Transmission Lines and E.M. Waves
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Lecture-6

Welcome, in the last lecture we investigated the standing wave patterns on a Transmission Line. We saw that the voltage standing wave pattern and the current standing wave patterns are staggered with respect to each other that is wherever there is a voltage maximum there is current minimum and wherever there is current maximum there is voltage minimum. We also introduced a very important parameter which is a measure of reflection on Transmission Line and that is the voltage standing wave ratio (VSWR) is the ratio of the maximum voltage seen on the Transmission Line to the minimum voltage seen on the Transmission Line. Higher the value of VSWR worse is the condition on the Transmission Line that is more reflection on Transmission Line.

Also we establish the bound on the VSWR that is the VSWR lies between 1 and ∞ and smaller the value of VSWR means better transmission less reflection on Transmission Line so more transfer of power to the load. We also establish the bounds on the impedance on the Transmission Line that for a given termination load on Transmission Line there is a maximum and minimum impedance which one can see and that value is characteristic impedance multiplied by the VSWR and the characteristic impedance divide by VSWR.

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When we do the impedance transformation on Transmission Line the impedances are bound by these two values the maximum value of impedance and the minimum value of impedance.

Today we will study the impedance transformation on a Loss-Less Transmission Line and then we will establish a very important characteristic of impedance transformation on Loss-Less Transmission Line. And then we will go to the calculation of power transfer to the load.

As we have seen for a general Transmission Line taking the ratio of voltage and current at any location on Transmission Line we get the impedance at that location.

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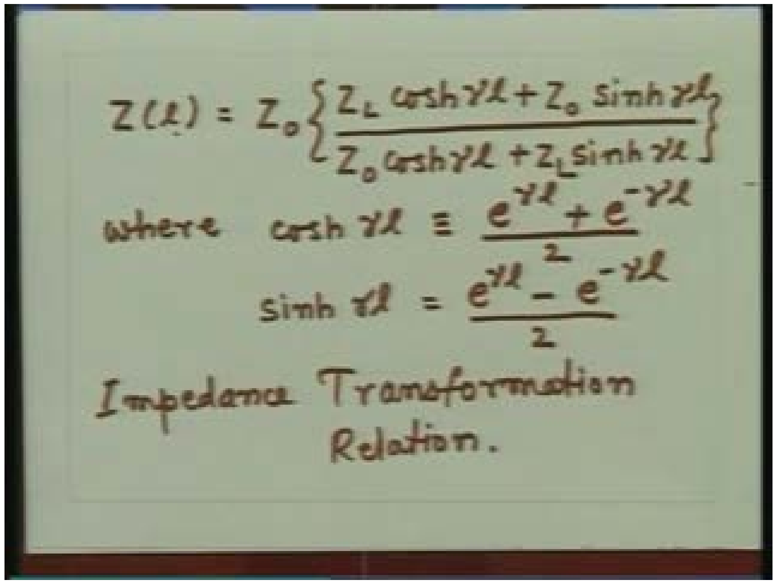

And we have seen the impedance is given by this Z at any location l is equal to the characteristic impedance multiplied by this impedance transformation term, where this quantity is the hyperbolic cosine and hyperbolic sine which are given in terms of the propagation constant γ and the length on the Transmission Line.

Now for a Loss-Less Transmission Line the propagation constant γ is $j\beta$ so if I substitute $\gamma = j\beta$ in this expression then I get the impedance transformation relationship for a Loss-Less Transmission Line. Also we have said that the absolute impedances do not have any meaning on a Transmission Line the impedances normalized to the characteristic impedance are the meaningful quantities so the same expression we have converted into a normalized impedances where we define the impedance by bar that is the actual impedance divide by the characteristic impedance that is the normalized impedance.

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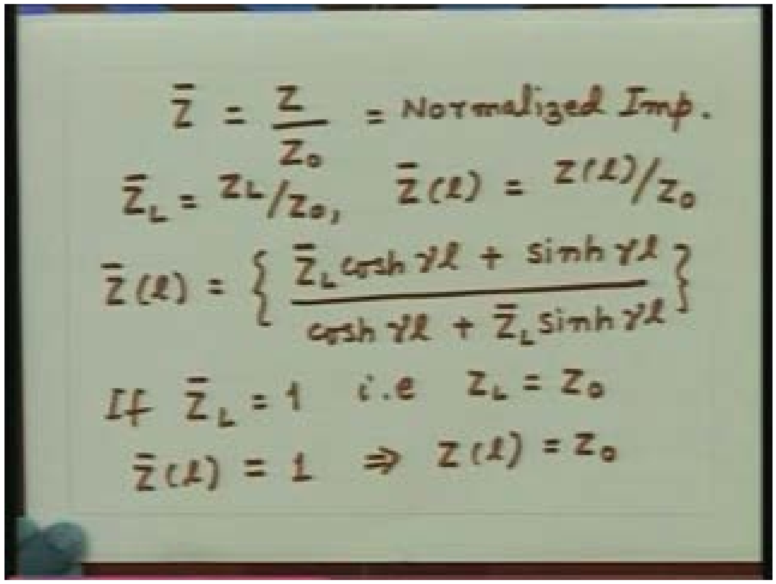

So every impedance which we see on Transmission Line the load impedance, the impedance at location l all of them are now normalized with respect to the characteristic impedance. Then the normalized impedance transformation relationship essentially is given by this. And as we have said when normalized value is equal to one that time the load impedance is equal to Z_0, similarly when the normalized impedance at location l is equal to one the value at that location is equal to Z_0. So either we can use the normalized impedance transformation relationship or we can use the normalized transformation relationship.

However, now for a lossless line we substitute for $\gamma = j\beta$ and get the relation for the Loss-Less Transmission Line. If you substitute $\gamma = j\beta$ for the Loss-Less Transmission Line the cosh γl is equal to the cosh βl that is nothing but equal to cos βl. Where as, the sinh γl is equal to sinh jβl that is equal to j sin βl.

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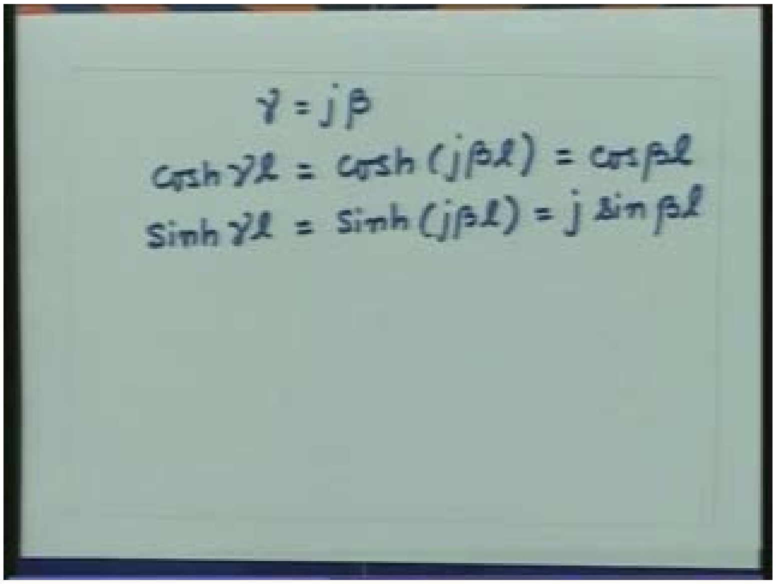

Substituting these for hyperbolic cosine and sine in the general expression which we had got last time we get now the impedance transformation relationship for the lossless line and that is now the impedance Z at the location l is equal to Z_0 the characteristic impedance of the line and characteristic impedance let me remind you again is a real quantity for a Loss-Less Transmission Line so this quantity is real. So Z(l) $= Z_0 \left\{ \frac{Z_L \cos\beta l + j Z_0 \sin\beta l}{Z_0 \cos\beta l + j Z_L \sin\beta l} \right\}$.

