Transmission Lines and E.M. Waves
Prof R.K. Shevgaonkar
Department of Electrical Engineering
Indian Institute of Technology Bombay

Lecture-7

Welcome, till now we analyzed the Transmission Line characteristics using analytical approach. There is another approach which is very elegant approach and that is the graphical approach for analyzing the problems of Transmission Line. In fact the graphical approach has many advantages firstly we all know that an image has much longer lasting impression than an equation or a text.

(Refer Slide Time: 01:08 min)

So if the characteristic of Transmission Line are represented in a form of an image it creates a much long lasting impression on the human mind, secondly we will see later that the graphical approach is much simpler compared to the analytical approach where the calculations can be reduced by a significant amount when we analyze the problems on Transmission Line, thirdly and most importantly the graphical approach is a very compact way of representing the impedance characteristics of Transmission Line.

So in fact whenever we are doing Transmission Line calculations even analytically it will be appropriate to keep the graphical representation in mind and any analytical result which we get on Transmission Line should always be cross checked with the graphical representation. So, one does not go at least conceptually wrong in solving the Transmission Line problems. The graphical approach for solving the Transmission Line problem essentially is for solving the impedance characteristic of Transmission Line. This approach does not give you the voltage and current solutions it gives you only the representation of impedances on Transmission Lines or the standing wave characteristics of Transmission Line, that means it can give you the impedance transformation relationship it can give you the calculation of the reflection coefficients the calculation of VSWR the location of the voltage minima, the location of voltage maxima and so on.

So today we developed a basic framework for analyzing the Transmission Lines by a graphical approach the basic idea here is to take the impedance which is normalized to the characteristic impedance and do transformation of this impedance into the complex reflection coefficient plane called the gamma plane. By doing this then we transform all the impedances from the impedance plane to the reflection coefficient plane and then we will see that this representation of impedances and reflection coefficient plane makes the calculation much simpler than if you carried out the calculation in the impedance plane.

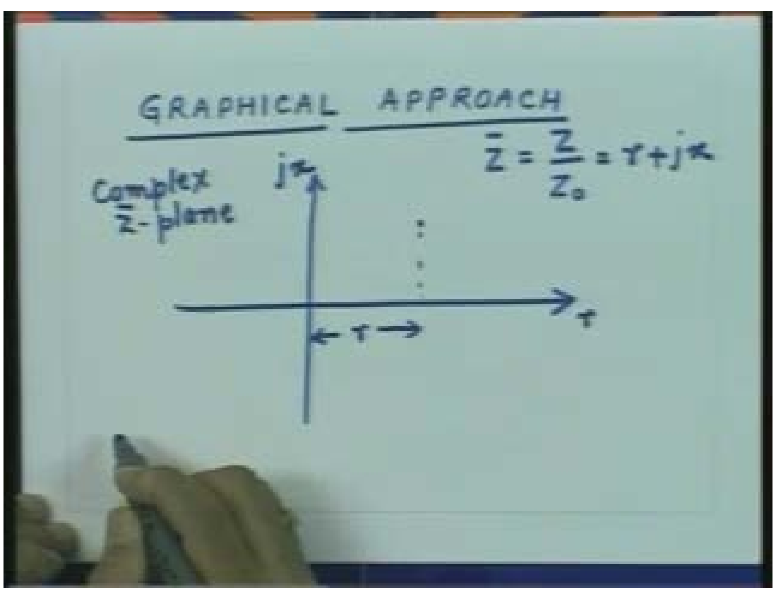
As we all know that if you take the passive impedances their resistive part is real and positive and the imaginary part could be positive or negative that means we are saying that the resistance in any impedance is always positive here we are not talking about negative resistances and the reactance could be positive that means it could represent inductance or it could be capacitive so it will be negative.

So if I plot a impedance point on the complex impedance plane then I get the impedance plane where the real axis represents the resistance and the imaginary axis represents the reactance. Before we get into this plotting first let us understand that the impedances in Transmission Line calculations always are the normalized impedances with respect to characteristic impedance. As we saw earlier the absolute impedances do not have much meaning in Transmission Line calculations the impedances always have to be normalized with respect to the characteristic impedance.

Now in our framework let us first make an assumption that the Transmission Line is a Loss-Less Transmission Line that means the characteristic impedance of this line is a real quantity and all impedance which we have are now normalized with respect to the characteristic impedance Z_0 so any impedance Z normalize is nothing but the actual impedance divided by the characteristic impedance Z_0. Let me write explicitly the real and imaginary part of this normalized impedance and let me denote by this small alphabets. The normalized impedance is represented by r + jx where r represents the resistive part of the normalized impedance and x represents the imaginary part of the normalized impedance.

Then in the complex z plane this is the complex z plane or the normalized z plane I can put any point in this plane whose this value is r so the real axis is normalized resistance r and imaginary axis is j times x, this value is jx.

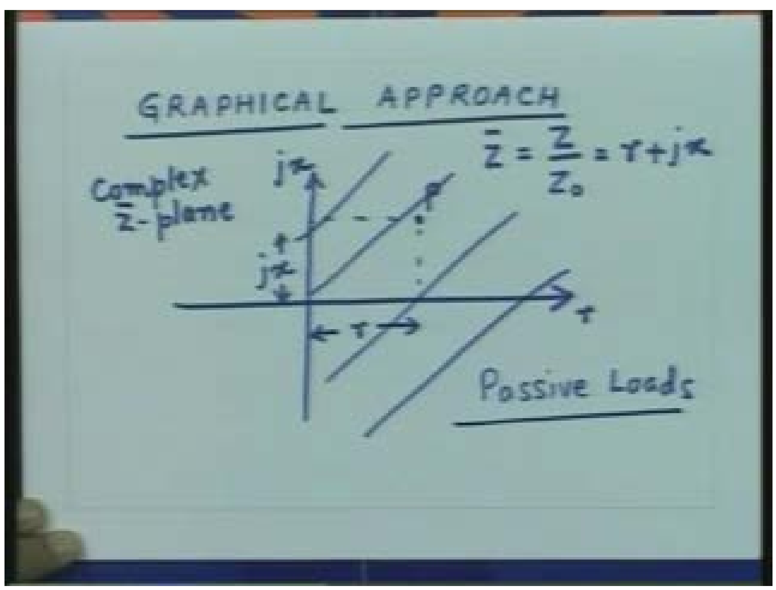
(Refer Slide Time: 06:10 min)



So here this point p represents an impedance which is having a normalized resistive value R and a normalized reactance value x. If I consider all possible passive loads for which the resistive part is always positive then any point in the right half plane of this complex z plane represents all possible loads, also the point which lie on the imaginary axis for which r = 0 represent purely reactive loads. That means if I consider all points on the imaginary axis and the point which are lying on the right half of the imaginary axis they cover all possible passive loads which one can realize on Transmission Line. So, essentially the entire impedance plane which is a right half of this represents the passive loads.

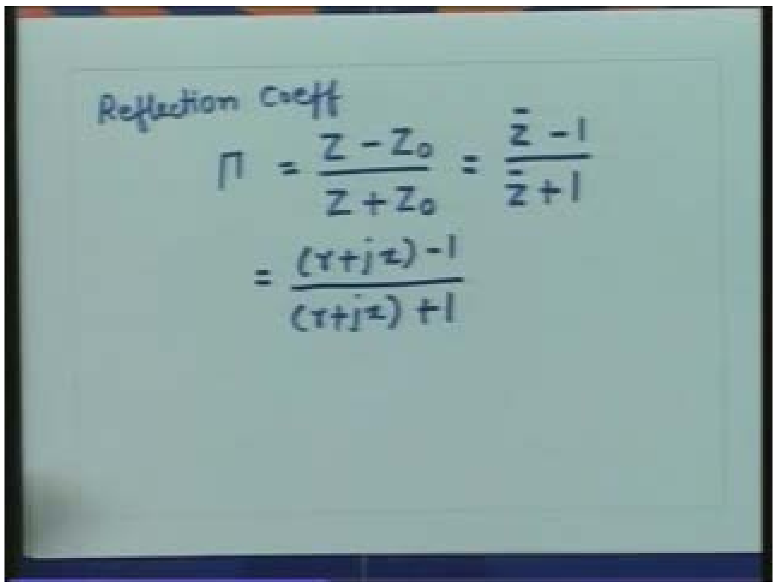
Now we know for every passive load we have a corresponding reflection coefficient.

