Foundations of Wavelets and Multirate Digital Signal Processing
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Demonstration Piecewise Constant Approximation of Functions

Today we will be talking about the projection of functions on different sub spaces and how can we use them to get a better and better approximation to that function.

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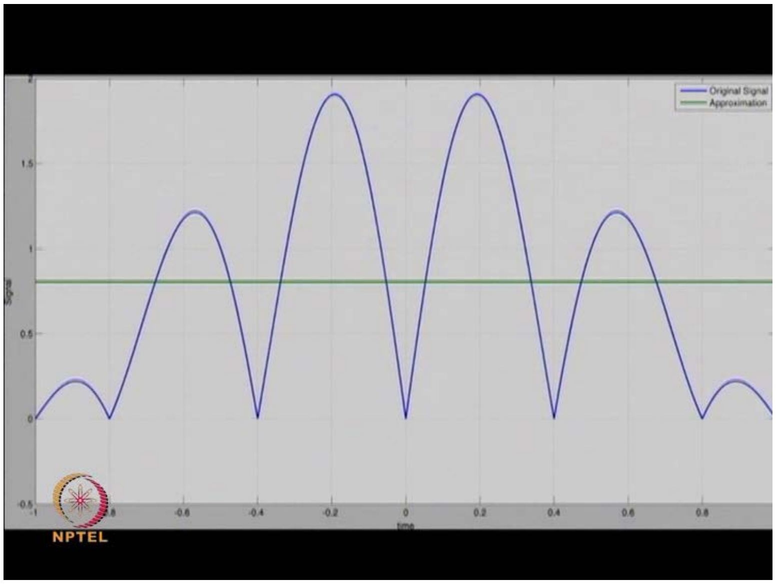

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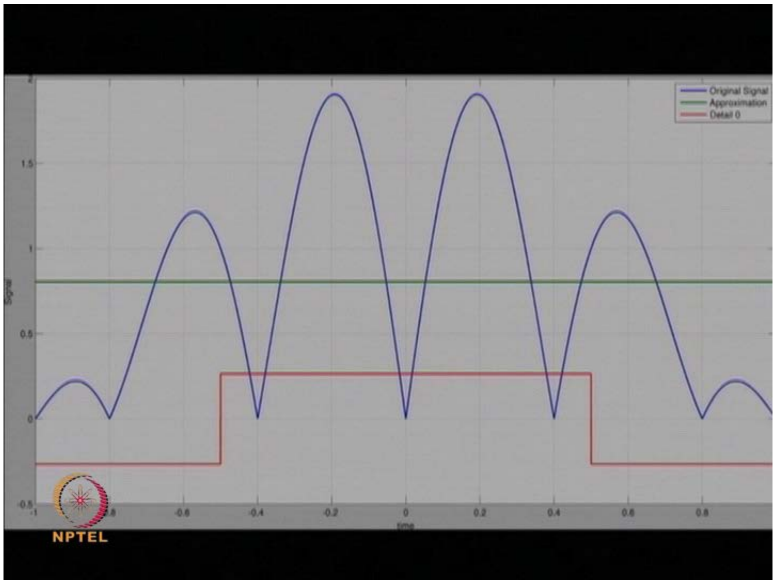

We start with a function which is the absolute value of 2 sign squares which is given by yt equal to sin of twice pie t plus sin of thrice pie t. This function is shown in this figure in blue.

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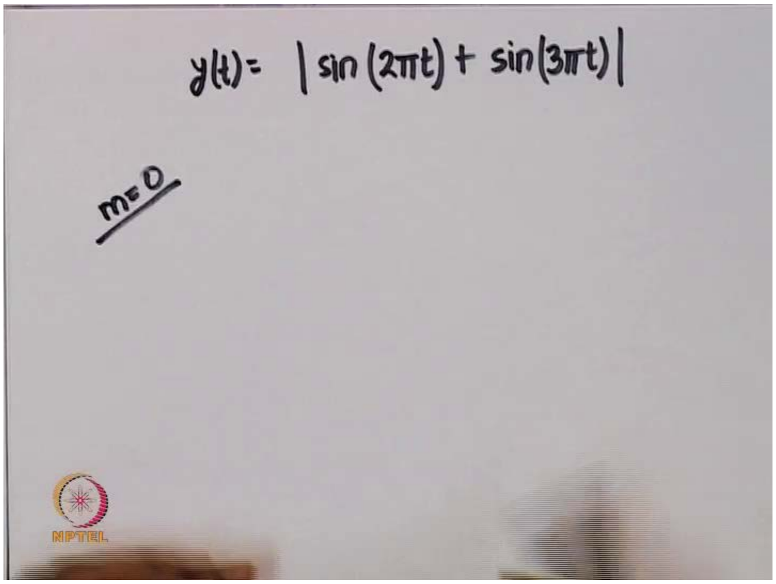