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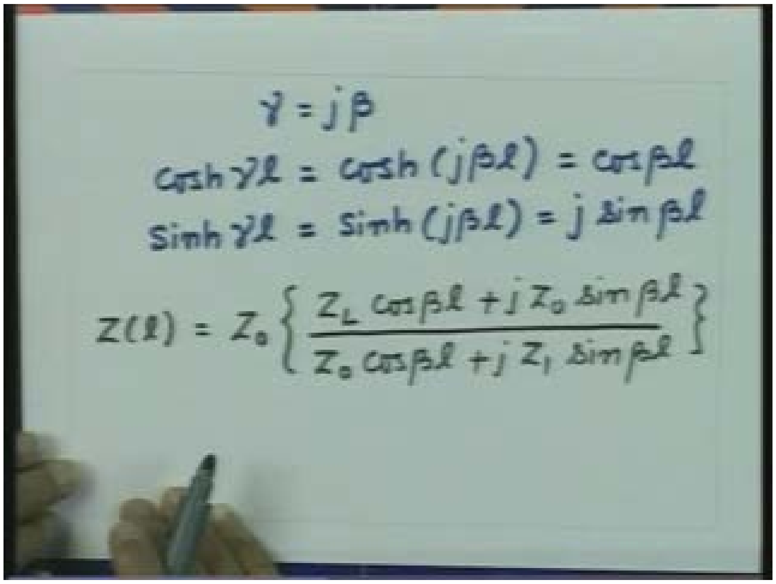

If I take Z_0 down here this quantity $\frac{Z(l)}{Z_0}$ will be normalized impedance, similarly I can take Z_0 common from the numerator and denominator so the same expression as we had obtained earlier in terms of normalized impedance will be equal to $\left\{\frac{\overline{Z}_L \cos\beta l + j \sin\beta l}{\cos\beta l + j\,\overline{Z}_L \sin\beta l}\right\}$.

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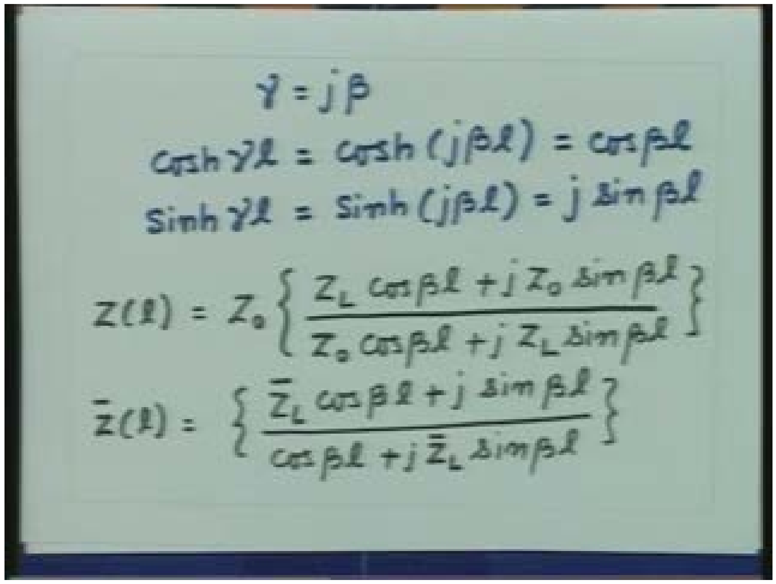

So either of the impedance transformation relationship can be used when we transform the impedance on a Loss-Less Transmission Line from one point to another this is the absolute impedance, this is the normalized impedance.

Once we get the impedance transformation relationship then we can establish some very important characteristic of impedance transformation on Transmission Line and that is we know that when you move on a Transmission Line by a distance of $\lambda/2$ the voltage characteristic the standing wave characteristic will repeat. So if you take a ratio of voltage and current at a location we expect that these characteristics would repeat at $\lambda/2$.

Similarly the special points are if I move a distance by $\lambda/2$ we will also see there is a special distance of $\lambda/4$ there is something very special happens and third characteristic is if I terminate the line into the characteristic impedance the impedance will always be equal to the characteristic impedance irrespective of the length of the line.

So we have three very important characteristics of the impedance transformation on a line. The first one that is the impedance value repeats every $\lambda/2$ distance.

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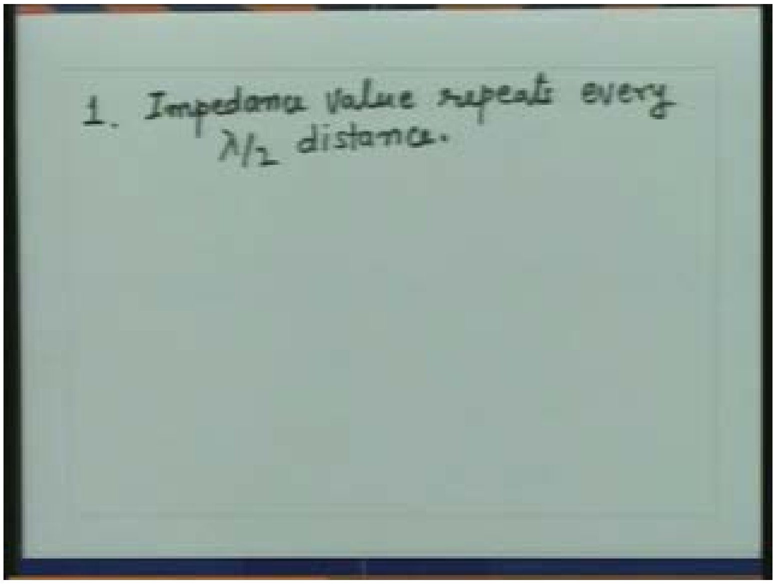

Going back to this expression here let us say I have a impedance value of Z(l) at some location on the line now if I move by a distance of $\lambda/2$ from here that means if I go to a location l + $\lambda/2$ then the impedance at that location would be where l will be replaced by l + $\lambda/2$

So, initially let us say at location l, impedance is equal to Z(l), what we want to find out is the impedance at Z(l) + $\lambda/2$ so I replace l by l + $\lambda/2$, if I use the normalized relation this will be equal to $\left\{\dfrac{\bar{Z}(l)\cos\beta\left(l+\lambda/2\right) + j\sin\left(l+\lambda/2\right)\beta}{\cos\beta\left(l+\lambda/2\right) + j\,\bar{Z}(l)\sin\left(l+\lambda/2\right)\beta}\right\}$.

Now $\beta\left(l+\lambda/2\right)$ = $\frac{2\pi}{\lambda}\left(l+\lambda/2\right)$ where $\beta=\frac{2\pi}{\lambda}$ and this again is equal to beta into βl plus λ will cancel so I will get only π.

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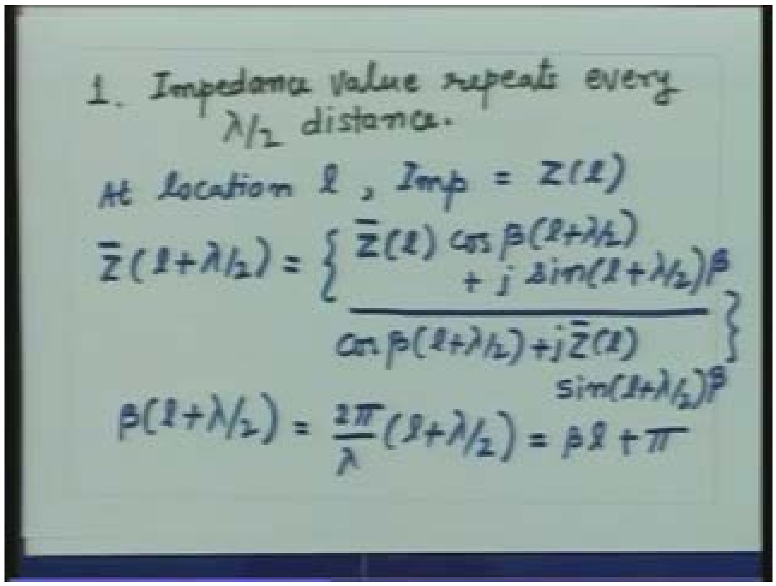

So $\cos\beta\left(l+\lambda/2\right)$ is nothing but $\cos\left(\beta l+\pi\right)$, similarly $\sin\left(l+\lambda/2\right)\beta$ is nothing but $\sin\left(\beta l+\pi\right)$.

