Transmission Lines and E. M. Waves
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Lecture – 60
Problems on Antenna and Radiation

In the last few lectures we developed the theory of radiation or antenna. In this session we solved some problems based on the theory which we have developed for radiation and antenna. So let us take a problem here.

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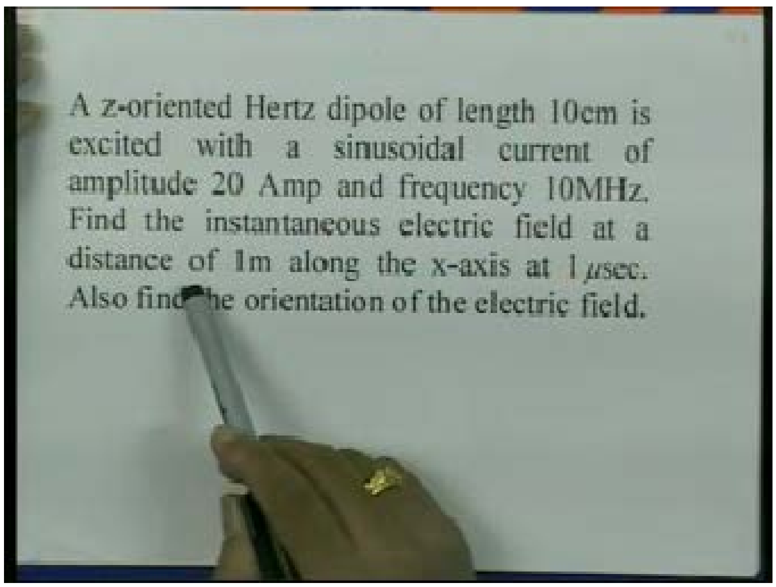

There is a z-oriented hertz dipole of length 10 centimeter excited with a sinusoidal current of amplitude 20 amperes and frequency 10 megahertz. Find the instantaneous electric field at a distance of 1 meter along the x axis at 1 microsecond. Also find the orientation of the electric field.

So this is a simple problem based on the theory which we have developed for the hertz dipole. So the problem which is given here is we have a hertz dipole, so let us write down the coordinate system. So let us say this is the direction is x, this is z, then the y will be oriented this way and we are have a hertz dipole here of length 10 centimeter that is 0.1 meters and the peak current is 20 amperes.

So we have here this is I into dℓ and then this Idℓ for this problem will be 20 into 0.1 that is equal to 2 amperes meter. So that is the current moment for for this dipole which is given here and then we require now for calculation of the electric field the quantity beta. So first we have to calculate the angular frequency omega then the radiation is going in the free space so the velocity is equal to velocity of light so we can get from there first the omega equal to 2pi into 10 megahertz so it is 10 to the power 6 so that is equal to 2pi into 10 to the power 7 radians per second and then the phase constant beta which is omega by c is equal to 2pi into 10 to the power 7 divided by 3 into 10 to the power 8 so that is equal to pi upon 15 radians per meter and the distance r is given as 1 meter from the antenna at a time which is 1 microsecond.

So here (Refer Slide Time: 4:44) we assume that at t equal to 0 the current is maximum in this hertz dipole and it will be flowing upwards so we can take the current variation as cosine of omega t and then we can calculate what will be the field after a time of 1 microsecond at a distance of 1 meter from the hertz dipole.

So once you get beta then we can get beta r that is equal to pi upon 15 into r which is 1 meter so that will be equal to pi upon 15 radians.

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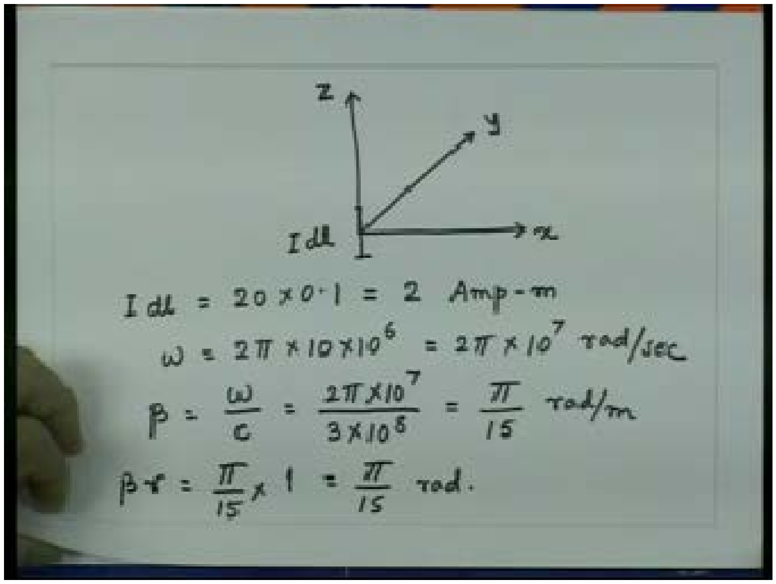

And we can just get this quantity beta t omega t that is equal to 2pi into 10 to the power 7 into 1 microsecond which is 10 to the power minus 6 so that is equal to 20pi radians.

So now we got all the basic quantities required for calculation of the electric field and in general we have the electric field which has three components: r, theta and phi and in this case we are looking for in the direction which is x axis so we want to find out the field in this direction. So for this the theta which is measured from the z axis that angle theta is equal to pi by 2. The r direction which is going outwards in this direction so that is same as r direction also so essentially we have the electric field E x which is equal to E r. The theta direction is going downward like that so in this case the theta component will be opposite to the z z axis so we have from here the theta component e theta equal to minus E z that is the way the E theta component will be oriented.

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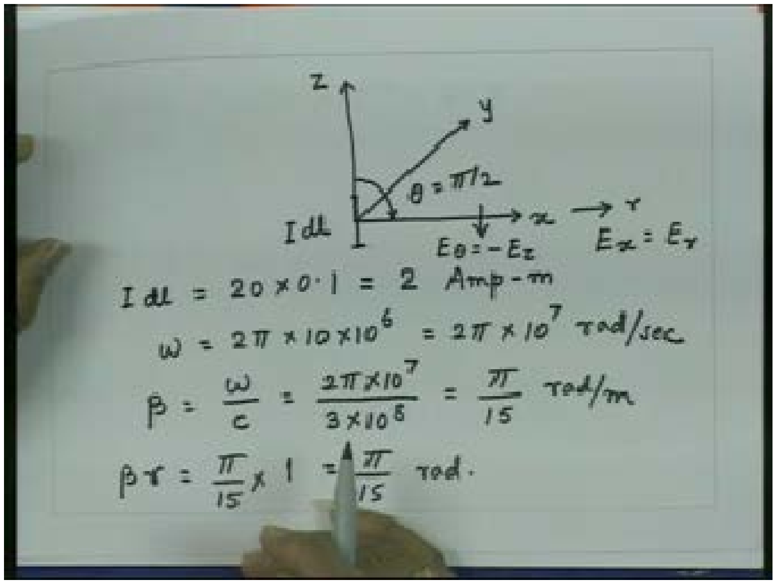

So now we can calculate using the expression which we have derived for the total field from the hertz dipole; we can get E r that is same as E x in this case which is equal to I 0, the I 0 is the this current that is same as this (Refer Slide Time: 7:17) so you got I 0 dℓ cos of theta divided by 4pi omega epsilon into beta upon r square minus j upon r cube and in this case since theta for this point which is 1 meter away along the x axis theta equal to pi by 2 so this cos theta will be equal to 0 so in this case E r equal to E x that will be equal to 0.

The theta component for the electric field E theta which is same as minus E z that is equal to I 0 into dℓ into sin theta upon 4pi epsilon and we want to find out now the electric field at some instant of time so we take the real part of e to the power j omega t minus j beta r multiplied by j beta square upon omega r plus beta upon omega r square minus j upon omega r cube. These are the expressions which you already derived when you were analyzing the hertz dipole (Refer Slide Time: 9:22). So now we can substitute different values here. The I 0 dℓ we have got these two, the beta r this pi upon 15, the omega t is 20pi. So if you substitute all these quantities in this and epsilon equal to epsilon 0 for the free space you get E theta which will be equal to 0.1932 volts per meter. And as we know for the hertz dipole the phi component of the electric field is 0 so we get E phi equal to 0.