Now we can get the approximation to this function for interval of size 1 that is for M equal to 0. The approximation has been shown in green. Note that the function basically looks very very similar on the both sides of the 0 because of the original function is symmetric about 0.

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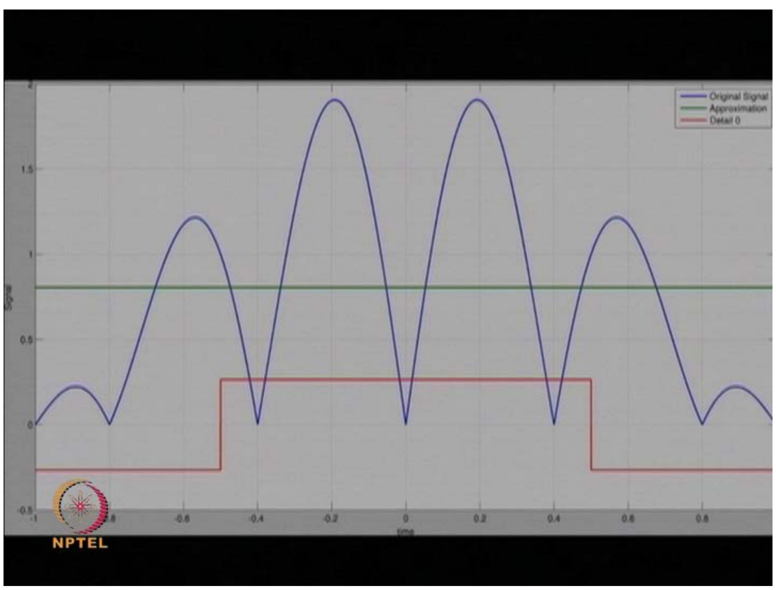

So now what we do is like we had the tale function to this function that is the projection of original single on to the W0 subspace and this is shown in red so. When we had the approximation and then detail function what we get is a better approximation to the original signal.

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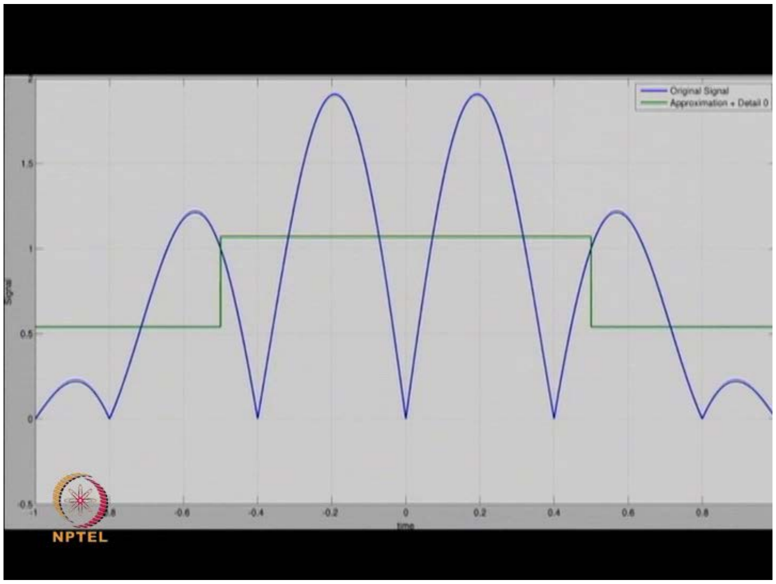

So which looks like this. Now we can add more and more finer details to the function that is we take, now the interval of size 2 to the power minus 1 that is half and we add the detail to the approximation that we just now got so which is shown in red.

