Foundations of Wavelets and Multirate
Digital Signal Processing
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Lecture - 1
Module - 2
Origin of Wavelets

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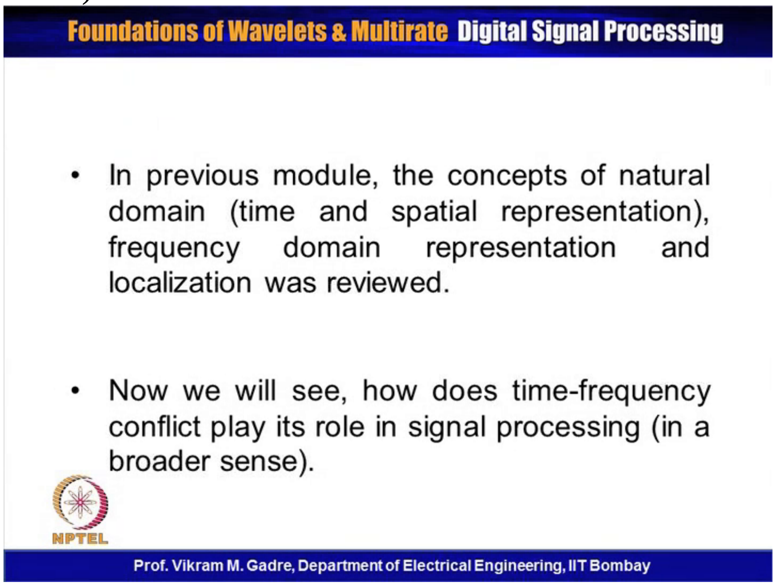

Now this is where the whole story starts and perhaps is the most fundamental inspiration for the scores and wavelets and multirate digital signal process. I first talk about where wavelets come from. Well Fourier transforms deal with waves sign waves to be more precise. We recognize the merits of sign waves. Sign waves have many nice properties, for one they occur naturally in many different circumstances.

For example, an electrical engineer recognizes the sign waves as naturally emerging from an electricity generation system when there is electromagnetic induction. If there is a perfectly circular rotating device in a magnetic field and if all is perfect in the generating system, we would be generating a perfect sign wave from the brushes. So sign wave is a good idealization to work with for an electrical engineer, but that’s not the only point. Sign waves are in some sense the most analytic the most smoothest possible periodic functions They also have the power of being able to express many other wave forms that means they form a good basis from which many other wave forms can be generated. They have many other nice mathematical properties.

If I take two sign waves with the same frequency possibly different amplitudes and phases, I would get back a sign wave of the same frequency of course of a third amplitude and phase. If I differentiate a sign wave I get back a sign of the same frequency and naturally if I make a combination of these operations, mainly differentiation or even integration for that matter and linear combinations and if I restrict myself to sign waves of a particular frequency, I remain within the domain of sign waves of that particular frequency. This is something beautiful about sign waves. It is not easy to find sign wave forms which obey this. And as I said before sign waves form a good basis so they form good building blocks for being able to express a wide variety of signals.

For all these reasons the sign waves has been very popular. In a first course on signals systems discrete time signal processing and what have you communication. But as I said right in the beginning of this lecture one of the reasons why we are not so happy with sign waves is that they need to last forever. Beginning from minus infinity and go all the way to plus infinity, otherwise if you trunked a sign wave, if it is one sided for example and if you look at what happens to an electric system when you apply a one sided sign wave. By one sided sign wave I mean a sign wave which starts from some point zero up to then and starts from some point and continues afterwards the sign wave… The response is very different from what would be the response for a sign wave that started at minus infinity, in general.

There would be transience which are not periodic and then all these beautiful properties of sign waves and their responses go away. So if I really wish to be able to apply the basic principles of signals systems and discreet time signal processing that I learnt in the basic course I need something unrealistic. I need a sign which lasts forever. How can I be more realistic in my demands?

By accepting that I cannot deal with waves but its more appropriate to deal with wavelets so that’s where the word wavelet comes from, small waves. Waves that don’t last forever. Functions that are not predominant forever. They are significant in a certain range of time perhaps only exist in a certain range of time and insignificant outside. So we have a certain support over which one might want to use them, one might want to consider them to exist and so on.