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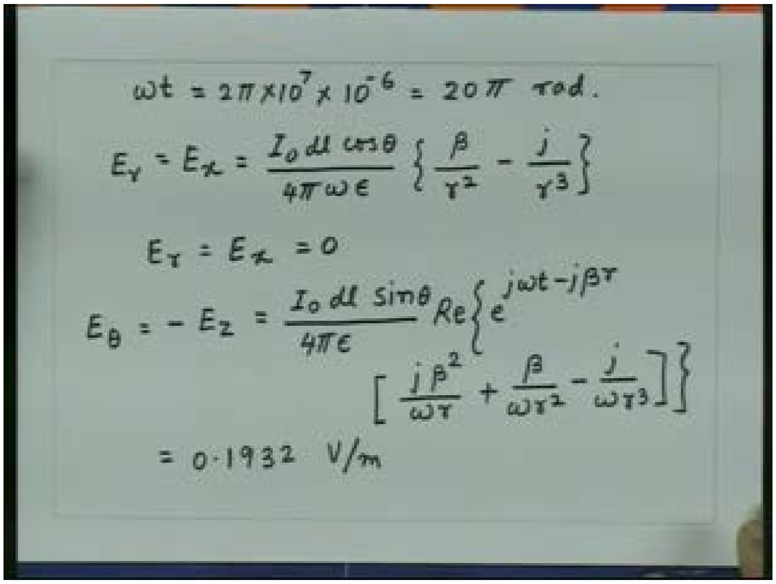

So in this case (Refer Slide Time: 10:13) if you move along the x axis then the r component is 0, the theta component which is same as the minus z component and the y component also is 0 which will be perpendicular to the plane of the paper. So by using the expression which we have derived now we can find out the total field at a distance and at some instant of time from the hertz dipole. Let us take the next problem.

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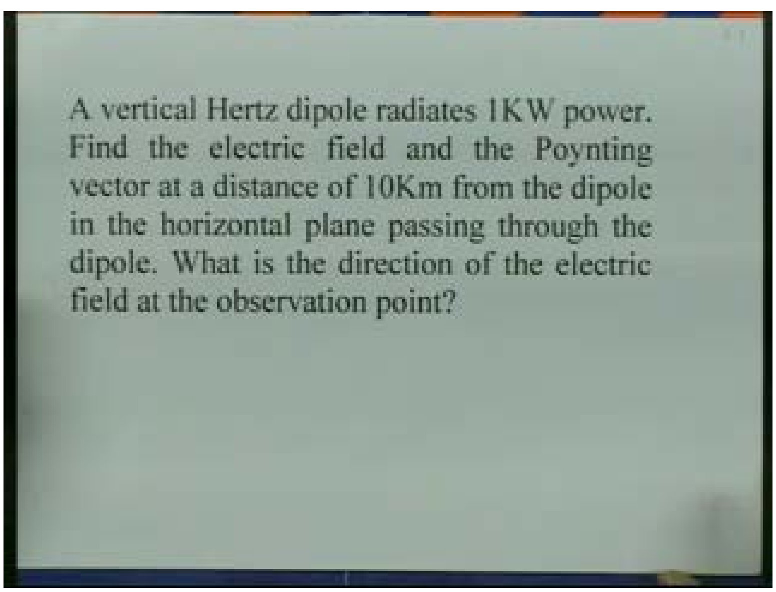

A vertical hertz dipole radiates 1 kilowatt of power. Find the electric field and the Poynting vector at a distance of 10 kilometer from the dipole in the horizontal plane passing through the dipole. What is the direction of the electric field at the observation point?

So in this case the total power which is radiated by the dipole is given and we have to find out the electric field at a distance of 10 kilometer. So the problem is we have the hertz dipole here, the horizontal plane which is passing through this which will be this so at a distance of 10 kilometer in this direction we have to find out the electric field.

As we know again the electric field here... if I take the standard coordinate system this is the z axis and if I take a plane which is perpendicular to the dipole then we have theta component which will be oriented like this (Refer Slide Time: 11:55) so the electric field is vertical at this location; and now by using the relation which we have derived we can find out what is the electric field at this location.

So first what we do we already derived the expression for the total power radiated by the hertz dipole so we know that the power radiated w is 1 kilowatt that is what is given that is equal to 40pi square I 0 square dℓ upon lambda whole square. now from here we can calculate this quantity I 0 dℓ upon lambda so you can invert this you get I 0 dℓ upon lambda that is equal to 1 kilowatt which is 10 to the power 3 watts divided by 40pi square square root so that is equal to 5 upon pi.

Now at this location the theta... this is z axis so theta in this case is again 90 degrees, this is pi by 2 so we can get the electric field magnitude of the electric field that will be equal to I 0 dℓ beta square upon 4pi epsilon omega into r. The omega upon beta we can separate out so this we can write as I 0 dℓ upon 4pi epsilon, we can write here beta upon omega into beta and omega upon beta is nothing but 1 upon velocity of light and this is 2i upon lambda so we can get this equal to I 0 dℓ upon 4pi epsilon 1 upon velocity of light multiplied by 2pi by lambda for beta.

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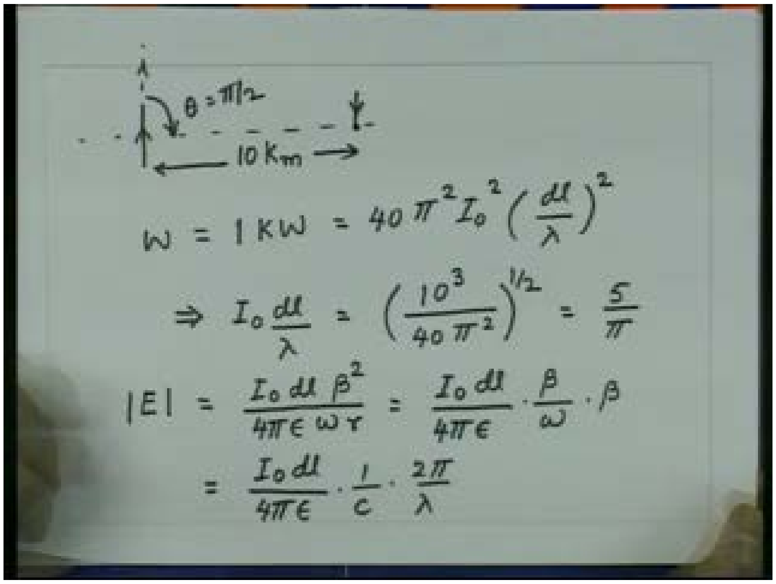

So, substituting now for this, we get mod of electric field that will be equal to 1 upon 2pi r into c into I 0 dℓ upon lambda. Now we have already derived the value for I 0 dℓ upon lambda which is 5 upon pi so if I substitute that I get the mode of electric field that is equal to 30 millivolts per meter.

The next thing which we have to find out in this problem is the horizontal plane passing through this. We want to know the electric field and the Poynting vector at a distance of 10 kilometer. so we found the electric field, now we have to find out what is the Poynting vector at that location so we know that Poynting vector P the magnitude of that that is equal to mod E square upon eta so that is equal to 30 into 10 the power minus 3 per millivolt square upon eta which is 120pi. So if I simplify this I get the power density which is 3 into 10 to the power minus 4 upon 40pi watts per meter square.