Now $\cos\left(\theta+\pi\right)$ = -cos θ and $\sin\left(\theta+\pi\right)$ = -sin θ. So this quantity $\cos\beta\left(l+\lambda/2\right)$ is nothing but –cos βl.

So if I substitute into this I get, $\cos\beta\left(l+\lambda/2\right)$ = $\cos\left(\beta l+\pi\right)$ = –cos βl

Similarly $\sin\left(l+\lambda/2\right)\beta$ = $\sin\left(\beta l+\pi\right)$ = –sin βl.

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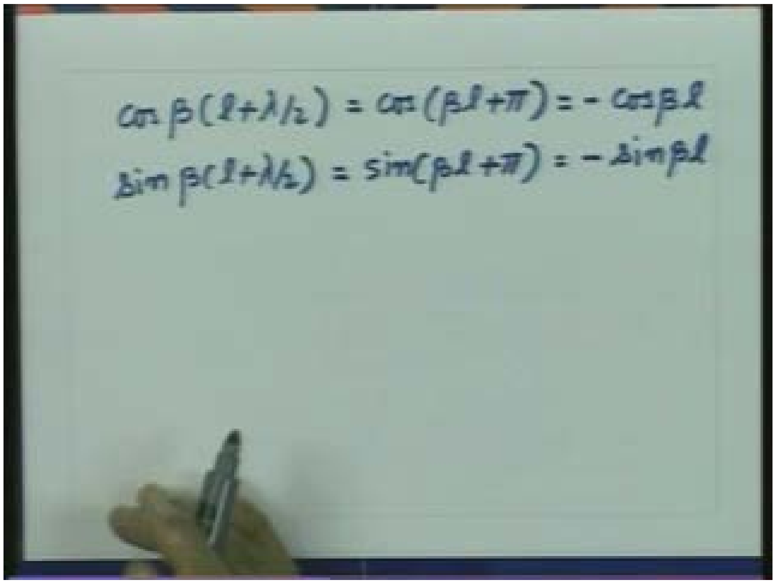

Substituting now this $\cos\beta\left(l+\lambda/2\right)$ and $\sin\left(l+\lambda/2\right)\beta$ into the impedance relation here we get the impedance at location $\left(l+\lambda/2\right)$ as $\bar{Z}\left(l+\lambda/2\right)=\left\{\frac{-\bar{Z}(l)\cos\beta l - j\sin\beta l}{-\cos\beta l + j\bar{Z}(l)\sin\beta l}\right\}$.

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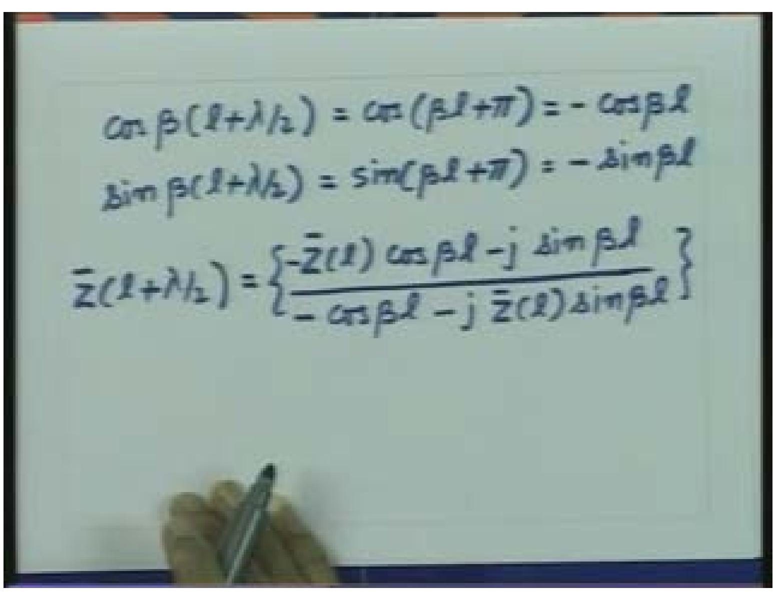

I can take minus common from numerator and denominator so all this quantity will be plus this is exactly same as the impedance Z(l) at location l on the Transmission Line so this is nothing but equal to normalize impedance at location l. That is very important characteristic on Transmission Line that is the impedance repeats itself over a distance of $\lambda/2$ or in other words the impedance transformation there is only memory of $\lambda/2$ distance on Transmission Line how many cycles of $\lambda/2$ have gone on Transmission line we will never be able to find out from the knowledge of the impedance.

So no matter what is the length of the Transmission Line essentially $|\lambda/2|$ is the special information which is available from the impedance transformation on Transmission Line. So this is one of the very important characteristic for a Loss-Less Transmission Line that every distance of $\lambda/2$ the impedance characteristic repeats.

The second characteristic is the impedance at a distance of $\lambda/4$ if I move by a distance of $\lambda/4$. Again if I know the normalized impedance at location l we want to find out what is the value of this impedance at l + $\lambda/4$ so the quantity β(l + $\lambda/4$) will be equal to βl plus beta is $\frac{2\pi}{\lambda}$ so this is $\frac{2\pi}{\lambda}$ into $\lambda/4$ so that is equal to βl + $\frac{\pi}{2}$.

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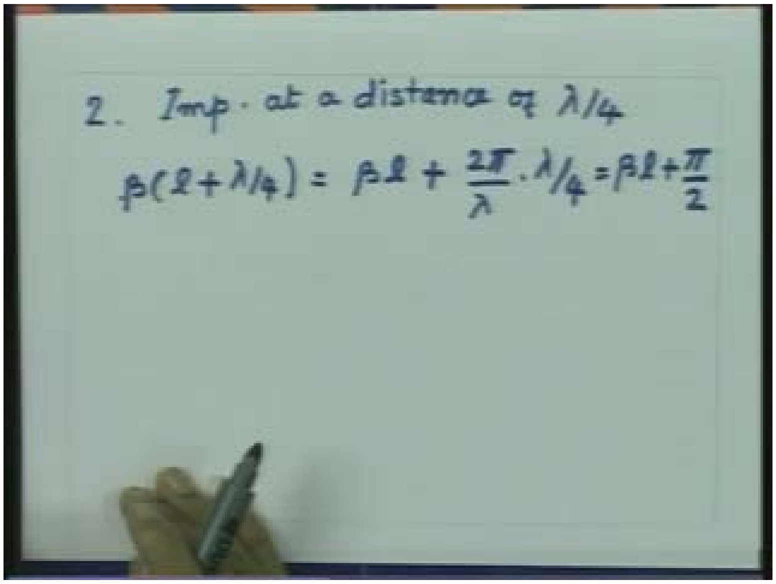

Now substituting this for βl + $\lambda/4$ in the transformation relation I get the normalized impedance at l + $\lambda/4$ which will be equal to Z(l) $\cos\beta\left(l+\lambda/4\right)$ which is nothing but $\cos\beta\left(l+\pi/2\right)$ which again will be -sin βl so this will be -sine βl + j $\sin\beta\left(l+\lambda/4\right)$ so that is $\sin\beta\left(l+\pi/2\right)$ so that will be equal to cos βl divided by -sine βl plus j $\bar{Z}(l)$ cos βl.

