Microwave Integrated Circuits
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Mod 02, Lec 09
Maximally flat (binomial) transformer Chebyshev transformer

Welcome to another module of this course 'Microwave integrated circuits'. In the previous module we have talked about 'Multi section transformers' and formula for input reflection percussions when a number of transmission line segments are connected in cascade. But I had also mentioned that it is just an analysis formula, here we will find out input reflection percussions there will be no wonder what the characteristic impedance and the length of the individual transmission line segments are.

But in design when we are into design in impedance transformer we have to do the reverse that is we have given a certain characteristics, certain impedance matching characteristics in the frequency domain and then we have to find out the individual, the characteristic impedance of individual transmissions, so the first step into achieving this is to write down certain prototype equations and then compare those equations with the formula that we derive, so the first prototype equation is that of what we call binomial transformer.

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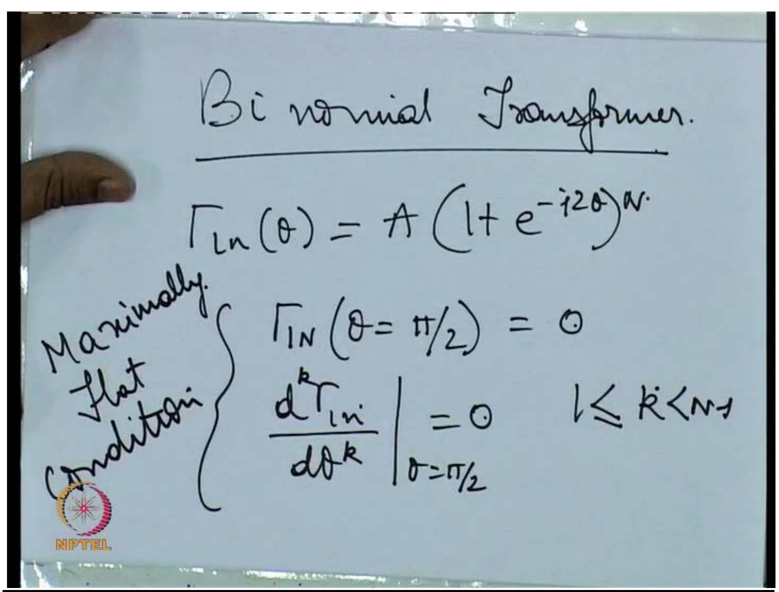

Now the formula for the binomial transformer, the input reflection prototype function is written like this. Now the interesting thing about this formula is that gamma in at theta is equal to 5 upon 2 is zero and D gamma in upon D theta K, or K between one and N minus one, is also equal to zero. This condition that is satisfied is called the maximally flat condition.

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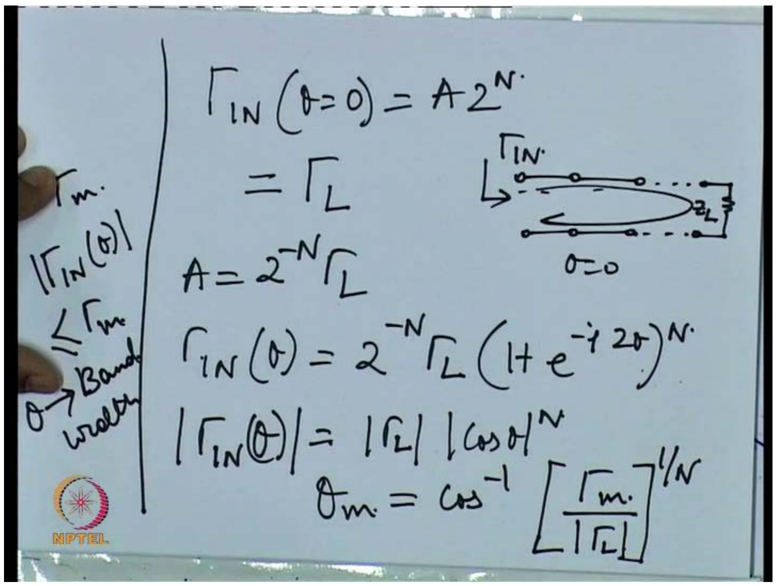

Now if we further analyze this equation that we just wrote down you see that gamma is what theta is equal to zero is given by this equation, we also know that when theta is equal to zero that is a DC condition and gamma E should be simply equal to gamma L the low reflection percussion. It is because we have a number of sections like this, the N number of sections with a low ZL Connector. Then gamma in or theta equal to Zero, this will a straight cut short and so that In that case gamma in at theta equal to zero will be equal to gamma L.

From this straight away write down an equation a is equal to this, now substituting this value in the, so then if we substitute the value of gamma A in the expression for gamma in, what we get is, if we try to find out the modulus or the magnitude of this function gamma in, then that comes out from this equation as Cos theta, so from here if we try to define the band width.

You recall how we define the band width in the previous lecture, we said that if we take a certain gamma M and for all values of theta or which gamma in theta magnitude is lesser or equal to gamma M then those values of theta respond to the band width, that is how we define the band width. So from here we can find out the value of theta M for which we get gamma M for which modular's of gamma and theta is equal to gamma M, so that value of theta is giving this equation is found out now.

And then once we know the value of theta M, we can find out the fractional N, fractional band width, delta F upon F0 as 2 minus 4 theta M upon 1. If we can, for a moment go back to slides on the monitor.

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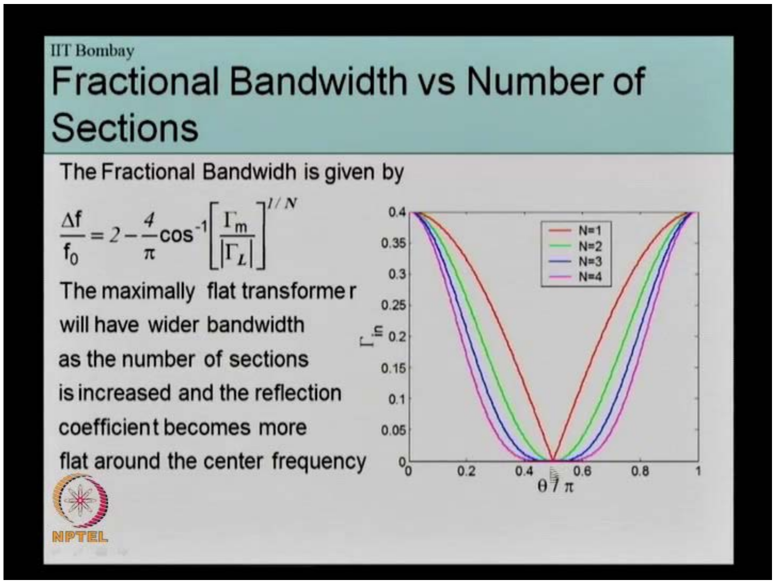

This function this maximally flat or binomial function that we just wrote for various of N is given like this and we see that for all values of N at theta is equal to 5 by 2, the value is zero.