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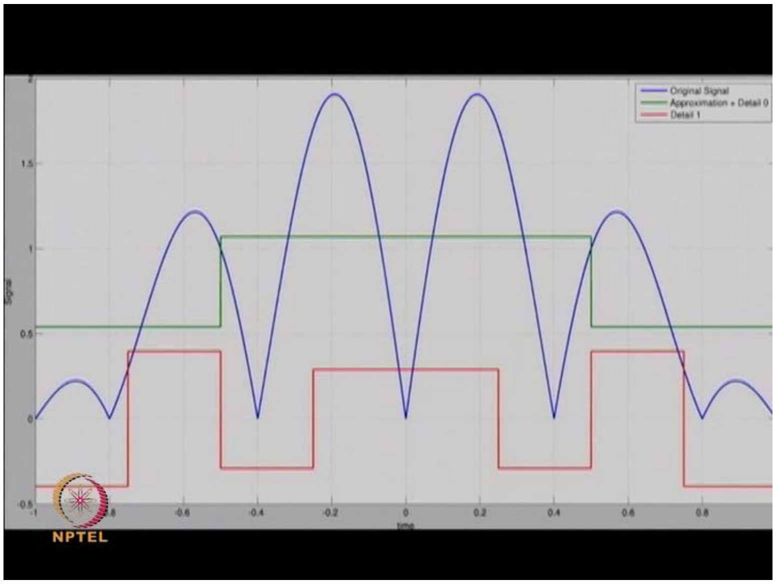

So when we add these two what we get is a much more better approximation to the original signal.

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Now we can add another finer detail that is the projection of signal on W 2 space…

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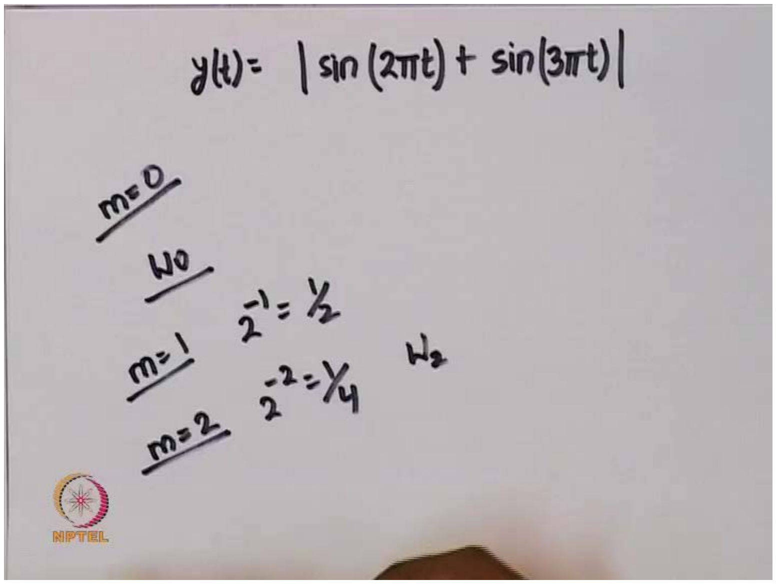

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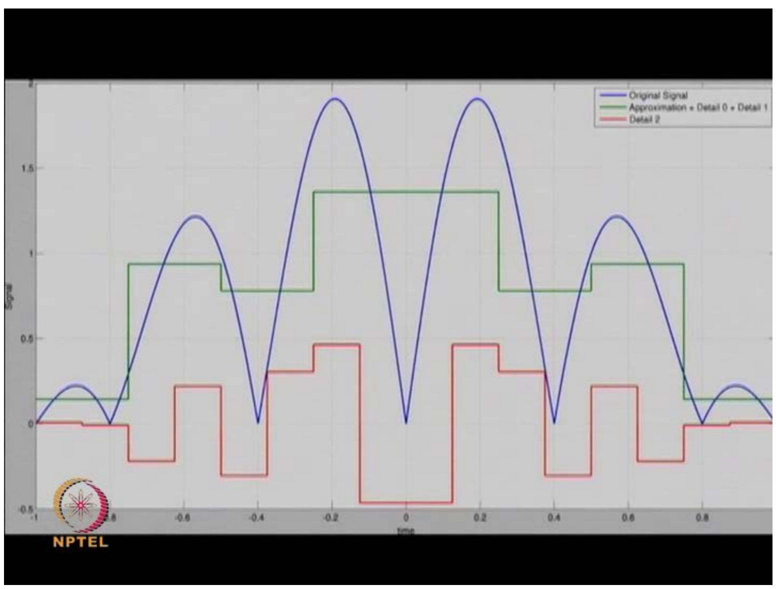

And which looks like this.