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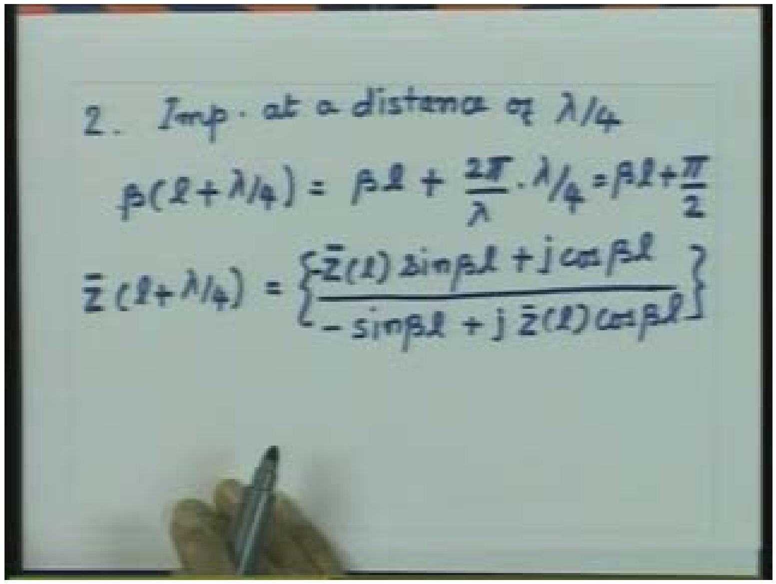

If I take the j common from here this will become $\bar{Z}(l)$ cos βl + j times sin βl, this will become cos βl + j $\bar{Z}(l)$ sin βl so that quantity is nothing but $\frac{1}{\bar{Z}(l)}$ of that.

So this is a very important characteristic of Transmission Line that every distance of $\lambda/4$ the normalized impedance inverts itself note the word normalize it is not the absolute impedance because absolute impedance if it inverts then dimensionally it will become admittance the normalized impedance does not have any unit it does not have any dimensions it is a dimensionless quantity. So for a distance of $\lambda/4$ the normalized impedance will invert itself so if I have a value of impedance at some location on Transmission Line if it is greater than Z_0 after a distance of $\lambda/4$ it will definitely going to be less than Z_0 because the normalized impedance will be the inverted value of the impedance at the previous location so by this a distance of $\lambda/4$ the normalized impedance will invert. Again the impedance will invert after $\lambda/4$ so impedance will become same that is what essentially the previous property that every distance of $\lambda/2$ the impedance is same.

So when we talk about the periodicity of the impedance on Transmission Line every $\lambda/2$ the absolute or normalized impedance repeats itself where as every distance of $\lambda/4$ the normalized impedance inverts itself and you will see later on when we talk about the impedance matching characteristics this property is used extensively for finding out the impedance transformation which can match impedances on the Transmission Line.

The third characteristic is the matched condition characteristic which you already discussed briefly that we talked in general Transmission Line so third characteristic is the matching condition on the Transmission Line and that is if the line is terminated in the characteristic impedance then the impedance seen at every point on Transmission Line is equal to the characteristic impedance.

So if I take $Z_L = Z_0$ that is $\overline{Z}_L = \frac{Z_L}{Z_0} = 1$, the impedance $\overline{Z}(l)$ at any location on Transmission Line will be equal to $\overline{Z}(l)$ which is 1 so $\left\{\frac{\cos\beta l + j\sin\beta l}{\cos\sin\beta l + j\sin\beta l}\right\}$ which is again equal to 1 so this is sinβl which is equal to 1.

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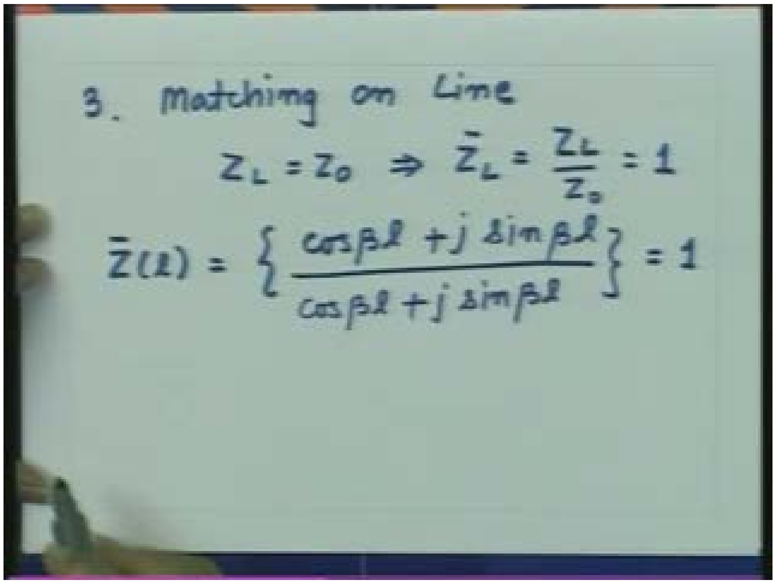

So irrespective of the length of the Transmission Line if the line is terminated in the characteristic impedance then the impedance seen at every point on Transmission Line is equal to the characteristic impedance and if you recall we had discussed this condition this because this is a very special condition what that means is once the line is terminated into the characteristic impedance one does not have to worry about the impedance transformation on Transmission Line you can use any piece of Transmission Line and the impedance at the input of the Transmission Line will be always same which will be equal to the characteristic impedance.

We also see that when the $Z_L = Z_0$ that time the reflection coefficient is zero so there is no reflected wave on Transmission Line you have only forward traveling wave on Transmission Line and as we have argued earlier forward traveling wave always sees an impedance which is equal to characteristic impedance so this result is not very new we have discussed this earlier when we were talking about the general Transmission Lines and that was if the line is terminated in the characteristic impedance then the impedance seen at every point on the Transmission Line is equal to the characteristic impedance.

These are three very important characteristics of Loss-Less Transmission Line. let me summarize it again, first the impedance transformation repeats of a distance of a $\lambda/2$ all impedance values repeat every $\lambda/2$ distance the normalized impedance inverts itself for every $\lambda/4$ distance and if the line is terminated in the characteristic impedance then the impedance seen at any point on the line is equal to the characteristic impedance.

With this understanding of the impedance transformation now we can go to the power transfer calculation on the Transmission Line. Initially when we start the discussion on the Transmission Line the purpose was to transfer the power from the generator to the load effectively. In the lossless case since there is no lossy element present on the line ideally the load should be such that the power taken from the generator is completely transferred to the load.

However, as we have seen if the impedance is not equal to the characteristic impedance then there will be always reflection on Transmission Line that means whatever energy the generators supplied that energy will reach to the load will not find a condition which is favorable for the maximum power transfer the part of the energy will get reflected back and this energy will come back and essentially will feed the generator back.

Now when we talk about the matching condition or the maximum power transfer condition there are two issues here, one is when the power is given by the generator it should be maximally transferred to the load it is the efficiency how the power get really transferred to the load and second issue is when the reflected power come back and hits the generator the generator actually is not capable of absorbing power it is delivering power so when this reflected power comes back with different amplitude and phase it affects the performance of the generator it changes its phase characteristic amplitude characteristic so it is desirable that generator should not see any power coming and hitting back so for these two purposes that the power whatever is given by the generator should be completely transferred to the load and also no power should come back and hit back to the generator we must make sure always that the impedance which the generator effectively sees equal to characteristic impedance so there is no reflection which will lead to the generator, also at the load end we do something so that the maximum power is transferred to the load.

These issues we will discuss little later but let us take a very general case at the moment and ask if I have a Transmission Line which is connected to a generator on one end and the load at the other end how much power will be delivered to the load. Again going back to the original voltage and current equations we can write down the power at the location of the load that means at l = 0. So what we discuss now is the power delivered to the load.

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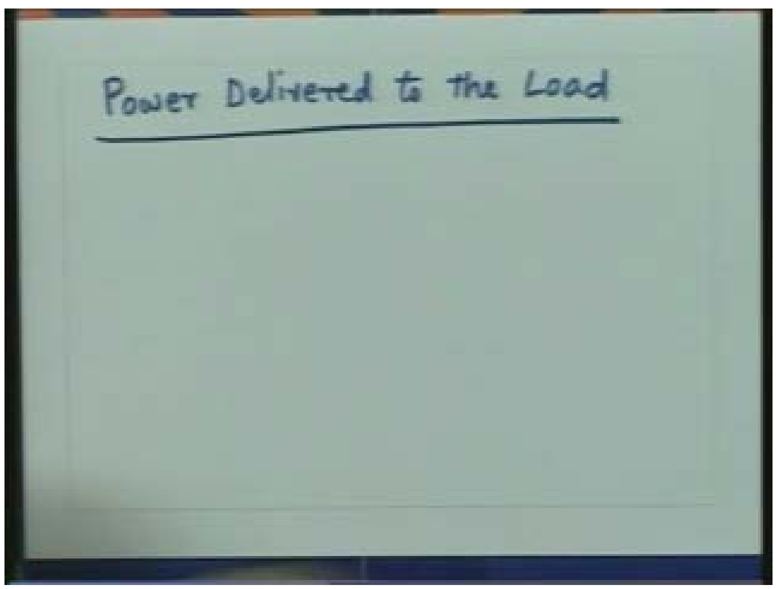

Let us start again with the basic equation. We have voltage equation the voltage at l = 0 is the load end that is $V(0) = V^+$ into $1 + \Gamma_L e^{-2\beta l}$ but l = 0 at the load end so this quantity will be $\{1 + \Gamma_L\}$ so this is the voltage at the load end this is we are talking about the voltage and current at the load end of the line.