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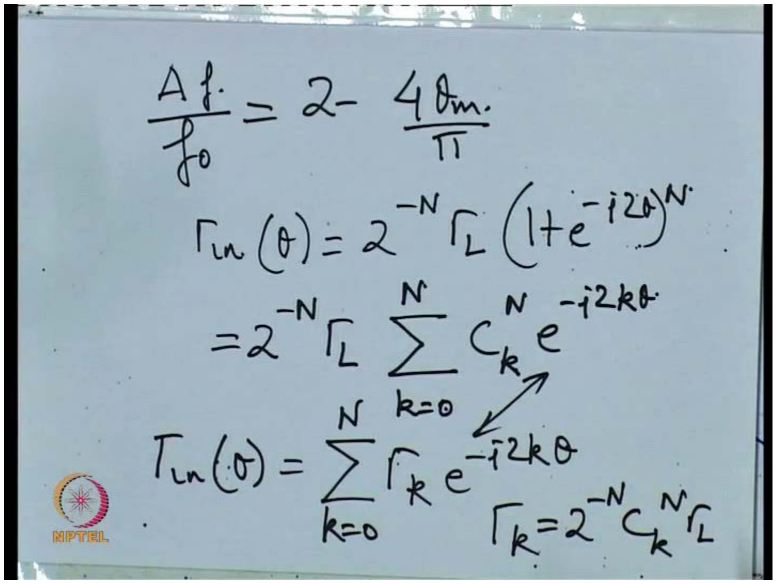

If we can back to our written slides, now how do we correlate this function with the formula or multiple section, for multiple sections of impedance matching network that we have derived in the previous class, how do we reconcile this formula with that. So the first thing we note that, which we note that gamma in theta is given by this, this is what we just derived and if we expand this even the binomial theorem, then we get an expression like this.

Now in the previous module we have seen that for multiple sections of transmission line the input reflection coefficient is also given by this formula. We see that these two equations are very similar, similar that we can find out an expression and write the values of gamma K like this, you see what we achieved here, we have written here an expression for gamma K in terms of the coefficient of the binomial function of the coefficient of the equation that we stated for the binomial transformer and so now gamma K is in terms of gamma L, we call that K refers to the sections of the multi section transformer.

So all the sections of the multi section transformer the gamma K values are now written down nicely., so we have done the reverse that we were trying to do that we said that we have a certain band width requirement, we have a certain gamma L, how can we design the ZK, once we find out the gamma K value, you can easily find out the ZK. Now some of the properties, since this is the equation what the gamma KS for the binomial transformer a few of the properties that we have to keep in mind are that…

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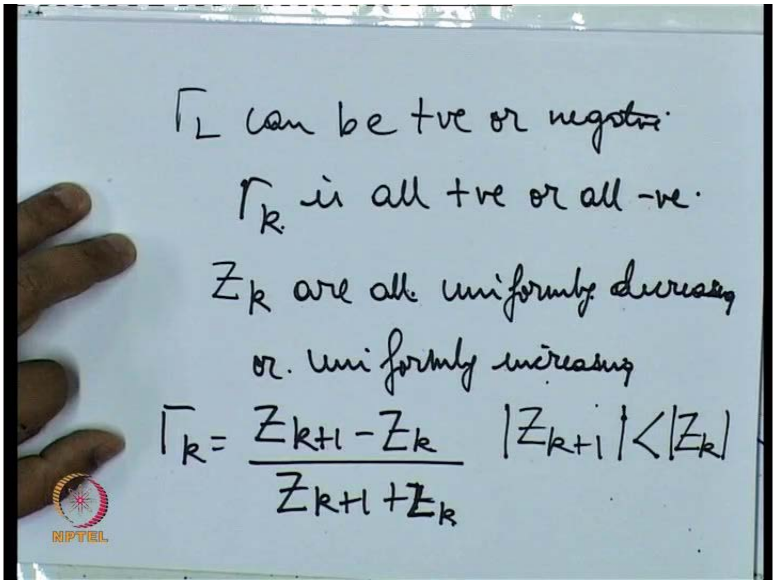

…this gamma L can be positive or negative, that is fine so then, so then this gamma K which is dependent on gamma L is all positive or all negative, if gamma K is all positive or all negative then means that the ZKs are all uniformly decreasing or uniformly increasing, why is that? Because gamma K recall is equal to, this is the formula for gamma K that we have found in the previous module.

If gamma K is negative, then ZK plus one is, I should write the modulars of ZK + 1 is lesser than ZK. In other words every subsequent stage has a lower characteristic impudence as compared to the preceding scale or if gamma K is positive then that means every subsequent stage has higher characteristic impudence as compared to the preceding one.