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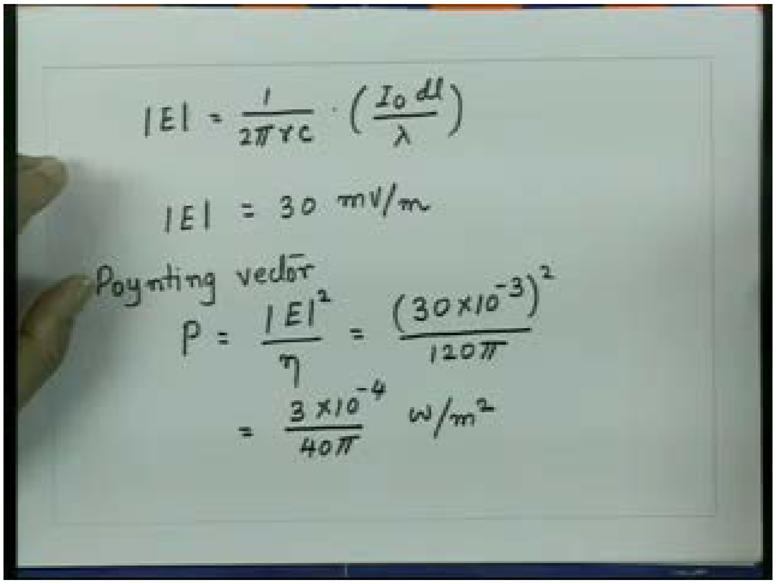

So this is again a very simple problem where the total power radiated by the hertz dipole is given and then we are asked to find out what is the electric field and the Poynting vector at a distance of 10 kilometer from the hertz dipole.

Let us now take some problem based on the dipole which is having a finite length. As we know the hertz dipole is infinitesimally small, the size is much much smaller than lambda whereas in practice we require dipoles which have substantial length.

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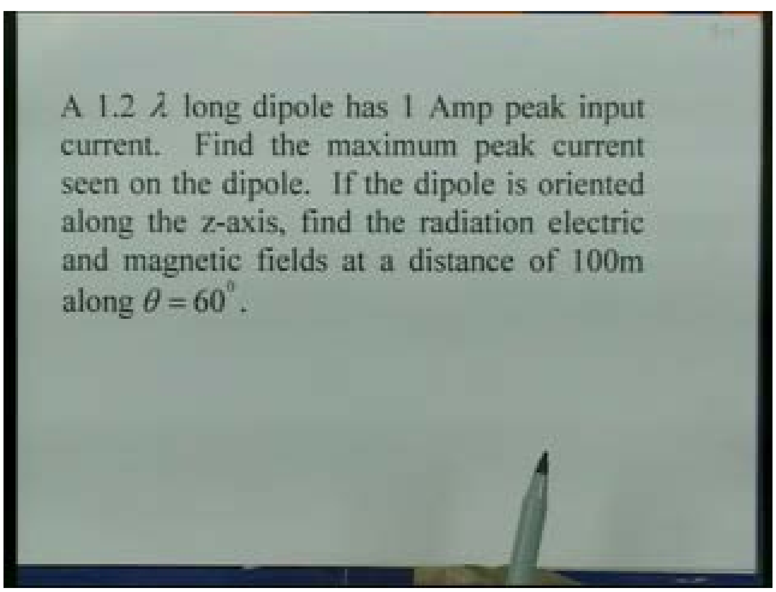

So in this case the problem is the 1.2 lambda long dipole has 1 Ampere peak input current. Find the maximum peak current seen on the dipole. If the dipole is oriented along the z axis find the radiation electric field and the magnetic field at a distance of 100 meter along theta equal to 60 degrees.

So, first of all we have to get the expression for the... field for the hertz the this dipole which is having a length of 1.2 lambda and then from there we can essentially calculate, substituting the parameter the field at an angle theta equal to 60 degrees at a distance of 100 meters.

Now as we know the dipole has a length of 1.2 lambda so h which is half this length for the dipole this we call as H so H is equal to total length of the dipole which is 1.2 lambda divided by 2 so that is equal to 0.6 lambda so this plane is little more than lambda by 2 so we get a current distribution which will look like that and that is the current which is the maximum current which we are going to see as I m and this will be the input current which will correspond to this value.

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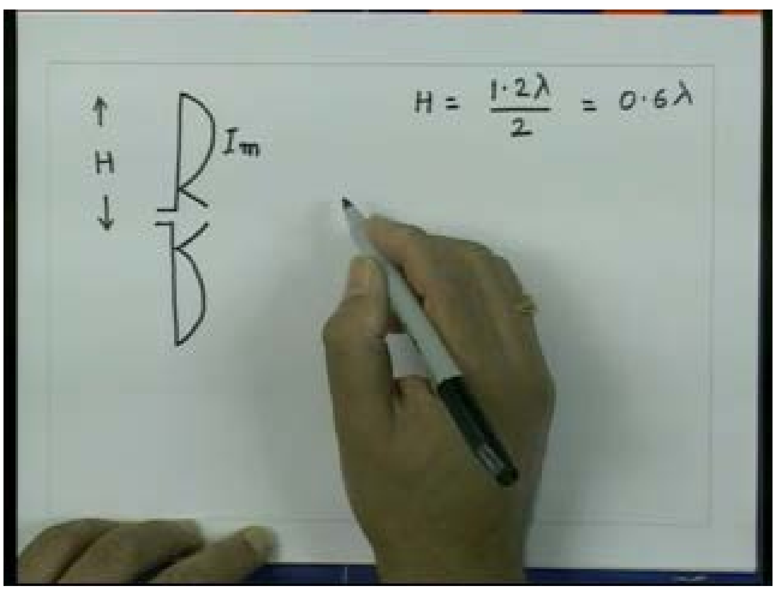

So now as we know the current I input current at this location so that is I in that is equal to I m sin of beta H and this current is given as 1 Ampere the input current peak current is 1 Ampere so this quantity is 1 Ampere that is equal to I m sin of beta which is 2pi by lambda into 0.6 lambda. So this quantity is 1.2pi, you will get I m is equal to sin of 1.2pi and the I maximum will be equal to 1 upon this quantity which will give 1 upon sin of 1.2pi which is equal to 1.7 Amperes.

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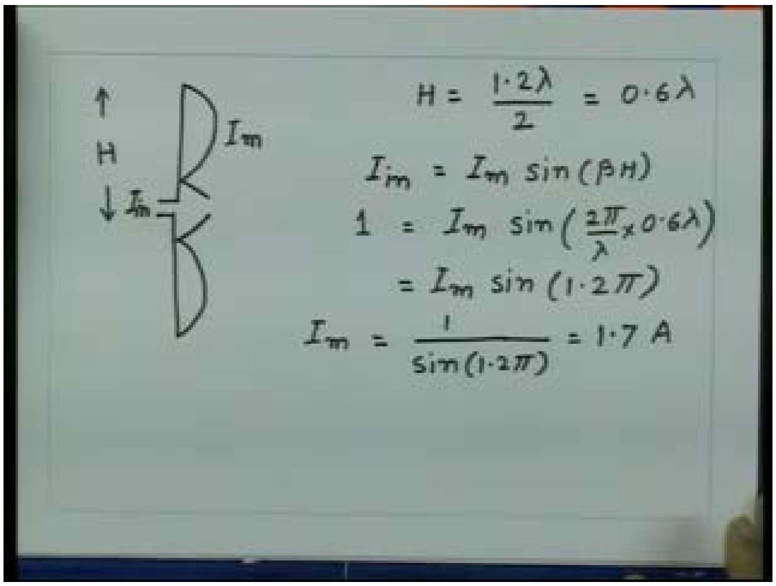

Once we get this quantity I m then we know that this dipole for the radiation field that is only theta component E theta and that is equal to 60 I m upon r where r is the distance measured from the center of the dipole, cosine of beta h cos theta minus cos of beta h upon sin of theta.

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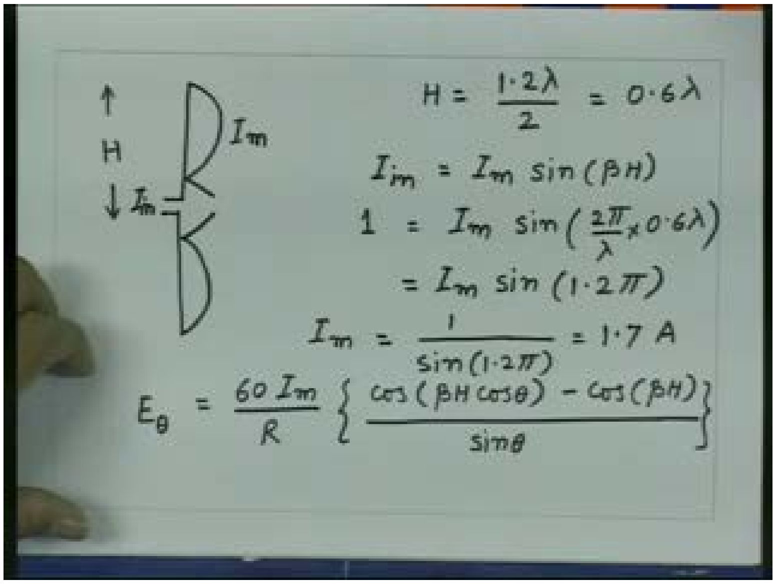

So now the distance is given as 100 meters, the theta is 60 degrees so we can substitute now for these parameters here. So here we are given that R is equal to 100 meters theta is equal to 60 degrees that is equal to pi by 3 so the theta component of the electric field will be equal to 60 I m which is 1.7 Amperes divided by 100 meters into cos of 1.2 pi that is beta H multiplied by cos theta which is 1 by 2 so this gives cos theta equal to 1 upon 2 so we can get here which is beta H cos theta that will be 0.6pi minus cos of 1.2pi upon sin of pi by 6 pi by 3 that is equal to root 3 by 2.