Similarly I have current at l = 0 at the load end that is $\frac{V^+}{Z_0}\{1 + \Gamma_L\}$.

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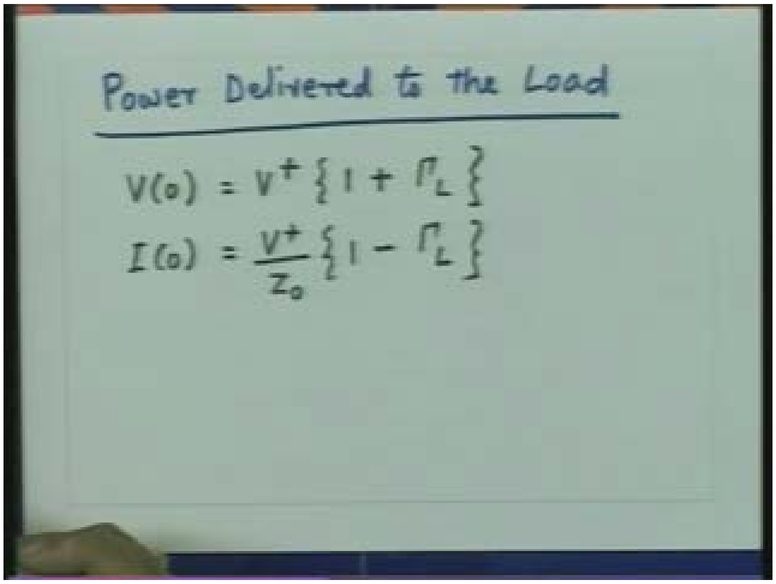

So from the general voltage and current relations on the Loss-Less Transmission Line I find out the voltage value at the load end, current value at the load end. So the power will be nothing but half real part of VI conjugate so the power delivered to the load p = ½Re{V(0) I*(0)}.

If I substitute for the V(0) and I(0) in this that will be equal to ½ V^+ multiplied by I conjugate which is V^+ conjugate divide by Z_0 conjugate where Z_0 is the real quantity so this will be equal to $\frac{|V^+|^2}{Z_0}$ multiplied by the conjugate of this so this will be equal to if I take the real part of this then that will be equal to $\{1 - |\Gamma_L|^2\}$. So once I know the impedance at the load I know the reflection coefficient and let me write there $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$ is the reflection coefficient so once I know the load impedance I can calculate the mod of reflection coefficient.

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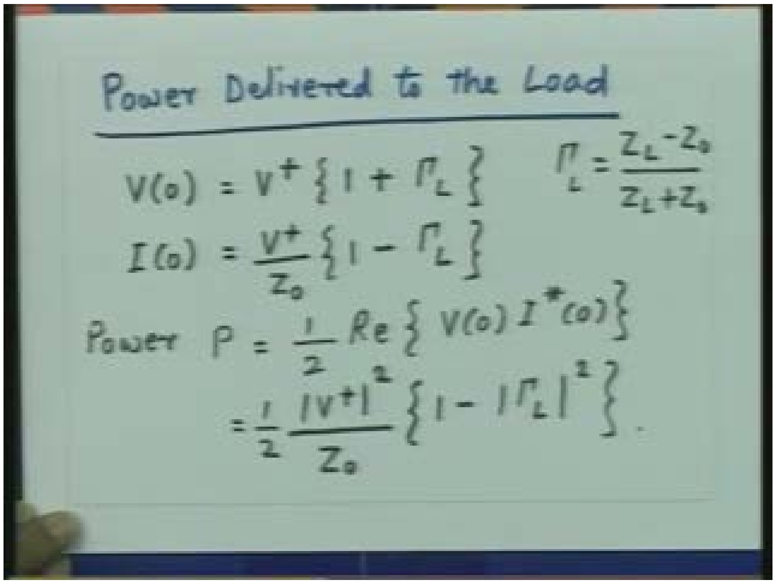

So the power delivered if I knew the value of V^+ then I can find out what is the power delivered to the load, how do we find out V^+ which we will discuss in a minute. However at this location if I know the amplitude of the forward traveling wave and if I know the load impedance I can calculate the value of the power delivered to the load.

Here we have calculated the power which is from the circuit point of view that if I know the voltage and current at a particular location I can apply the relation that the power delivered at that location is VI* conjugate the real part of that and from there we get this power delivered to the load. We can use a little different argument to come to the same answer and that is on the Transmission Line the power was supplied by the generator in the form of a traveling wave which was going towards the load and as we already said that the traveling wave always sees an impedance which is equal to characteristic impedance. So if the traveling wave had an amplitude V^+ it is as if this wave is supplying a power to Z_0 which is the real quantity in a lossless case.

So one can say that we have now a wave which is going in the forward direction which is having an amplitude V^+ this wave always sees an impedance which is equal to Z_0 so the power carried by this wave will be half of ½ $\frac{|V^+|^2}{Z_0}$. So the power carried by forward wave P_{for} will be equal to ½ $\frac{|V^+|^2}{Z_0}$ because this is the voltage V^+ which is traveling on this it is seeing an impedance equal to the characteristic impedance Z_0 which is the real quantity.

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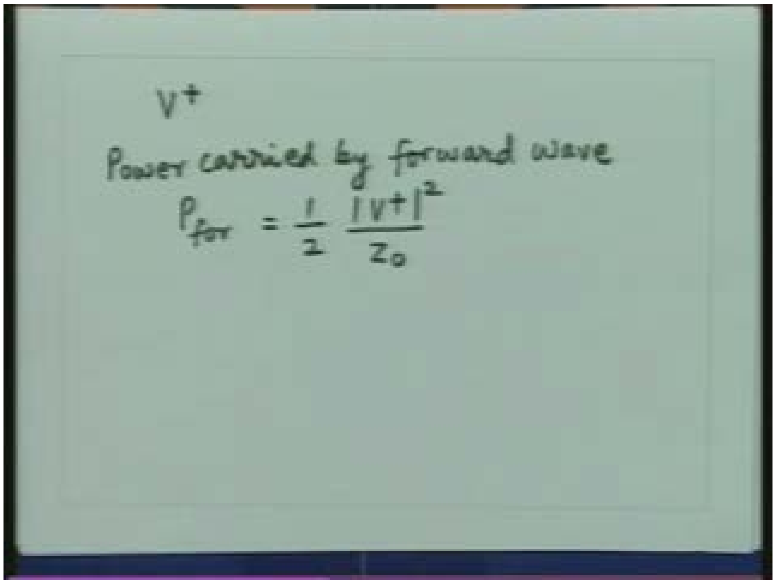

So the power delivered to the forward wave is ½ $\frac{|V^+|^2}{Z_0}$ where ½ is the factor for the rms value of the power.

Now when this voltage wave reaches to the load part of energy is going to travel back and there will be again a traveling wave which is traveling backwards with an amplitude of V^-. So since this wave also sees the impedance which is equal to the characteristic impedance the power carried by this wave will be nothing but $\frac{|V^-|^2}{Z_0}$.

So we get the power reflected by the load in the backward wave that is P_{ref} = ½ $\frac{|V^-|^2}{Z_0}$.