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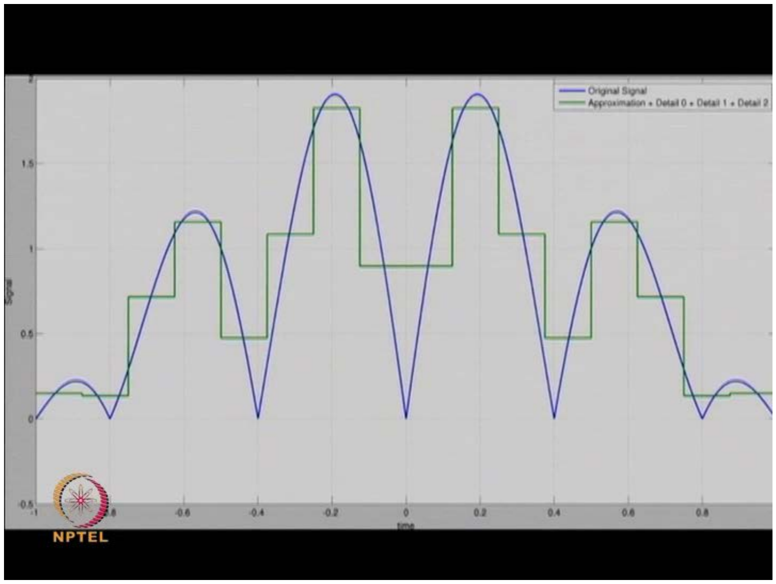

When we add these two what we again get is like even more better approximation to the signal which looks like this which is shown in green.

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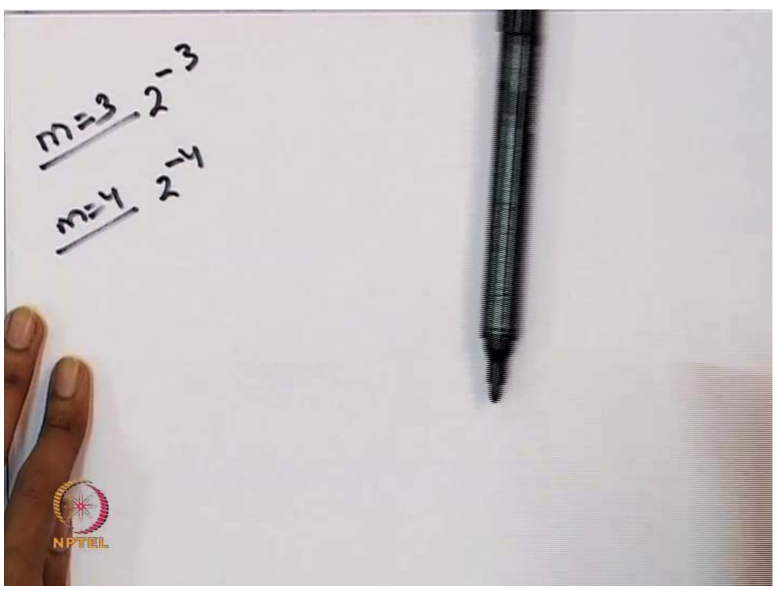

Now in the similar way we can go for more and more finer approximations that is for the intervals of size 2 to the power minus 3 2 to the power minus 4 that is for M equal to 3 M equal to 4.