So, simplifying this we can get the value for the electric field that is approximately equal to 0.09 volts per meter. Once I know the electric field then I can find out the magnetic field; since I know the relation between the electric and magnetic field that is H phi for the radiation field which is equal to E theta upon the intrinsic impedance of the medium which is 120pi so this will be H phi will be equal to 0.09 upon 120pi that is equal to 0.24 milli Amperes per meter.

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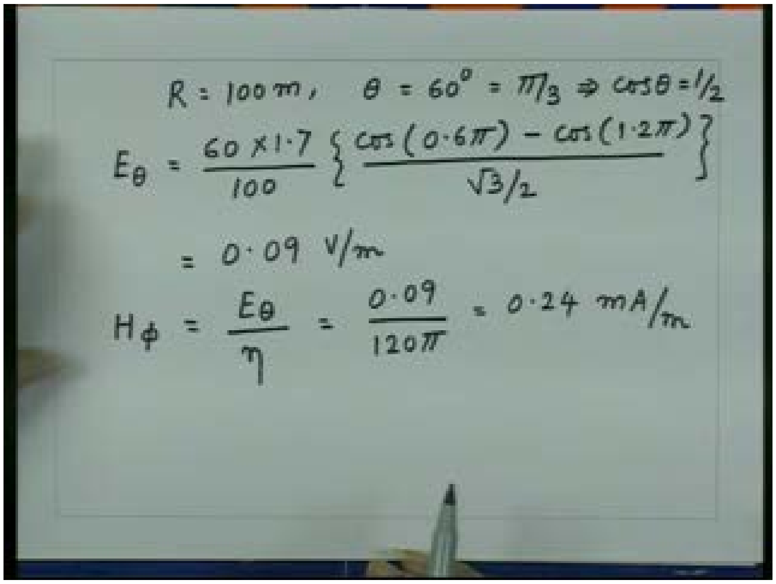

So in this case we were given the finite length of the dipole which is 1.2 lambda then the input current is given and then we are supposed to find out what is the maximum current which we will see on the dipole and also the electric field at an angle theta equal to 60 degrees so this is 100 meters and this angle is 60 degrees; at this point we were supposed to find the electric field and magnetic field (Refer Slide Time: 23:42).

So that is what we did which gives the expression for E theta, substitute these values then we get the value for E theta; once we get E theta then we can find out H phi which is E theta upon the intrinsic impedance at the medium.

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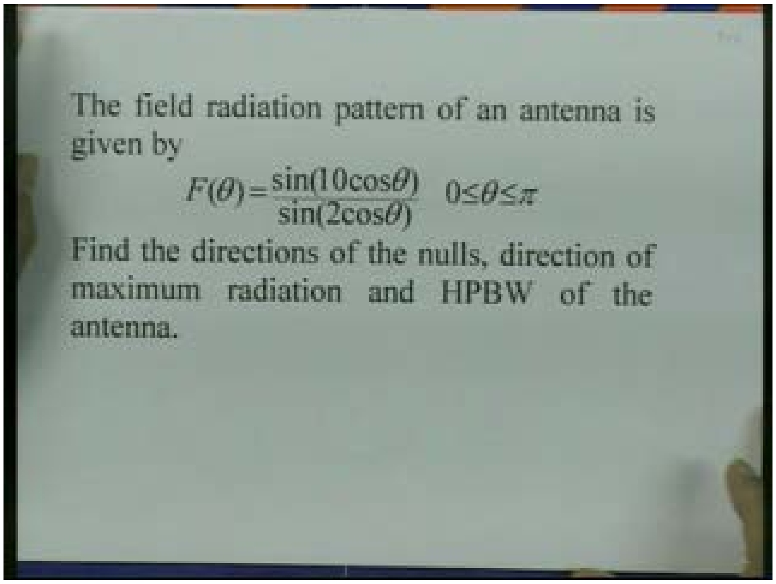

Let us now take a problem which is a general problem for the antenna and that is the field variation pattern of an antenna is given by F theta which is equal to sin of tan cos theta divided by the sin of two cos theta where theta varies from 0 to pi. Find the directions of the null, direction of the maximum radiation and also the half power beam width of the antenna.

So without specifying what kind of antenna this is essentially the radiation pattern directly has been given and we are we have to find out now the direction of the null, direction of maximum radiation and the half power beam width.

Now as we know the nulls correspond to f theta equal to 0 so if I take this function and equate that to 0 you will get the directions thetas for which the field will go to 0. So the nulls of this antenna would correspond to f theta is equal to 0 that is sin of 10 cos of theta that should be equal to 0 provided the denominator is not equal to 0.

So for those angle for which this quantity (Refer Slide Time: 25:35) goes to 0 that will not represent a null but otherwise for all those values of theta for which the numerator goes to 0, you will get a null for the radiation pattern. So provided sin of 2 cos theta is not equal to 0.

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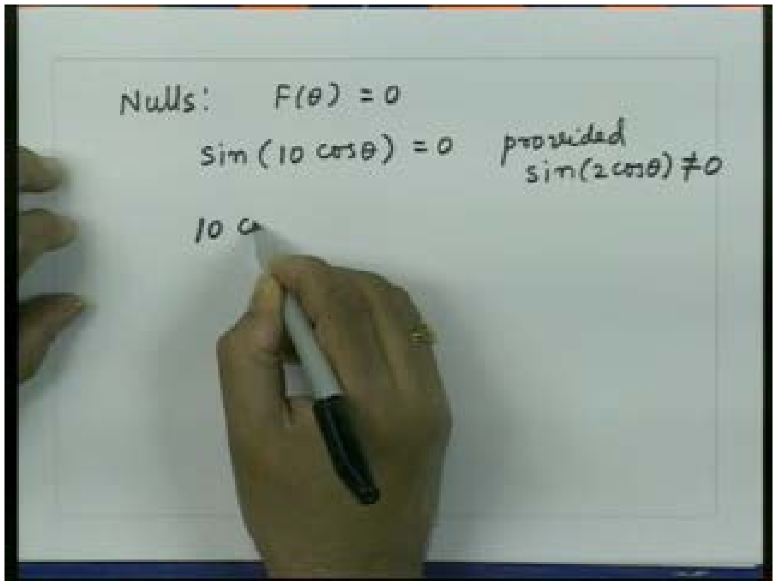

So from here essentially we get 10 cos of theta that will be equal to plus minus m pi where m is an integer. So m is equal to 1, 2, 3 and so on. So we get cos theta null that is equal to plus minus m pi upon 10.

Now, since cos of this angle the magnitude of that should be always less than or equal to unity essentially that that means this quantity m pi upon 10 should be less than or equal to 1. So we have a finite number of angles because we have this quantity should be less than or equal to 1 so we get nulls only for those values of m for which m pi upon 10 is less than 1. So we get nulls only for values of m which satisfy m pi upon 10 less than or equal to 1.