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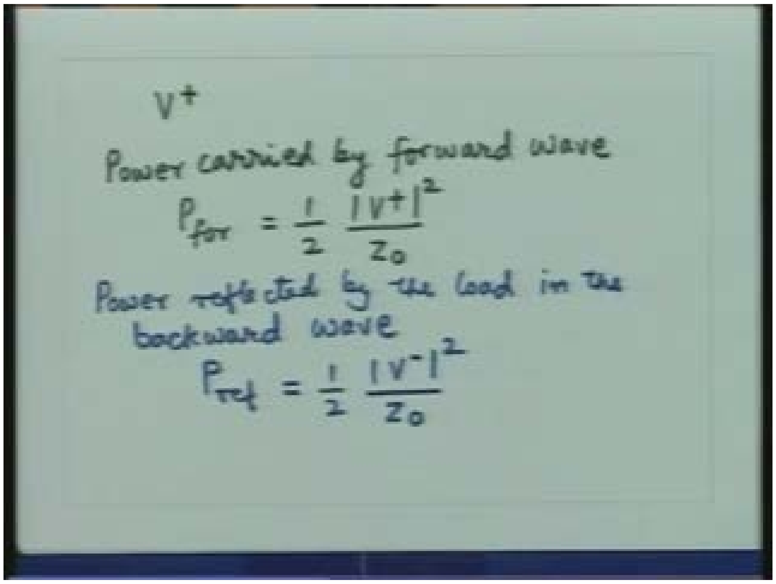

So $P_{forward}$ was the power which was carried towards the load, $P_{reflected}$ is the power which was taken away from the load so difference of these two powers is the one which is delivered to the load. The net power which is delivered to the load is p is $P_{forward}$ - $P_{reflected}$ that is equal to $\frac{1}{2}\left\{\frac{|V^+|^2}{Z_0} - \frac{|V^-|^2}{Z_0}\right\}$ and $\frac{V^-}{V^+}$ as we know is the reflection coefficient at the load.

So this quantity is nothing but if I take $|V^+|^2$ common this will be $1 - |\Gamma_L|^2$

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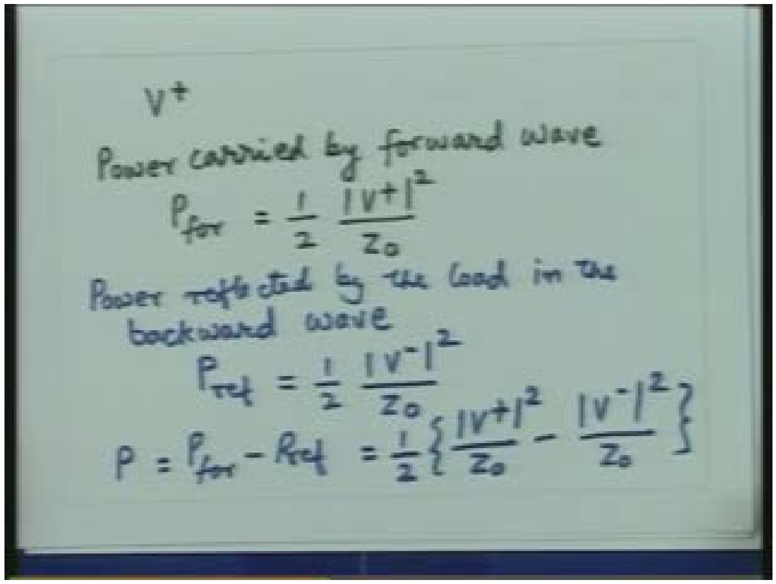

This relation is exactly same as what we have derived earlier for the power delivery. So what we see is when we do the power calculations on the Transmission Line either I can go by the circuit concept to find out the voltage and current at that location and then simply take the power delivered will be half real part of VI* conjugate or I can talk in terms of the traveling waves find out how much power was carried by the wave and how much power was taken back in the form of a reflected wave, difference at the two power will be the power delivered to the load. So by using either of the concepts one can find out what is the power was supplied to the load.

This is the story for the real power that is the power which is actually supplied to the load. One can ask in general suppose I want to calculate the complex power at a particular location how does that reflect or in general I can ask the question that if I calculate the power flow at any particular location on Transmission Line not necessarily at the load end what does that indicates. So if I get the voltage and current at any arbitrary location on Transmission Line that is V(l) at some location l is $V^+ e^{j\beta l}\{1 + \Gamma_L e^{-j2\beta l}\}$.

You have derived the same relation last time for the Loss-Less Transmission Line the current I(l) will be $\frac{V^+}{Z_0}$ $e^{j\beta l}\{1 + \Gamma_L\, e^{-j2\beta l}\}$.

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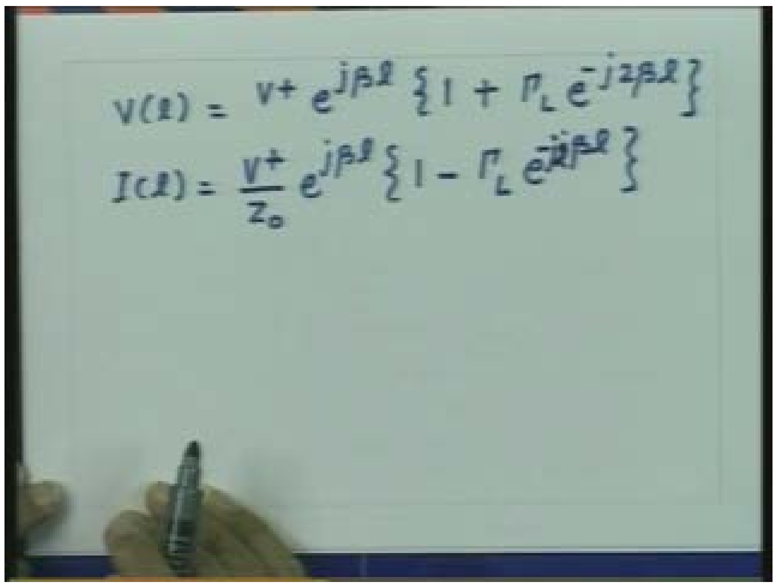

Now again as I did in the previous case at the load point I have the voltage and current so the total power the complex power P at that location l is equal to ½ (V I*) I can substitute from here for (V I*) and I get ½ $\frac{|V^+|^2}{Z_0}$ $\{1 + |\Gamma_L|^2 + 2I_m\,(\Gamma_L\, e^{-j2\beta l})\}$. It is simple algebra you substitute the value of V and I in the expression here you get the complex power which will be essentially given by this.

Now this quantity is just the imaginary part so the real part of the power which tells you the actual power delivered at that location is equal to this, this is the power which is the imaginary part of the power what is called the reactive power. So here we have a resistive power and here we have a reactive power.

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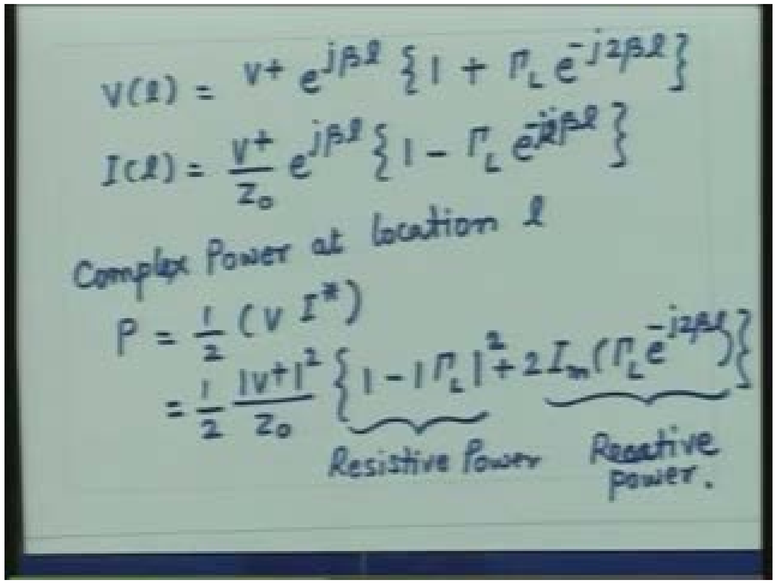

So at a general location on Transmission Line the power is complex and of course that will be complex at the load end also. But the interesting thing to note here is this resistive power at location l is exactly same as the power which we have got at the load end which is this power. Since the line is lossless the resistive power at any location in the line is same as the resistive power which we will see at the load and that make sense because if the line is completely lossless then there is no absorption of power anywhere on the line but at the load because the load is the one where you have a resistive component and the power can be absorbed at that location.

