

**Foundation of Wavelets and Multirate
Digital Signal Processing
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Lecture Number 6
Module No. 2
Hard Analysis Filter Bank in Z-domain**

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Foundations of Wavelets & Multirate Digital Signal Processing

- Filter bank can be thought of as a discrete time system that separates the input signal into multiple components.
- This is done in the analysis part of the filter bank.
- In this module system function of filters in the analysis part of Haar filter bank is derived.


NPTEL

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Now, I shall just recapitulate a few concepts from Discrete Time Processing to refresh our memories and enable our discussion. Recall that if we have a sequence y of n , its Z transform is described by Y of Z .

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$$y[n] \xrightarrow{\text{Z transform}}$$
$$Y(Z) = \sum_{n=-\infty}^{+\infty} y[n] Z^{-n}$$

Converges in a region
of complex Z .


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Normally we use the small letter to denote the sequence in time and the corresponding capital letter to denote the sequence in Z domain or also in the frequency domain.

And Y of Z is submission and going from minus to plus infinity y n Z raise to power minus n. Now we must remember that this convergence in a region of the Z plane, it doesn't converge all over the complex Z plane and that region in which it converges is called the region of convergence. So a Z transform is always defined by an expression and a region of convergence. Outside the region of convergence the expression has no meaning.

Recall also that both of them are absolutely necessary to complete the Z transform. If we only give the expression and do not specify the region of convergence, there is a possibility that 2 sequences could correspond to that expression, 2 or more in fact. Therefore, it is only after we specify the region of convergence, that the Z transform is completely specified. A few points that we noted in connection with Z transform.

We are also familiar with several properties of the Z transform. For example, when we shift a sequence, the Z transform is multiplied by an appropriate power of Z. So let us recall that property of the Z transform. So if y n has the Z transform and you know having the Z transform we shall denote by script Z line this.

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$$y[n] \xrightarrow{Z} Y(Z), R_Y$$

$$y[n-D] \xrightarrow{Z} Z^{-D} Y(Z), \text{at least } R_Y$$

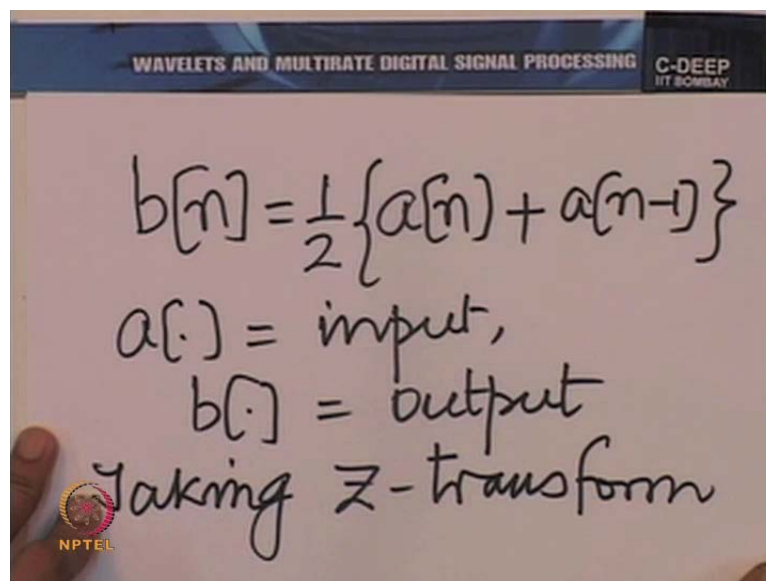
So when we when we write this, what we mean is that y of n has the Z transform given by Y of Z with a region of convergence R.

Maybe if you want to be specific, we could say region of convergence of capital Y. So if y of n has Z transform given by Y Z with a region of convergence R y, then y of n minus D has a Z transform given by Z raise the power minus t Y of Z and the region of convergence is at least R y if not more. Sometimes, the region of convergence might expand a little beyond R y.

The Z transform is a linear operator, so if we take a linear combination of sequences, the same linear combination occurs in the Z domain. As far as the regions of convergence go, the regions of convergence are at least t intersection of the region of the convergence of the individual sequences which are linearly combined if not more.

So when we have an operation being done on sequences and the corresponding Z transform is recorded, it is possible the region of convergence might expand beyond the intersection of the region of convergence of the sequences. Anyway, using this let us transform the filters that we had a minute ago into the Z domain. Let me put down the 2 filters explicitly once again.

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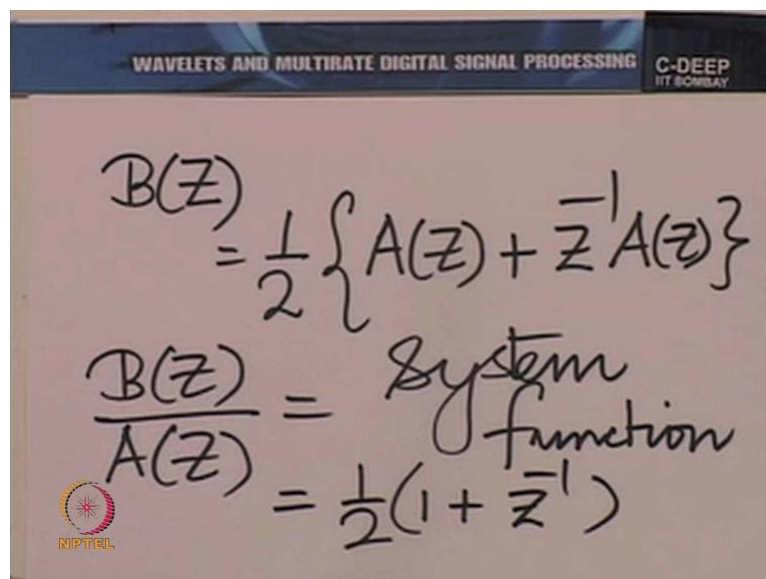
$$b[n] = \frac{1}{2} \{a[n] + a[n-1]\}$$

$a[\cdot] = \text{input},$
 $b[\cdot] = \text{output}$
 Taking Z-transform

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I have the first filter given by b n is half a n plus an minus 1 where a is the input and b the output. If we take the Z transform on both sides, we get what is called the system function of this filter. So let us calculate that system function.