A much more realistic assumption and that really is what we call a wavelet. Not a wave but a wavelet. For example you could if you wish think of truncating a sign wave to a rectangular region that means, suppose we take a sign wave to last from 0 to 1 millisecond as an example. It could be an example of wavelet, later we will see this is not a very good example but yes in principal a wavelet, a wave that doesn't last forever. A simplistic explanation of what wavelets means.

But that's not the whole story. Our whole objective was to talk about other domain too, so going back to the example of the audio signal. If I thought of the audio signal as comprising of many sign waves to come together to form an audio piece then I wish to be able to do something simultaneously in two domains, and that is the key idea here. So for example to put it in plain language, I should be able to say well there was this two seconds audio clip out of which there were 5 notes being played. Each note was played for different intervals of time.

Maybe the first note was played for 0.4 seconds the second note was played for 0.7 seconds, the third note only for 0.2 seconds and so on. So I need to be able to segment in time but when I am talking about being able to identify notes I am also talking about being able to segment in frequency and low and behold that is where the conflict arises. A very basic principle in nature says, if I wish to be able to segment in time and frequency simultaneously, I am going to run into trouble. Nature does not allow it beyond a point and that is something very fundamental.

It pops up in many different manifestations in different subjects. In modern physics they call it uncertainty, the uncertainty of position and momentum. In signal processing we call it uncertainty, the uncertainty of the time and frequency domain. So to put it simply though not very accurately the shorter you play a note the more difficult it is to identify it. Not very far from intuition.

If you play a note for a long time and listen to it for a long time you are likely to be able to identify it better. Common sense tell us that, but what common sense does not tell us is that you can never quiet go down to identifying one particular frequency precisely. So if I wish to be able to come down to a point on the time axis then I need to spread all over the frequency axis and if I wish to be able to come down to a point on the frequency axis I need to spread all over the time axis. That is of course the strong version of this restriction but there is a weaker and a little more subtle version and that is as follows. Even if I am not quiet interested in coming down to a point on the time axis.

I am content with being in a certain region, as I said in the first 0.4 seconds out of the two seconds clip, I was playing note number 1 that means some frequency number 1. I would be able to say this at least to a certain degree of accuracy that is what I am trying to point out here. What the principle of uncertainty tells us is that this can be done to a certain degree of accuracy. You can identify that note to a certain degree of accuracy. Well what uncertainty also tell us in a more subtle form is that if I even choose to relax to a certain region of time, so I say well in this region of time tell me the region of frequencies which were predominant, even then there is a restriction on simultaneous length of measure of the time and frequency regions.

And of course they have a tussle with one another. The smaller I make that time region the larger that frequency region becomes that means the more I want to focus in time the less I am able to do so in frequency. This is indeed something that rouses a lot of thought It may seem something far from our interests at first glance but when we look at it carefully we realize it is something very fundamental to what we often desire. That is what I am now going to explain to you with a couple of more examples.

We live in an age where we use mobile telephones, in fact more fundamentally digital communication. What are we asking for in digital communication when we look at it from a signal or system perspective, or a transformed domain perspective or time and frequency perspectives? Going right down to brass tacks what we are asking for in digital communication is I should be able to transmit a sequence of bits binary values 0 or 1. And how do I transmit the sequence of binary values… I choose maybe 1 of 2 possible wave forms in the simplest scheme for corresponding to 0 I have 1 wave form corresponding to bit 1 I have a different wave form. To make life simple the 2 wave forms have the same time interval.

So for example we talk about, I mean all of us hear about computer networks and they talk about the speed of the network. So they say well this network can operate at the speed of 1 megabit per second. What does that mean it means that in 1 second I can transmit 10 raised the power of 6 bits, so you have 1 milli, 1 micro second to allow each bit to be transmitted.

Give it a thought. Here we are talking about time, what are we saying about frequency. Now let's quote the mobile communication context, I have so many different mobile operators obviously each operator will want its own privacy. So what is being communicated on the network of operator 1 should not interfere with what is being communicated on network of operator 2. Now where is the separation going to occur, not in time. After all there are many different people simultaneously using mobiles bought from both of the operators so the separation cannot be in time.