So wherever we are seeing on the Transmission Line it is telling you a power flow but this power flow essentially is the power which is ultimately going to get delivered to the load. So the resistive part of the power which is telling you the actual power flow should be independent of the location on the line because this is the power which is ultimately given to the load connected to the Transmission Line.

If you look at the reactive part of the power however this is the function of l and the reactive part power tells you essentially the energy is stored at different locations on

Transmission Line so now we are having two things when we calculate the power on Transmission Line there is a resistive power which tells you the flow of power at any particular location on Transmission Line and this flow of power is exactly same as what power would be delivered to the load. However the reactive power tells you the energy storage at different locations on line and that depends upon the value of voltage and current at that location and since because of standing wave the voltage and current is varying along the Transmission Line the energy storage is different at different location on Transmission Line.

So what we see in general is the reactive power will vary along the length of the line however the resistive power which is the power delivered to the load will be independent of the location of the Transmission Line and that is a very important characteristic. So no matter where you calculate the real power at the load end or generator end or in the middle of the Transmission Line for a Loss-Less Transmission Line this power will be exactly same as what is ultimately delivered to the load. Having done this now one can come to the final question of the analysis of the transmission line that everything we have done now the impedance transformation relationship we developed we also analyzed the power flow on the Transmission Line but we have not evaluated final arbitrary constant of the voltage or current expression and that is V^+ and that we did not do so far that is because we were always taking the relationship which were for the impedances and for that we had a ratio of voltage and current and the absolute value of V^+ did not play any role.

However as we see now when we do the power flow calculation on Transmission Line we require the knowledge of V^+ because now we cannot talk about the relative quantities we have to absolutely find out the voltages and currents at the load end or at any other location on the line and therefore we now have to absolutely evaluate the quantity V^+ because without knowledge of V^+ we will not be able to tell the absolute power delivered to the load for a given generator and the load conditions.

So now from the boundary condition by connecting the generator and the load impedance we calculate the final arbitrary constant of the voltage expression that is V^+ and then completes the analysis of the voltage or current on Transmission Line. So now what we discuss is we discuss evaluation of the arbitrary constant V^+.

Let us take the general case I have a generator here which is having a voltage V_s then I have a Transmission Line which is of length l and then I connect a load impedance to this Transmission Line which is equal to Z_L.

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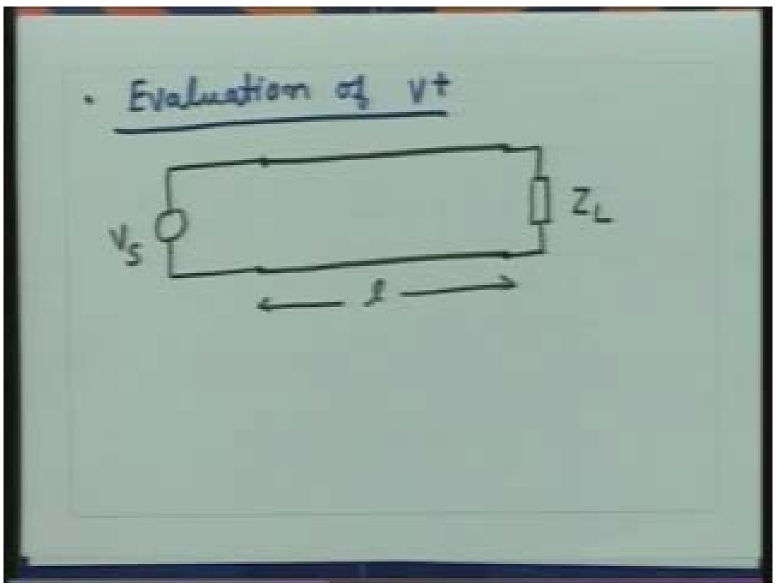

The characteristic impedance of this line is Z_0 which is given a priori, some voltage wave is going to travel on this Transmission Line which will get reflected from here it will come back so on the Transmission Line we will have the standing waves so there will be impedance transformation from Z_L to some value when see from the generator. And now by using that impedance transformation relationship and matching the boundary condition at the input we want to find out what is this quantity V^+ for the given circuit.

So first thing what we can do is we can transform this impedance at this location and then treat this circuit as the lump circuit. Let us also make it little general let us say this voltage source is having a internal impedance which is equal to Z_s so let us say I connect an internal impedance here of the voltage source which is given as Z_s. Let us say I have the input terminal which is given by some A, A' and these are the locations which are denoted by B, B'.

Once I know this Z_L and I know this length then I can find out what is the transform impedance at this location A, A'. Let us say this impedance if I see from here that is given as Z'_L so Z'_L is the transformed impedance seen at the generator end for the length of this line l as the terminating load Z_L.

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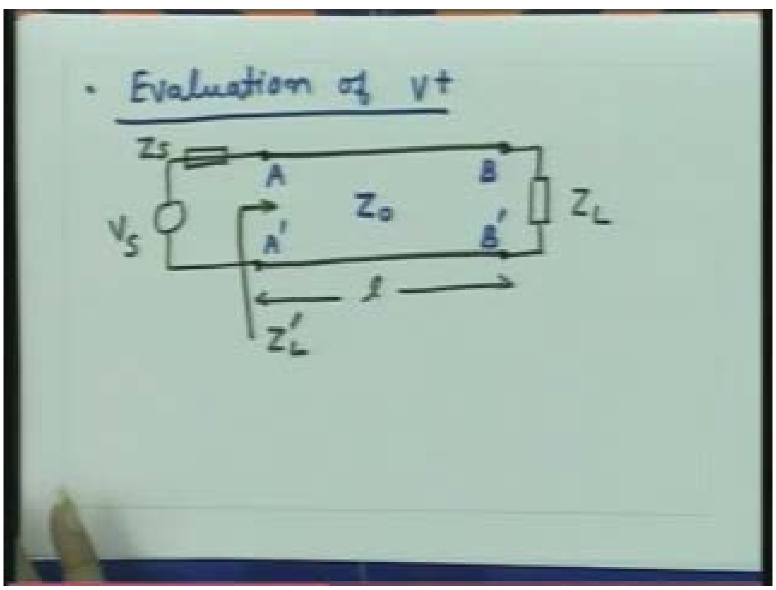

Once I transform the impedance then the whole circuit is the lumped circuit at this location I do not have to worry about the distributed elements. so now this circuit is equivalent to having a voltage source which is V_S, the internal impedance for the voltage source that is Z_S and connected to a effective impedance which is Z'_L, once I get that then I can find out what is the voltage this is the location now A, A' so by using the lumped circuit analysis I can find out the voltage at this location and also I can find out the current at this location so the voltage here we call as V_A and we call this current as I_A in this location.

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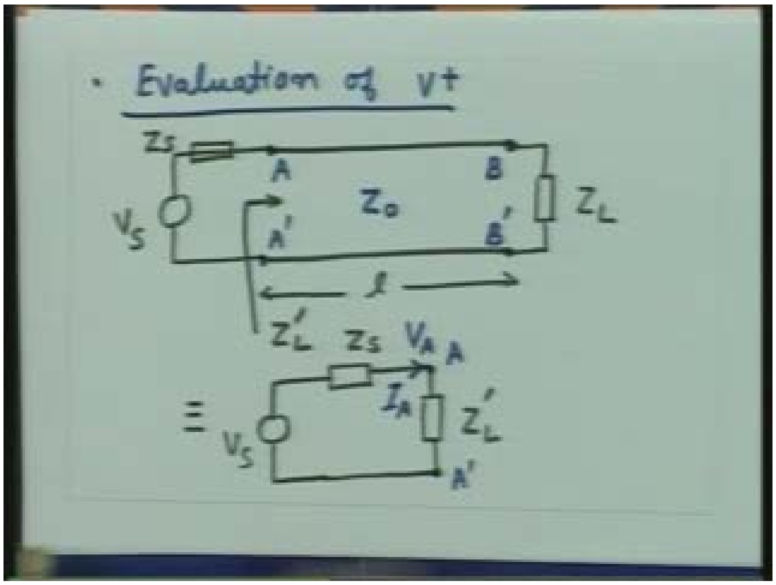

I can put the lumped circuit analysis and find out what is V_A. So, V_A will be $\frac{Z'_L}{Z'_L + Z_S} \cdot V_S$ and I_A will be $\frac{V_A}{Z'_L}$.

