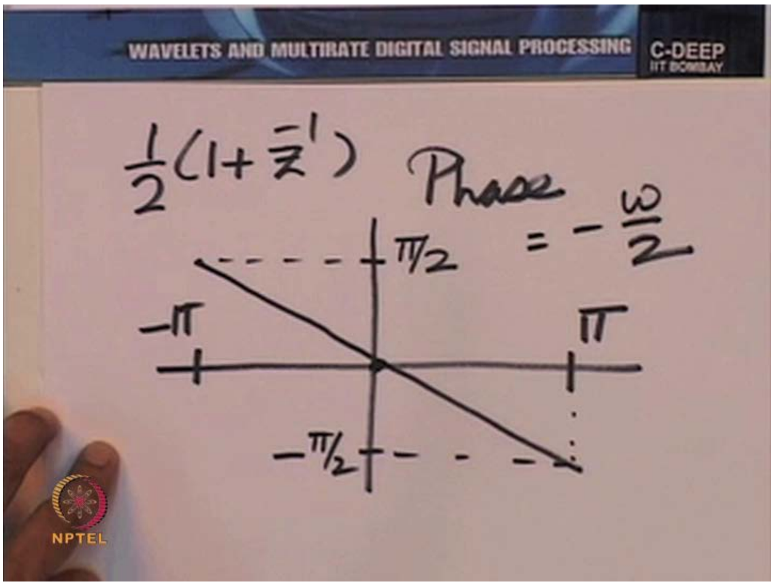

This starts at + pie by 2 here and goes up to - pie by 2 there, passes through the origin of course, it is a straight line.

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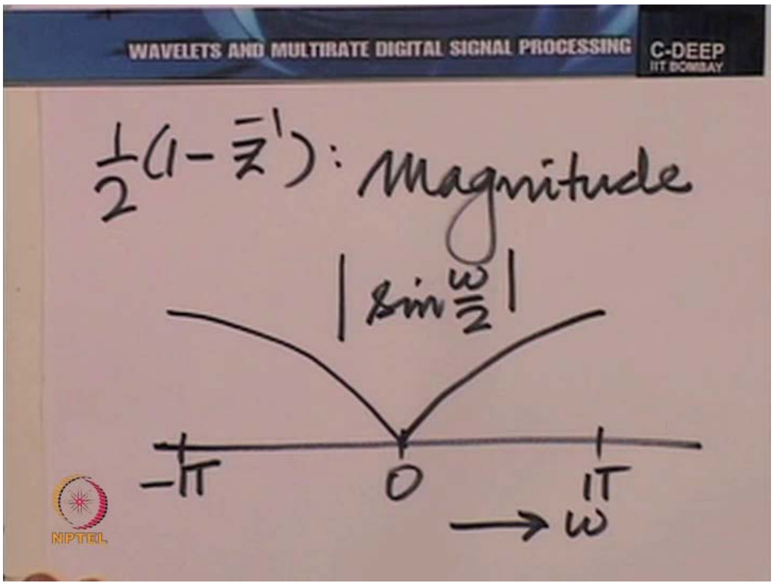

For the 2nd filter, the overall magnitude response looks like this.

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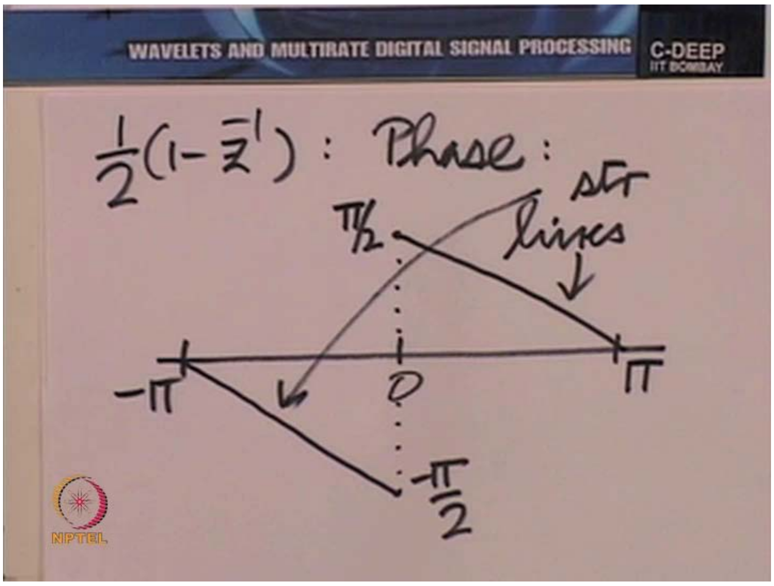

