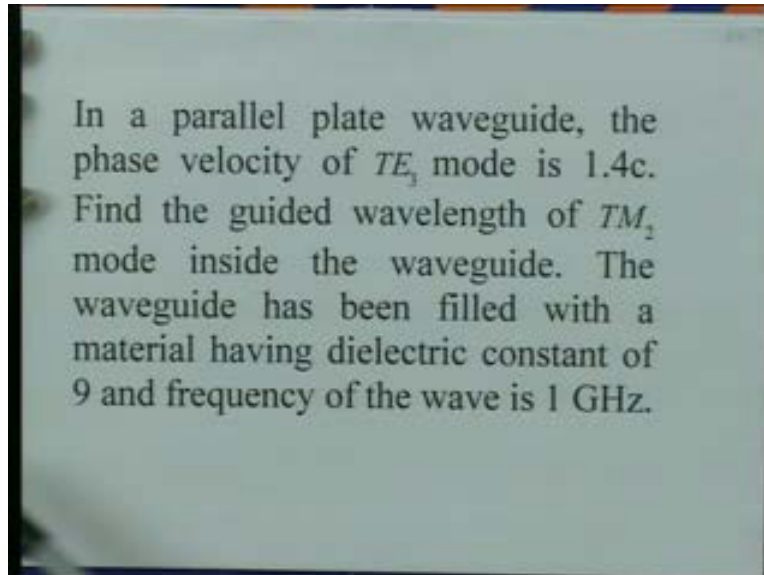


Transmission Lines and E. M. Waves
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Lecture – 59
Problems on Waveguides

In this session we solve few problems related to the waveguides. So we will take two problems which are based on parallel plane waveguide and then we will take two problems which are based on the rectangular waveguide.

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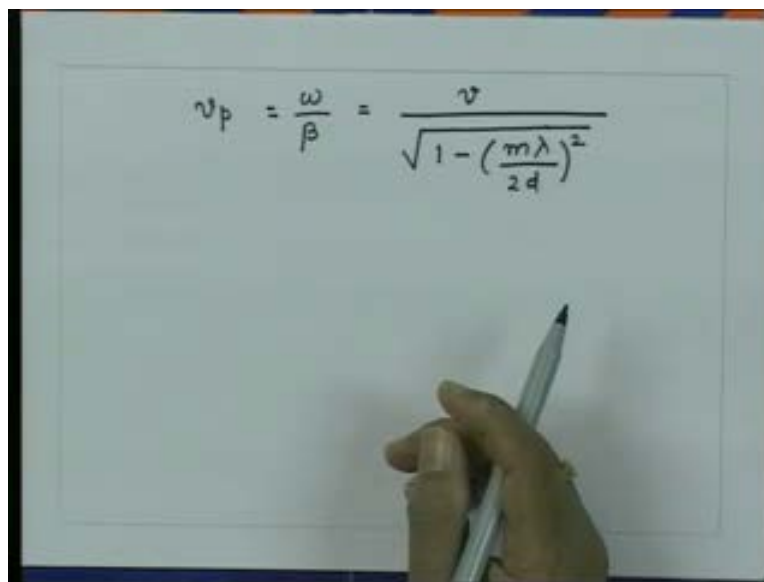


Let us consider a problem here that in a parallel plate waveguide the phase velocity of TE_3 mode is $1.4c$ where c is that velocity of light. Find the guided wavelength of TM_2 mode inside the waveguide. The waveguide has been filled with a material having dielectric constant of 9 and the frequency of the wave is 1 Gigahertz. So in this case we are given the velocity of TE_3 mode and as we know the velocity is related to the size of the wave guide. We can find out from this information what is the size of the waveguide and once we know size of the waveguide then we can find out the phase velocity for any other mode so in this case for TM_2 mode and once we know the phase velocity and the

frequency then we can find out the guided wavelength for this mode which is TM₂ mode.

So as we know the phase velocity for any mode v_p is given as ω by β where β is the phase constant in the direction of wave propagation inside the waveguide and that is equal to the velocity of light in the medium which is filling the waveguide divided by $\sqrt{1 - (m\lambda / 2d)^2}$ where λ is the wavelength and v is the velocity filling the medium. So this velocity in this case since the waveguide is filled with air this v is c so we have essentially the information of this velocity which is $1.4c$ and then we have the information which is this one which is c .

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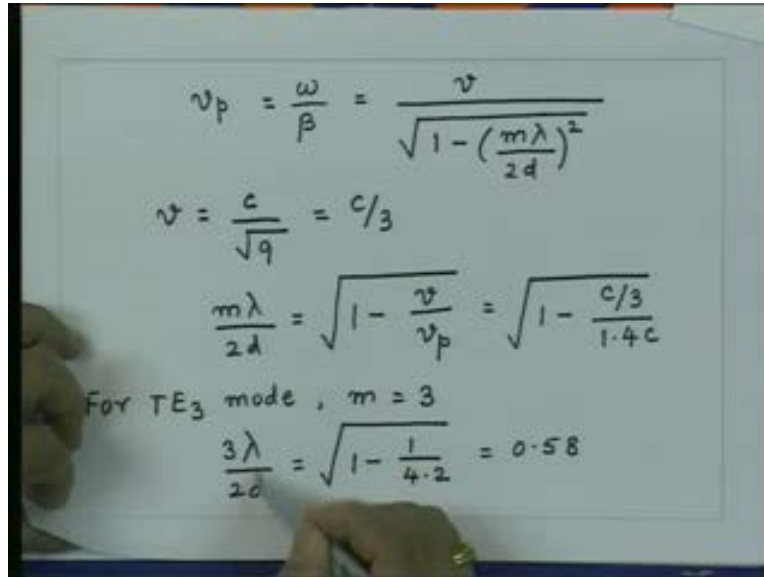


$$v_p = \frac{\omega}{\beta} = \frac{v}{\sqrt{1 - \left(\frac{m\lambda}{2d}\right)^2}}$$

So now if I substitute these quantities into this relation, first of all this velocity now, since the dielectric constant of the material which is filling the waveguide is 9 this velocity v will be equal to the velocity of light c divided by square root of ϵ_r which is 9 so that is equal to c upon 3. Then by inverting this relation we can get this quantity here which is $m\lambda$ upon $2d$ that will be equal to square root of $1 - v$ divided by v_p that is equal to square root of $1 - c$ upon 3 this velocity divided by $1.4c$. Solving this essentially we get this quantity here and in this case this velocity is given for the mode

which is TE 3 mode so m is equal to 3; so for TE 3 mode m is equal to 3 so we can substitute here so we get 3λ upon $2d$ that is equal to square root of this quantity which is $1 - \frac{1}{4.2}$ that will be equal to 0.58.

(Refer Slide Time: 5:56)



The image shows a whiteboard with handwritten mathematical derivations. The equations are as follows:

$$v_p = \frac{\omega}{\beta} = \frac{v}{\sqrt{1 - \left(\frac{m\lambda}{2d}\right)^2}}$$

$$v = \frac{c}{\sqrt{9}} = c/3$$

$$\frac{m\lambda}{2d} = \sqrt{1 - \frac{v}{v_p}} = \sqrt{1 - \frac{c/3}{1.4c}}$$

For TE₃ mode, $m = 3$

$$\frac{3\lambda}{2d} = \sqrt{1 - \frac{1}{4.2}} = 0.58$$

So, from here we can find out this quantity λ upon d so we get λ upon d that is equal to 2×0.58 upon 3 . Now once we know this quantity λ upon d then we can find out the phase velocity for TM 2 mode. So the phase velocity for TM 2 mode is the v_p will be equal to v divided by square root of $1 - \dots$ since now in this case m is equal to 2 so this will be 2 upon λ upon $2d$ whole square. So we can substitute for this λ upon d into this expression from here so this is 2 will cancel so this will be equal to the v divided by square root of $1 - \text{this quantity}$ which is 1.16 divided by 3 whole square and v is nothing but the velocity of light in that medium which is filling the waveguide so this is nothing but c by 3 so this is equal to c by 3 which is c is 3×10^8 meters per second so this quantity v here (Refer Slide Time: 7:49) is equal to 3×10^8 meters per second divided by 3 that is equal to 10^8 meters per second.