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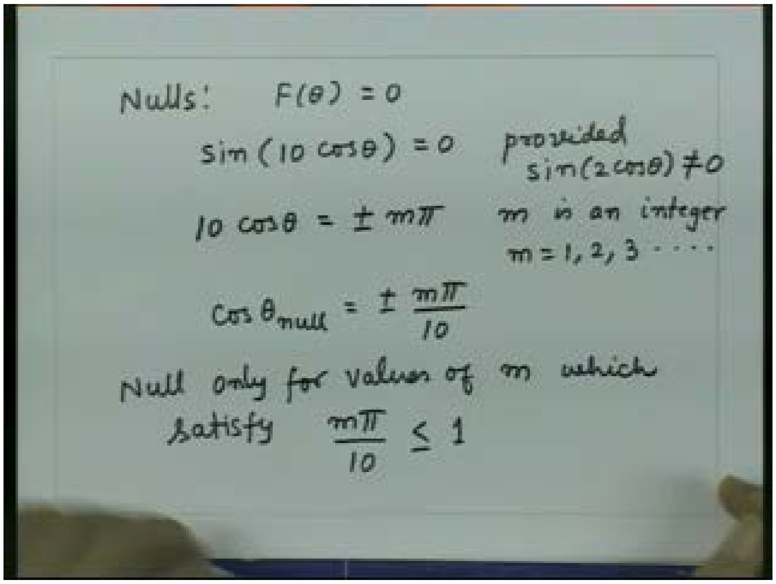

That means this quantity here m can go only 1, 2, 3 and if m becomes 4 then this quantity is greater than 1. So essentially the possible values of m we we get m equal to 1, 2 and 3. So substituting now this value for m equal to 1, 2, 3 we get the the nulls that would correspond to eta null that will be cos inverse of plus minus pi upon 10 cos inverse 2pi upon 10 and cos inverse 3pi upon 10 and as it is given theta lies in the range 0 to pi so from here we can find out the corresponding values of theta so we get theta nulls that is equal to 19.53 degrees, 51.07 degree, 71.69 degree, 108.31 degree, 128.93 degrees and 160.47 degree. So the radiation pattern has six nulls in the range theta equal to 0 to pi.

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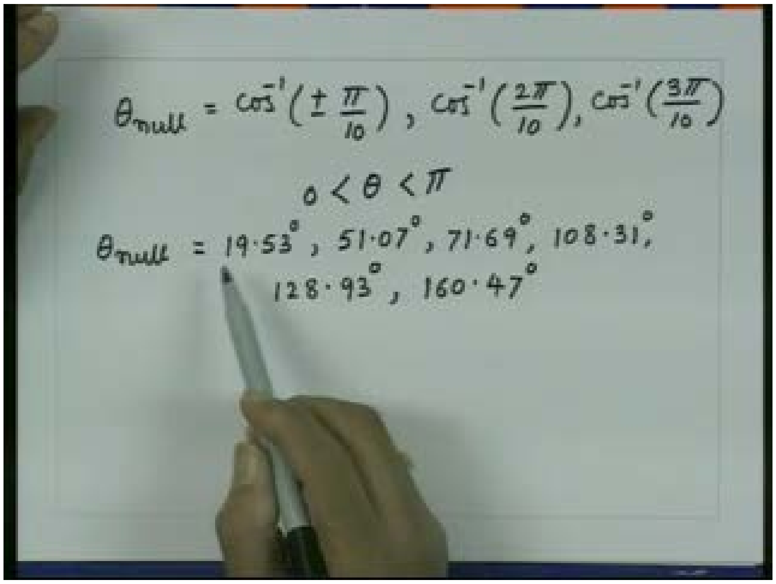

The angle corresponding to theta equal to pi by u pi by 2 the denominator of F theta also goes to 0 making theta equal to pi by 2 as a direction of maximum radiation. So if I look at here, if I put theta equal to pi by 2 this quantity is 0, this quantity is 0; theta equal to pi by 2 this quantity is 0. So we may get a null corresponding to theta equal to pi by 2 here for that does not represent a null because for that a denominator also goes to 0 and then that direction essentially is the direction of maximum radiation. So we get direction of maximum radiation that is theta max is equal to pi by 2.

Now to find out the half power beam width of the antenna essentially you have to take this field expression, find out what is the magnitude maximum magnitude which is going to get for this field then you find out the angle for which the field will reduce to 1 over root 2 of its maximum values and the angular difference between the these two angles essentially will give half power beam width.

So in this case we have the beam if I measure let us say theta going from 0 to pi then we have a beam which looks like that which is theta equal to 0, this is theta equal to pi and we have a direction of maximum radiation that is theta equal to pi by 2.

So we have to find out now the half power beam width. That means we have to find out these two angles: (Refer Slide Time: 31:38) this is angle theta 1, this is angle theta 2 at which the field reduces to 1 over root 2 of which is this maximum value.

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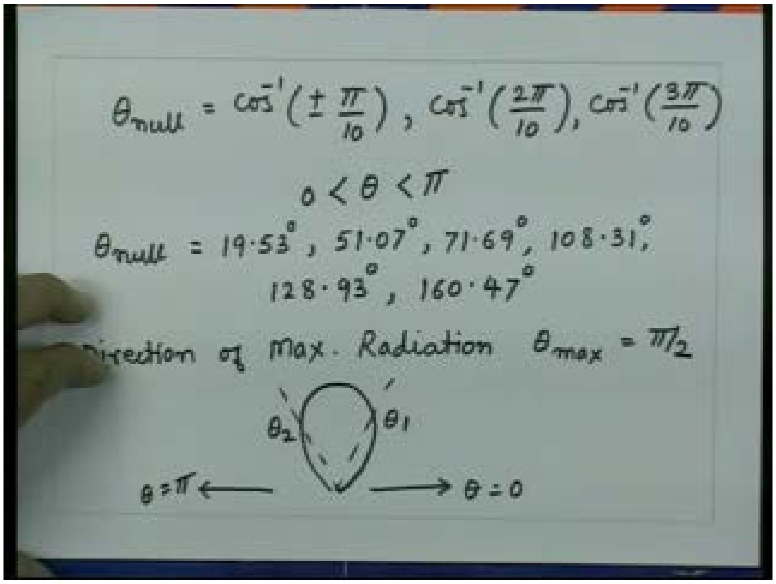

First we have to find out the maximum value of this expression (Refer Slide Time: 31:50) and since that theta equal to pi by 2 this is 0 by 0 so essentially you have to calculate this by taking a limit when theta tends to pi by 2. So for this we can expand this... so for maximum value of the function F theta maximum that would be the limit when theta tends to pi by 2 of this expression which is sin of 10 cos theta divided by sin of 2 cos theta. Expanding this sin function we get this is limit theta tends to pi by 2 10 cos theta minus 10 cos theta cube upon factorial 3 plus and so on. This is the expansion of the sin function so here you get 2 cos theta minus 2 cos theta whole cube upon factorial 3 plus and so on.

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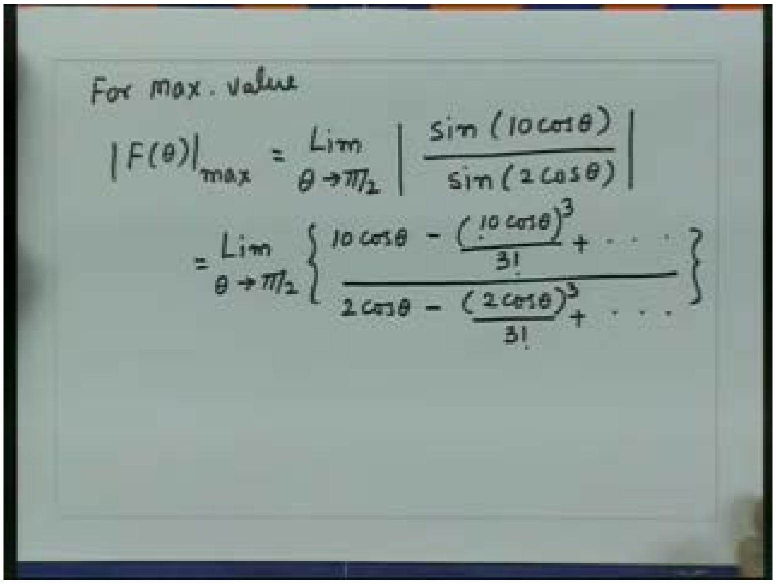

Now, taking this cos theta common from numerator and denominator and then when theta tends to 0 cos theta will... theta tends to pi by 2 cos theta will tend to 0 so all these terms will go to zero except this term which will give the value equal to 5. So the maximum value of the radiation pattern is 5. So, for half power points mod of f theta should be equal to 5 upon root 2. So in this you have the maximum value, this is equal to 5 the radiation pattern (Refer Slide Time: 34:29) this point now is 5 upon root 2, this is 5upon root 2.