(Refer Slide Time: 07:36 min)



As we have seen earlier, the reflection coefficient $\Gamma = \frac{Z - Z_0}{Z + Z_0}$ where Z_0 is the real quantity for a lossless line, Z in general could be complex if I write this expression in the normalized impedances I can take Z_0 common from numerator and denominator so this will be $\frac{\bar{Z} - 1}{\bar{Z} + 1}$. Writing explicitly for real and imaginary parts this reflection coefficient can be written as $\frac{(r + jx) - 1}{(r + jx) + 1}$.

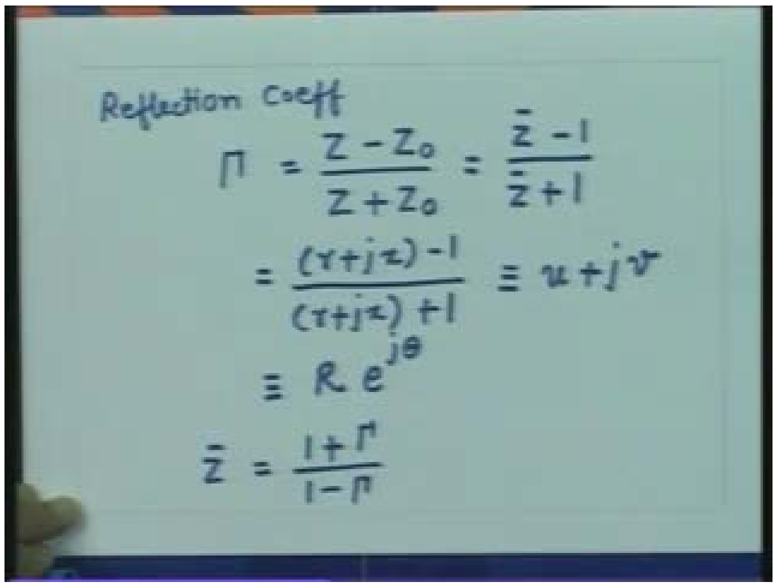
(Refer Slide Time: 08:36 min)



So in general the reflection coefficient will have a real value and it will have an imaginary value this can either be written in the Cartesian form or can be written in the polar form. So let us say the real part of the reflection coefficient is denoted by u and the imaginary part is denoted by v so reflection coefficient can be written as u + jv.

If I represent the reflection coefficient into the polar form this is also equal to the magnitude of the reflection coefficient R and some angle θ. So the reflection coefficient gamma can be represented by a Cartesian number which is u + jv or it can be represented like a polar number which having a magnitude R and an angle θ, using this expression now we have one to one correspondence between the reflection coefficient and the normalized impedance. So if you know normalized impedance we can find out the reflection coefficient I can invert this relationship and get the normalized impedance Z that is equal to $\frac{1+\Gamma}{1-\Gamma}$. So if I know the value of normalized impedance I can get uniquely the value of reflection coefficient and if I know the value of complex reflection coefficient then I can uniquely find the value of the corresponding impedance.

(Refer Slide Time: 10:20 min)



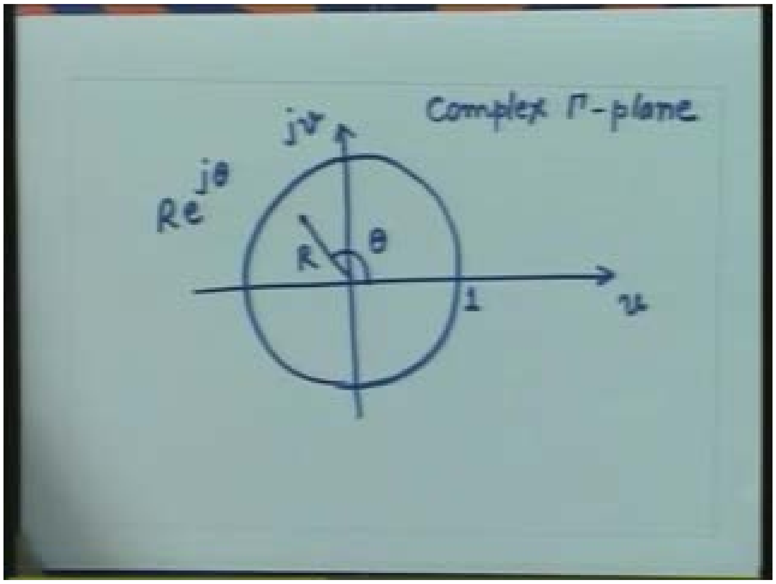
So this transformation between the reflection coefficient and the normalized impedance is a one to one transformation.

Now if I plot the reflection coefficient for all passive loads as we have seen earlier for passive loads the magnitude of the reflection coefficient is always less than or equal to one. We saw when the impedance is purely reactive or short circuit or open circuit that is the time the magnitude of the reflection coefficient becomes equal to one otherwise for any other impedance the magnitude of reflection coefficient is always less than one. That means if I plot the reflection coefficient into the complex reflection coefficient plane which we can call it as the complex gamma plane this is now my complex gamma plane where the real axis is u, the imaginary axis is jv and here we are marking the complex reflection coefficients so this is complex gamma plane.

Since for all passive loads the magnitude of the reflection coefficient will be less than or equal to one all impedance point have a corresponding point of reflection coefficient which lies within the unity circle in the complex gamma plane. So if I draw a unity circle whose radius is equal to one all passive loads reflection coefficient will lie within this circle so any point if I take within this circle then that will have a corresponding passive load value or if I take any passive load value it will have a corresponding reflection coefficient point inside this unit circle so this radius is equal to one.

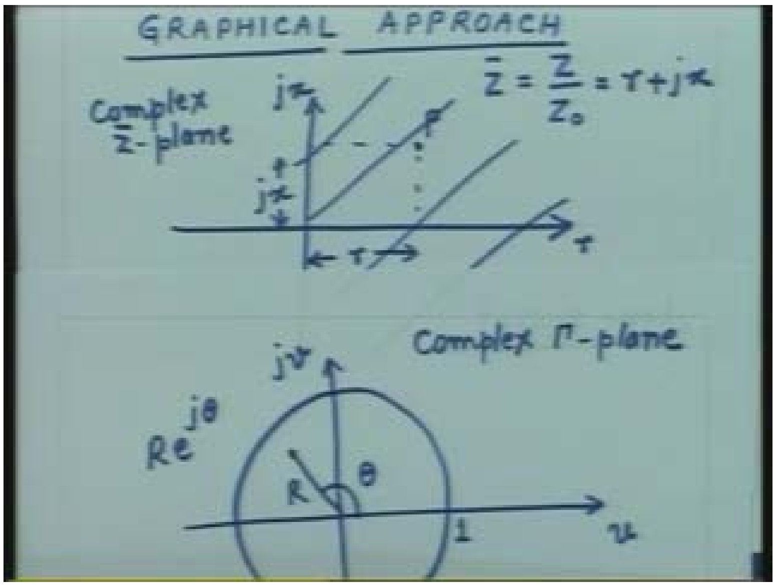
So a reflection coefficient point here has a magnitude as we mention if I represent in a polar form this distance from the center of the complex gamma plane gives you the magnitude of the reflection coefficient and the angle of this sector from the real u axis that gives you the angle θ so this point is the complex reflection coefficient point denoted by $R\ e^{j\theta}$ is having equivalent value of u and v so the complex reflection coefficient point can be marked on the gamma plane.