So substituting here for v we get 10^8 divided by this quantity $1 - \frac{1.16}{3}$ whole square that is that will be equal to 1.229×10^8 meters per second.

Once we know this velocity phase velocity and the frequency then we can find out the guided wavelength of this. So λ_g for this will be equal to v_p divided by the frequency which is 1 Gigahertz so 1 Gigahertz so this is equal to 1.229×10^8 divided by 10^9 which is 0.122 meters.

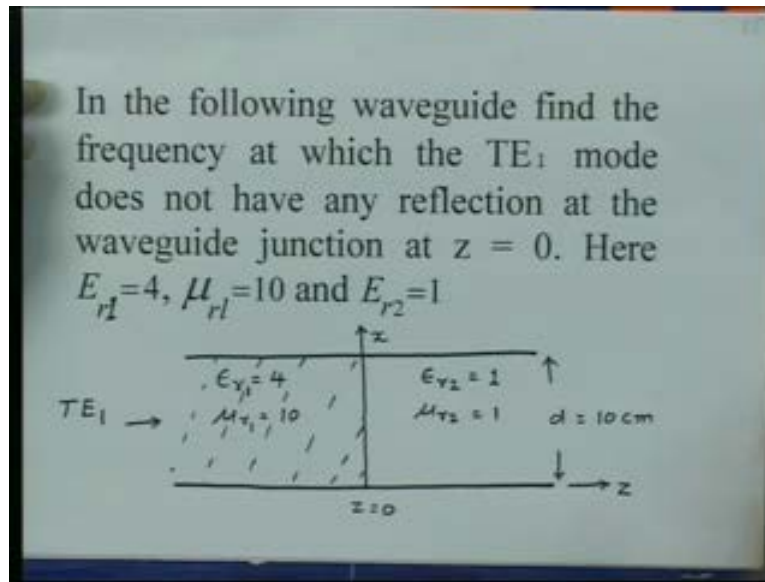
(Refer Slide Time: 9:03)

The image shows a handwritten derivation on a slide. It starts with the equation $\frac{\lambda}{d} = \frac{2 \times 0.58}{3}$. Below this, it states "Phase velocity for TM₂ mode is". Then it calculates the phase velocity $v_p = \frac{v}{\sqrt{1 - (\frac{2\lambda}{2d})^2}} = \frac{v}{\sqrt{1 - (\frac{1.16}{3})^2}}$. This is simplified to $v_p = \frac{10^8}{\sqrt{1 - (\frac{1.16}{3})^2}} = 1.229 \times 10^8 \text{ m/s}$. Finally, the guided wavelength is calculated as $\lambda_g = \frac{v_p}{1 \text{ GHz}} = \frac{1.229 \times 10^8}{10^9} = 0.122 \text{ m}$.

So in this problem the characteristics for a particular mode are given which is TE 3 mode and we are asked to find out the properties or the characteristic for TM 2 mode. And since these are related through the size of the waveguide and the frequency, essentially we can find out from here the the characteristics of TM 2 mode after getting the dimension of the waveguide, that is what precisely we have done in this.

Let us consider another problem which is again a problem on parallel plane waveguide. But this waveguide now is divided into two parts. So essentially we are having here parallel planes which are given like this and half the waveguide is filled with one dielectric material and half the waveguide is left empty.

(Refer Slide Time: 9:56)



So the problem here is as follows: in the following waveguide find the frequency at which the TE_1 mode does not have any reflection at the waveguide junction at z equal to 0. So we have here parallel plane waveguide, this is z equal to 0, so, for z less than 0 the waveguide is filled with a material with the dielectric constant 4 and relative permeability 10 and for z greater than 0 the dielectric constant is 1 and the relative permeability is 1 and the width of the waveguide or height of the waveguide is equal to 10 centimeter.

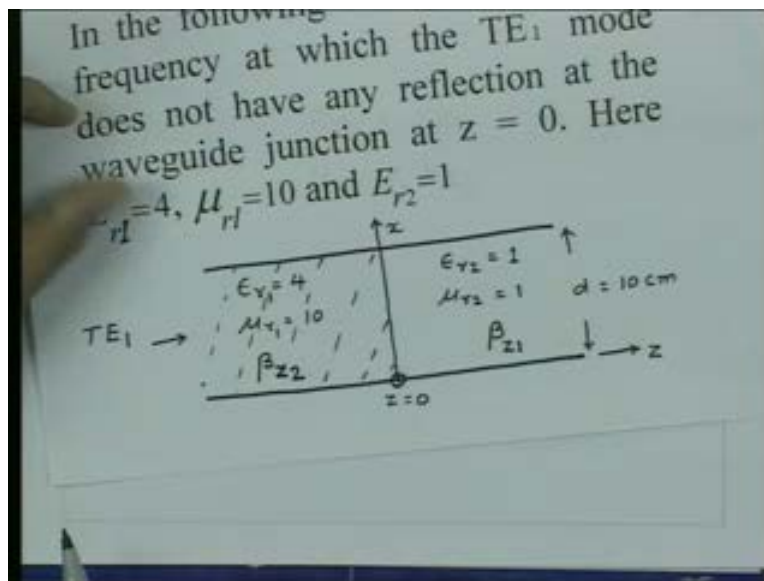
So now you have to find out the frequency for which there will not be any reflection from from here. What that means is essentially that we have a traveling wave which is coming here in TE_1 mode and since there is no reflection... in this region also we have only forward traveling wave, in this region also we have only forward traveling wave so the problem essentially can be solved by matching the boundary condition at z equal to 0 for the mode which is in this region for z less than 0 and for the region which is z greater than 0.

So first we get the the fields for TE_1 mode and essentially made the boundary conditions. Now as we know for TE_1 mode we get the field components which will be the y component which will be like this the y direction is coming out of the paper so we

have the y component for the electric field and for the magnetic field will have a z component and the x component. So, for matching the boundary condition then the y component which is this it should be continuous across the boundary and also since we have normal component of the magnetic field that should be continuous so we can essentially apply the continuity of the z component of magnetic field and the y component of the electric field for the mode.

So, at z equal to 0 we have got E_y 1 is equal to E_y 2 and H_z 1 is equal to H_z 2. Now the nodal fields will have different phase constants in z direction because now the medium which is filling the waveguide is different for z less than 0 and greater than 0 so let us say that the phase constant in this is given by β_{z1} and the phase constant in this is given by β_{z2} .

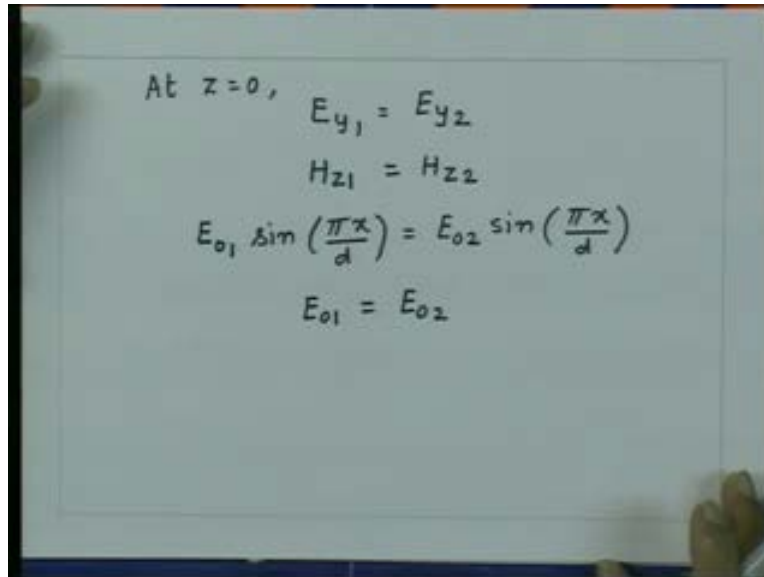
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This is the phase constant for TE 1 mode in this region and in this region, let us call this as 1 (Refer Slide Time: 13:10), this region is 1 and this is 2. So now we can write down the field expressions here. So we know the the field expression if I write down at z equal to 0 we will get this which corresponds to some $E_0 \sin$ of πx upon d that will be equal

to $E_{02} \sin(\pi x/d)$ which essentially will give that the amplitude of these two E_{01} is equal to E_{02} that you get from the continuity of the y component.