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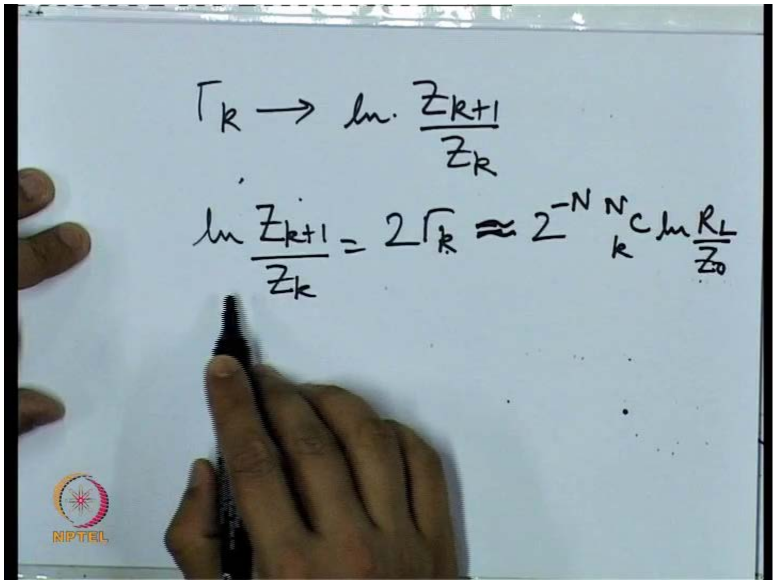

So then this is how we can do the design, and if we use that same approximation using that logarithmic series that we have done in the previous lecture then we can write this gamma K in terms of LN ZK+ 1 upon ZK so we can simply have you know that LN ZK+1 upon ZK is equal to twice of gamma K, this in turn is nearly equal to…

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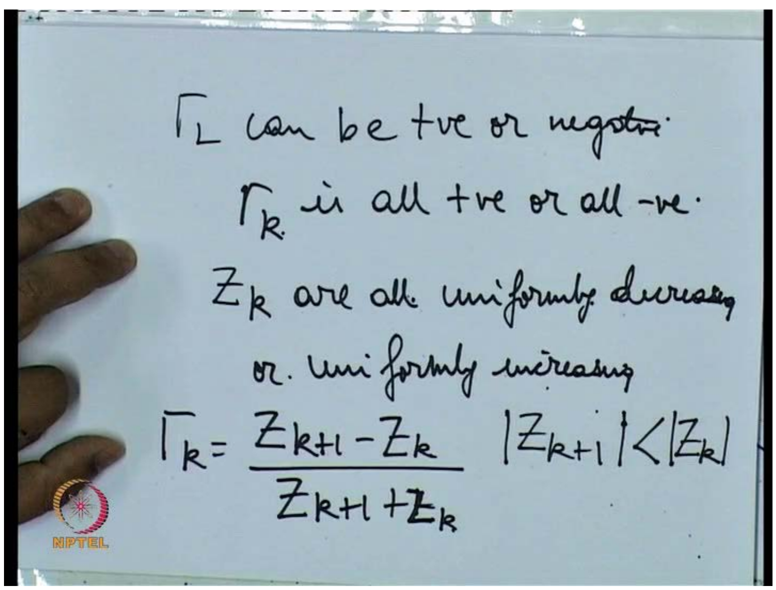

Now from here we can get the reason we do this approximation is because if we take this expression for gamma K then we will get more equations than there are variables.

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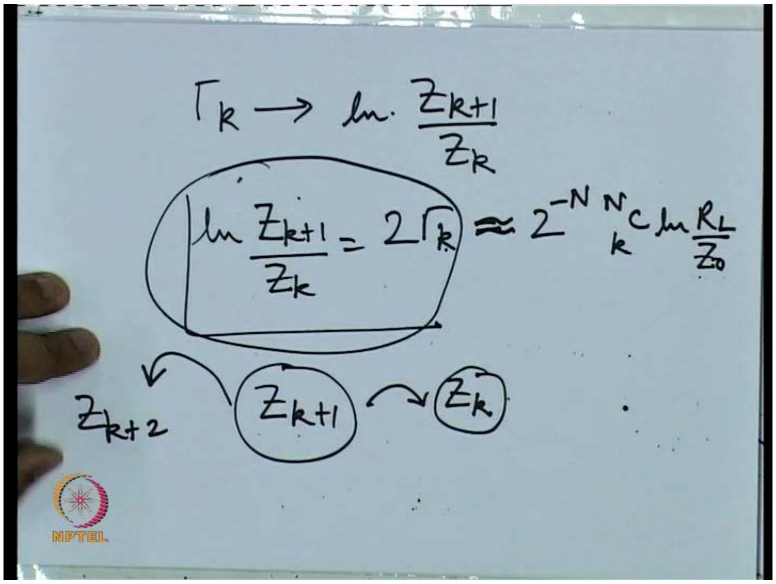

But on doing this approximation, gamma K that is this particular approximation, we reduce the number of equations and we also get consistent solutions, so using this formula and with this LHS and this in the RHS we can easily find out ZK plus one in terms of ZK and then we do this for each of the stages, so first we find out ZK then we find out ZK + 1, from ZK + 1 value we find out ZK + 2 and so on. So this is how the design takes place.

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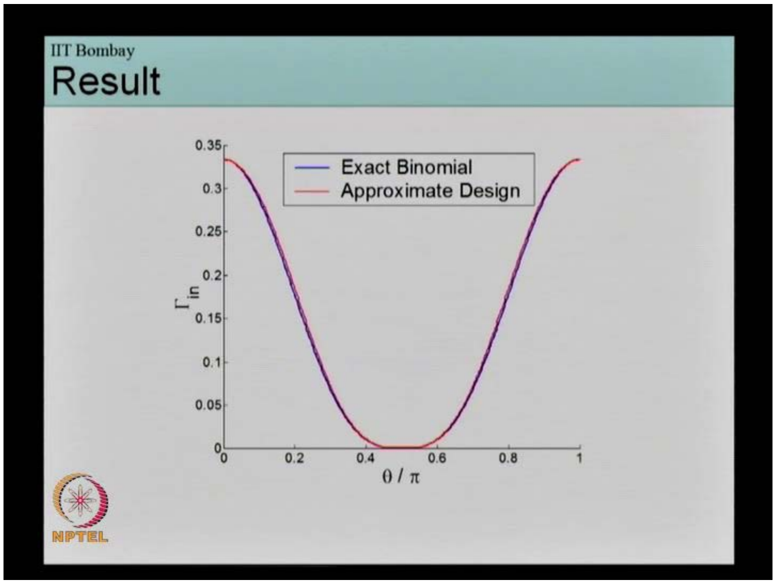

Here is a P section binomial transformer the input reflection coefficient that we design, we see that there is a slight, you there is slight mismatch between the exact binomial function represented by the blue line and this red line representing what we actually implemented.