(Refer Slide Time: 13:15 min)



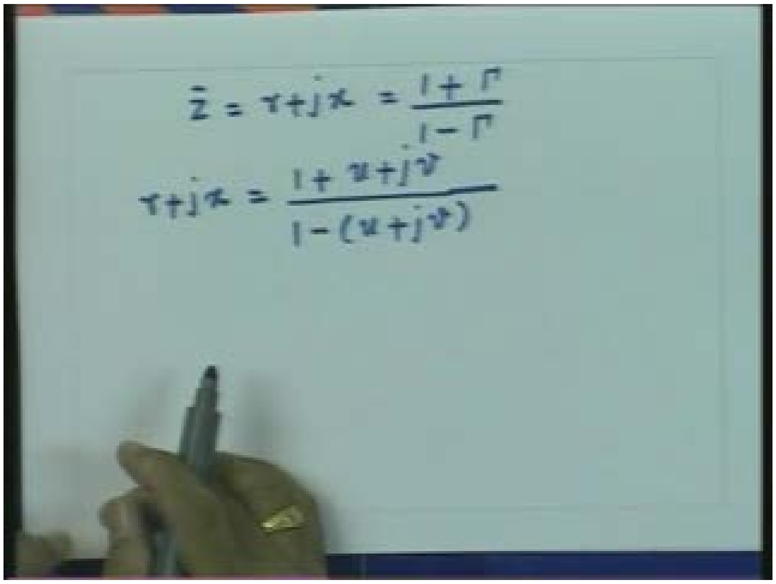
Now we have two planes you are having a complex z plane and you are having a complex gamma plane and as we know since there is a one to one transformation between these two planes every point on this plane essentially is mapped to a point on this plane. So first exercise we do is we try to map all possible impedances on to the gamma plane and then we essentially create a graphical structure where the analysis is carried out in the complex reflection coefficient plane.

(Refer Slide Time: 14:00 min)



So for doing this transformation let me go back again to the original relation which I had and that is the impedance I can write as normalized impedance $\bar{Z} = r + jx$ that is equal to $\frac{1+\Gamma}{1-\Gamma}$, substituting for $\Gamma = u + jv$ I get $r + jx$ that is equal to $\frac{1+(u+jv)}{1-(u+jv)}$, I can rationalize this function and separate out the real and imaginary parts.

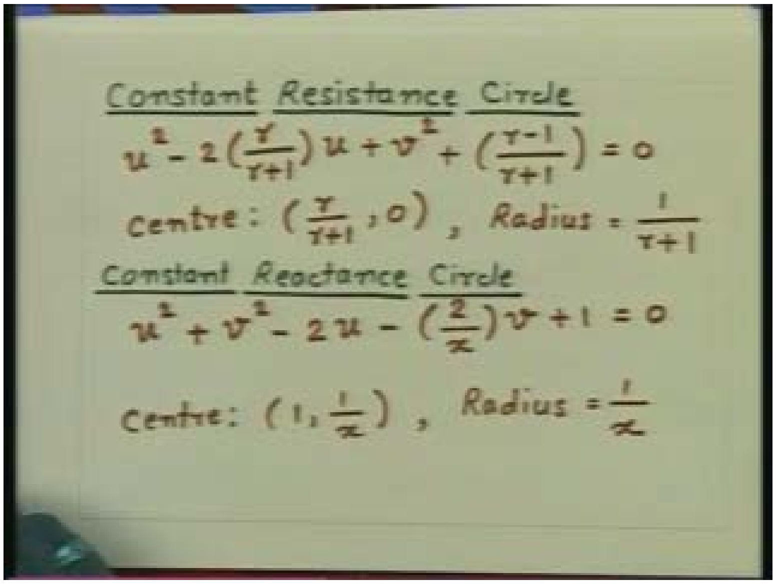
(Refer Slide Time: 14:57 min)



So I will get r equal to some function of u and v, similarly x equal to some function of u and v. So from this relation I will get two equations on the complex gamma plane for a given value of r or x. Essentially when I map the impedances on the complex gamma plane I will see two set of curves on the complex gamma plane one will correspond to a given value of r other will correspond to a given value of x.

So if I separate out the real and imaginary parts essentially I will get the two equations and these equations will be given by this.

(Refer Slide Time: 15:49 min)



This is equation corresponding to the real part of this equation. So if I take r equal to real part of that and rearrange I will get a equation which will be this, similarly if I take x equal to imaginary part of this and rearrange the terms I will get a equation which will look like that.

Now if I look at this equation immediately it strikes to us that both equations represents circles on the complex gamma plane so I get a circle on the complex gamma plane for any given value of r. Similarly for any value of x I get a circle on the complex gamma plane. What then this represents is that for a given value of r if I take different values of reactance's or x then I am going to move on this circle. It is a locus of all reactance point for a given value of r that is the reason we call these circles as the constant resistant circles.

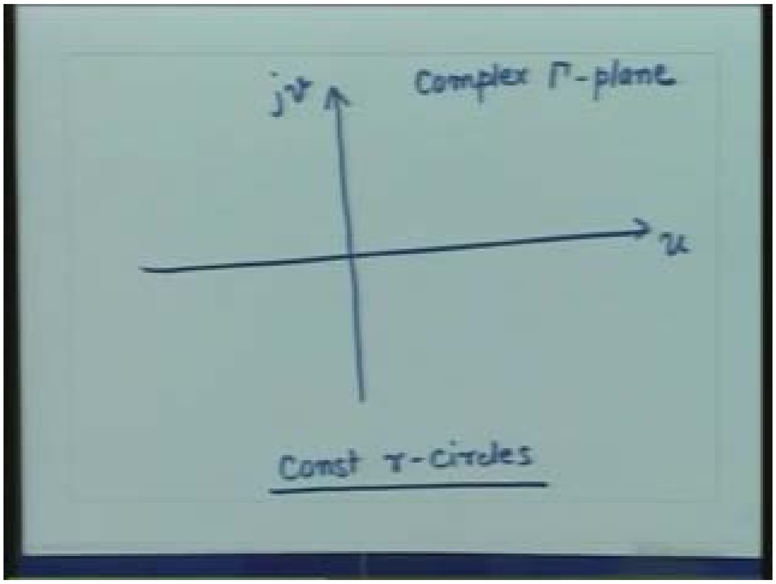
So this circle represent different values of reactance's but all the point which are lying on this circle they have the same resistance value. Similarly when I go to this circle it is the locus of the points of different resistance and all of them have the same reactance value. So these circles are called the constant reactance circles.

Once we realize that then one would like to see graphically how this circle would look like on the complex gamma plane. So to draw the circles we can get the center and the radius of the circles so for a constant resistance circle the center is r divided by $(\frac{r}{r+1},0)$ that means the center of all these constant resistant circles lie on the real gamma axis lie on the u axis and the radius of this circle is equal to $\frac{1}{r+1}$ where r will vary from zero to infinity.

Similarly when I go to the constant reactance circle I have a center which lies at $(1, \frac{1}{x})$ that means this center lies on a vertical line passing through u = 1 and its radius are at these circles will be given by $\frac{1}{x}$.