(Refer Slide Time: 13:59)



At $z=0$,

$$E_{y1} = E_{y2}$$

$$H_{z1} = H_{z2}$$

$$E_{01} \sin\left(\frac{\pi x}{d}\right) = E_{02} \sin\left(\frac{\pi x}{d}\right)$$

$$E_{01} = E_{02}$$

For the z component as we know this is $E_{01} \sin(\beta_1 z)$ upon $\beta_1 z$ where β_1 is the phase constant in a medium which is filling this waveguide for z equal to z less than 0. So, if I take a medium which is of infinite extent and whose dielectric constant is 4 and relative permeability is 10 then β_1 will be the phase constant of uniform plane wave in that medium; that multiplied by $\sin(\pi x/d)$ that will be equal to $E_{02} \sin(\beta_2 z)$ upon $\beta_2 z$ that is again common on both sides so we get from this boundary condition $\beta_1 = \beta_2$.

So this equation (Refer Slide Time: 15:06) we are getting from here, this is this comes from here and this equation is coming from the magnetic fields from here. So, from the first equation we have got $E_{01} = E_{02}$ so this will cancel and this $\sin(\pi x/d)$ that is again common on both sides so we get from this boundary condition $\beta_1 = \beta_2$.

(Refer Slide Time: 15:30)

At $z=0$,

$$E_{y1} = E_{y2}$$

$$H_{z1} = H_{z2}$$

$$E_{o1} \sin\left(\frac{\pi x}{d}\right) = E_{o2} \sin\left(\frac{\pi x}{d}\right)$$

$$E_{o1} = E_{o2}$$

$$\frac{E_{o1}}{\eta_1} \left(\frac{\beta_{z1}}{\beta_1}\right) \sin\left(\frac{\pi x}{d}\right) = \frac{E_{o2}}{\eta_2} \left(\frac{\beta_{z2}}{\beta_2}\right) \sin\left(\frac{\pi x}{d}\right)$$

Now we can write down for beta 1 and eta 1, so from here we get by substituting the medium parameters we get beta 1 eta 1 equal to omega square root mu 0 mu r 1 epsilon 0 epsilon r 1 into eta 1 which is square root of mu 0 mu r 1 divided by epsilon 0 epsilon r so we get from here this will be omega mu 0 mu r 1. Similarly, we can calculate this quantity here which is beta 2 eta 2 that will be... since this medium (Refer Slide Time: 17:04) the relative permittivity is 1 relative permeability is 1 essentially this quantity if we substitute mu r equal to 1 epsilon r equal to 1 we will get beta 2 eta 2 that will be nothing but omega mu 0.

So substituting now for these two in these expression essentially we get omega mu 0 beta z 1 is equal to omega mu 0 mu r 1 beta z 2; again mu 0 omega will cancel, these two and this quantity now we can write down in terms of the phase constant of the medium beta 1 and the phase constant of the medium beta 2. So from here we can get square root of beta 1 square minus pi upon d whole square.

In general we have here m pi upon d but since we are talking about a mode which is TE 1 mode that is what it is incident on this (Refer Slide Time: 18:22) so you put m equal to 1 that will be equal to mu r 1 square root of beta 2 square minus pi upon d whole square.

(Refer Slide Time: 18:45)

$$\frac{\beta_{z1}}{\eta_1 \beta_1} = \frac{\beta_{z2}}{\eta_2 \beta_2}$$

$$\beta_1 \eta_1 = \omega \sqrt{\mu_0 \mu_{r1} \epsilon_0 \epsilon_{r1}} \sqrt{\frac{\mu_0 \mu_{r1}}{\epsilon_0 \epsilon_{r1}}} = \omega \mu_0 \mu_{r1}$$

$$\beta_2 \eta_2 = \omega \mu_0$$

$$\omega \mu_0 \beta_{z1} = \omega \mu_0 \mu_{r1} \beta_{z2}$$

$$\sqrt{\beta_1^2 - \left(\frac{\pi}{d}\right)^2} = \mu_{r1} \sqrt{\beta_2^2 - \left(\frac{\pi}{d}\right)^2}$$

Keep in mind we are essentially assuming here; keep in mind that the TE 1 mode which is coming here and on this side also only the TE 1 mode propagates. It is possibly general that many other modes might propagate in this region. However, we are saying that since the field variation in this has to be identical to the field variation which is going to get excited in this for matching the boundary condition continuity of the electric field the field distribution in this region must be identical in the transverse direction to the field distribution in a region 1 in region 1 and that is the reason we are saying that the mode which is going to propagate at this will also be TE 1 mode and that is why the index for this also will be equal to 1.

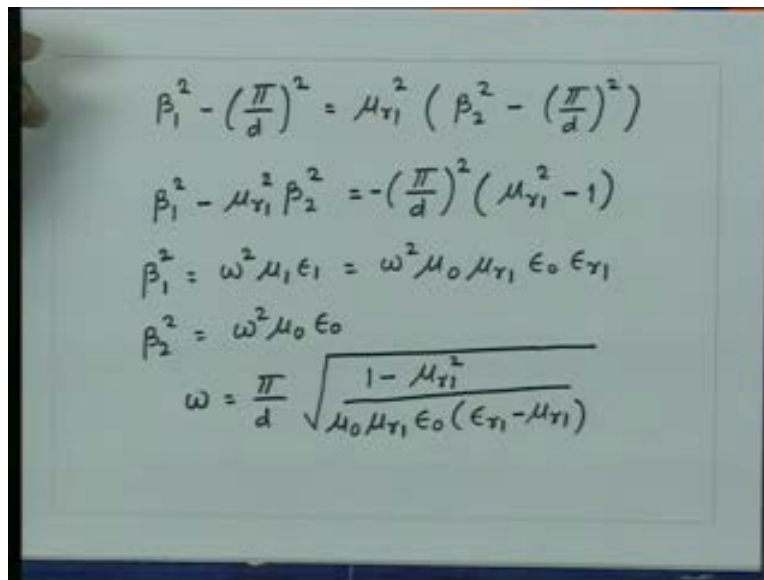
once we once we get that then we can simplify these expressions and we get from here $\beta_1^2 - \left(\frac{\pi}{d}\right)^2$ that is equal to $\mu_{r1}^2 \left[\beta_2^2 - \left(\frac{\pi}{d}\right)^2\right]$. So from here we can get $\beta_1^2 - \mu_{r1}^2 \beta_2^2$ that will be equal to $\left(\frac{\pi}{d}\right)^2 (\mu_{r1}^2 - 1)$ with a minus sign.

Now noting that this quantity here β_1 is equal to $\omega \sqrt{\mu_0 \epsilon_0 \epsilon_{r1}}$ that is $\omega \sqrt{\mu_0 \mu_{r1} \epsilon_0 \epsilon_{r1}}$ and β_2 square is equal to $\omega^2 \mu_0 \epsilon_0 \epsilon_{r2}$

square $\mu_0 \epsilon_0$. We can substitute into this expression here and from here we can find out this quantity ω which will be equal to π upon d square root of $1 - \mu_r$ 1 square upon $\mu_0 \mu_r 1 \epsilon_0 \epsilon_r 1 - \mu_r 1$. So this is the frequency at which we have a continuity of the field and therefore we will not have any reflection from from this boundary here which is at z equal to 0 (Refer Slide Time: 22:16).

So even if the medium properties inside the waveguide suddenly change from these parameters to free space still there will not be any reflection from this continuity and wave will be launched inside this waveguide without any reflection.