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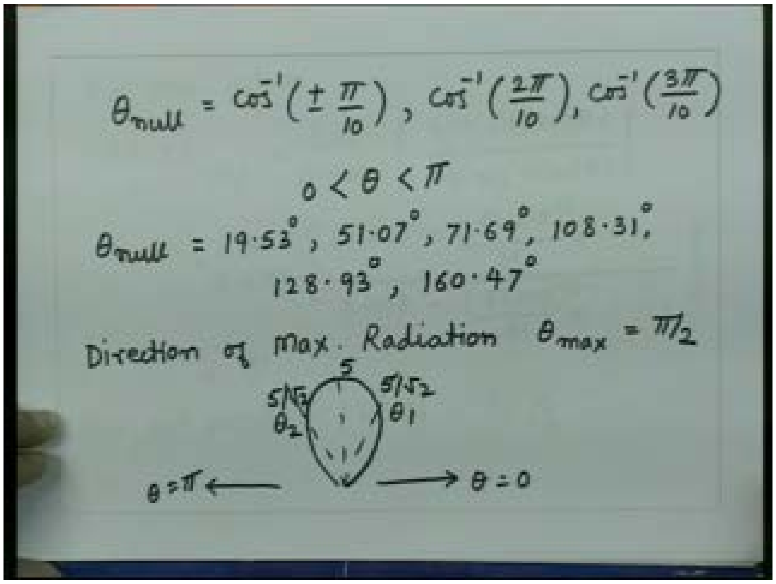

Now this thing cannot be solved analytically. This expression should be now equated to 1 upon root 2, 5 upon root 2 and this thing has to be solved numerically. So we have to get the values for these two angles theta 1 and theta 2 essentially by solving that expression numerically.

So we solve numerically the expression sin of 10 cos theta upon sin of 2 cos theta that is equal to 5 upon root 2 and there we get the value two angles theta 1 that is equal to 82.08 degrees and theta 2 is equal to 97.93 degrees. So the half power beam width for this antenna is equal to theta 2 minus theta 1 that is approximately equal to 16 degrees.

So, depending upon the radiation pattern one can solve the half power beam width analytically or if the expression is non-analytically solvable then essentially you find out the angles at which the field reduces to root 2 of its maximum value by using numerical techniques and then one can find out the half power beam width of this antenna.

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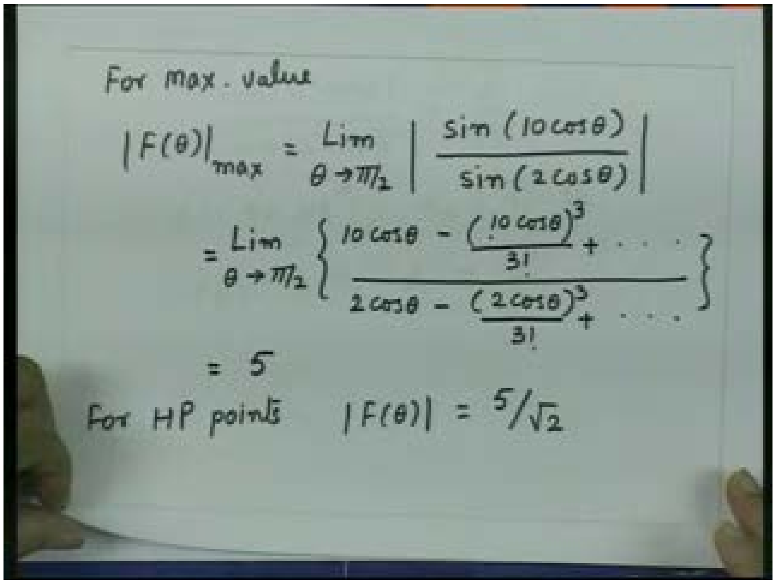

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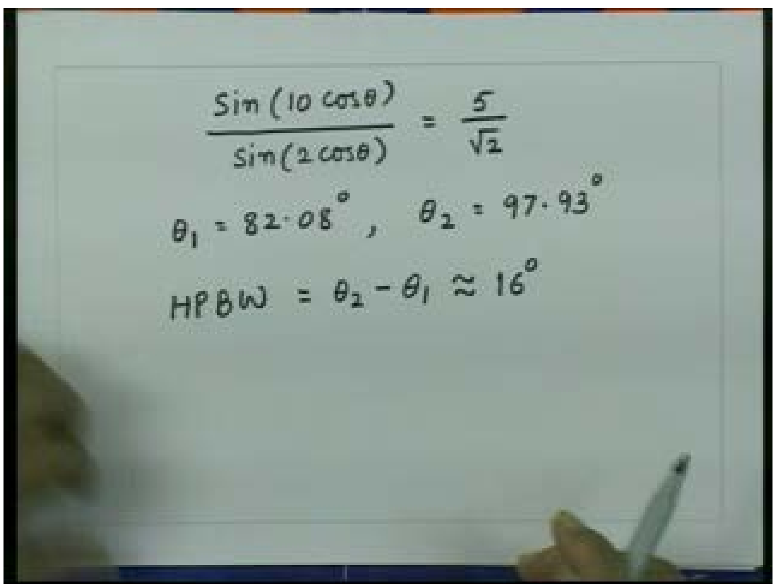

Let us take one more problem which is now related to the directivity of the antenna.

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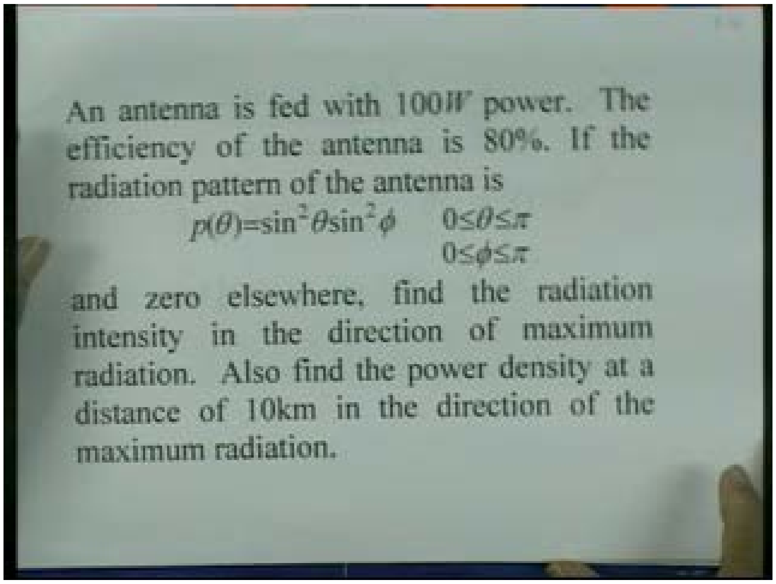

There is an antenna which is fed with 100 watt of power. The efficiency of the antenna 80percent. if the radiation pattern of the antenna is given as p theta equal to sin square theta sin square phi where theta varies from 0 to pi and phi varies from 0 to pi and 0 elsewhere find the radiation intensity in the direction of maximum radiation. Also find the power density at a distance of 10 kilometer in the direction of maximum radiation.

So this problem, there are various points to be noted. Firstly the antenna is not radiating full power that the efficiency which is 80 percent that means whatever power is input to the antenna only 80 percent of that power is radiated in the space. Secondly, since we have a radiation pattern which is given by this the power is not uniformly distributed in all directions so the radiation intensity in the direction of maximum is increased compared to the isotropic antenna and that increase essentially is related to the directivity of the antenna. So we have to calculate the directivity of the antenna, we can calculate the radiation intensity for an isotropic antenna once we know the power radiated by the antenna and then from there we can calculate the electric field or the power density at a given distance.

So first we wrote power radiated W that is equal to power input multiplied by the antenna efficiency. So it is given the input power to the antenna is 100 watts and the efficiency of the antenna is 80 percent that is 0.8 so the total radiated power is 80 watts.