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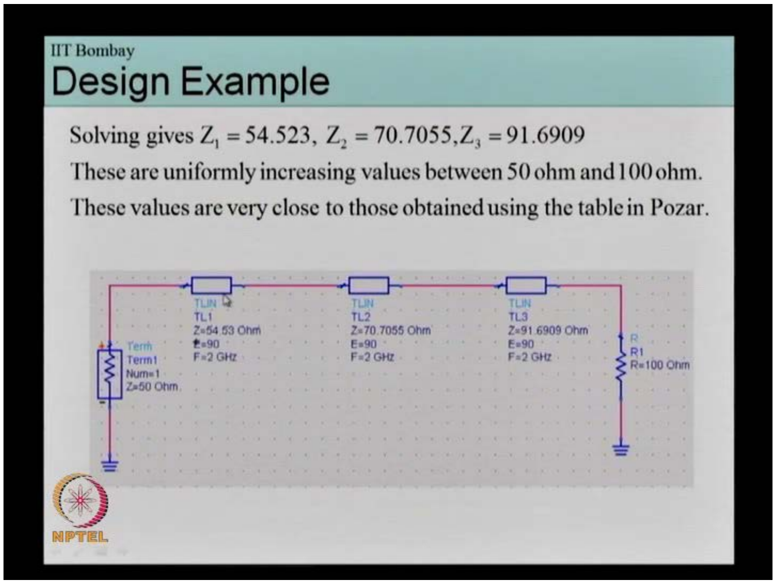

So this is what we actually implemented on tool called ADS this is the three section of our impudence matching network, these are the values for Z1, Z2 and Z3 this we obtained using the binomial equation that we have derived, ah what we notice is that first of all one thing that I should mention for binomial transformers is that they are symmetric. That is the first stage is similar has the same characteristic impudence as the last two.

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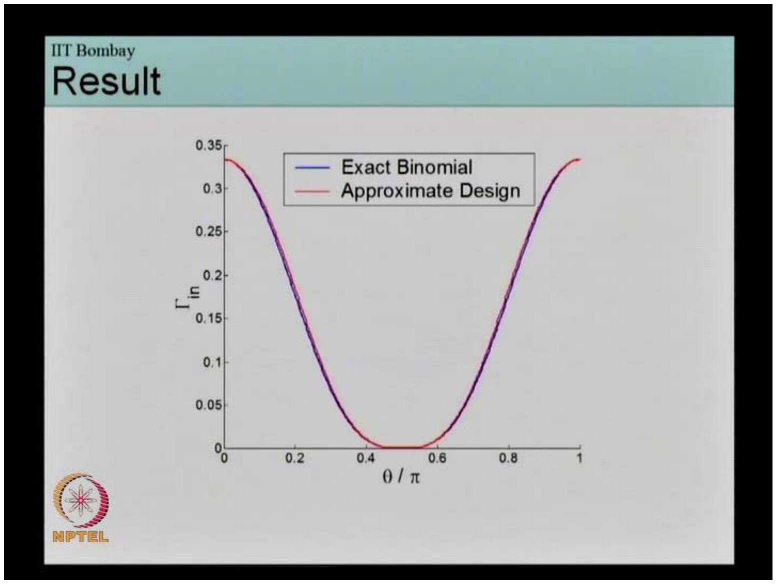

The other thing is when we do this design we see that there is a slight mismatch between the exact binomial formula and this design that we have implemented on the circuit and mismatch is because…

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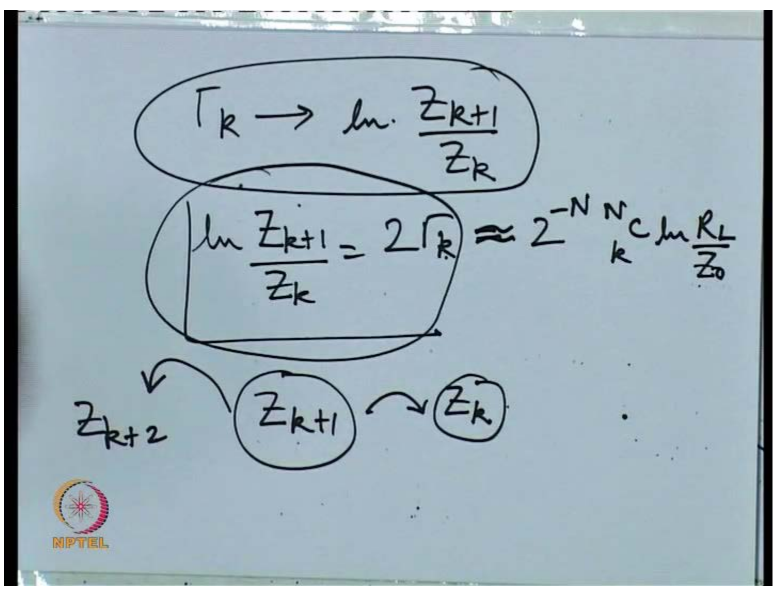

if we can come back to the written slide for a moment is because of this approximation, this logarithmic approximation that we give, because of this, there is a slight mismatch, so that was all about design using the binomial transformer.

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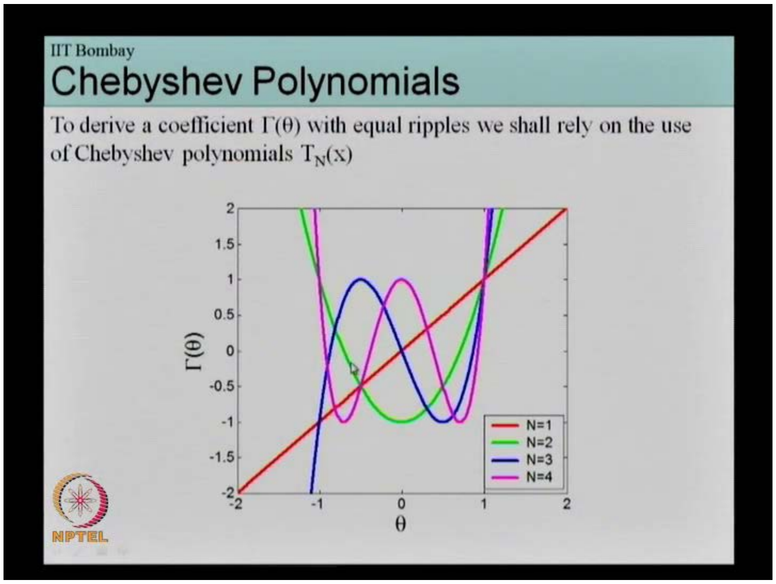

Then we will consider one more transformer which is Chebyshev, so let us see what is a Chebyshev. So the Chebyshev so in order to understand the Chebyshev transformer, these are the Chebyshev polynomials. If we can see it properly for various orders or values of N as we call, these are the Chebyshev polynomials.