(Refer Slide Time: 22:30)



$$\begin{aligned}\beta_1^2 - \left(\frac{\pi}{d}\right)^2 &= \mu_{r1}^2 \left(\beta_2^2 - \left(\frac{\pi}{d}\right)^2 \right) \\ \beta_1^2 - \mu_{r1}^2 \beta_2^2 &= -\left(\frac{\pi}{d}\right)^2 (\mu_{r1}^2 - 1) \\ \beta_1^2 &= \omega^2 \mu_1 \epsilon_1 = \omega^2 \mu_0 \mu_{r1} \epsilon_0 \epsilon_{r1} \\ \beta_2^2 &= \omega^2 \mu_0 \epsilon_0 \\ \omega &= \frac{\pi}{d} \sqrt{\frac{1 - \mu_{r1}^2}{\mu_0 \mu_{r1} \epsilon_0 (\epsilon_{r1} - \mu_{r1})}}\end{aligned}$$

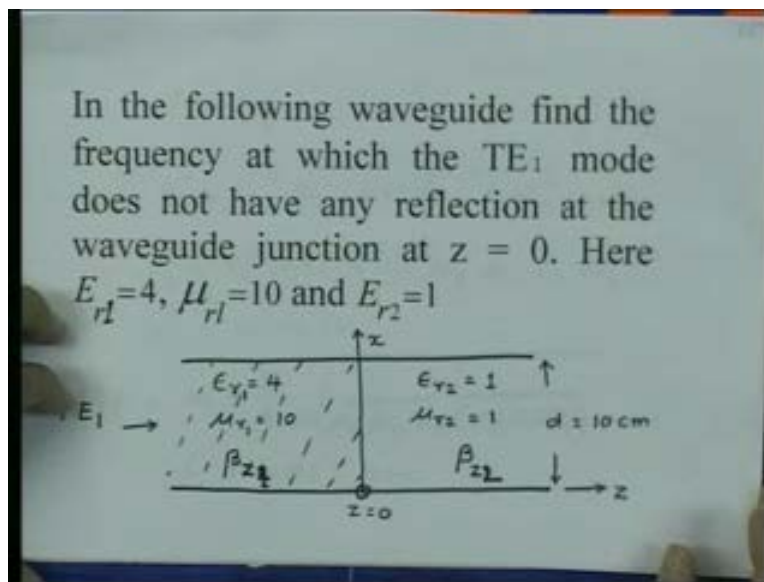
So this is the frequency at which essentially the wave can be launched without any reflection even if we are having a medium discontinuity inside the waveguide. Substituting now for these parameters $\mu_r 1$ equal to 10 $\epsilon_r 1$ equal to 4, all that we can solve for the frequency so this ω will be 2π into frequency, so we can calculate from here the frequency that will be equal to 1.927 Gigahertz.

In fact as we have seen for parallel plane waveguide analysis that the modal propagation can also be seen as the reflection of the uniform plane wave on this conducting planes so

we can visualize this TE₁ mode as if the plane wave is going like that (Refer Slide Time: 23:31), reflected between these two plane and we have a sustained propagation, then we know that this plane wave essentially is going to see this dielectric boundary and we know that there is no reflection for the for a uniform plane wave from a dielectric boundary when the wave is launched at the Brewster angle.

So for this if I can visualize this TE₁ mode as superposition of the uniform plane waves then I can find out an angle at which the wave is incident at the Brewster angle at this interface and then from there I can calculate the frequency for which we will get no reflection on the boundary.

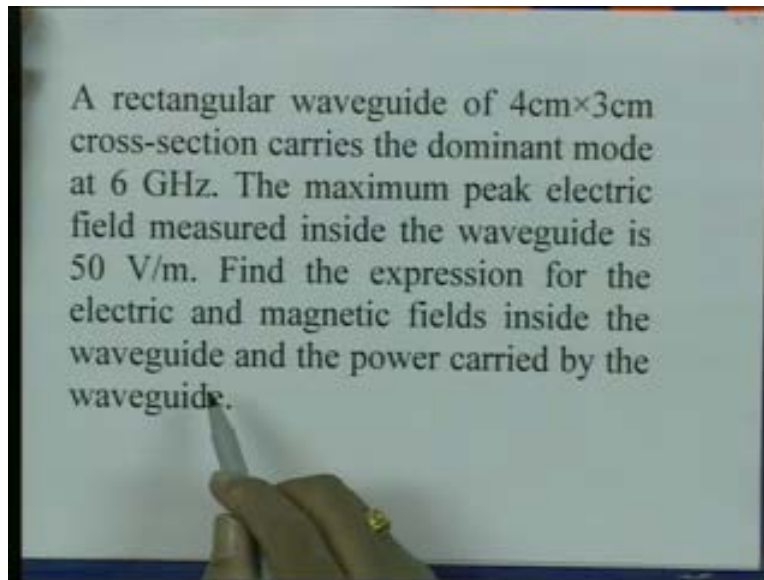
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So this problem can be solved in fact in two ways: one is taking the fields for a particular mode and then matching the boundary conditions at z equal to 0. Second way is that visualizing this TE₁ field by superposition of uniform plane waves and then applying the concept of Brewster angle; and both the methods are acceptable and both the methods essentially give the same value.

Let us now consider a problem which is based on rectangular waveguide. Let us consider a rectangular waveguide of cross section 4 centimeter into 3 centimeter cross section and that carries the dominant mode and as we know the dominant mode of a waveguide is TE₁₀ mode the frequency of the mode is 6 Gigahertz. The maximum peak electric field measured inside the waveguide is 50 volts per meter. Find the expression for the electric and magnetic fields inside the waveguide and the power carried out by the waveguide.

(Refer Slide Time: 25:02)



So first we have to find out the field expressions. Once we know the field expressions then we can find out what is the poynting vector, once we get the pointing vector we can integrate the pointing vector over the cross section of the waveguide and then we can get the total power guided by the waveguide.

So problem essentially, first is to find out the fields given the condition that the maximum electric field is 50 volts per meter and then find out the pointing vector and then the power carried by the mode.

So in this problem the waveguide is a rectangular waveguide. So a in this case is 4 centimeter, for writing in meters this is 0.04 meters and b for this is 0.03 meters and the

mode which is the dominant mode of the waveguide that is TE₁₀ mode. So for this mode waveguide h the parameter which we have defined for the waveguide that is nothing but π upon a so this is equal to π upon 0.04 . Once we know this then we can find out what is the phase constant for this mode on this waveguide, so that is β that will be equal to square root of $\omega^2 \mu \epsilon$ minus π upon a whole square.

The waveguide is filled with air, nothing has been given about what material is filling the waveguide so we assume that waveguide is filled with air so the relative permeability and relative permittivity for the media for this medium filling the waveguide are unity and the frequency is 6 Gigahertz. So we can write here ω will be equal to 2π into 6 Gigahertz so 6 into 10 to the power 9 radians per second; μ is equal to μ_0 which is equal to 4π into 10 to the power minus 7 Henry per meter and ϵ will be equal to ϵ_0 which is 1 upon 36π into 10 to the power minus 9 Farad per meter and π upon a is given here.

(Refer Slide Time: 28:17)

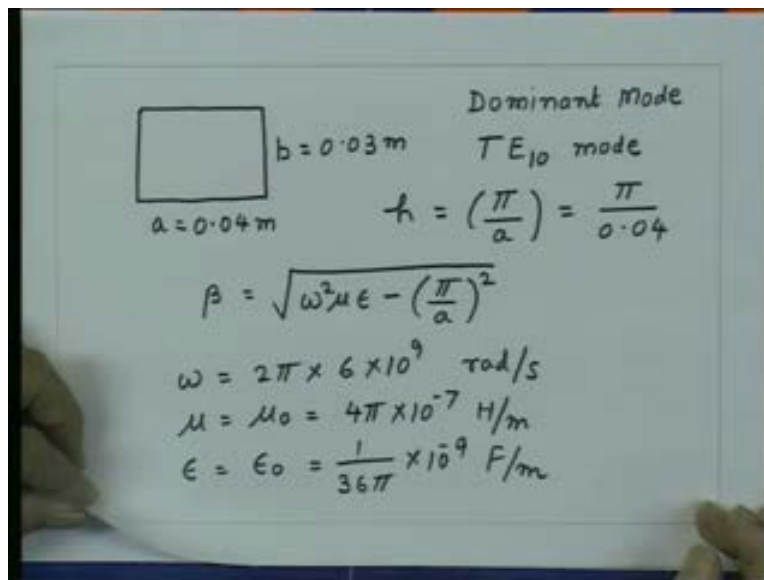


Diagram of a rectangular waveguide with dimensions $a = 0.04\text{m}$ and $b = 0.03\text{m}$.