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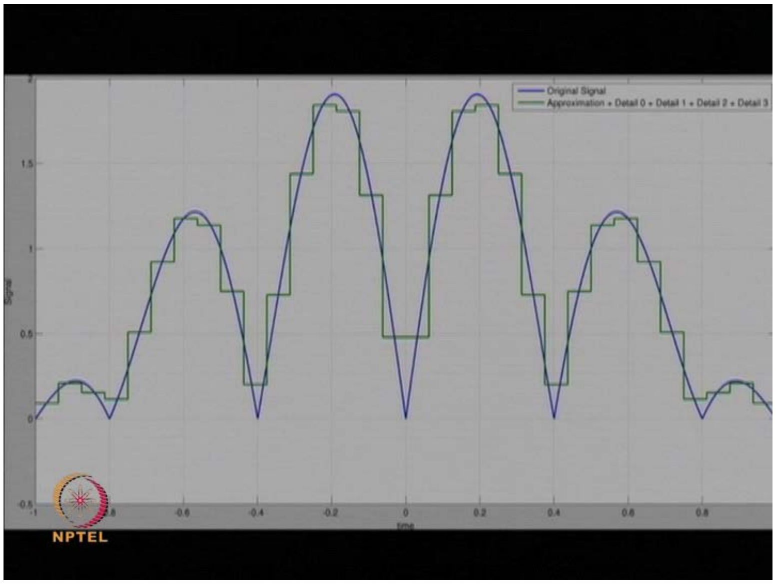

What we get is like a much more better approximation of the original signal that we had started

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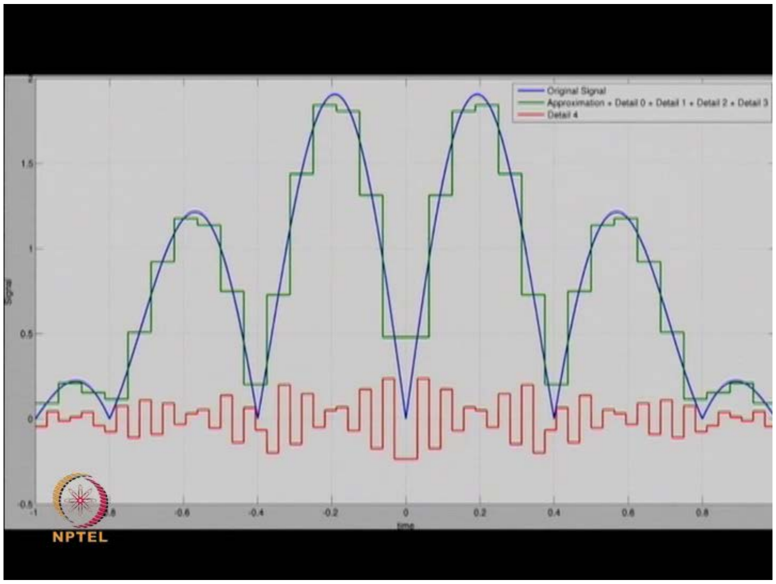

This looks like this adding another detail.

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So let us look at another signal function which is a triangular function. The approximation to this function is shown in green and the original function has been shown in blue. When we take the projection of the blue function on the W 0 subspace it looks like this.

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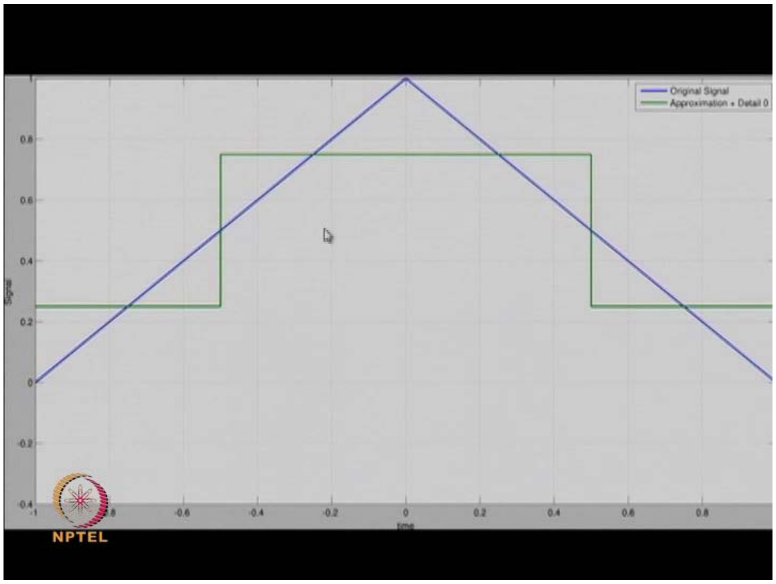

And when we add this detailed function to the green approximation what we obtain is a better approximation to that triangular function which looks something like this.