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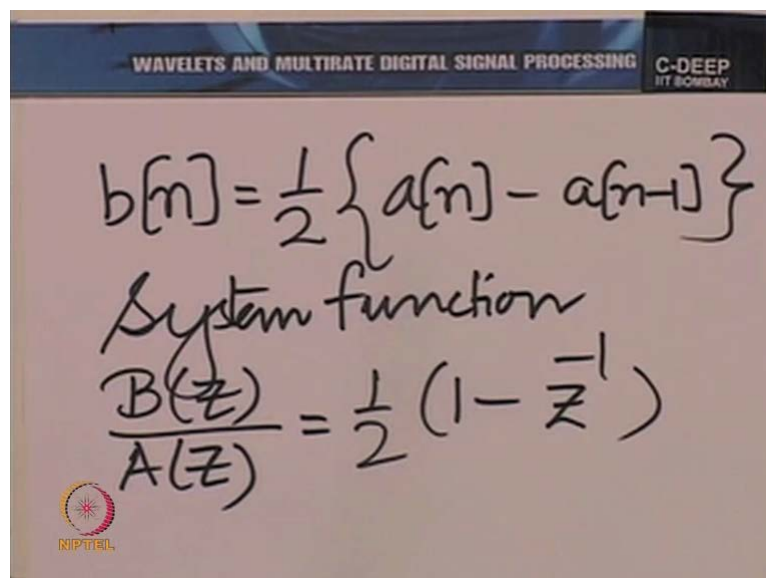
$$B(Z) = \frac{1}{2} \{ A(Z) + Z^{-1} A(Z) \}$$
$$\frac{B(Z)}{A(Z)} = \text{System function}$$
$$= \frac{1}{2} (1 + Z^{-1})$$

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We get B Z is half A Z plus Z inverse A Z whereupon B Z by A Z can be obtained and this is called the system function of the filter. And the system function is easily seen to be half 1 plus Z inverse.

In a similar manner we can calculate the system function of the second filter. That is described by b of n is half a n minus a n minus 1.

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$$b[n] = \frac{1}{2} \{ a[n] - a[n-1] \}$$

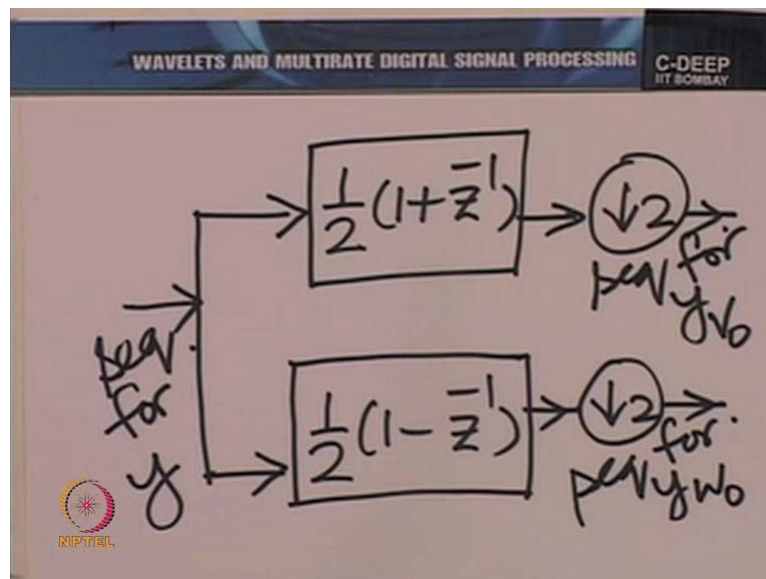
System function

$$\frac{B(Z)}{A(Z)} = \frac{1}{2} (1 - Z^{-1})$$

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And there the system function would be B Z by A Z given by half 1 minus Z inverse. So all in all, we have a pair of filters followed by a pair of decimator and the structure looks like this, half into 1 plus Z inverse here, half into 1 minus Z inverse there and a decimator.

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So here we have the sequence for y , here we have the sequence for y_{v0} and here we have the sequence for y_{w0} . Coming out here, here and here respectively, now this is what decomposes or analyses the function $y(t)$ into its component. And therefore, this pair of filters followed by the decimation operations is referred towards the analysis filter bank corresponding to the Haar multiresolution analysis.

Now we should also bring out the synthesis filter bank for the same multiresolution analysis. In other words, if we have the projection on V_0 and the projection on W_0 , how do we reconstruct the $y(t)$ function? In other words, if I have the sequences corresponding to y_{v0} , the sequences corresponding to y_{v0} and y_{w0} , the projections on V_0 and W_0 . And if we wish to reconstruct the sequence corresponding to y , what are the things that we need to do?