Dominant Mode
TE₁₀ mode

$$h = \left(\frac{\pi}{a}\right) = \frac{\pi}{0.04}$$

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{\pi}{a}\right)^2}$$

$$\omega = 2\pi \times 6 \times 10^9 \text{ rad/s}$$

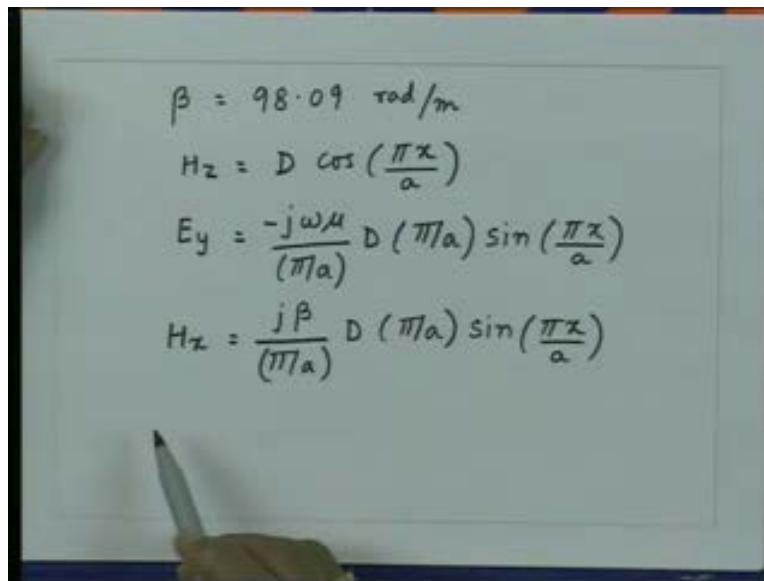
$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\epsilon = \epsilon_0 = \frac{1}{36\pi} \times 10^{-9} \text{ F/m}$$

So if I substitute all these values inside in this expression we get the phase constant for this mode β that is equal to 98.09 radians per meter.

Now if I look at the expressions which are given for the TE 01 mode inside this waveguide we have got the expressions that H_z is equal to d where d is some constant $\cos$ of πx upon a E_y that is equal to minus j omega mu divided by π upon a d into $\sin$ of πx upon a and H_x is equal to j beta upon π upon a d into $\sin$ of πx upon a and the other field components for TE 10 mode are 0.

(Refer Slide Time: 30:01)



Handwritten equations on a whiteboard:

$$\beta = 98.09 \text{ rad/m}$$

$$H_z = D \cos\left(\frac{\pi x}{a}\right)$$

$$E_y = \frac{-j\omega\mu}{(\pi/a)} D \left(\frac{\pi}{a}\right) \sin\left(\frac{\pi x}{a}\right)$$

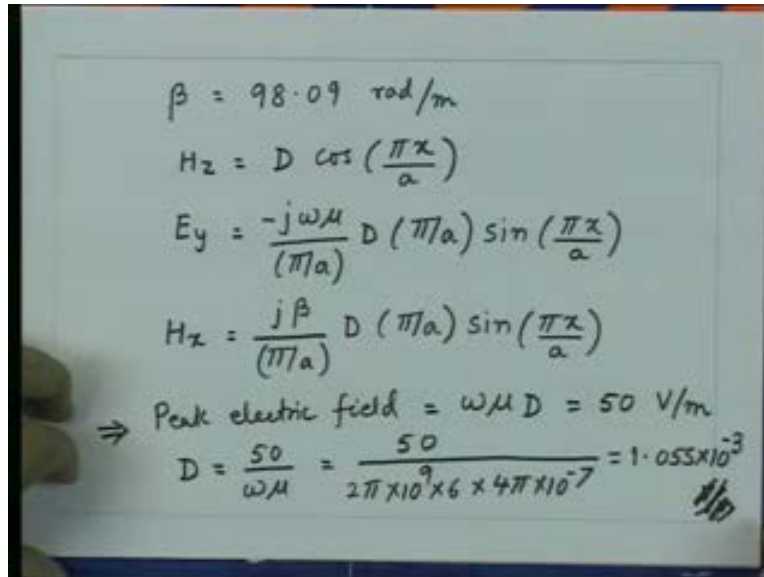
$$H_x = \frac{j\beta}{(\pi/a)} D \left(\frac{\pi}{a}\right) \sin\left(\frac{\pi x}{a}\right)$$

So now what we see here that this is the constant in terms of which we had derived all the field components. So E_y was in terms of d , E_x was in terms of d ; however, now this parameter d is not given, what is given is the maximum electric field the peak electric field which we can measure inside the waveguide is 50 volts per meter. That means this quantity (Refer Slide Time: 30:32) which is a coefficient of this that quantity is 50 volts per meter.

So essentially what this will cancel so what that essentially means is that the peak electric field that is equal to omega mu into d that is equal to 50 volts per meter. Since I know the frequency, I know the permeability which is free space permeability, I can find out the value of d which is 50 divided by omega mu so this is 50 divided by 2π into 10 to the

power 9 into 6 so 6 Gigahertz multiplied by 4pi into 10 to the power minus 7 that will be equal to 1.055 into 10 to the power minus 3.

(Refer Slide Time: 31:56)



Handwritten mathematical derivations on a whiteboard:

$$\beta = 98.09 \text{ rad/m}$$

$$H_z = D \cos\left(\frac{\pi x}{a}\right)$$

$$E_y = \frac{-j\omega\mu}{(\pi/a)} D \left(\frac{\pi}{a}\right) \sin\left(\frac{\pi x}{a}\right)$$

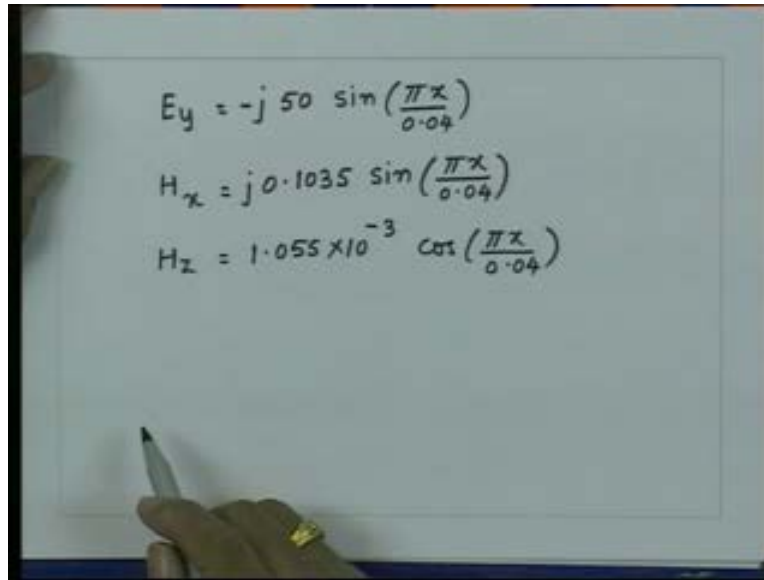
$$H_x = \frac{j\beta}{(\pi/a)} D \left(\frac{\pi}{a}\right) \sin\left(\frac{\pi x}{a}\right)$$

⇒ Peak electric field = $\omega\mu D = 50 \text{ V/m}$

$$D = \frac{50}{\omega\mu} = \frac{50}{2\pi \times 10^9 \times 6 \times 4\pi \times 10^{-7}} = 1.055 \times 10^{-3} \text{ A/m}$$

So once I get the value of d then I can substitute into these expressions and I can find out the the field components so I get now the field components E y that is equal to minus j 50 sin of pi x upon 0.04, H x will be equal to j 0.1035 sin of pi x upon 0.04 and H z will be equal to 1.055 into 10 to the power minus 3 cos of pi x upon 0.04. So I substituted the value of d in these three expressions (Refer Slide Time: 33:00). In fact from here this is the magnetic field so that dimension of d should be amperes per meter.