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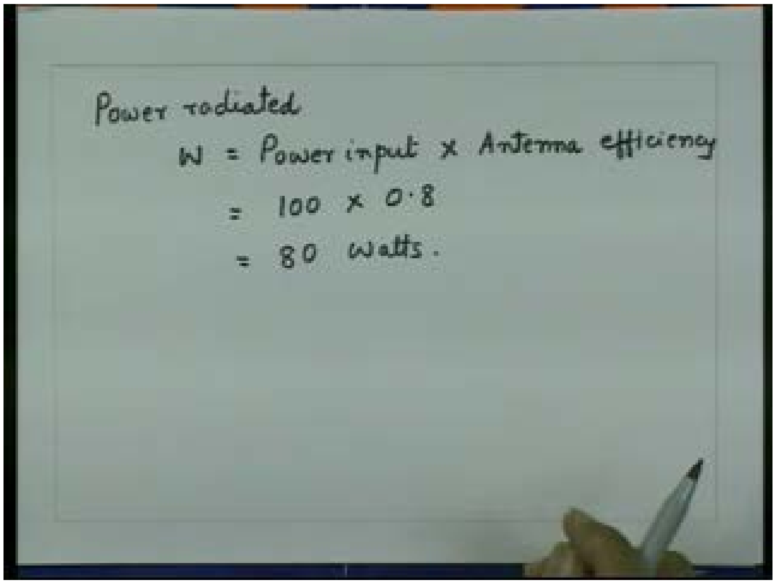

Now, if this power is given to the isotropic antenna, then it will create uniform distribution of the power and the radiation intensity will be, if I distribute this power uniformly over 4pi solid angle then the power per unit solid angle is the radiation intensity. So in this case essentially we get the radiation intensity for an isotropic antenna and that radiation intensity we call as the average radiation intensity so that U average is equal to 80 divided by 4pi solid angle 4pi watts per st radian.

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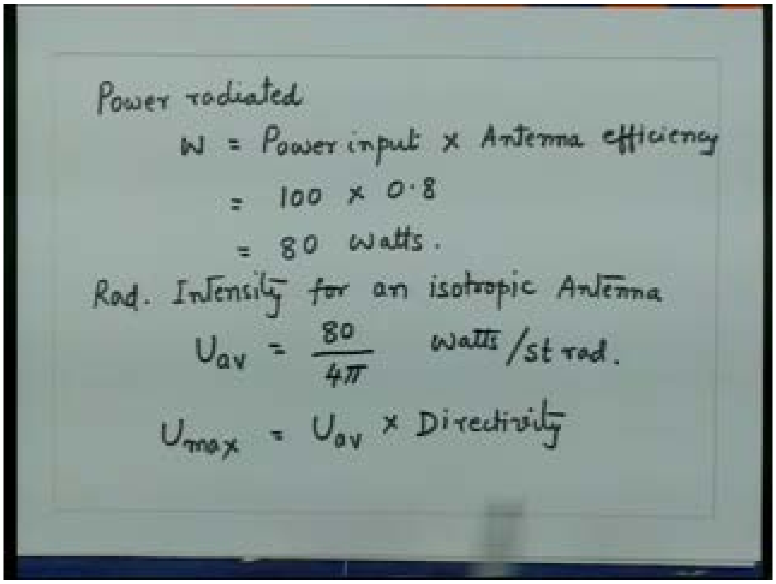

Now we know the maximum radiation intensity which we can get from the antenna is the average radiation intensity multiplied by the directivity of the antenna so U maximum that is equal to U average multiplied by the directivity of the antenna.

So now to find out the maximum radiation intensity first we have to find out the directivity of the antenna from the radiation pattern which is given here. And note here the radiation pattern which is given here is P theta that is the radiation pattern straightaway for the power.

So now we can go and use this radiation pattern to find the directivity the directivity of an antenna which is equal to 4pi upon the power pattern p theta integrated over 4pi solid angle so this we can write down explicitly in terms of the spherical coordinates and this is 4pi divided by pi which varies... the radiation pattern is nonzero only over this range of pi which is 0 to pi and the same is to for theta that the radiation pattern is nonzero only over this range which is 0 to pi so we have a limit here for phi which is equal to 0 to pi, for theta also we have limit 0 to pi and the power radiation pattern is given by theta sine square phi so this is sin square theta sin square phi and the incremental solid angle is sin theta d theta d phi.

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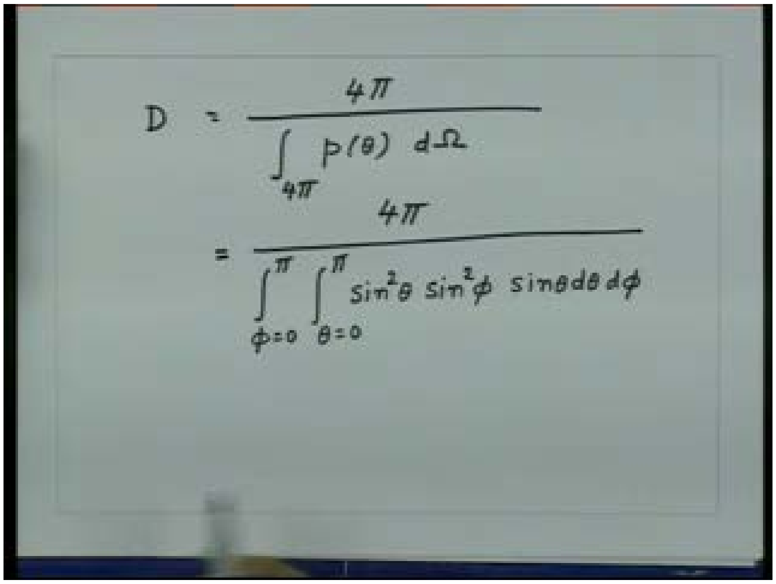

So this will give us 4pi divided by 0 to pi for phi because this sin square phi d phi 0 to pi sin cube theta d theta.

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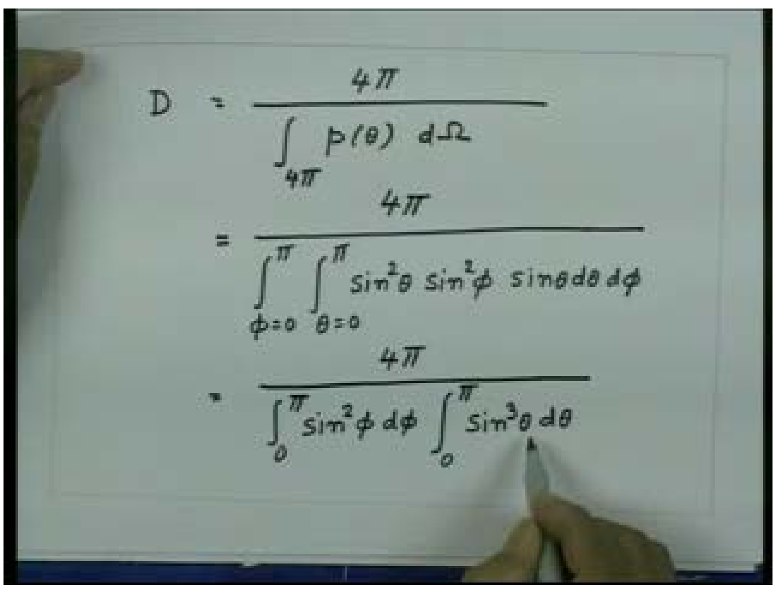

This integral we can solve by expanding this in cosine of 2 phi and this one we can write as 1 minus cos square theta and sin theta so these two integrations can be written as... this is 4pi from 0 to pi this you can write as 1 minus cos of 2 phi upon 2 d phi so the sin square phi we can write like that and for theta we can write here the sin square theta into sin theta and sin square theta we can write as 1 minus cos square theta so this we can get 0 to pi 1 minus cos square theta sin theta d theta.