Essentially mod sin omega by2. And the phase response looks like this, starts at pie by 2 there and goes to 0, it is a straight line segment. Here it would start, that is interesting, you know the phase on this side between - pie and 0 needs to be the negative of the phase between 0 and pie, so it will be a mirror image.

These are lines, straight lines. So, we call this pseudo-linear phase. Now one point needs to be understood, there is the peculiarity situation at the point omega equal to 0 here, the phase is both pie by 2 and - pie by 2, how can this be possible?

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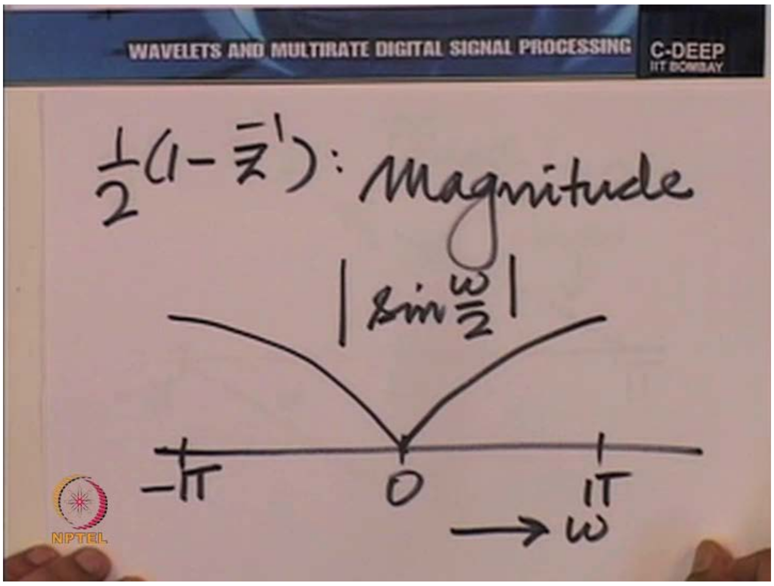

Well the answer comes from the magnitude response, the magnitude response that that point omega equal to0 is0. When the magnitude response is 0 at a point, the phase response has no meaning, the phase response could be anything. So, you see it means that sin wave at omega equal to 0 is anyway being destroyed, so what consequence is the phase response, that is why there is an ambiguity in phase or a discontinuity in phase at the point omega equal to 0 in this phase response.

A small detail but important when we try to understand this filter bank completely.

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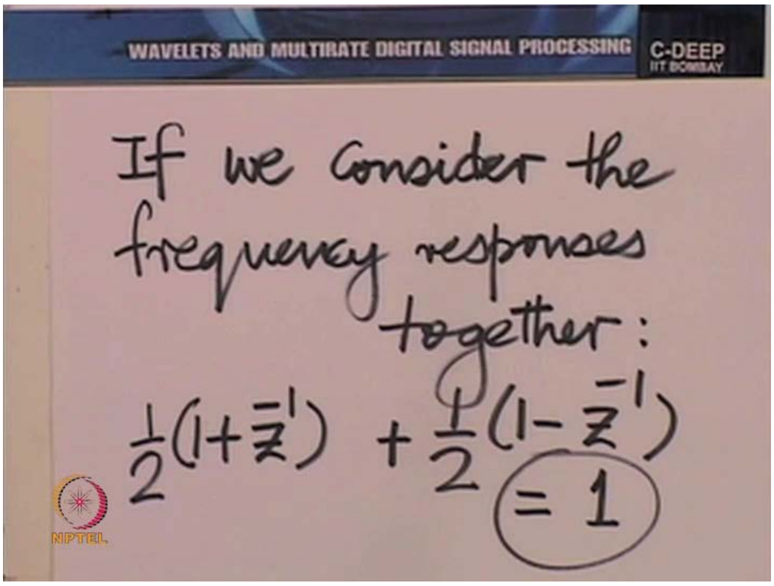