(Refer Slide Time: 33:18)



A photograph of a whiteboard with three handwritten equations. The first equation is $E_y = -j 50 \sin\left(\frac{\pi x}{0.04}\right)$. The second equation is $H_x = j 0.1035 \sin\left(\frac{\pi x}{0.04}\right)$. The third equation is $H_z = 1.055 \times 10^{-3} \cos\left(\frac{\pi x}{0.04}\right)$. A hand holding a pen is visible at the bottom left of the whiteboard.

$$E_y = -j 50 \sin\left(\frac{\pi x}{0.04}\right)$$
$$H_x = j 0.1035 \sin\left(\frac{\pi x}{0.04}\right)$$
$$H_z = 1.055 \times 10^{-3} \cos\left(\frac{\pi x}{0.04}\right)$$

So this is now the fields which we get for this waveguide for the dominant mode and once we know this field then we can write down the Poynting vector. So we have got Poynting vector p that is equal to $1/2$ for real part of E cross H conjugate so this one H_z and E_y they will not have any real part because this is having j this is nothing but j so the product of these two will give the power which will be imaginary so this essentially would be equal to $1/2$ into E_y into H_x so this is 50 into 0.1035 into $\sin^2 \pi x$ upon a .

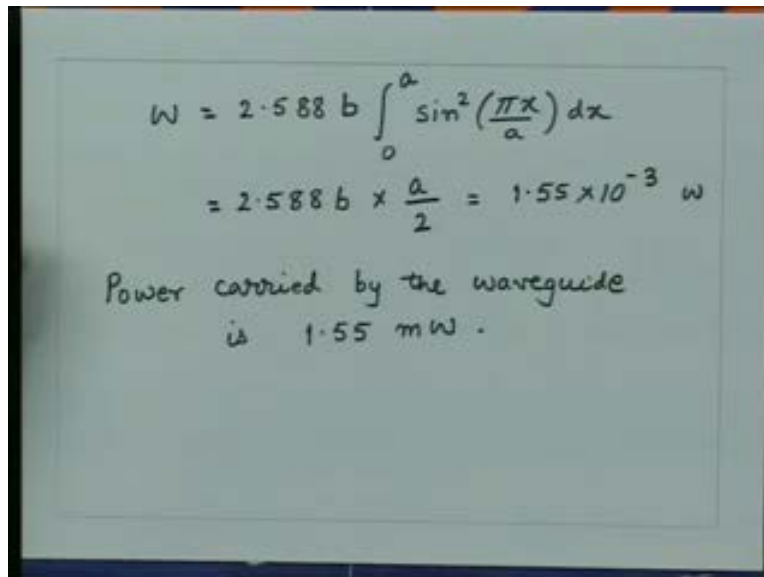
Once I know this Poynting vector then I can find out the total power guided by the waveguide w that will be equal to the integrated poynting vector over the cross section so x will be 0 to a y equal to 0 to b Poynting vector $dx dy$. So I can substitute for P inside this and put a and b you will get the total power carried by the waveguide w that will be if you if you solve this you will get 2.588 so it is 2.588 . The Poynting vector does not vary in y direction (Refer Slide Time: 35:30) so this integral with respect to y will simply b so that will be equal to b integral 0 to a $\sin^2 \pi x$ upon a into dx .

So substituting now, solving this and substituting the values for a and b we get here 2.588 b multiplied by integral of this is a by 2 so that gives us 1.55 into 10 to the power minus 3 watts. So the power carried by the waveguide is 1.55 milliwatts.

So see what we have done in this this problem. In this problem (Refer Slide Time: 36:44) the waveguide cross section was given, the frequency of operation is given, and the maximum peak electric field is given, and then from here we are supposed to find out what is the power flow inside the waveguide.

So first we calculate from here the fields; and in this for TE 10 mode for dominant mode the fields are having component which is E y, H x and H z so we calculate the field components which is E y H x and H z. Put a constraint that the peak amplitude for the electric field is 50 volts per meter from there we calculate this unknown parameter d, once we get d we substitute into the expressions and we get the three component components of the fields and then we calculate the Poynting vector which is nothing but E y multiplied by H x and then we integrate the poynting vector to get the total power carried by the waveguide.

(Refer Slide Time: 37:44)

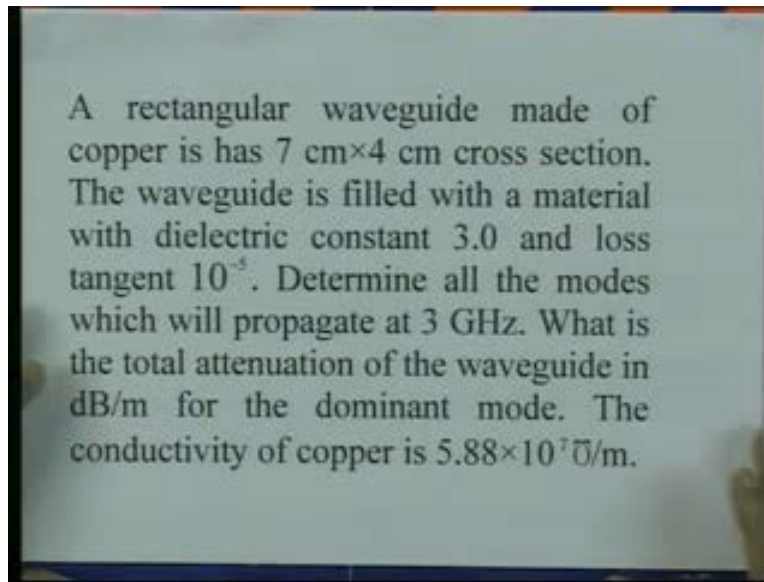


The image shows a handwritten derivation on a piece of paper. It starts with the equation for power W, which is 2.588 b times the integral from 0 to a of sin squared of (pi x over a) dx. This is then simplified to 2.588 b times a over 2, resulting in 1.55 times 10 to the power of -3 W. Below this, it states that the power carried by the waveguide is 1.55 mW.

$$W = 2.588 b \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx$$
$$= 2.588 b \times \frac{a}{2} = 1.55 \times 10^{-3} \text{ W}$$

Power carried by the waveguide
is 1.55 mW.

(Refer Slide Time: 37:58)



Let us take one more problem here that is a rectangular waveguide made of copper has 4 centimeter into 7 centimeter cross section. The waveguide is filled with a material with dielectric constant 3 and this dielectric material is lossy so it has a loss tangent of 10^{-5} to the power minus 5. Determine all the modes which will propagate at 3 Gigahertz. What is the total attenuation of the waveguide in db per meter for the dominant mode? The conductivity of copper is 5.88×10^7 mhos per meter.

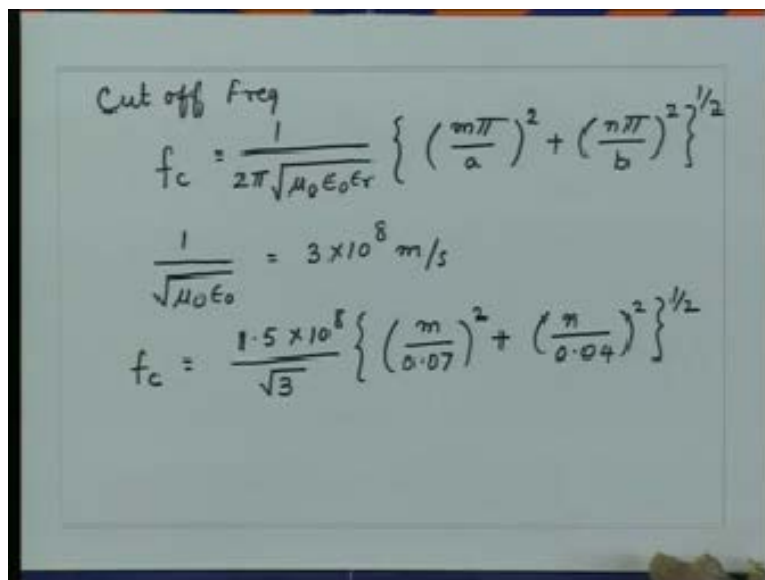
So in this case there are three things we have to find out. First: we have to find out all the modes which will propagate inside the waveguide at 3 Gigahertz. Second: the attenuation constant of the waveguide which will come from two components one is because of lossy dielectric because it has a loss tangent of 10^{-5} to the power minus 5 and the finite conductivity of the walls because here the conductivity is not infinite so there will be conductor losses in the waveguide walls. So we have to calculate the attenuation constant for the dominant mode of the waveguide.