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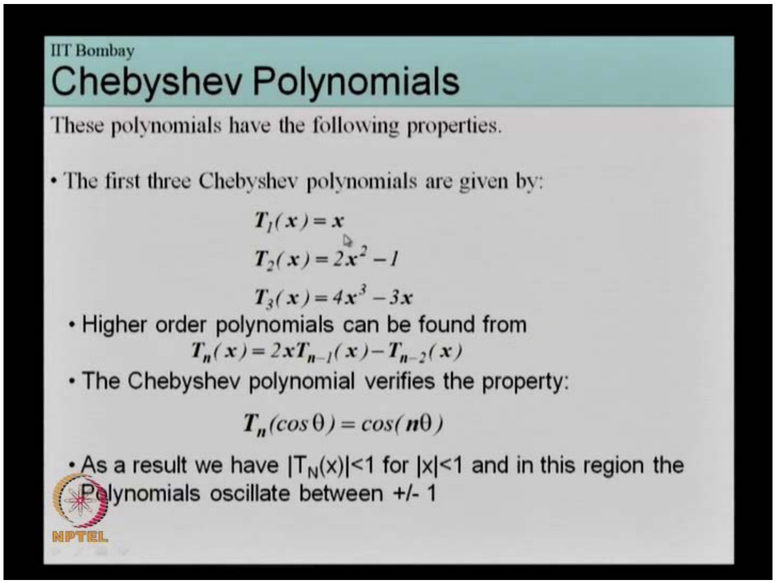

And the expression for this Chebyshev polynomial is given by these equations T1X represents the first order, T2 the second order and then T3 the third order.

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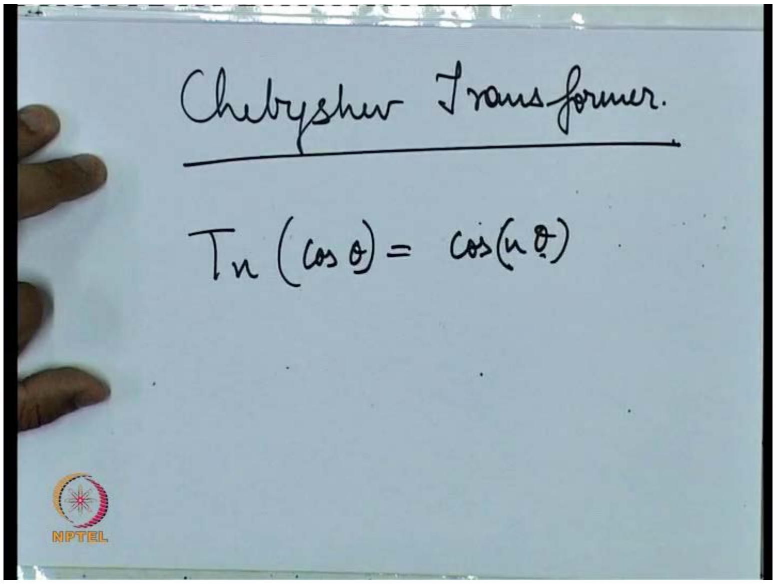

Now these Chebyshev polynomials they verify the property TNX, TN Cos theta is equal to cos N3.

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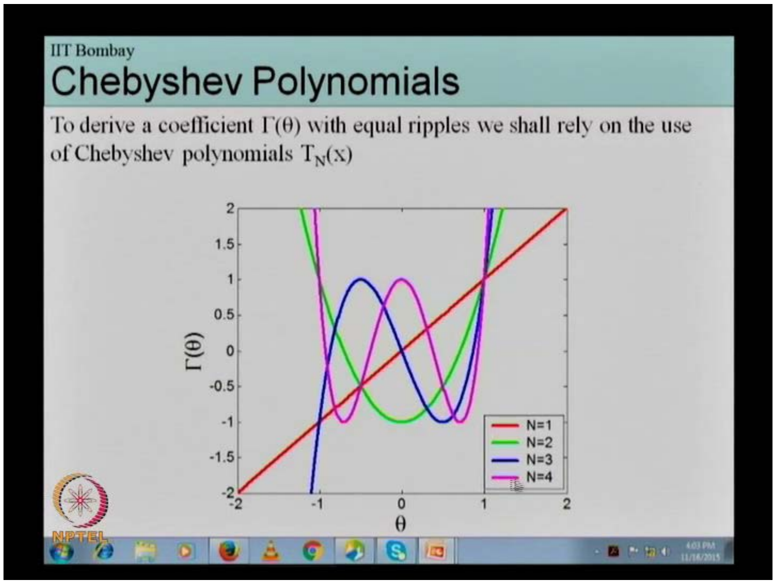

If you can for a moment go back to our slides, monitor slides what you see is for theta these are the chevsif polynomials once again, for theta between minus one and plus one the value of the Chebyshev polynomials represented by this gamma is also confined to this range, that is minus one, plus one.