So first we graphically draw these two sets of circles and see how they look like. So let us first take the constant resistance circle so this is our complex gamma plane u and jv and I am drawing the complex gamma plane and what we are drawing is the constant r circle or constant resistance circles.

(Refer Slide Time: 19:15 min)

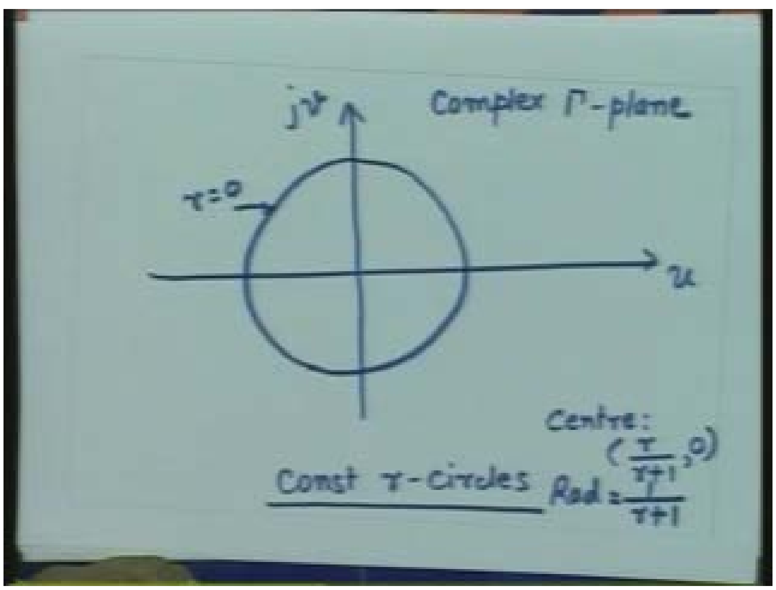


So as we saw the center for this circle is $\frac{r}{r+1}$ so the center is $(\frac{r}{r+1}, 0)$ and the radius for this circle is equal to $\frac{1}{r+1}$.

The constant resistance circles have a center which is $(\frac{r}{r+1}, 0)$ and its radius is $\frac{1}{r+1}$.

Now let us try to plot this circle for different values of r. Let us vary the value of r from zero to infinity and then see how these circles will get plotted on this complex gamma plane. If I take r = 0 the center this quantity becomes zero so I get a center which is (0, 0) which is the origin of complex gamma plane and the radius of the circle is equal to one for r = 0. The unity circle in the complex gamma plane itself represents a constant resistance circle corresponding to r = 0 so this circle corresponds to r = 0.

(Refer Slide Time: 20:57 min)

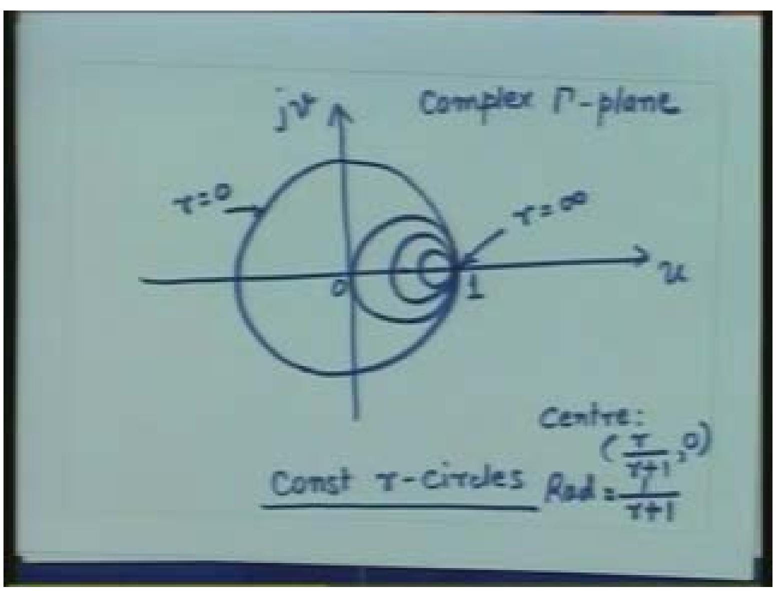


Let us take another value of r and the special value of r will be r = 1 which will correspond to $Z = Z_0$ so if I put r = 1 in this then this is 1 and r plus one will be equal to 2 so this will be (½, 0). so the center of this circle will lie halfway between these two points because this is zero origin and this is unity circle and is equal to one. The center for r = 1 circle lies halfway here and the radius is equal to half so this circle is having a radius equal to this and the center here so this circle passes through origin and will look like that.

If I further increase the value of r one thing we will note is the center shifts towards the right and the radius of the circle becomes smaller and smaller. If I take the extreme value that is r = ∞ then this center will become one so this will be (1, 0) and the radius will become equal to zero.

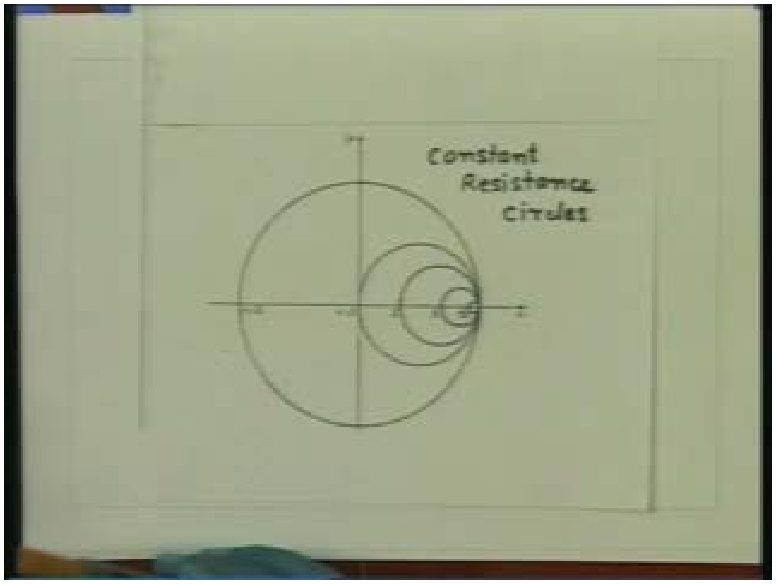
So for r = ∞ the circle will degenerate into a point at this location so if I take different values of r this circle essentially lie one within another like with increasing value of r and at this location represents that r =∞. In fact one can see from this equation for the constant resistance circle that all this curves here are pass through the point u = 1, v = 0 so all this circle which we have drawn here are pass through this point u = 1, v = 0, similarly even these constant reactance circles also pass through the same point u = 1, v= 0. So this point is rather a special point because both the set of curves whether you take a constant resistance circle or constant reactance circle they pass through this special points so u = 1, v = 0 is the location from which the circles essentially are going to pass.

(Refer Slide Time: 23:37 min)



So if I show you these circle which are constant resistance circles these circle essentially look like that.

(Refer Slide Time: 23:46 min)



So just to give you few numbers here this circle which is the outermost circle that corresponds to r = 0, this circle which passes through the origin that is r = 1. Then as you go for higher and higher value of r this is r equal to two circle five circles ten circles and the final that is point here is equal to one, equal to zero point that correspondents to r equal to infinity. The constant resistance circles are one within another and they the r = 0 gives you the largest circle and the circle becomes smaller and smaller as r increases and the final part r = ∞ the circle becomes 0.5,

So this is the one set of curves which will correspond to the constant resistance of the impedance, the another set of circles which are the constant reactance circles.