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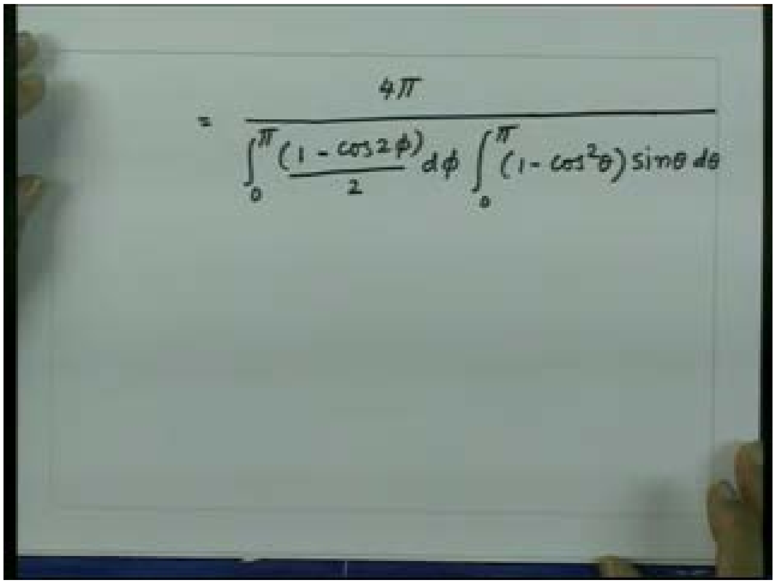

Now in this integral we can substitute for cos theta equal to t so sin theta d theta will become dt, this integral is now very simple to solve so you get finally a value which is 4pi divided by pi upon 2 that is 2 upon 3 this value. So you get a directivity for this antenna which is 12.

Once you know the directivity then now I can calculate the U maximum which is the average radiation intensity which is 80 upon 4pi steradian multiplied by directivity which is 12 so from here we can get 80 upon 4pi into directivity 12 so that gives us 240 upon pi watts per steradian that is the maximum radiation intensity which we get.

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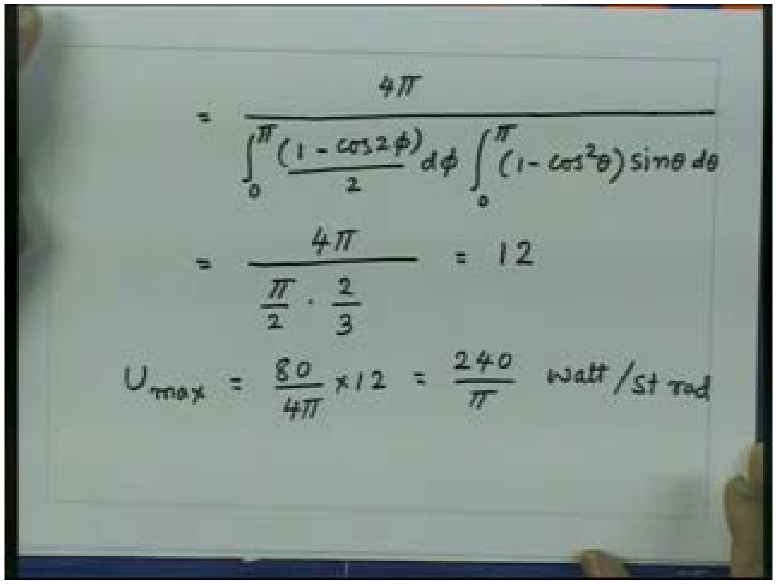

Now the Poynting vector in that direction at a distance we can calculate because we know that the the power density P is the radiation intensity divided by r square. And it is given... you have to find out this power density at a distance of 10 kilometer in the direction of the maximum radiation. So since we have found out the radiation intensity in the direction of maximum radiation and r is 10 kilometer we can substitute into this so this is 240 upon pi divided by 10 kilometer so which is 10 into 0 to the power 3 whole square so that would give us 0.764 microwatt per meter square.

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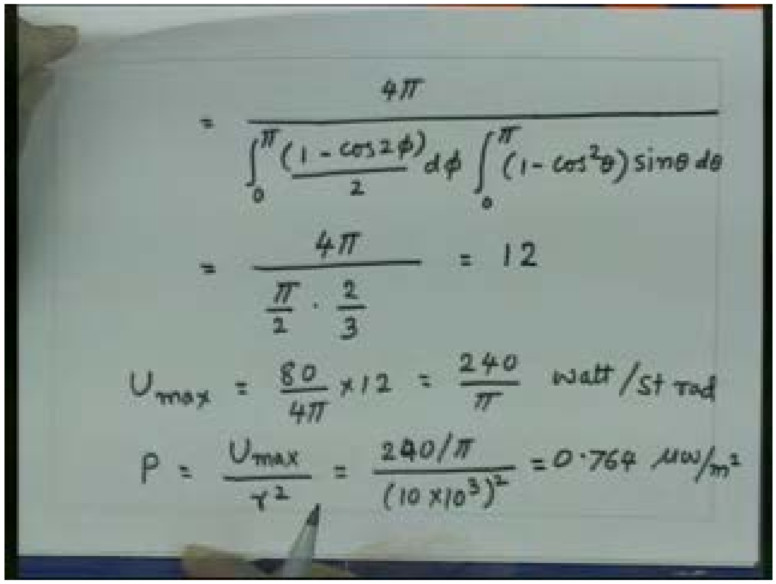

Once we know the the power density then we can find out the electric field at the point because the power density P is mod of E square divided by eta so E electric field will be equal to square root of power density into eta is equal to square root of 0.764 microwatts per meter square so this is 10 to the power minus 6 for microwatt into eta which is 120pi. So simplifying this essentially we can get the expression or the value of the electric field at that distance.

So see in this problem, what is given? The total input power is given, the efficiency of the antenna is given, the power radiation pattern is given and we are asked to find out the maximum radiation intensity, the maximum power density at a distance of 10 kilometer and also the maximum electric field.

So first we find out what is the power radiated by the antenna, then we find out the directivity of the antenna and then using this relation (Refer Slide Time: 47:57) we find out the maximum radiation intensity which is average radiation intensity multiplied by directivity and once you get maximum radiation intensity then you can calculate the power density which is maximum radiation intensity by r square and once we get the power density then we can find out the electric field which is square root of power density multiplied by the intrinsic impedance of the medium. So these are some representative problems essentially which we see in practice.

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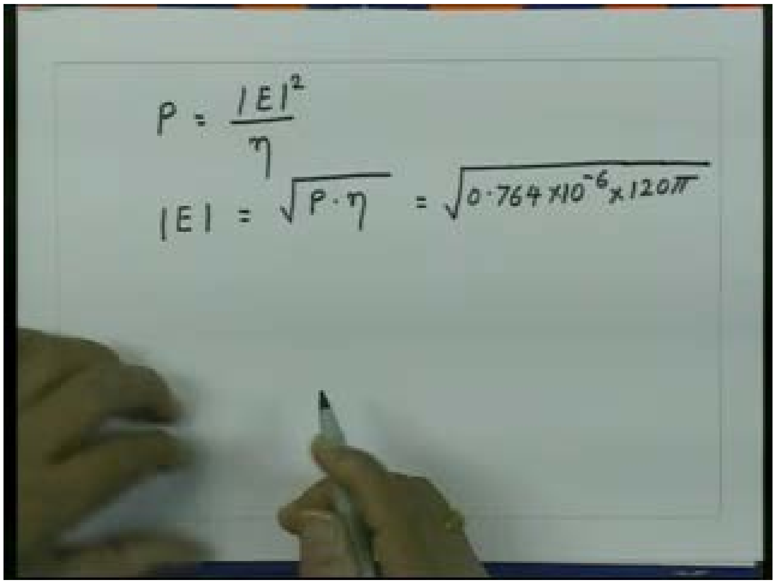

So if we systematically follow the analysis which we have developed for the antenna or the radiation then the problem essentially can be split into small small parts and one can solve the complicated problems by using a systematic approach.

So once we understand the theory properly for the radiation and if we can physically visualize this radiation pattern then the solution of the antenna problems rather becomes extremely simple. So these problems which we have discussed today essentially give us some feel on how the radiation related problems will be or will be solved in practice. In fact these are the problems which we normally encounter in designing, for example, the microwave links or for designing the satellite communication antennas or the radar antennas and so on. So this essentially completes our discussion related to the radiation or the antenna.