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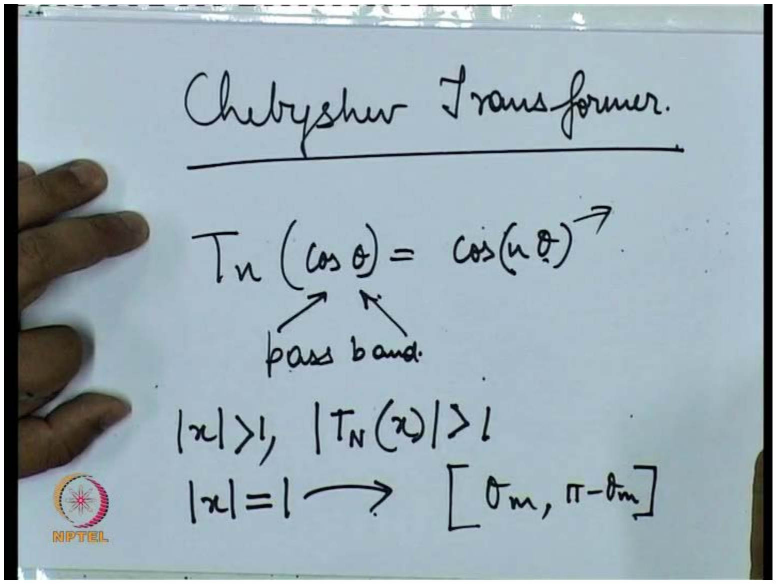

So coming back to our written slides, so then that you see that this when the argument of this Chebyshev polynomials is between minus one if we, when the argument of this Chebyshev polynomial us between minus one and plus one, the output is also between minus one and plus one. So then from here what we can see is that if we map our range you know that if we map our pass band as we call into this range then the output of Chebyshev polynomials since that is also restricted, then that also will be limited because of this restriction.

So in a one way if we choose say our X, for modulus X greater than one we have the modulus of the Chebyshev polynomial greater than one and if we restrict suppose say our X between minus one to plus one so that modulus of X is one and this is mapped to the end points of our band width theta M, pi minus theta M, then we can achieve the impudence match, this is the logic, this is the infusion behind the Chebyshev polynomial.

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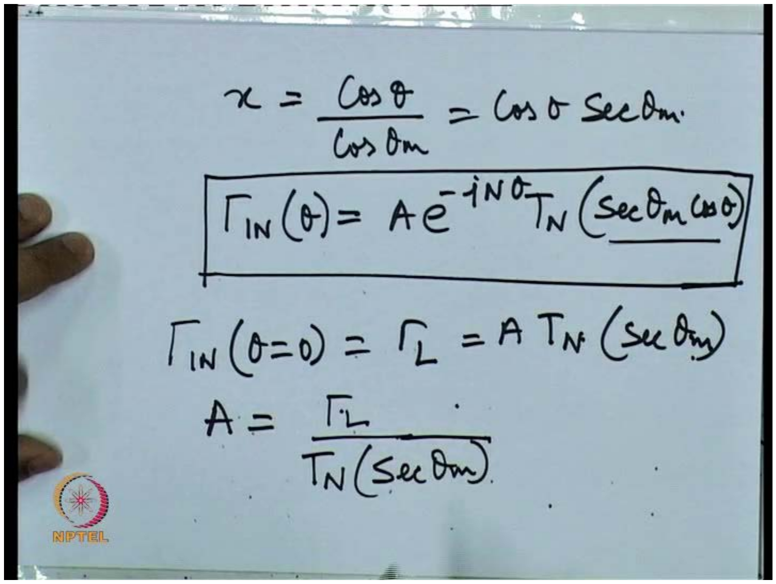

Suppose we choose our X, this X that we are talking about, which we are mapping the argument of the Chebyshev function as cos theta upon cos theta M which is equal to cos theta, sec theta M, then if we define our input reflection prototype function as, like this the first that we note is that our argument is between minus one to plus one and the terms which are outside this, which are outside the Chebyshev function are served to scale the gamma in value to the appropriate limits.

So suppose this is a prototype function just like for the binomial expression of the binomial transformer we had a prototype function, so this Chebyshev transformer this is our prototype function so again just like we did it like the binomial transformer if we take gamma in theta is equal to zero and that should be equal to gamma L and that substituting theta is equal to zero and this expression we get this is equal to ATN sec theta M from which we can derive a value expression for A as gamma L upon TN sec theta M.

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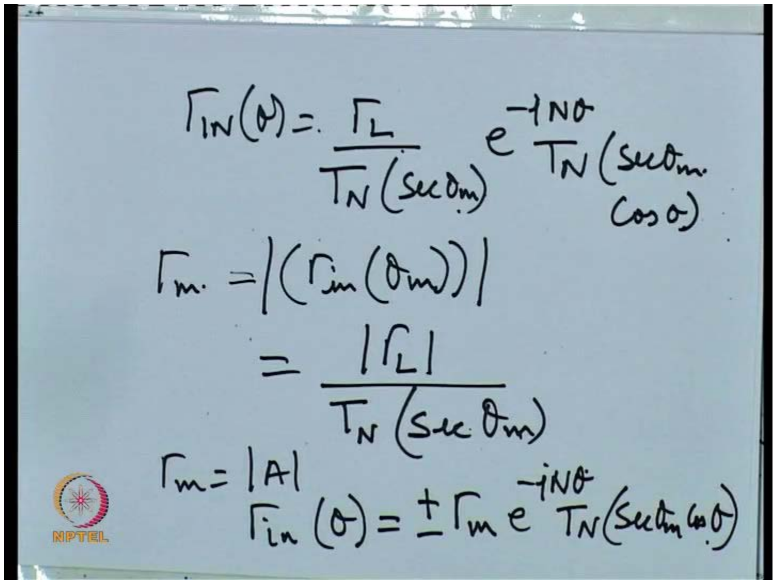

From which we can then straight away write gamma in theta is equal to gamma L upon TN sec theta M, E is to minus J and theta, TN sec theta M cos theta, again if we define a gamma M value that is the maximum allowable magnitude of the input reflection coefficient then that is equal to gamma in of theta M. And the expression for that will be… This is the limit and then this is the value we see that gamma M is simply equal to the magnitude of A so then we can also write gamma in theta as equal to plus minus gamma M.

So this is the complete expression for gamma In theta and we see that this input reflection coefficient is dependent on both gamma M and theta M.