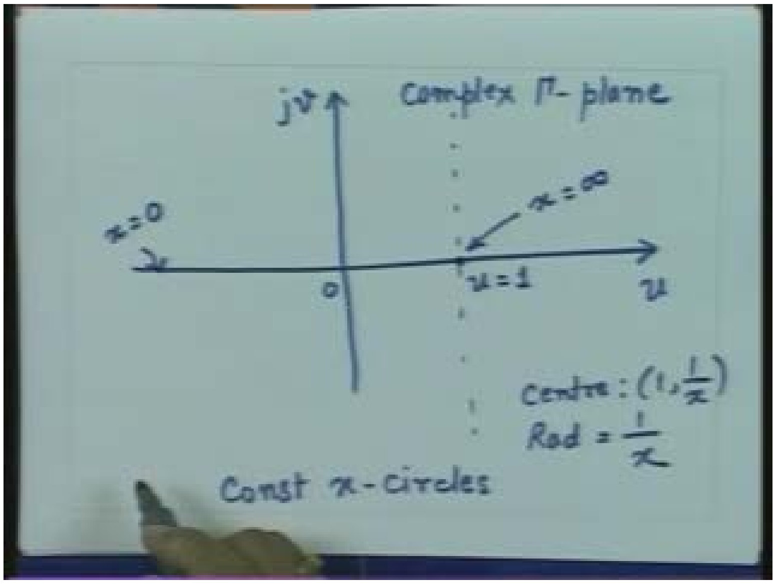
Here let us draw the constant reactance circle this is u, this is jv and again this is complex Γ-plane and these are the constant x-circles or constant reactance circles. As we saw the center for this circles is $(1, \frac{1}{x})$ and radius of this circle is equal to $\frac{1}{x}$. Again let us substitute different values of x and plot this circle on the complex Γ-plane. This is origin this is v = 0 if I put x = 0 then the center of this circle is (1, ∞) and the radius of this circle is also ∞.

Now the radius of this circle is equal to infinity means this circle has become a straight line but I mentioned earlier we know this circle pass through this point u = 1, v = 0 that means now I have a circle which passes through u = 1, v = 0 point and it is a straight line so the real axis of the complex Γ-plane corresponds to x = 0. So all the reactance value zero correspond to this line so this line itself corresponds to x = 0.

Now consider the two types of reactances the positive reactance which is inductive or the negative reactance which is capacitive. Let us say first I take the inductive reactance that means x is positive. So the center lies on $(1, \frac{1}{x})$ that means it always lies on a line which is u = 1 as x becomes larger so as I increase the value of x the center which was at infinity that now comes down and the radius also becomes smaller and smaller. So initially I had a circle which was this straight line now as I increase the value of x I will get a circle which will have a smaller radius and so on and so forth.

Taking the extreme value when x = ∞ at this point this quantity will be zero so the center will become (1, 0) which is this point and the radius of this circle will be equal to zero. So this point again represents x = ∞ so it is interesting that this circle r = ∞ also corresponded to this point and this point also represent x = ∞.

(Refer Slide Time: 28:26 min)

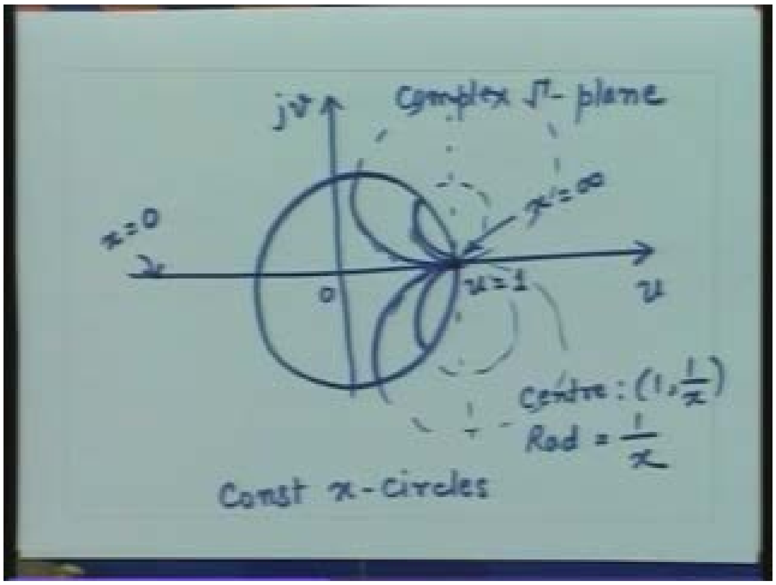


Now if I plot the various circles now again first thing we should note here that we are only interested in reflection coefficients which lie within the unit circle because they only correspond to the passive loads. If any portion of this constant reactance circle lie outside the unity circle is not of our relevance because that portion of the curve does not represent a passive load. Although the constant reactance circle fill the entire space only that portion on the constant reactance circle is useful to us which lies within the unity circle of the complex Γ-plane.

Now if I draw these circles these circles will essentially look like that they are the constant reactance circles they becomes smaller and smaller like this so only this portion of this curve is meaningful to us. So all positive value of x at inductive reactances their center will always lie on this line but on the upper half on the complex Γ-plane and that is the reason this circle will be always lying above the u axis. These curves which are within the unity circle represent the reactive parts corresponding to the passive loads.

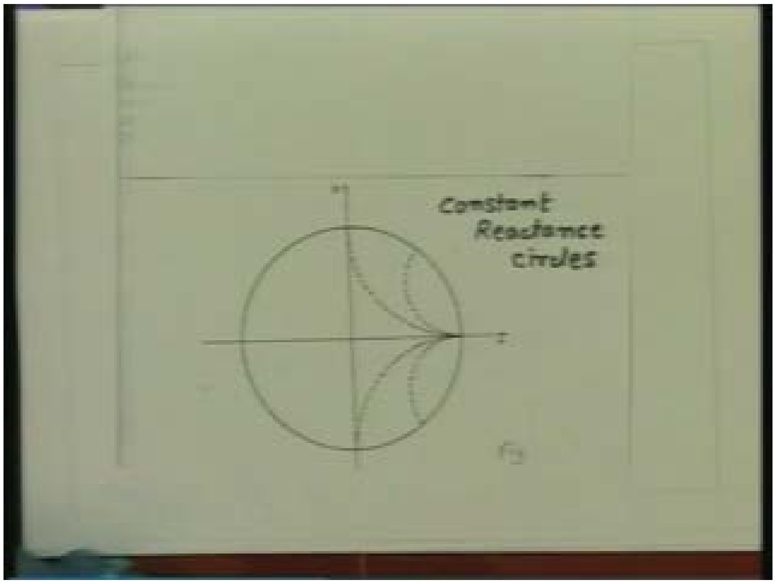
Similarly I can take negative values of x as I will get mirror image of these circles on the lower half. So I will get the set of curves which will look like that and again we take only those portions of the curve which lie within the unit circle because they are the one which will correspond to the passive loads.

(Refer Slide Time: 30:26 min)



If I take different values of x and plot this the curve essentially will look like that, this horizontal lines here corresponds to x = 0 except that this extreme point is on the unit circle over that point corresponds to x = ∞.