Anyway, coming back to these magnitude responses now, if we consider the 2 magnitude responses together, in fact let us 1st consider the frequency responses together, both magnitude and phase. A very important property emerges, you see if suppose we add them, so suppose you take, you know I do not even need to substitute Z equal to E raised to the power J Omega, let me keep it as it is. So, half into 1+ Z inverse + into 1 - Z inverse, it is very easy to see this is equal to 1. A very interesting consequence, what does it physically mean?

It physically means that if I were to send a sample sine wave frequency omega into one of these filters and then the other, and if I took these 2 samples sine wave from the 2 filters and put them together by adding, you would get back during the sine wave. So, sine wave at frequency Omega is Split by these 2 filters in such a way that the part can simply come together and reconstruct the sine wave as it is. Now let us look at something more interesting.

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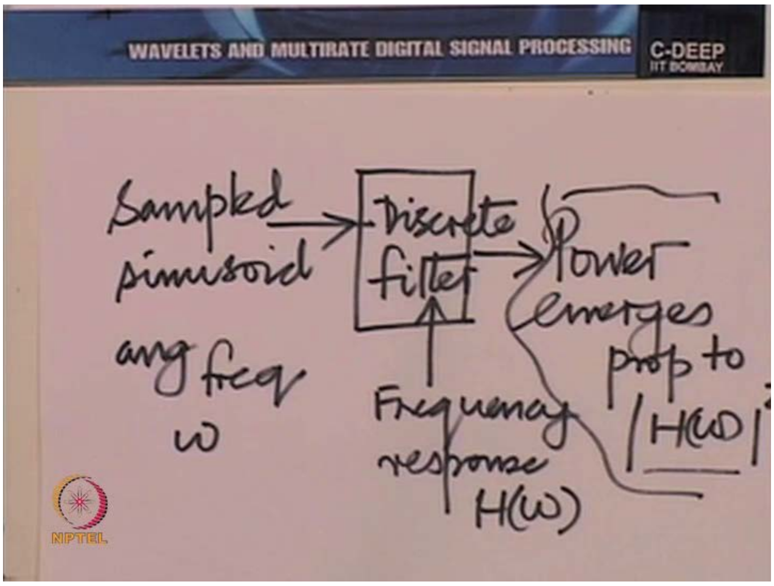

What can we say about the power? So, recall that if you give a sampled sine wave, sampled sinusoid to a discrete time filter, let us say angular frequency is omega and the frequency response here is H of omega, then the power that emerges from here is proportional to mod H omega square. So, in other words, whatever is the power of example sinusoid with angular frequency equal to omega here is multiplied by mod H omega square where it emerges at the output. So, the squared magnitude of a frequency response is indicative of the change in power of the sine wave when it goes for that discrete time filter.

What can we say about the power change of a sine wave where it goes through either of these 2 filters here? Let us see.