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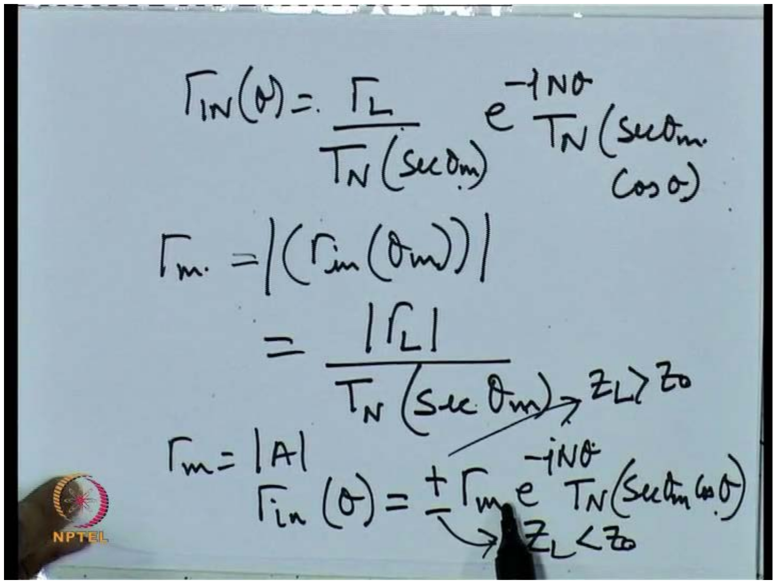

And we know that this positive sign is for ZL greater than Z0 that is when gamma L is positive and this negative sign is for ZL lesser than Z0, that is when gamma L is negative, now how do from here to the design, the nice expressions that we have derived for our binomial transformer, how do we do that.

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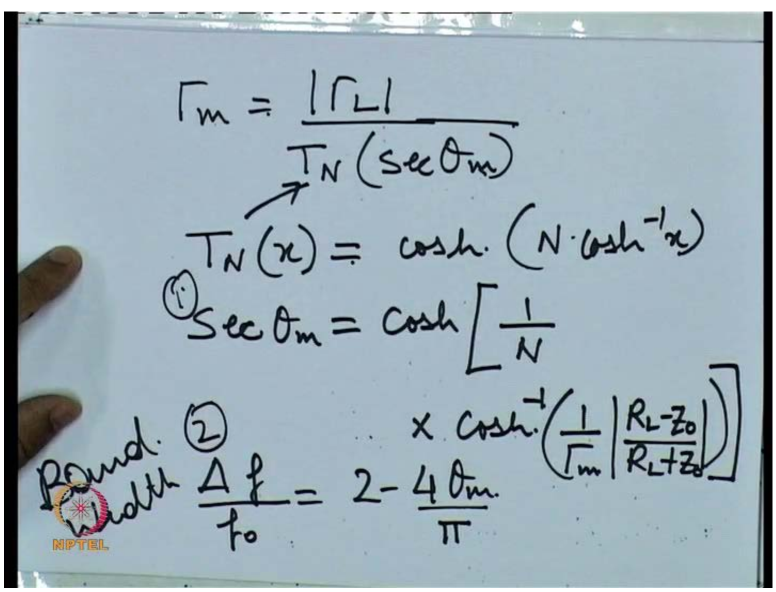

So first thing is let us try to find an expression for the band width we saw that gamma M is equal to gamma L upon TN sec theta M, ah using an alternate definition of the Chebyshev polynomials, the side and state but this is also an expression for this polynomial. From here I can write substituting this here I can write, get an expression for sec theta M as follows? And from this once we find out the value of theta M we can find out an expression for the band width, the given a certain value of gamma M, we first find out the value of theta M, and from this using this formula and once we find out the value of theta M, we find out the fractional band width.

So this is how the design goes, first this is step 1, this is step 2 and then we just simply equate you know the values of the Chebyshev polynomial with our expression for the input reflection coefficient of a multi stage impudence matching network, but then the expansion for the Chebyshev polynomial is not so simple as we saw as the case for the binomial, so let us see how we can do that.

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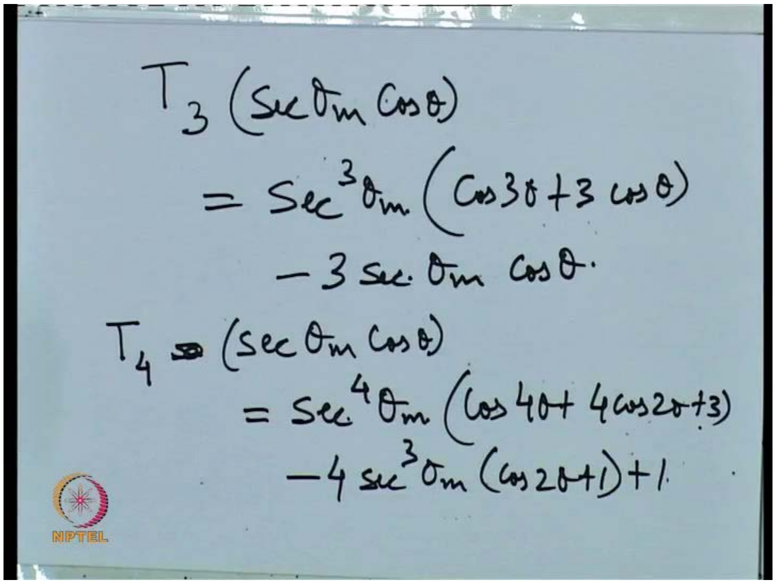

See that, if we go by the various polynomial orders the third order Chebyshev polynomial can be written like this. So it is not, so straightforward as that for the binomial, polynomial but here we have to find an expression for expansion for each and every order itself, there is no general formula using the combinatorial functions as we saw for the binomial expansion, the third T 4 sec theta M cos theta can be expanded like this...

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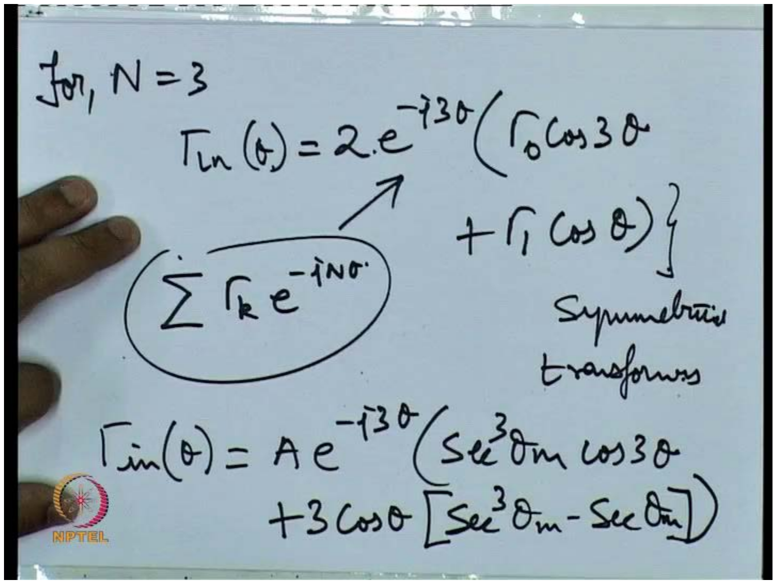

So these are the expansions and what we say N equals to 3, gamma in theta using these expressions that I had derived in the previous slide, you can write gamma in theta is equal to, this is the expansion from the earlier expansion that we had derived, this when expanded for third order we get like this, ah this is of course only true for symmetrical transformers and using the Chebyshev expansion that we just saw… Gamma in theta can be given like this.