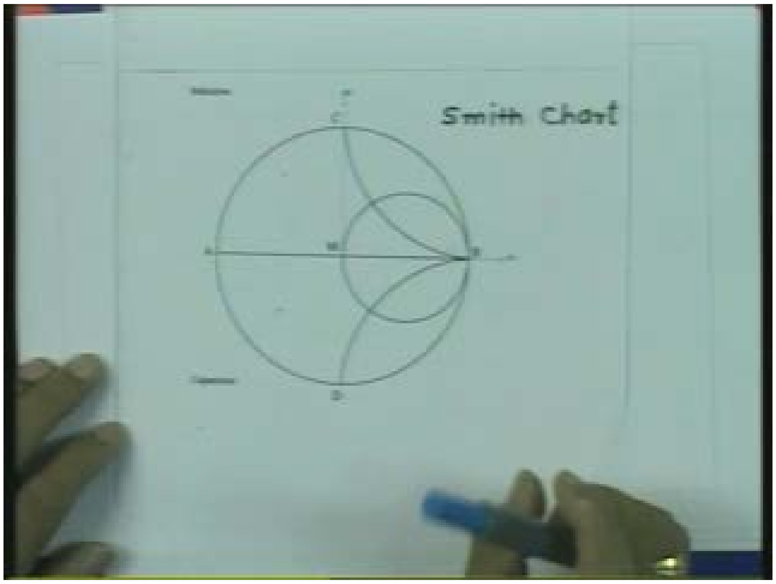
(Refer Slide Time: 31:04 min)



These circles are for higher values of x the upper half of this represents the positive value of x and the lower half of this represents the negative values of x that means if I am talking about reactances then upper half of this circle represents the inductive reactances and the lower half represents the capacitive reactances.

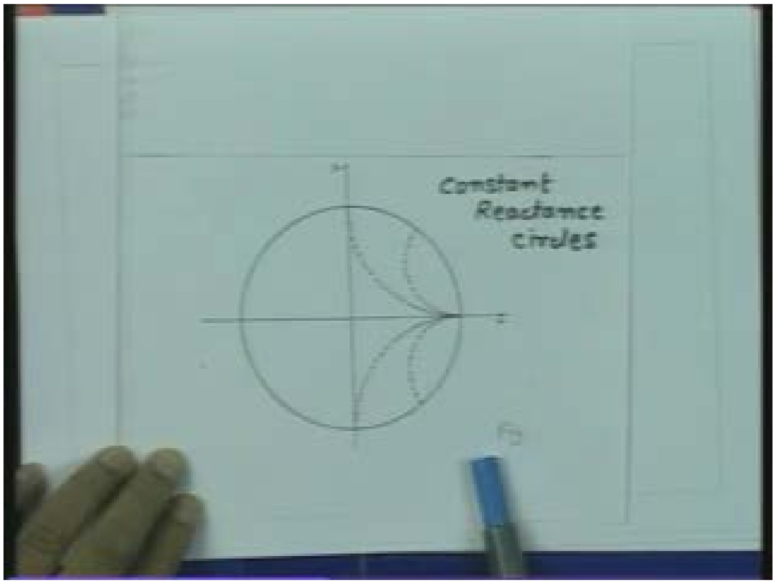
Now I have got two sets of circles, the constant resistance circle within the unity circle of the complex Γ-plane and I have got the constant reactance circle again plotted within the unity circle of the complex Γ-plane. Now if I want to identify the impedance point I can superimpose these two set of circles and essentially I have created a coordinate system for plotting the impedance point on the Γ-plane that is what essentially we do. We take these two sets of circles and superimpose within the unity circle of the complex Γ-plane. Just for the clarity we have drawn here only very few circles one is this outermost circle is the unity circle, this is the u axis, this is the v axis and then this circle corresponds to $r = 1$, these circles corresponds to $x = 1$ and the horizontal line here as we saw corresponds to $x = 0$, this point here is $x = \infty$, this point also represents $r = \infty$.

(Refer Slide Time: 32:41 min)



Now let us see the gauss characteristics of these two sets of circles, firstly all constant resistance circles have their centers lying on the positive real axis of the complex Γ-plane all the circles pass through the point u = 1, v = 0, as the value of r increases the center shifts towards right from the origin to u = 1 and the radius decreases so this point here corresponds to r = 0 or this circle corresponds to r = 0 and this point corresponds to r = ∞.

(Refer Slide Time: 33:41 min)

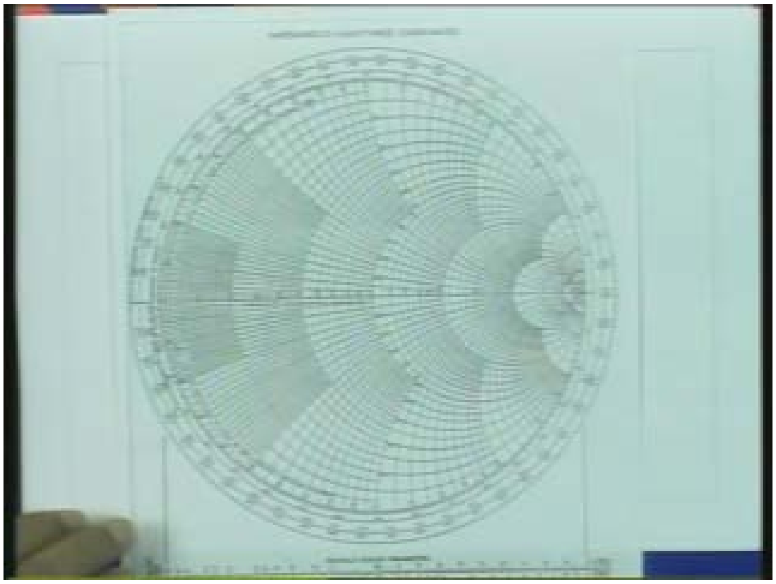


Similarly you summarize the characteristics of constant reactance circles. The constant reactance circle lie on the vertical line passing through v = 1 and the upper half of this circle represents the positive reactances and the lower half represents the negative reactances. The center of the circle approaches towards u axis and the radius of the circle goes on reducing as the reactance value increases so the size of the circle reduces and ultimately when $x = \infty$ the size of circle becomes equal to zero so this point again represent $x = \infty$.

Now the superposition of these two circles as I mentioned is the coordinate system for impedance on the complex Γ-plane that is what called the smith chart and smith chart is a very powerful tool for analyzing the Transmission Line problems. Essentially we have created a graphical tool where impedances are represented on the complex Γ-plane. And now without going back to the impedance plane I can directly mark the impedances on the complex Γ-plane or on the smith chart from these two sets of circles.

Normally this smith charts are readily available for calculations so one does not have to draw this smith charts. Having understood that the smith chart is the superposition of constant resistance and constant reactance circles we get a figure something like this, this is called a smith chart.

(Refer Slide Time: 35:20 min)

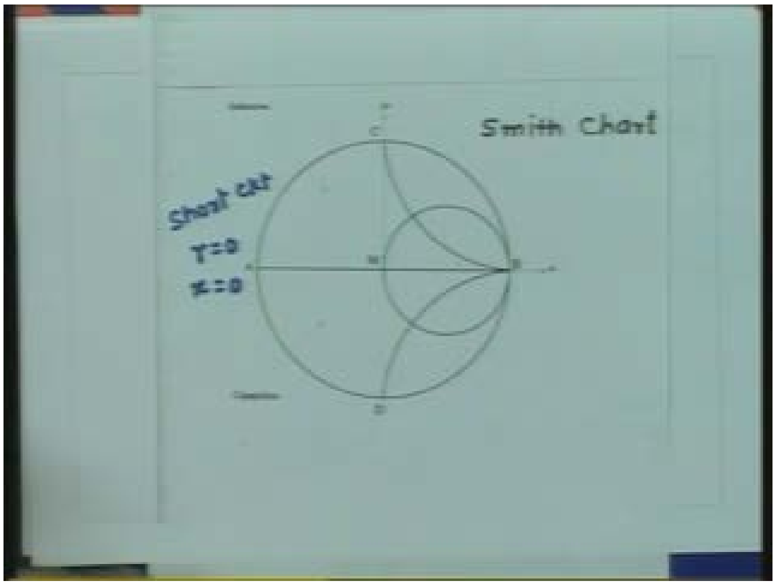


So as we can see here this circle is the unity circle, this is the real axis of the gamma which is u axis, the center here is the origin of the complex Γ-plane. These circles which are flaring like that, they are the constant reactance circle and this circle which are one within another represent the constant resistance. .