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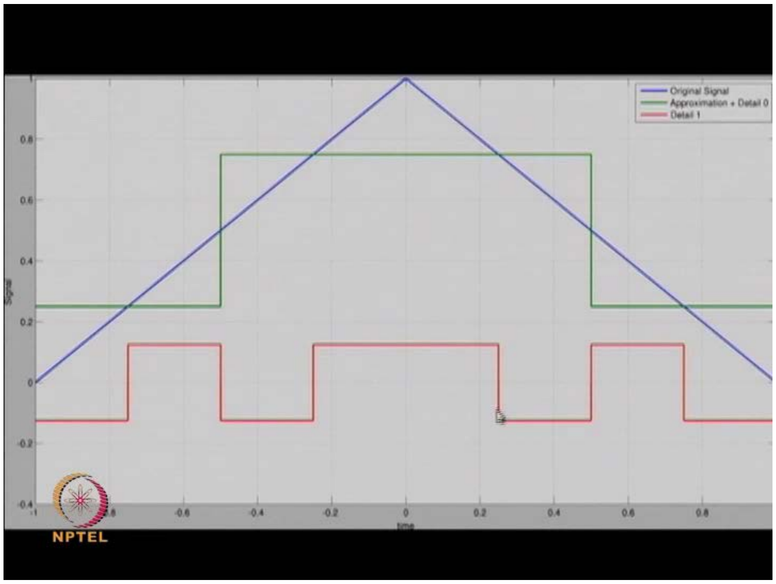

When we take the projection of the blue function on W1 subspace that is on the interval of size 2 to the power minus 1 that is an interval of size half. What we get is a function shown in red which is a detail function on W1 subspace.

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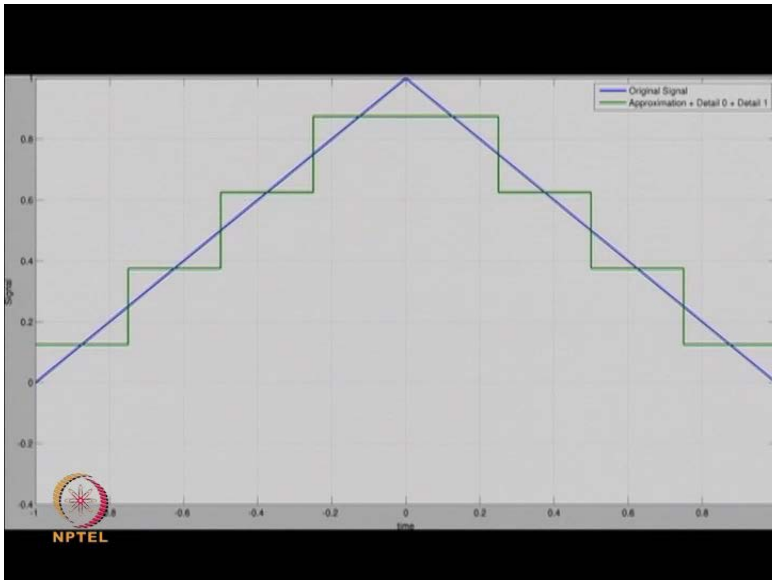

When we add these 2 functions we get even better approximation of the blue function which is shown something like a staircase.