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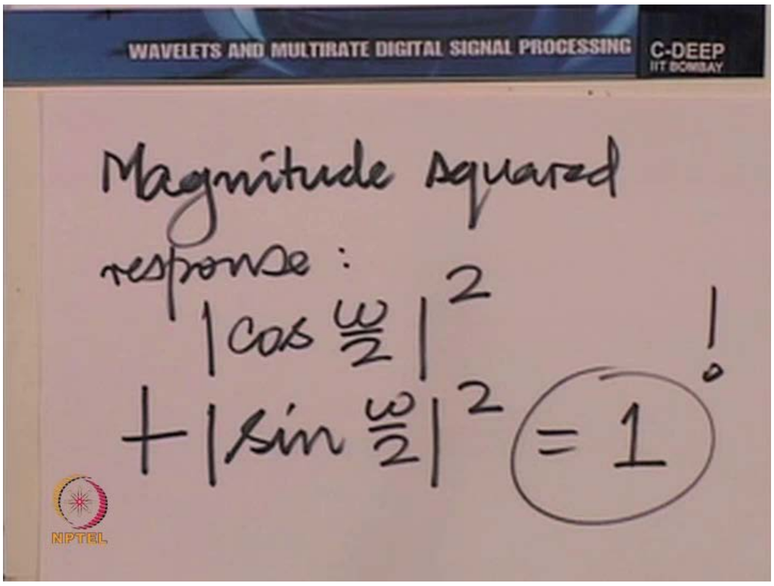

So, that means in other words were asking the question, what is the magnitude squared response? In the 1st one it is mod cos omega by2 the whole squared and in the 2nd one it is mod sin omega by2 the whole squared. And lo and behold, when you add these, you get one as well, that is a very very interesting observation.

Not only does it happen that when you add the 2 responses together, you know we literally took 1+ Z inverse by2 and 1 - Z inverse by2 in respective of that, in fact I said that you did not even need to substitute Z equal to E raised to the power J Omega, you just added the system functions together and you got 1. So, if you put a sine wave frequency Omega, I mean the example sinusoid of frequency, angular frequency omega and looked that the corresponding Emerging sine wave on the top branch and the lower branch and just added them together, you will get back the original sine wave.

Not only that, what we have just shown is that when you multiply, if you look at the power emerging from the upper branch and the power emerging from the lower branch, the powers also add. So, in fact you have 2 kinds of complementarity in the filters, this is something very very interesting here.

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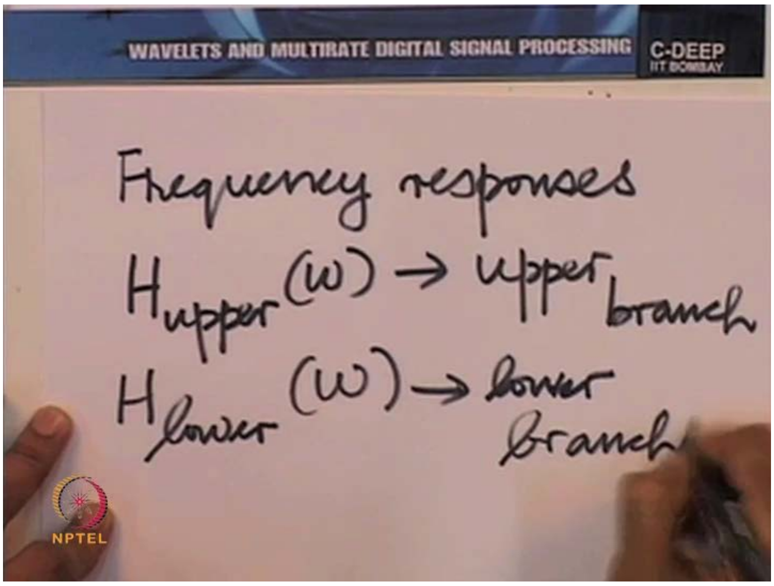

So, if you call the frequency responses of the upper branch and lower branch respectively as,