Now this is the expansion for the third order Chebyshev polynomial and this is the expansion for the expression for the input reflection coefficient for a third order impudence transformation network and provided the network is symmetrical, so then once we have derived these two expressions, this one and this one we have to equate the two.

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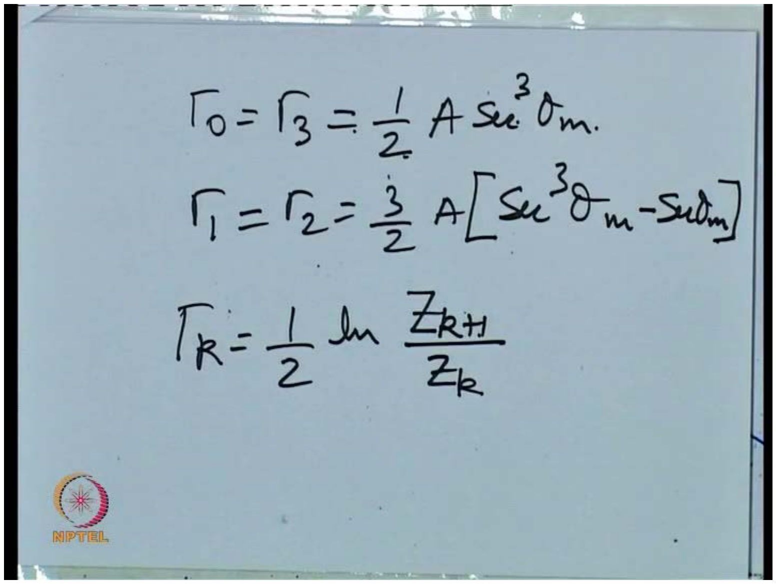

And once we equate the two, we get gamma zero is equal to gamma three, this is because of the symmetry, and gamma one is equal to gamma two, so these once we of course find out the values of gamma zero, gamma one, gamma two and gamma three, then we apply this formula, from which we can, from which once we know gamma zero then we can find out Z1, once we know gamma one we can find out Z2 and so on.

So then this is the complete design procedure, ah so in summary I want to say that for this Chebyshev polynomial the expansion is not so straight forward, then for each order we have to have an individual expansion and it is somewhat complicated, the binomial expansion on the other hand is much simpler, it is we have a straight forward formula for the binomial expansion and it is much easier to handle and to design of course, the question then is why do we have this Chebyshev transformer in the first place.

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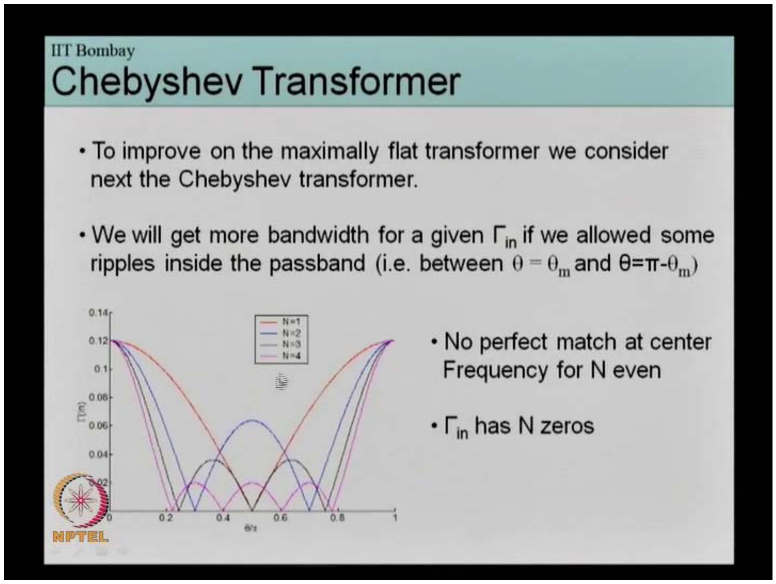

To answer that question if we can go back to the monitor slides for a moment. Ah see these are the Chebyshev polynomial expansions for various order, if the red one is for N equal to one , then this N equal to 1 polynomial is very similar to the binomial expansion then the blue one represents N equal to 2 black one N equal to 3 and violet or purple one represents N equal to 4 Chebyshev polynomial.

The advantage of the Chebyshev polynomial is that, first I should say the disadvantage, the disadvantage is that there are multiple ripple, it is not a maximally flat expression, but then because of the presence of ripple, we can because we are allowing some ripples in the past band, we can allow our band width to be further expanded. For example say if our gamma M is along this line for N equal to 4, this purple line, say our gamma M is like this, then because we are allowing some ripples never of course crossing the value of gamma M, because we are allowing these ripples we can give more allowance to our band width.

So for providing for more ripples in the past band we are enabling a wider band width, that is the reason we go for Chebyshev polynomial, but the binomial, polynomial or the monomial transformer has a very flat response, has a very flat characteristic of the past band in the impudence matching band width, but then the problem is because we are making it so flat we are not allowing any ripple in the past band, and that’s why our band width is somewhat constrained as compared to a Chebyshev polynomial .

So that is a paid off between Chebyshev polynomial or the Chebyshev transformer or the binomial transformer. Binomial transformer has a very flat response but lesser band width, Chebyshev polynomial because it is aligned from ripple; it has a wider band width. In the next module we shall be covering some special ‘impedance transformation network’ known as ‘Paper’.

Thank you.