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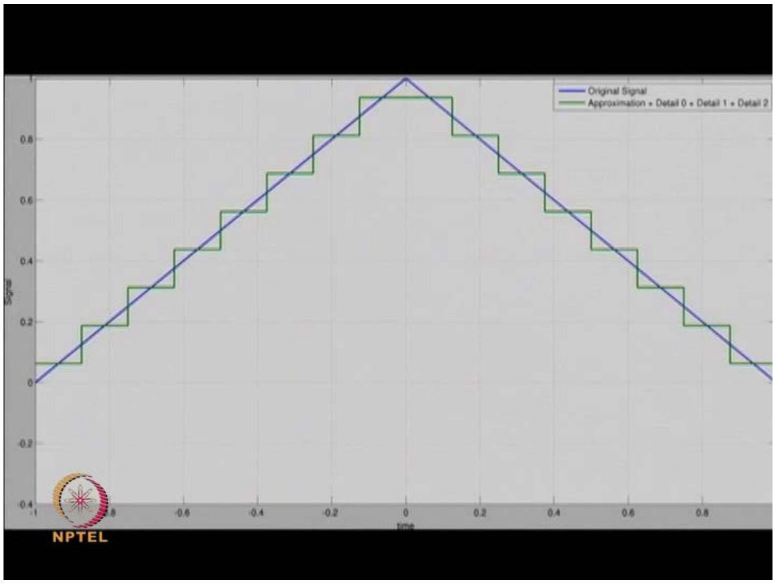
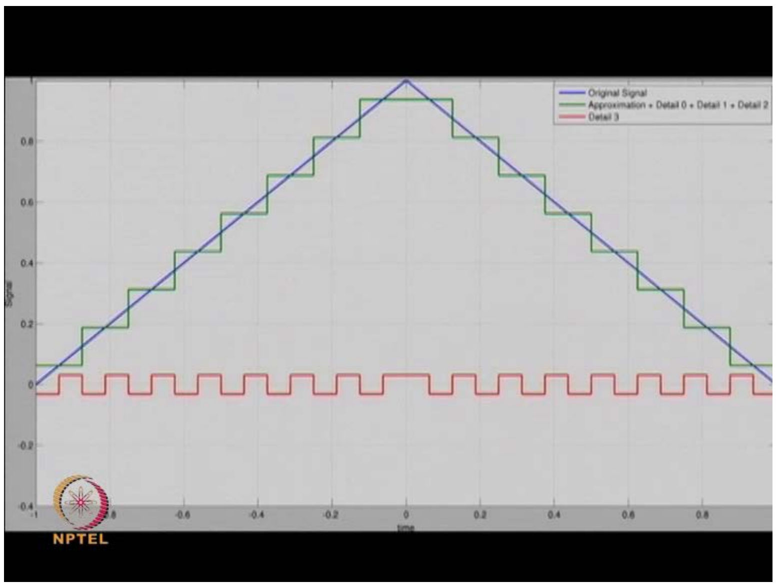
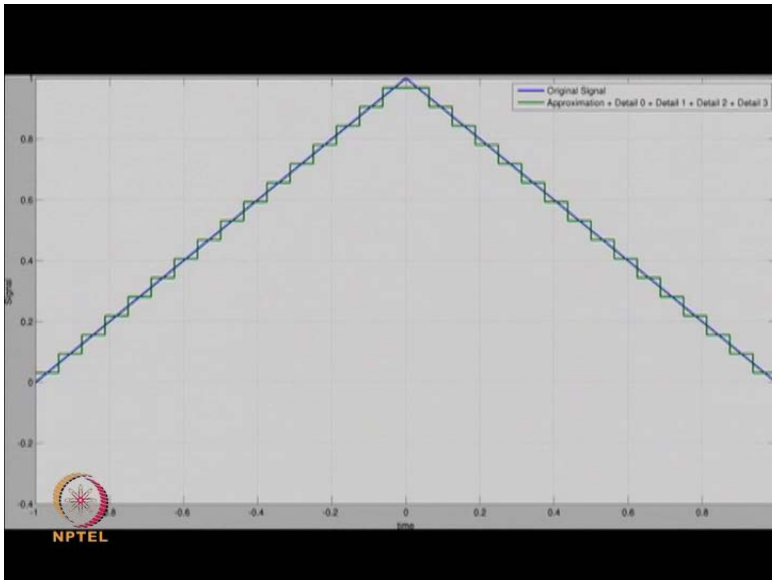



Original Signal
Approximation + Detail 0 + Detail 1
Detail 2
Signal
time
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
NPTEL


Original Signal
Approximation + Detail 0 + Detail 1 + Detail 2
Detail 3
Signal
time
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
NPTEL



Now, when we add more and more details to the function say this we get better approximation to the blue function again.

Adding finer and finer details results in getting even better approximations to the original signal. So this was another example in which we had the approximation of the signal on different subspaces. So what we have done is essentially that we have got a much more better approximation by combining more and more finer details. In this way we can construct a much better and better approximation of the function that we started with by adding more and more details. Thank you!