We may argue that the separation can be in space, so in 1 region you may have mobiles from 1 operator in another region in space I mean mobiles from the other operator. So far so good, but that's also not always true It is very common to see mobiles purchased from different operators operating in the same room. So there is not separation in time no separation in space. So where is the separation then?

The separation has to be in a domain which is not so easy to see, but once we have done a course on signals and systems reasonably easy to understand and that domain is frequency. So we say well operator 1 has this bandwidth allocated to him, operator 2 has other bandwidth allocated to him. Now when we say this bandwidth maybe a certain region of the frequency access of size, let us say 2 mega hertz. When we say this region of 2 mega hertz is allocated to operator 1 and another region of 2 mega hertz is allocated to operator 2, are we not talking about a segmentation in a different domain. In fact there we are talking about simultaneous segmentation.

We have segmentation in time because you want to transmit different bits in different time segments and you want to have separation in frequencym because what is transmitted by operator 1 should not interfere with what is transmitted by operator 2. So here is a very common, though not so obvious example of simultaneous design of localization in time and frequency. Other than the audio example which is of course little more obvious, a little easier to understand. This example is equally common at least in scenario today but perhaps not so easy to understand. But a little reflection makes it very clear to us. There is a desire to localize in two domains, simultaneously.

Well let us go to a bio medical example. Very often when one analyses an electro cardio graphic wave form. What one wishes to indentify are the features in the ECG signal. Now I do not intend to go into the medical details. But there are different segments in a typical ECG signal. They are often indexed by letters P, Q and so on. Without meaning to focus on specific details of an ECG signal let us try and understand the connection to time and frequency localization. When we talk about an ECG signal all features are not the same length and time, some features are kind of shorter some are longer. In fact to go away from an ECG signal bio, medical engineers often talk about what are called evoked potentials.

So we provide stimulus to bio medical system or to a bio physical system and we evoke a response and the wave form corresponding to the response is called the evoked potential. It can be studied as an electrical signal. Now the evoked potential again typically has quicker parts in the response and slower parts in the response. Naturally we expect the slower parts of the response would be predominantly located if you think of the frequency domain in the lower ranges of frequency, and the quicker parts of the evoked potential wave form will be located in the higher ranges of frequency.

Now here is an example of time frequency conflict. Suppose I wish to be able to isolate the quicker parts. Is it alright simply to isolate the higher frequency content in a certain signal and which comes from an evoked potential and suppress the lower frequency part. Well, you see, if we try to suppress the lower frequency part then we have already suppressed the slower parts of the response and if we try and suppress the higher frequencies in a bid to emphasize the slower parts of the response we have suppressed the quicker parts of the response.

So, if we think conventionally in terms of the frequency domain maybe high pass filtering or low pass filtering, nothing works for us. If we do high pass filtering then we have effectively suppressed the slower parts of the response and if we do lower pass filtering we have suppressed the quicker parts of response. So we need a different paradigm or a different perspective on filtering.

We need to identify in the different parts of the time axis which regions of the frequency axis are predominant and therefore in a certain sense identify different parts of the frequency axis to be emphasized in different time ranges. This is another perspective again on the time frequency conflict and all this is going to lead us in the direction of building up this course on the wavelets. We shall of course understand some of these concepts a little better as we progress in the lectures, but for the time being I have given you these 3 examples with the intent of bringing before you perhaps not completely but at least in a way to inspire your imagination the whole idea of time frequency conflict, or more generally the conflict between 2 domains, domains of analysis and representation of a signal and of course then going further even of a system.

In a first course we understand the domains very well; we understand there is a time domain. We understand there is a frequency domain We do well because we keep them apart. It makes life easier, but what we are trying to bring out through these 3 examples the audio example, the digital communication example and the bio medical wave form example. Whether it is the electrocardiographic wave forms or the evoked potential wave form, what we are trying to bring out is that one normally needs to consider the two domains together time and frequency and when we try and do so there is a certain very fundamental conflict that we have to deal with.

That conflict called uncertainty appears as I said in different manifestations in different subjects and we are going to look at that principle the uncertainty principle as a reply to signal processing in great depth at a certain stage in this course. But before that we are going to consider one particular tool for analyzing signals analyzing situations with a recognition that we need to be local and not global.

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