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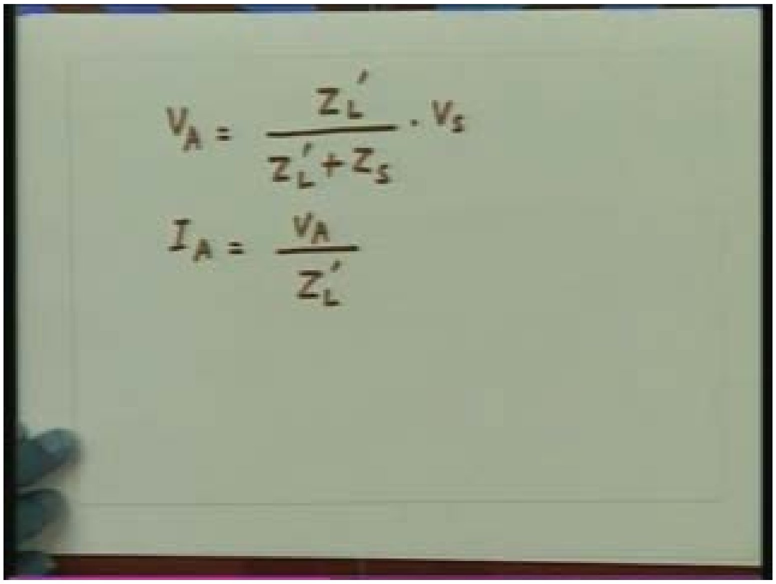

So from the lump circuit analysis at the generator end I know the voltage and current at the input end of the Transmission Line. I can write down the voltage and current at the input end of the Transmission Line using the Transmission Line equations. So, now knowing the load end impedance termination Z_L and a distance l from that essentially I have to get the voltage and current at a distance l from the load end. Now I know from the distributed elements then that the V at input end of the line V_A is nothing but V at a distance l from the load which is equal to $V^+ e^{j\beta l}\{1 + \Gamma_L e^{-j2\beta l}\}$ and the current I_A which is nothing but I at the location l from the load that is $\frac{V^+}{Z_0} e^{j\beta l} \{1 + \Gamma_L e^{-j2\beta l}\}$.

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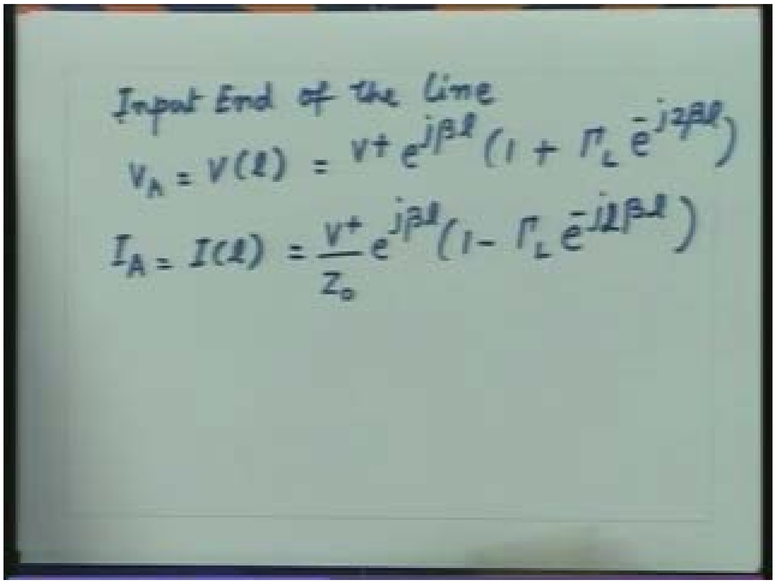

So now I know the value of V_A from two sides, one is from the lumped element side the V_A is given by this from the distributed elements on transmission line the V_A is given by this. I can equate these two values of V_A and I_A and from here I can solve for the unknown quantity which is V^+ where every other quantity is known here. the propagation constant β is known, the length of the line is known, the Γ which is related to the load impedance that reflection coefficient is known, the characteristic impedance is known so substituting the V_A and I_A in this two equations I can finally solve for the quantity which is the V^+ quantity so I get the final expression for the V^+ and that is $V^+ = \frac{Z'_L V_S e^{-j\beta l}}{(Z_S + Z'_L)(1 + \Gamma_L e^{-j2\beta l})}$.

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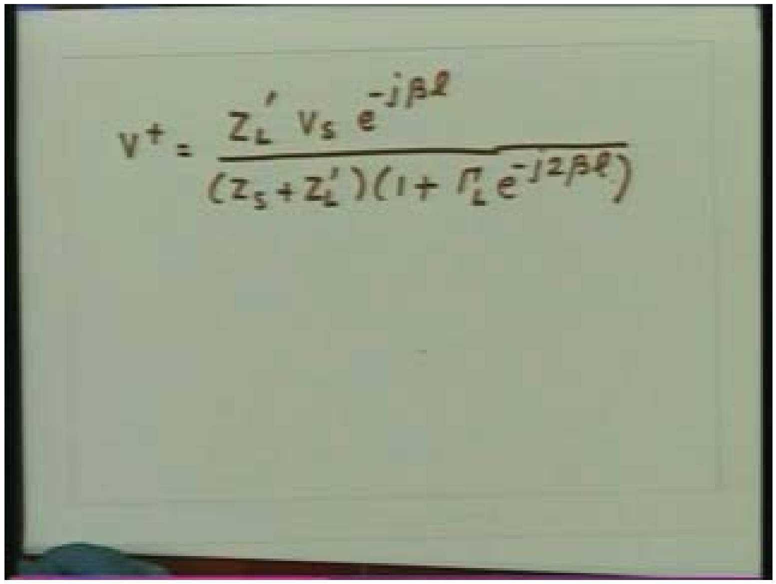

Once I know the value of V^+ then my problem is completely solved I can substitute that in to the power calculation and I can find out what is the power delivered to the load or what is the power at any particular location on the line.

Now this is a complete solution to the voltage and current on the Transmission Line so let me summarize what we have done so far in the Transmission Line.

To start with we made a case that when we increase the frequency the concept of lump element is not adequate because the space has to be brought into picture and then we introduce the concept of the distributed elements, then in the framework of distributed elements we wrote down the voltage and current relations taking the limit the either sides of the circuits tends to zero so that the model is valid at any arbitrary high frequency. We got the differential equation for voltage and current we solved the voltage and current equations and then we got a general solution for voltage and current on Transmission Line. Then we impose the boundary conditions that were the impedance boundary conditions on Transmission Line and then from there we evaluated certain arbitrary constants on Transmission Line.

We define very important parameters on Transmission Line what are called the reflection coefficients and the voltage standing wave ratios. Then we studied the impedance transformation relationship on a Transmission Line we took a special case that was a Low-Loss or a Loss-Less Transmission Line and then we found some important characteristics of the Loss-Less Transmission Line, further we investigated the power flow on Transmission Line and calculated how much power will be delivered to the load and ultimately we found out the final unknown arbitrary constants which was V^+ on Transmission Line so that we can now calculate the absolute power delivery to the load for given conditions.

So this now essentially completes the first part of analysis of Transmission Line. Here on wards we will go to more applications of Transmission Line, we will go and discuss a graphical representation of Transmission Line or graphical tool for analyzing the problems on Transmission Line and later on we will go to applications of Transmission Line at high frequencies.