Now if I want to plot or mark a point an impedance on the complex Γ-plane this is a coordinate system which I am having now and this is orthogonal coordinate system so you can take the corresponding value of r and x and find the intersection of these two circles corresponding to that constant value r and the constant value x and the intersection point will represent an impedance for that value of r and x. so now every time without going into the transformation I can read from this itself the value of r and x and directly mark a impedance point on the complex Γ-plane. So this is an extremely powerful tool called a smith chart and now we will see this smith chart can be used for doing various types of Transmission Line calculations.

However before we go into this analysis of the Transmission Line let us first identify some special points on the smith chart. So let me take a simplified version of a smith chart only very a few circles just drawn for clarity the smith chart will look something like this and let me mark some very specific points as a landmark points on the smith chart. Let us say we are considering the impedances which are given by r + jx where r is a resistance and x is reactance. This point here is a, as we see any point lying on the outermost circle will represent r = 0, any point lying on this horizontal axis represents x = 0 so the intersection of these the horizontal axis and the unit circle which is the point a corresponds to r = 0, x = 0 so here we have r = 0,x = 0 and from impedance point of view when r = 0, x = 0 this load is nothing but the short circuit so on the smith chart this point a represents a short circuit point.

(Refer Slide Time: 38:35 min)



Similarly if I go to the other point here b as we saw the two sets of circle degenerate into a point and that is this point so this point corresponds to r = ∞, it also corresponds to x = ∞ so this point here is r = ∞, it is also x = ∞. That means the impedance is infinity so this point is nothing but the open circuit point, you have this circuit which is the open circuit point.

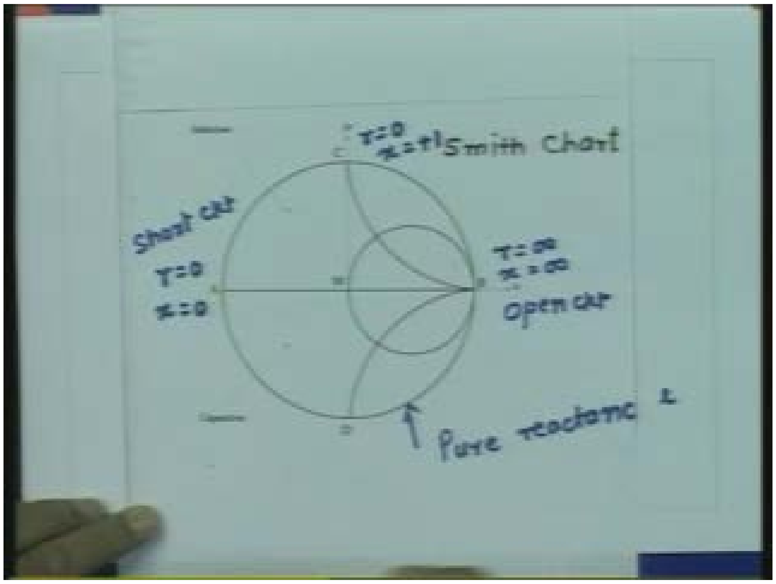
So the left most point on the smith chart corresponds to the short circuit load, the right most point on the smith chart corresponds to the open circuit load. All the points which are lying on the outermost circle represents pure reactances so any point on the outermost circle is pure reactance because for these points r = 0. You have points which are lying on the outermost circle for the x value changes but r is always equal to zero so they essentially represent the pure reactances.

As we saw the upper half of this circle corresponds to the positive value of x and positive value of x means inductive reactances. So the unity circle which is in the upper half corresponds to the purely inductive reactance or purely inductive loads. Similarly, the points which are lying on the lower half of this circle represent purely capacitive loads.

Now if I go inside this circle I have some value of r here but in the upper half of this circle the x value will be always positive that means all the points in the upper half of this circle represent the inductive loads and the points which are on the lower half of this circle represent the capacitive loads. So just looking at the point once the point is marked on the smith chart visually I can immediately tell whether the load is capacitive load or the load is the inductive load.

Again here we can take a special point this circle passes through the upper most point of the smith chart has a value x = 1 so this point here c corresponds to r = 0 because it is lying on the outermost circle for which r = 0 and its x value is equal to 1. So this point corresponds to r = 0, x = +1 so this point represents a pure reactance whose magnitude is equal to the characteristic impedance of the line.

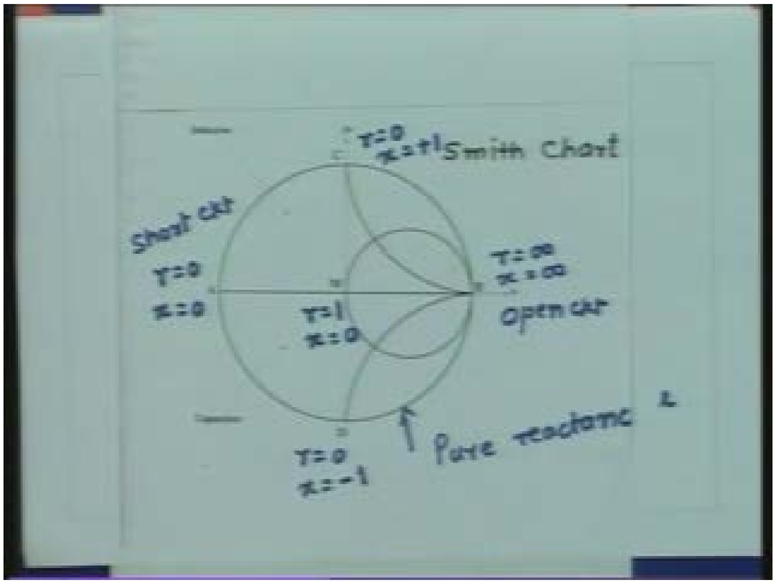
(Refer Slide Time: 41:54 min)



Similarly the lowermost point b corresponds to r = 0 and x = -1 so this point represents a purely capacitive reactance whose magnitude is equal to the characteristic impedance of the line.

There is one more special point and that is the center of the smith chart. The point m which is the origin of the smith chart or the origin of the complex Γ-plane is the intersection of the r equal to one circle and x equal to zero line. So this point here represents r = 1 and x = 0 that means at this location the impedance is equal to the characteristic impedance of the line and that point is of greatest interest to us because that point represent the matched condition on the Transmission Line.

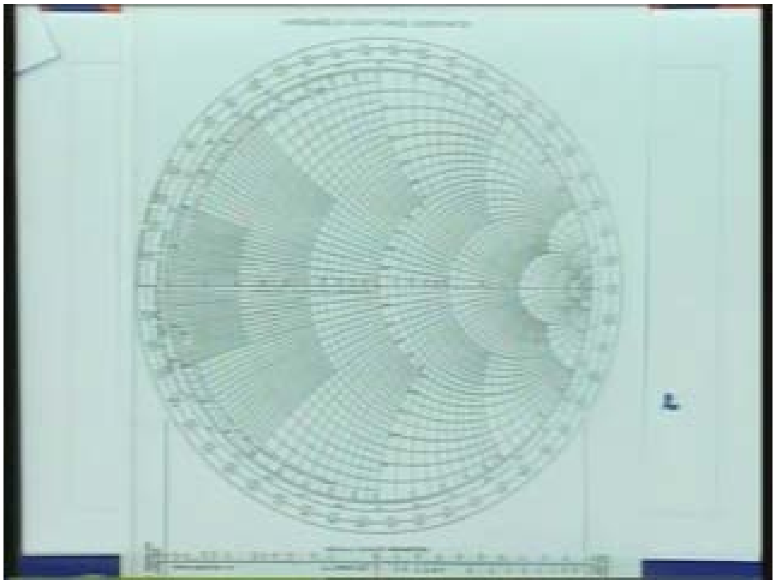
(Refer Slide Time: 43:07 min)



So when the impedance lies on the center of the smith chart which corresponds to the magnitude of the reflection coefficient zero then this point represent the matched condition on the Transmission Line.