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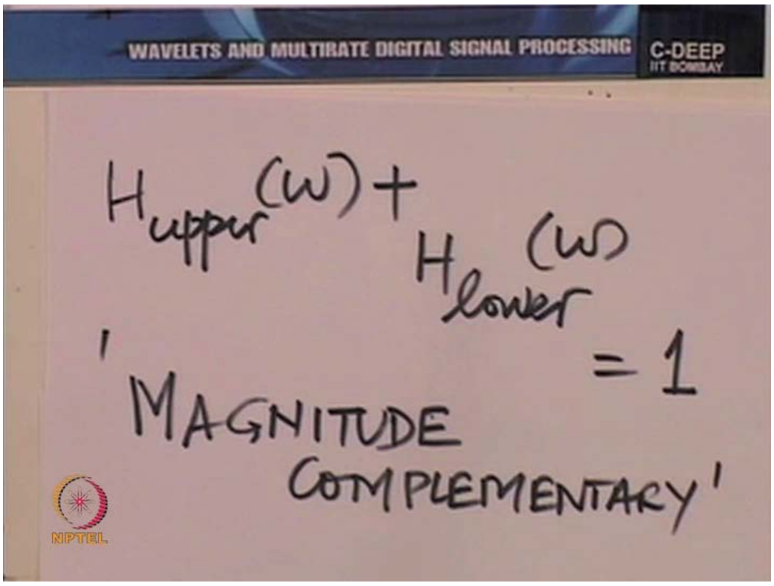

H upper omega and H Lower omega, upper branch, lower branch. Then 2 properties are immediately satisfied, H upper omega + H Lower omega is equal to1. This is called magnitude complimentary property.

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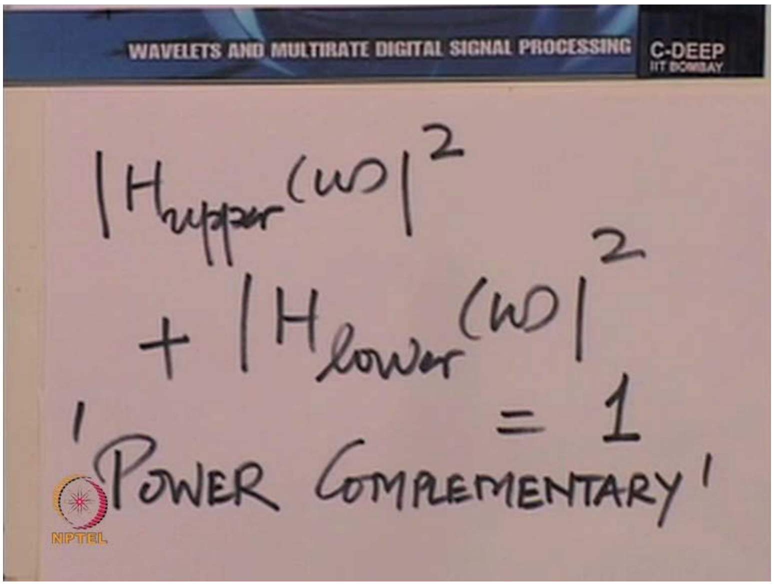

And in addition, mod H upper omega squared mod H lower omega squared is identically equal to1, this is called the power complimentary property. A very very interesting result.

The Haar analysis filter bank is both magnitude complimentary and power complimentary. In fact I leave it to you to study the synthesis filter bank and come to a similar conclusion, the filters are magnitude complimentary and power complimentary. Whatever it be, this is something striking. Now you see what I mean when I said filters have individual properties and collective properties, magnitude complementarity and power complementarity our collective properties. The lowpass and the high pass nature, if you recall, the 2nd filter that we had was high pass because it emphasise higher frequencies and deemphasise the lower frequencies.

So, lowpass and high pass properties are individual properties, the magnitude and the power complimentary properties are collective properties. So, we have filters with individual and collective properties forming 2 filter banks, the analysis filter bank and the synthesis filter bank.

Now, you know the idea of the filter bank is very deeply entrenched in multiresolution analysis. In subsequent lectures, we shall study this connection even further. So, for today, we shall conclude the lecture here by noting that we have already established even more deeply the frequency domain behaviour of the filter bank that we brought out in the previous lecture. Thank you.