So let us see how do we proceed for this. Firstly, to find out how many modes will propagate at a given frequency we essentially find out the cutoff frequency and then we find then if the cutoff frequency is less than the frequency of operation then the mode will

propagate, if the cutoff frequency is higher than 3 Gigahertz then the mode will not propagate.

So we know for a rectangular waveguide the cutoff frequency f_c that is equal to $\frac{1}{2\pi \sqrt{\mu_0 \epsilon_0 \epsilon_r}}$; here the dielectric material we are talking about so the relative permeability is equal to 1. Also, since the loss tangent is very small for calculation of cutoff frequency we consider the dielectric constant almost real and that is equal to 3 so the ϵ_r is equal to 3 multiplied by $m\pi$ by a whole square plus $n\pi$ by b whole square the power $1/2$. That is the expression for the cutoff frequency and we know this quantity $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$ is nothing but the velocity of light in vacuum so $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$ is equal to 3×10^8 meters per second. Substituting this we get f_c the cutoff frequency of a mode that is this to be this divided by 2 will give me 1.5×10^8 divided by square root of ϵ_r , this square root of 3, this $m\pi$ (Refer Slide Time: 41:35) will cancel with this π so this will give m upon a which is for this waveguide which is 7 centimeter and b is 4 centimeter so this will be 0.07 whole square plus n upon 0.04 whole square square root.

(Refer Slide Time: 42:00)



Handwritten equations for the cutoff frequency of a rectangular waveguide:

$$\text{Cut off freq}$$

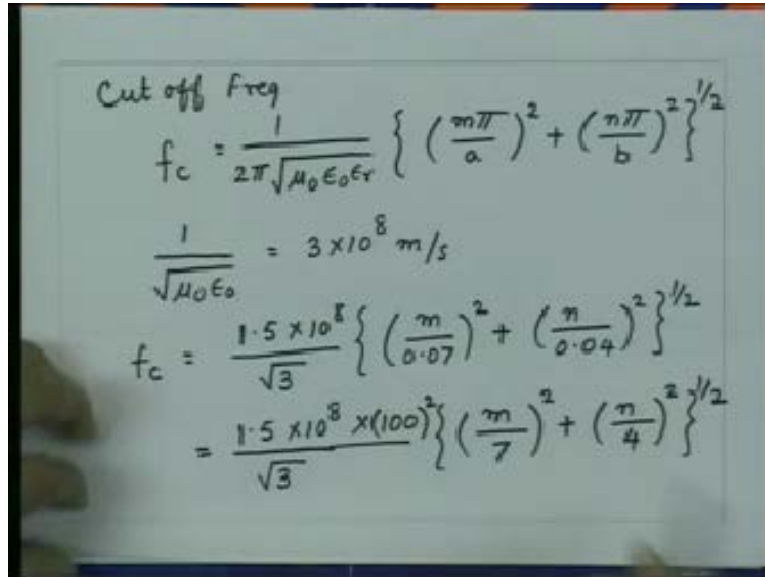
$$f_c = \frac{1}{2\pi \sqrt{\mu_0 \epsilon_0 \epsilon_r}} \left\{ \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right\}^{1/2}$$

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$$

$$f_c = \frac{1.5 \times 10^8}{\sqrt{3}} \left\{ \left(\frac{m}{0.07} \right)^2 + \left(\frac{n}{0.04} \right)^2 \right\}^{1/2}$$

So, simplifying this we get simply 1.5 into 10 to the power 8 divided by root 3 into 100 into m upon 7 whole square plus n upon 4 whole square square root this should be 100 square here. So from here essentially we get the condition for m and n.

(Refer Slide Time: 42:55)



Handwritten derivation of the cutoff frequency formula for a rectangular waveguide:

$$\text{Cut off freq}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu_0\epsilon_0\epsilon_r}} \left\{ \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right\}^{1/2}$$

$$\frac{1}{\sqrt{\mu_0\epsilon_0}} = 3 \times 10^8 \text{ m/s}$$

$$f_c = \frac{1.5 \times 10^8}{\sqrt{3}} \left\{ \left(\frac{m}{0.07} \right)^2 + \left(\frac{n}{0.04} \right)^2 \right\}^{1/2}$$

$$= \frac{1.5 \times 10^8 \times (100)^2}{\sqrt{3}} \left\{ \left(\frac{m}{7} \right)^2 + \left(\frac{n}{4} \right)^2 \right\}^{1/2}$$

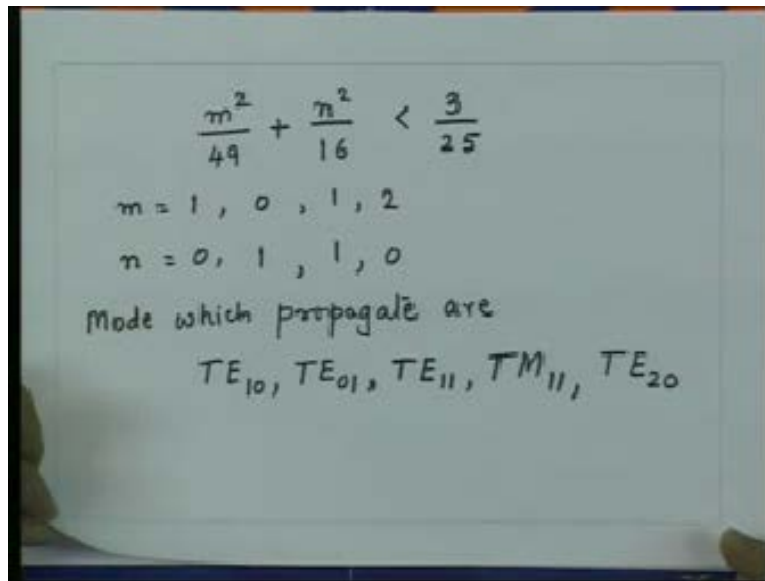
So if m square upon 49 plus n square upon 16, if the frequency f_c is less than the frequency which is 3 Gigahertz then the mode will propagate. So what that means is if this this quantity here where substitute here for the frequency then we get that if this quantity (Refer Slide Time: 43:32) is less than 3 upon 25 then the modes will propagate.

So now we can try for different values of m and n and as we know m and n both are integers so we can start with all possible combinations of m and n for which this condition is satisfied. So we can take m is equal to 1 and equal to 0, we can take m equal to 0 n equal to 1, we can take m equal to 1, n equal to 1, we can take m equal to 2, n equal to 0 and beyond this this condition will not be satisfied so essentially we got the modes which would be corresponding to TE or TM with the indices for m which is going from 0, 1, 2 and the second index which is going from 0 and 1.

Now as we know, for TE case one of the indices nonzero will still give me the mode whereas for the transverse magnetic case both the indices have to be nonzero so the modes which will propagate inside the waveguide are corresponding to this you have TE 10, then you have TE 01, then you have TE 11 but for both the indices non zero even the TM mode will be existing so you have got TM 11 and then here again we will get TE 20.

So at 3 Gigahertz inside this waveguide these five modes will propagate: TE 10, TE 01, TE 11, TM 11 and TE 20.

(Refer Slide Time: 45:46)



Handwritten mathematical derivation on a whiteboard:

$$\frac{m^2}{49} + \frac{n^2}{16} < \frac{3}{25}$$

$m = 1, 0, 1, 2$
 $n = 0, 1, 1, 0$

Mode which propagate are
 $TE_{10}, TE_{01}, TE_{11}, TM_{11}, TE_{20}$

The second part which is asked in this problem is to find now the attenuation constant for the dominant mode and that is the TE 10 mode.