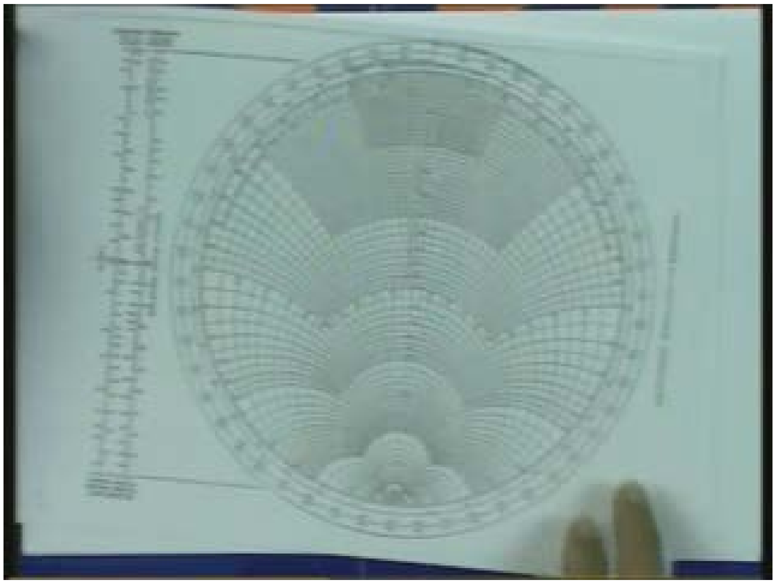
Once you understand the various specific points on the smith chart then we can go back and mark the reflection coefficient on this smith chart. So the smith chart now has given you the coordinate system which is in terms of r and x which is a very complex coordinate system but the complex reflection coefficient axis is not drawn on the smith chart it is understood that whenever we are having a smith chart the horizontal line passes through the center of smith chart is the real gamma axis where the vertical line is the imaginary gamma axis. Also it should be kept in mind that if I look at the smith chart properly since this is a complex Γ-plane it should always be held like this the most clustered portion of the circle should be on the right hand of the user because the real axis of complex Γ-plane is in this direction and the positive imaginary axis of complex Γ-plane is in vertical direction.

(Refer Slide Time: 44:37 min)

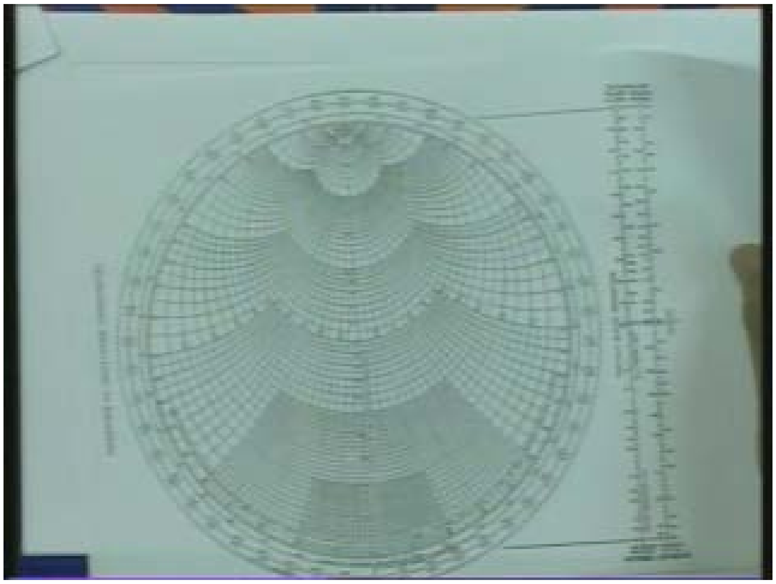


So we should not use the smith chart like this or we should not use the smith chart like this.

(Refer Slide Time: 44:39 min)

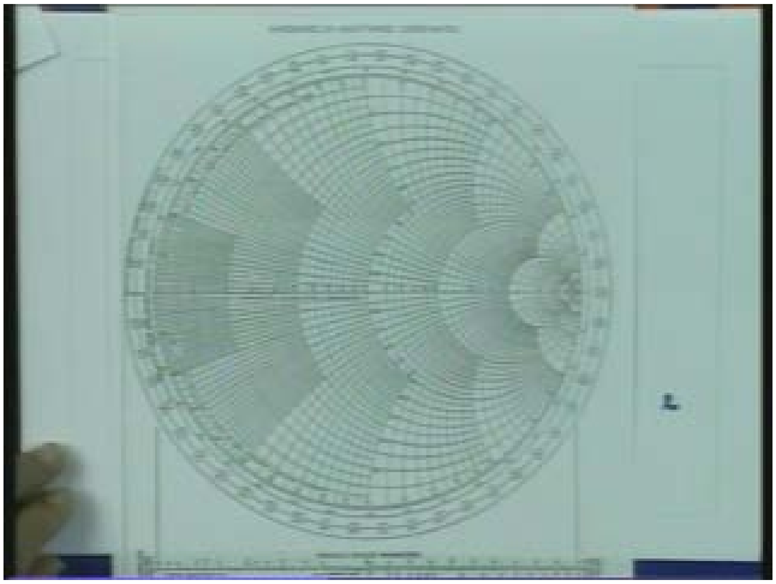


(Refer Slide Time: 44:41 min)



We should keep the smith chart like this so that the complex reflection coefficient axis is properly defined.

(Refer Slide Time: 44:50 min)



So normally on the smith chart the reflection coefficient axis is not drawn but the user must remember that the smith chart is nothing but the set of constant resistance and constant reactance circle drawn on the complex Γ-plane.

Once I get that then I can do simple calculations of conversion from reflection coefficient to the impedances or vice versa in a very simple manner. I can just mark a point or impedance on the smith chart and then treat this thing as a complex Γ-plane and find out the coordinates of that point in the complex Γ-plane which will give me the complex reflection coefficient. So the conversion of impedances to the complex reflection coefficient and vice versa has become extremely simple once the smith chart is available to the user.

So for analyzing the Transmission Line problem the smith chart is a very handy tool and we will see later that many complex impedance transformation problems can be solved by using the smith chart.

However, before we get into that let us try to look at not only the impedances but the admittances. Till now we have done the analysis assuming that the impedances are represented by the only resistance and reactive part. Many times when the Transmission Lines or impedances are connected in parallel it turns out to be more handy to do the calculation in terms of the admittances rather than in terms of impedances. So it will be worthwhile to first see how the smith chart gets modified in terms of admittances and then how do we do conversion from the impedances to the admittances and vice versa.

So let us quickly look at that instead of impedances I had admittances for the calculation then how the transformation between the reflection coefficient and the admittance will would go. Before we get into that however we have to define the characteristics of Transmission Line in terms of admittances so to start with you have to define the characteristic admittance of a Transmission Line then we have to take any arbitrary admittance then define the normalized admittance with respect to the characteristic impedance of the Transmission Line and then we can find out the graphical representation of the transmission line in terms of the normalized admittances.

So next time when we meet we first take the normalized admittances, find out the reflection coefficient in terms of normalized admittances establish relationship between the real and imaginary part of admittances and then get the smith chart and try to find out the relationship between the admittance smith chart and the impedance smith chart.

Thank you.