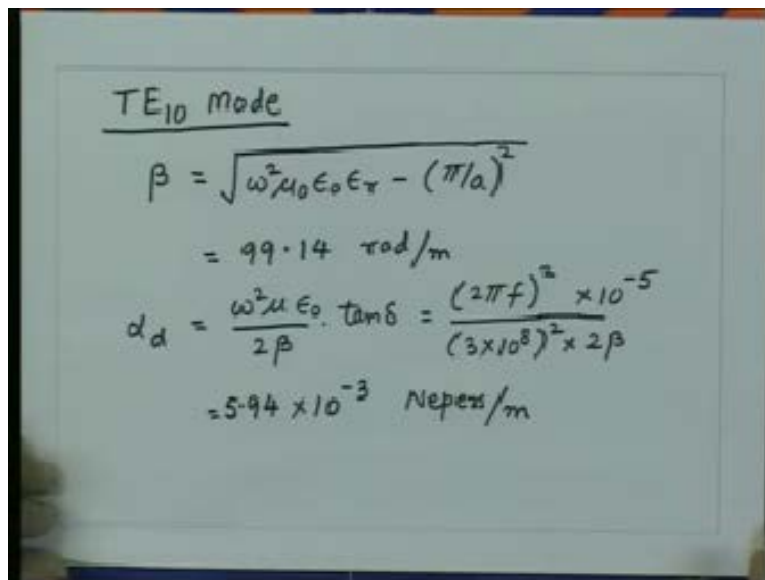
Since the loss is coming from two components: one is dielectric loss, other one is the conductor loss and since the losses are small what we do is we calculate these losses one at a time and then we calculate total attenuation constant which will be the sum of the two attenuation constants.

So first we calculate beta for the TE₁₀ mode and that is square root of omega square mu epsilon₀ epsilon_r minus pi upon a whole square. If I substitute the values for epsilon_r which is 3 and a which is the dimension of the waveguide that is 7 centimeter so it is 0.07 I get the propagation constant which is 99.14 radians per meter.

Now the attenuation constant due to dielectric loss as we have derived alpha_d that is equal to omega square mu epsilon₀ upon 2 beta into tangent of delta tan delta I can substitute here which is 2pi into frequency square and this quantity here mu epsilon₀ that will be 1 upon velocity square so this will be 3 into 10 to the power 8 square into 2 beta into tan delta which is 10 to the power minus 5 that is what is given here that the medium has a loss tangent of 10 to the power minus 5.

So I can substitute these values for all this and then I can solve for alpha_d which will be equal to 5.94 into 10 to the power minus 3 Nepers per meter.

(Refer Slide Time: 48:36)



Handwritten calculations for the TE₁₀ mode:

$$\beta = \sqrt{\omega^2 \mu_0 \epsilon_0 \epsilon_r - (\pi/a)^2}$$

$$= 99.14 \text{ rad/m}$$

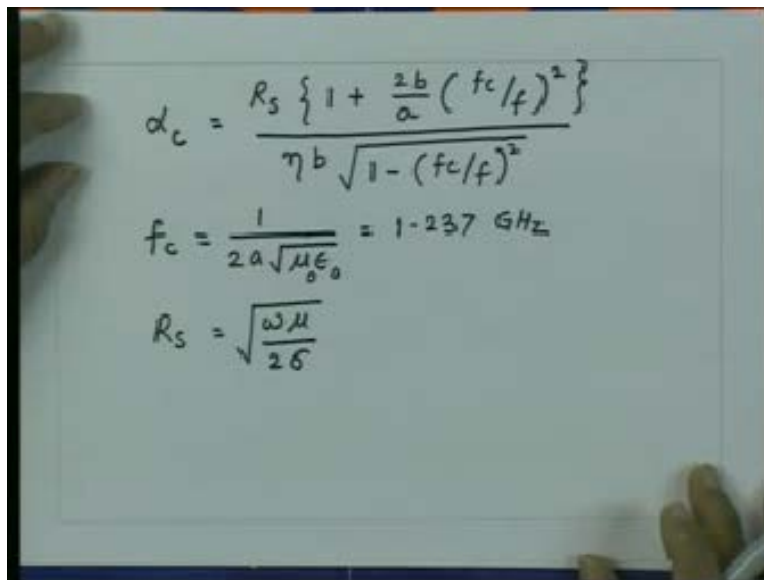
$$\alpha_d = \frac{\omega^2 \mu_0 \epsilon_0 \tan \delta}{2\beta} = \frac{(2\pi f)^2 \times 10^{-5}}{(3 \times 10^8)^2 \times 2\beta}$$

$$= 5.94 \times 10^{-3} \text{ Nepers/m}$$

Then we can calculate the conductor loss; the attenuation constant due to the finite conductivity of the walls and that as we know is given as the surface resistance into 1 plus 2 b upon a f c upon f whole square divided by eta b square root of 1 minus f c upon f

whole square and f_c we know as the cutoff frequency for the TE₁₀ mode which is nothing but $\frac{1}{2a} \sqrt{\mu \epsilon_0}$ and in this case it is $\mu_0 \epsilon_0$ so that will be equal to 1.237 Gigahertz and the surface resistance is square root of $\omega \mu$ upon 2σ so I can substitute the value of the conductivity per copper which is 5.88×10^7 to the power 7, I know the frequency which is 3 Gigahertz so I can calculate this quantity which is a surface impedance; I can substitute into this so we have all the parameters we know the cutoff frequency, we know the frequency of operation, we know the dimension of the waveguide a and b , we know the surface impedance and we know the η which is the intrinsic impedance which is 120π for the free space.

(Refer Slide Time: 50:24)



$$\alpha_c = \frac{R_s \left\{ 1 + \frac{2b}{a} \left(\frac{f_c}{f} \right)^2 \right\}}{\eta b \sqrt{1 - \left(\frac{f_c}{f} \right)^2}}$$

$$f_c = \frac{1}{2a \sqrt{\mu_0 \epsilon_0}} = 1.237 \text{ GHz}$$

$$R_s = \sqrt{\frac{\omega \mu}{2\sigma}}$$

So, by substituting these values essentially we can calculate this quantity α_c which is equal to 0.438×10^{-3} nepers per meter.

Then since the losses are small, the total attenuation constant α is approximately equal to α_d plus α_c and that is that will be equal to the attenuation because of dielectric which is 5.94 so 5.94×10^{-3} plus 0.438×10^{-3} nepers per meter so we can add these and we find out the total attenuation constant for for this waveguide.

(Refer Slide Time: 51:41)

Handwritten mathematical derivations for waveguide attenuation constant:

$$\alpha_c = \frac{R_s \left\{ 1 + \frac{2b}{a} \left(\frac{f_c}{f} \right)^2 \right\}}{\eta b \sqrt{1 - \left(\frac{f_c}{f} \right)^2}}$$

$$f_c = \frac{1}{2a\sqrt{\mu_0\epsilon_0}} = 1.237 \text{ GHz}$$

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

$$\alpha_c = 0.438 \times 10^{-3} \text{ Nepers/m}$$

Total atten const

$$\alpha \approx \alpha_d + \alpha_c = 5.94 \times 10^{-3} + 0.438 \times 10^{-3}$$

Then this attenuation constant now is the nepers per meter. So if I multiply this quantity by 8.68 I will get the attenuation constant in dbs per meter. So essentially see what we have done in this problem.

Firstly though you were asked to find out how many modes will propagate inside this waveguide. So we so at a given frequency thus the dimension of the waveguide or given the dielectric constant of the medium the waveguide is given and we are asked to find out how many modes will propagate. So first you find out the cutoff frequency for modes, then you find out a condition for m and n for which the cutoff frequency will be less than the frequency of operation which is 3 Gigahertz so we get this condition, from here (Refer Slide Time: 52:28) then we find out what are the indices which can satisfy this condition and then by using the knowledge that TE modes can exist given when one of the indices is not zero whereas for TM mode both the indices have to be nonzero we find these modes which can propagate inside this waveguide.

from here then we calculate the... for TE₁₀ we can calculate the phase constant, then by using the expression which we have derived we find out the attenuation constant due to dielectric and then we find out the attenuation constant due to the finite conductivity of

the waveguide walls and the sum of the two essentially gives the total attenuation constant.

So these are some of the representative examples which you will see for the waveguide. Typically the waveguide dimensions are given, the frequency of operation is given and we have to find out the modal characteristics of the waveguide, that is how many modes will propagate; and also we have problems where we have a dominant mode which is propagating on the waveguide and then we would be interested in finding out how much power will flow, what will be field expressions for that mode, what be the Poynting vector, what will be loss and so on.

So these once we see these problems then on the similar lines other problems related to the waveguide also will be solved very easily.